

# CHAPTER 6

## Section 6.4

1. (a)

$$\lim_{n \rightarrow \infty} \frac{n^2 + 1}{n^3 + 1} = \lim_{n \rightarrow \infty} \frac{(1/n^2) + 1/n^3}{1 + 1/n^3} = \frac{0}{1} = 0$$

(b)

$$\lim_{n \rightarrow \infty} \frac{\ln n}{n} = \lim_{n \rightarrow \infty} \frac{d(\ln n)/dn}{d(n)/dn} = \lim_{n \rightarrow \infty} \frac{1/n}{1} = \frac{0}{1} = 0$$

(c)

$$\lim_{n \rightarrow \infty} \frac{n}{2^n} = \lim_{n \rightarrow \infty} \frac{1}{2^n \ln 2} = \frac{1}{\infty} = 0$$

(d)

$$\begin{aligned} \lim_{n \rightarrow \infty} n \ln \left( 1 + \frac{1}{n} \right) &= \lim_{n \rightarrow \infty} \frac{\ln(1 + 1/n)}{n^{-1}} = \lim_{n \rightarrow \infty} \frac{d[\ln(1 + 1/n)]/dn}{d(n^{-1})/dn} \\ &= \lim_{n \rightarrow \infty} \frac{1/(n^2 + n)}{1/n^2} \\ &= \lim_{n \rightarrow \infty} \frac{1}{1 + 1/n} \\ &= \frac{1}{1 + 0} = 1 \end{aligned}$$

(e)

$$\lim_{n \rightarrow \infty} s_n = 1 \text{ for } n = 1, 2, 3, \dots \implies \lim_{n \rightarrow \infty} s_n = 1$$

2. (a)

$$\overline{\lim}_{n \rightarrow \infty} \cos n\pi = 1 \qquad \underline{\lim}_{n \rightarrow \infty} \cos n\pi = -1 \qquad (1)$$

(b)

$$\overline{\lim}_{n \rightarrow \infty} \sin \frac{1}{5} n\pi \approx 0.951 \qquad \underline{\lim}_{n \rightarrow \infty} \sin \frac{1}{5} n\pi \approx -0.951$$

(c)

$$\overline{\lim}_{n \rightarrow \infty} n \sin \frac{1}{2} n\pi = \infty \qquad \underline{\lim}_{n \rightarrow \infty} n \sin \frac{1}{2} n\pi = -\infty$$

3. (a) A sequence

$$s_n = 1 + \cos n\pi$$

has limits

$$\overline{\lim}_{n \rightarrow \infty} s_n = 2 \qquad \underline{\lim}_{n \rightarrow \infty} s_n = 0$$

(b) A sequence

$$s_n = -n^2 \sin^2 \left( \frac{1}{2} n \pi \right)$$

has limits

$$\overline{\lim}_{n \rightarrow \infty} s_n = 0$$

$$\underline{\lim}_{n \rightarrow \infty} s_n = -\infty$$

(c) A sequence

$$s_n = n$$

has limits

$$\overline{\lim}_{n \rightarrow \infty} s_n = \underline{\lim}_{n \rightarrow \infty} s_n = \infty$$

4.