George Mason University

Time Domain Analysis and Design of Control Systems

Report from laboratory experiment 2 conducted on 02 February 2016
As part of ECE 429 Control Systems Lab
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09 February 2016

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Objective
The objective of this lab is to design and analyze control systems to meet time domain specifications by using cascade compensation.

Experiment
The system plant is defined as:

$$G_p(s) = \frac{s+20}{s(s+0.4)(s+5)}$$

and requires the following closed-loop specifications:

$$Steady\ State\ Error: \quad (E_{ss}) = 0.01$$
*for a ramp input
$$Maximum\ Percent\ Overshoot: (PO) = 10\%$$
*for a step input
$$Settling\ time: \quad (t_s) = 15\ s$$
*to within 2% of final value for a step input

A lead-lead, lag-lag, or lead-lag compensator must be designed to satisfy the above specifications. The restraints for any given stage of compensation are:

$$\alpha_{lead} = \frac{z_{cd}}{p_{cd}} > 0.05\ and$$

$$\alpha_{lag} = \frac{z_{cg}}{p_{cg}} < 20$$

Step 1: Find the closed-loop poles to satisfy percent overshoot and settling time.
Derivation of ζ from 10% percent overshoot specification:

$$PO(\%) = 100e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$
$$\zeta = \frac{|\ln\frac{PO}{100}|}{\sqrt{\pi^2+\ln^2\frac{PO}{100}}}$$
$$\zeta = 0.59116$$
$$*\ for\ PO = 10$$

Derivation of $\omega_n$from 15s settling time specification:

$$t_{s(2\%)} = \frac{4}{\zeta\omega_n}$$
$$\omega_n = \frac{4}{\zeta t_s}$$
$$\omega_n = 0.45109$$

$$* \, for \; \zeta = 0.59116 \; and \; t_s = 15s$$

The closed loop poles will be based on ζ and ω_n:

$$s_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$
$$\boldsymbol{s_d = -0.2667 \pm j0.3638}$$

<u>**Step 2: Determine lead or lag compensation based on necessary phase compensation.**</u>

The phase of the compensated open loop system should be -180 degrees. Evaluate the system G_p at the roots s_d to obtain the current phase:

```
s = tf('s');
k = 1;
Gp_ol = k*(s+20)/(s*(s+0.4)*(s+5));
sd = [-0.2667+0.3638*1i -0.2667-0.3638*1i];
freqresp(Gp_ol,sd)
```

$$G_p(s_d) = -22.4287 \pm 7.9250i$$

```
angle(freqresp(Gp_ol,sd))
```

$$current \; phase(deg.): \phi = \angle G_p(s_d) - 360$$
$$\phi = -3.4812 \; rad. = -199.4629 \; deg.$$

Calculate the necessary phase compensation:

$$\phi_c = -180 - \phi$$
$$\boldsymbol{\phi_c = 19.4630}$$

If:

$$\boldsymbol{\phi_c > 0 \rightarrow Lead \; Compensator}$$
$$\phi_c < 0 \rightarrow Lag \; Compensator$$

<u>**Step 3: Design lead compensator**</u>

The lead compensator should contribute the amount of phase calculated in the previous step at the desired closed loop pole location.

$$G_{c\,Lead} = k\frac{s - z_{cd}}{s - p_{cd}}$$

with the constraint:

$$\alpha_{lead} = \frac{z_{cd}}{p_{cd}} > 0.05$$
$$\alpha_{lead} = z_{cd} > 0.05(p_{cd})$$

One possible location is to cancel the pole closest to zero of G_p.

$$z_c = -0.4$$

The new zero adds phase to the system:

$$\phi_{zc} = \angle(s_d - z_c)$$
$$\phi_{zc} = 1.2196\ rad. = 69.8780\ deg.$$

The phase needed by the pole to compensate the new phase is:

$$\phi_{pc} = \phi_{zc} - \phi_c$$
$$\phi_{pc} = 69.8780 - 19.4630$$
$$\phi_{pc} = 50.4149$$

The new pole location is:

$$p_c = real\{sd\} - \frac{imag\{sd\}}{\tan \phi_{pc}}$$
$$p_c = -0.2667 - \frac{0.3638}{\tan 50.4149}$$
$$p_c = -0.5674$$

