拓扑学（模块2）

BIT, Fall 2020

Michael Ehrig
micha.ehrig@outlook.com
michael.ehrig@bit.edu.cn

Assignment 1

We use the notations and conventions from the course, e,g. a map refers to a continuous map, a space refers to a topological space, S^1 denotes the unit circle, and D^2 the unit disc.

Exercise 1. Let X be a space.

(a) Show that X is contractible if and only if id_X is nullhomotopic.

(b) Assume that X is contractible and there exists $A \subset X$ together with a retraction $r : X \to A$. Show that A is contractible.

(c) Show that the following are equivalent

(i) X is contractible,

(ii) every map $f : X \to Y$ (for a space Y) is nullhomotopic,

(iii) every map $f : Y \to X$ (for a space Y) is nullhomotopic.

Exercise 2. Let X be a space. Show that the following are equivalent

(a) Every map $f : \mathrm{S}^1 \to X$ is nullhomotopic,

(b) every map $f : \mathrm{S}^1 \to X$ can be extended to a map $\widetilde{f} : \mathrm{D}^2 \to X$,

(c) for any $x_0 \in X$ it holds $\pi_1(X, x_0)$ is trivial.

Exercise 3. Show that there exists no retraction $r : X \to A$ for the following situations

(a) $X = \mathbb{R}^n$ and A an arbitrary subspace homeomorphic to S^1.

(b) $X = \mathrm{S}^1 \times \mathrm{D}^2$ and $A = \mathrm{S}^1 \times \mathrm{S}^1$ the boundary of X.

(c) $X = \mathrm{D}^2 \vee \mathrm{D}^2$ and $A = \mathrm{S}^1 \vee \mathrm{S}^1$ for the pointed spaces $(\mathrm{D}^2, (1, 0))$ and $(\mathrm{S}^1, (1, 0))$.
Hint: One can argue with the methods from the course why A has a non-trivial fundamental group without the use of the van Kampen theorem.

(d) $X = \mathrm{D}^2/_\sim$, for $(1, 0) \sim (-1, 0)$, and $A = \pi(\mathrm{S}^1)$ the image of the boundary of D^2 under the quotient map π.
Hint: There is a natural way to identify A with $\mathrm{S}^1 \vee \mathrm{S}^1$.

(e) X the Möbius strip from the course, i.e. $X = I \times I/_\sim$ where $(0, s) \sim (1, 1 - s)$, and $A = \pi(I \times \{0\} \cup I \times \{1\})$ for π the quotient map.
Hint: Find a subspace B in X such that $B \cong \mathrm{S}^1$ and B is a deformation retract of X to obtain $\pi_1(X)$.

Due date: November 20, 2020
submission in English either by email or on paper