拓扑学（模块2）

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Assignment 3

We use the notations and conventions from the course, e,g. a map refers to a continuous map, a space refers to a topological space, S^1 denotes the unit circle, and D^2 the unit disc.

Exercise 1.

(a) Let $[v_0, v_1, v_2]$ be a 2-simplex. Define $X = [v_0, v_1, v_2]/\sim$ with the equivalence relation given by $v_0 \sim v_1 \sim v_2$. Compute the simplicial homology of X.

(b) Let $[v_0, v_1, \ldots, v_n]$ be an n-simplex. For $1 \leq k \leq n$ and $\underline{i} = (0 \leq i_1 < i_2 < \ldots < i_k \leq n)$ denote by $\varphi_{\underline{i}} : \Delta^k \to [v_{i_1}, v_{i_2}, \ldots, v_{i_k}]$ the canonical homeomorphism from the standard k-simplex to $[v_{i_1}, v_{i_2}, \ldots, v_{i_k}]$ (as defined in the course).

Now define $X = [v_0, v_1, \ldots, v_n]/\sim$ where for any $\underline{i} = (0 \leq i_1 < i_2 < \ldots < i_k \leq n)$, $\underline{j} = (0 \leq j_1 < j_2 < \ldots < j_k \leq n)$, and $x \in [v_{i_1}, v_{i_2}, \ldots, v_{i_k}]$ we set $x \sim \varphi_{\underline{j}} \circ \varphi_{\underline{i}}^{-1}(x)$, i.e. we identify all k-simplices contained as iterative faces in $[v_0, v_1, \ldots, v_n]$ via the canonical homeomorphisms. Compute the simplicial homology of X.

Exercise 2. Let $r : X \to A$ be a retraction of a space X to a subspace A and $i : A \to X$ the inclusion. Show that $i_* : H_n(A) \to H_n(X)$ is injective and $r_* : H_n(X) \to H_n(A)$ is surjective for all $n \geq 0$.

Exercise 3. Let $A = \mathrm{S}^2$ and $D = \{(x, 0, 0) \mid -1 \leq x \leq 1\}$ inside $\mathbb{R}^3$.

(a) Compute the simplicial homology of $X = A \cup D$.

(b) Compute the simplicial homology of the simply-connected covering space $\widetilde{X}$ of X.

As a reminder: There is an explicit construction of $\widetilde{X}$ in Assignment 2 - Solutions, Exercise 4(c). Let $\Theta_k : \mathbb{R}^3 \to \mathbb{R}^3$ be the translation defined by $\Theta_k(x, y, z) = (x + k, y, z)$. Define $A_k = \Theta_{2k}(A)$, for $k \in \mathbb{Z}$ even, and $A_k = \Theta_{2k}(D)$, for k odd, and set

$$\widetilde{X} = \bigcup_{k \in \mathbb{Z}} A_k \subset \mathbb{R}^3.$$

Due date: December 18, 2020
submission in English either by email or on paper, include name and student ID number