Checking compensator constraint:

$$\alpha_{lead} = \frac{z_{cd}}{p_{cd}} > 0.05$$
$$\alpha_{lead} = \frac{0.4}{.5674} = 0.7049 > 0.05$$

Knowing that the magnitude of the open loop system at any point on the root locus must equal 1, the gain K can be calculated:

$$|G_{cLead}(s_d)|\ |G_p(s_d)| = 1$$
$$1 = \left|K \frac{(s+0.4)}{(s+0.5674)}\right| * \left|\frac{(s+20)}{s\ (s+0.4)\ (s+5)}\right|\ for\ s = s_d = \ -0.2667 \pm j0.3638$$

```
Gp_ol =(s+20)/(s*(s+0.4)*(s+5));
Glead = (s+0.4)/(s+0.5674);
sd = [-0.2667+0.3638*1i -0.2667-0.3638*1i];
abs(freqresp(Gp_ol, sd)).*abs(freqresp(Glead, sd))
```

$$1 = K * 19.5272$$
$$K = 0.05121$$

Resulting in the following the lead compensator:

$$\boldsymbol{G_{cLead} = \frac{0.05121(s+0.4)}{(s+0.5674)}}$$

Step 4: Design lag compensator for steady-state error.
To satisfy the steady-state error specification a lag compensator must be designed:

$$G_{cLag} = k_c \left[\frac{1}{\alpha} * \frac{(s + z_c)}{(s + p_c)} \right]$$

With the constraint for a single stage:

$$\alpha_{lag} = \frac{z_{cg}}{p_{cg}} < 20$$

$\alpha_{lag_{total}}$ is:

$$\alpha_{lag_{total}} = \frac{k_{vDesired}}{k_{vCurrent}}$$

Where $k_{vCurrent}$ is:

$$k_{vCurrent} = \lim_{s \to 0} s * G_{cLead}(s) * G_p(s)$$
$$k_{vCurrent} = \frac{(0.05121)(20)}{(0.5674)(5)}$$
$$k_{vCurrent} = 0.3610$$

And $k_{vDesired}$ is:

$$k_{vDesired} = \frac{1}{E_{ss}}$$
$$k_{vDesired} = 100$$

Plugging in $k_{vCurrent}$ and $k_{vDesired}$, $\alpha_{lag}$is obtained:

$$\alpha_{lag_{total}} = 276.9967$$

The value α for each lag compensator is given by the equation:

$$\alpha_{stage} = \sqrt[n]{\alpha_{lag_{total}}}$$

Where n is the number of stages.

Letting n = 2:

$$\alpha_{stage} = \sqrt[2]{276.9967} = 16.64$$

Which is within the requirements of $\alpha_{stage} < 20$.

Letting the zero location be much less than the desired closed loop pole location:

$$Z_c = -\frac{real\{s_d\}}{5000}$$
$$Z_c = -0.00005334$$

Design the pole to be smaller than the zero by a factor of α_{stage} :

$$P_c = \frac{Z_c}{\alpha_{stage}} = -\frac{0.00005334}{16.64} = 0.0000032055$$

Resulting in the 2-stage lag compensator:

$$G_{cLagStage} = \left(\frac{s + 0.00005334}{s + 0.0000032055}\right)$$

$$G_{cLag} = (G_{cLagStage})\hat{}n$$

$$G_{cLag} = \left(\frac{s + 0.00005334}{s + 0.0000032055}\right)^2$$

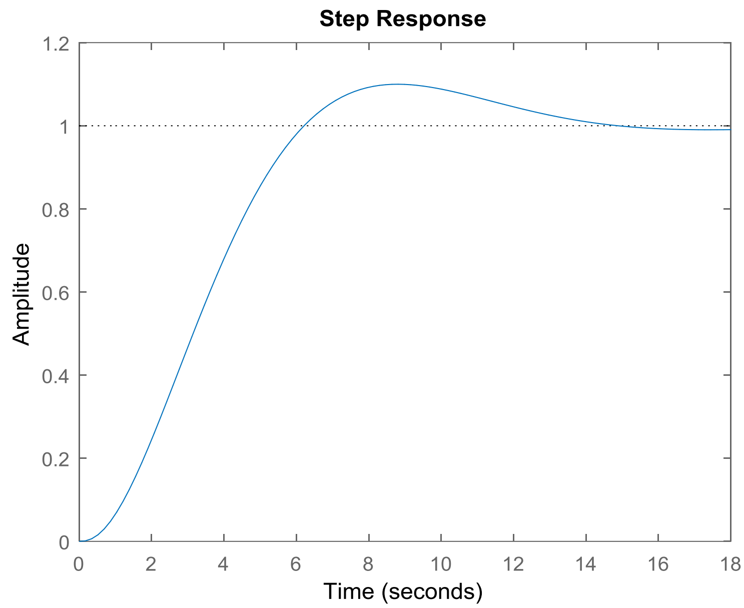
<u>**Thus the final compensated system using a lead-lag compensator is**</u>:

