拓扑学（模块2）

BIT, Fall 2021

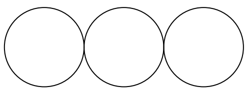
Hints and instructions for the exam

Since there are no notes allowed in the exam, here are some pointers as to what is included in the exam:

- **Definitions:** Know the definitions! There will be questions to give or complete definitions, e.g. "A deformation retraction from X to A is ...".
- **Examples:** Look at and think about examples for the different properties that were discussed in the lecture and on assignment sheets. There will be questions asking for examples of specific property, e.g. "Give an example for a path-connected space X such that $\pi_1(X)$ is a finite group.". You will not need to prove that your example satisfies the property, but the claim must follow with the results from the course.
- **Computing fundamental groups:** Look at the methods that were discussed in the course to compute the fundamental group. The most important one being the van Kampen theorem.
- **Computing simplicial homology:** Look at the examples and calculations for computing simplicial homology used in the course.
- **Computing singular homology:** Look at the methods used to compute singular homology in the course when one cannot use simplicial homology instead. This especially includes the use of the long exact sequences of homology to obtain isomorphisms between different homology groups. The most important ones being the long exact sequence of reduced homology using the quotient space and the one for relative homology.

The following are common sources of mistakes:

(a) **The wedge sum:** Note that the wedge sum of spaces is the quotient where the base points in all spaces are identified. That means that



is not equal to $S^1 \vee S^1 \vee S^1$. In this case one can define a homotopy from one to the other, but in general glueing together more than two spaces at different points is not homotopic to the wedge sum of the spaces.

(b) **van Kampen Theorem:** If applying the van Kampen theorem to a space X with a covering $\{A_\alpha\}$, make sure that each A_α satisfies the prerequisites, especially that they are **open and path-connected** and **every** A_α needs to contain the base point x_0.

(c) **Simplicial homology:** In the definition of a Δ-complex make sure that the restriction of a simplicial simplex to a face is, up to identification with the standard simplex, exactly equal to another element of the Δ-complex. For 1-dimensional simplices this means for example that the orientation coming as a face of a 2-simplex has to agree with the orientation of the 1-simplex itself.