拓扑学（模块2）

BIT, Fall 2020

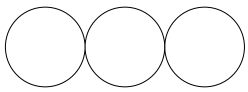
Hints and pointers for the exam

The following are some hints what will be important to complete the exam.

- **Definitions:** Know the definitions! There will be questions to give or complete definitions, e.g. "A deformation retraction from X to A is ...".
- **Examples:** Look at and think about examples for the different properties that where discussed in the lecture. There will be a question asking for examples of specific property, e.g. "Give an example for a path-connected space X such that $\pi_1(X)$ is a finite group.".
- **Computing fundamental groups:** Look at the methods that were discussed in the course to compute the fundamental group. The most important one being the van Kampen theorem.
- **Computing simplicial homology:** Look at the examples and calculations for computing simplicial homology used in the course.
- **Computing singular homology:** Look at the methods used to compute singular homology in the course when one cannot use simplicial homology instead. Especially look at the different long exact homology sequences that were discussed in the course. This includes the one using the homology of a quotient space as well as the one using relative homology groups.

The following are common mistakes:

(a) **The wedge sum:** Note that the wedge sum of spaces is the quotient where the base points in all spaces are identified. That means that



is not equal to $S^1 \vee S^1 \vee S^1$. In this case one can define a homotopy from one to the other, but in general glueing together more than two spaces at different points is not homotopic to the wedge sum of the spaces.

(b) **van Kampen Theorem:** If applying the van Kampen theorem to a space X with a covering $\{A_\alpha\}$, make sure that each A_α satisfies the prerequisites, especially that they are **open and path-connected** and **every** A_α needs to contain the base point x_0.

(c) **Simplicial homology:** In the definition of a Δ-complex make sure that the restriction of a simplicial simplex to a face is, up to identification with the standard simplex, exactly equal to another element of the Δ-complex. For 1-dimensional simplices this means for example that the orientation coming as a face of a 2-simplex has to agree with the orientation of the 1-simplex itself.