$$G(s) = G_{cLead}(s) * G_{cLag}(s) * G_p(s)$$

$$\boldsymbol{G(s) = \left(\frac{0.05121(s + 0.4)}{(s + 0.5674)}\right)\left(\frac{s + 0.00005334}{s + 0.0000032055}\right)^2\left(\frac{(s + 20)}{s(s + 0.4)(s + 5)}\right)}$$

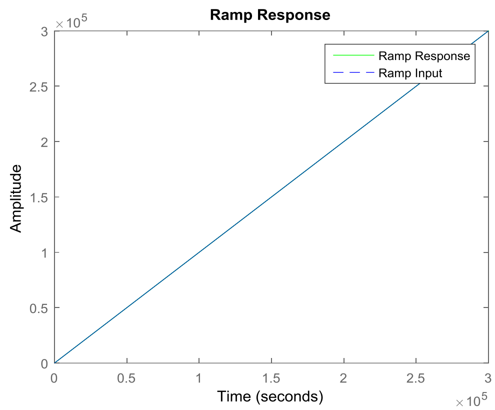
MATLAB simulation of the System

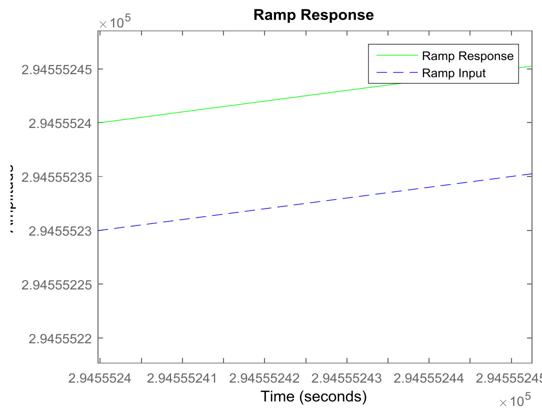
```
%Define transfer function
s= tf('s')
k=1;

%Plant function (gp)
gp = k*(s+20)/(s*(s+0.4)*(s+5))

%Lead compensator (gclead)
gclead = 0.05121*(s+0.4)/(s+0.5674)

%Lag compensator (gclag)
gclag = ((s+0.00005334)/(s+0.0000032055))^2

%Total system(g)
g_ol = gclead*gclag*gp
g_cl = feedback(g_ol,1)

%Step response
step(g_cl)
stepinfo(g_cl)

%Ramp response
figure(2)
t=0:1:300000;
plot(t,t,'g')
hold on
step(g_cl/s,t,'--b')
title('Ramp Response')
legend('Ramp Response', 'Ramp Input')
```




```
ans =

       RiseTime: 4.0839
   SettlingTime: 13.3102
    SettlingMin: 0.9050
    SettlingMax: 1.0999
      Overshoot: 9.9900
     Undershoot: 0
           Peak: 1.0999
       PeakTime: 8.8093
```

The step response of the compensated system satisfies the time domain specifications of a settling time under 15 sec. and a maximum percent overshoot of 10%.





Upon observing the Y-axis, it can be noted that the error is:

$$E_{ss} = RampInput(t_0) - RampResponse(t_0)$$
$$E_{ss} = 294{,}555.24 - 294{,}555.23 = 0.01$$

Which satisfies the steady-state error requirement of 0.01.

Conclusion

Given specifications for a system in the time domain, it is often necessary to design compensators to satisfy these requirements. This lab required the design of a lead compensator and two lag compensators that had to be placed in series to comply with the compensator constraints. Finally, the compensated system was simulated and analyzed in the time domain to ensure the specifications were met.