CHAPTER 6

Section 6.4

1. (a)

$$\lim_{n\to\infty}\frac{n^2+1}{n^3+1}=\lim_{n\to\infty}\frac{(1/n^2)+1/n^3}{1+1/n^3}=\frac{0}{1}=0$$

(b)

$$\lim_{n\to\infty}\frac{\ln n}{n}=\lim_{n\to\infty}\frac{d(\ln n)/dn}{d(n)/dn}=\lim_{n\to\infty}\frac{1/n}{1}=\frac{0}{1}=0$$

(c)

$$\lim_{n\to\infty}\frac{n}{2^n}=\lim_{n\to\infty}\frac{1}{2^n\ln 2}=\frac{1}{\infty}=0$$

(d)

$$\begin{aligned}\lim_{n\to\infty} n\ln\left(1+\frac{1}{n}\right)=\lim_{n\to\infty}\frac{\ln(1+1/n)}{n^{-1}}&=\lim_{n\to\infty}\frac{d[\ln(1+1/n)]/dn}{d(n^{-1})/dn}\\&=\lim_{n\to\infty}\frac{1/(n^2+n)}{1/n^2}\\&=\lim_{n\to\infty}\frac{1}{1+1/n}\\&=\frac{1}{1+0}=1\end{aligned}$$

(e)

$$\lim_{n\to\infty}s_n=1 \text{ for } n=1,2,3,\cdots \implies \lim_{n\to\infty}s_n=1$$

2. (a)

$$\overline{\lim}_{n\to\infty}\cos n\pi=1 \qquad \underline{\lim}_{n\to\infty}\cos n\pi=-1 \tag{1}$$

(b)

$$\overline{\lim}_{n\to\infty}\sin\frac{1}{5}n\pi\approx 0.951 \qquad \underline{\lim}_{n\to\infty}\sin\frac{1}{5}n\pi\approx -0.951$$

(c)

$$\overline{\lim}_{n\to\infty}n\sin\frac{1}{2}n\pi=\infty \qquad \underline{\lim}_{n\to\infty}n\sin\frac{1}{2}n\pi=-\infty$$

3. (a) A sequence

$$s_n=1+\cos n\pi$$

has limits

$$\overline{\lim}_{n\to\infty}s_n=2 \qquad \underline{\lim}_{n\to\infty}s_n=0$$

(b) A sequence

$$s_n = -n^2 \sin^2\left(\frac{1}{2}n\pi\right)$$

has limits

$$\overline{\lim}_{n\to\infty} s_n = 0 \qquad\qquad \underline{\lim}_{n\to\infty} s_n = -\infty$$

(c) A sequence

$$s_n = n$$

has limits

$$\overline{\lim}_{n\to\infty} s_n = \underline{\lim}_{n\to\infty} s_n = \infty$$

4. Let a sequence $s_n = 1/n$ be given. Now this sequence converges, since

$$s = \lim_{n\to\infty} \frac{1}{n} = 0$$

Hence, for every $\epsilon > 0$ an N can be found such that

$$|s_n - s| < \frac{\epsilon}{2}$$

for all $n > N$. Hence, for all $m, n > N$

$$|s_m - s_n| = |s_m - s + s - s_n| \leq |s_m - s| + |s - s_n| < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$$

and so condition (6.10) is satisfied.

5.