George Mason University

Ball and Beam Experiment
Design and Test Controller using MATLAB/SIMULINK

Report from laboratory experiment D.1 conducted on 22 April 2016
As part of ECE 429 Control Systems Lab
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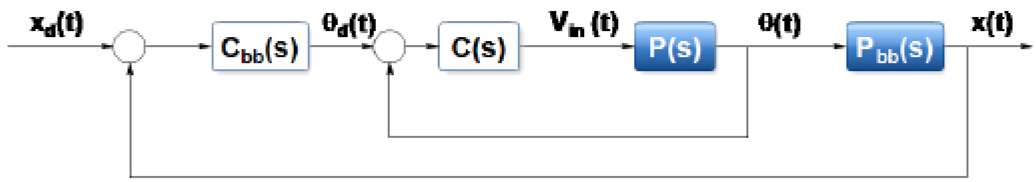
Objective

The objective of this lab is to understand the system of the servo motor and the ball balancer. Once gaining a sufficient understanding, then the student shall design a controller for the position control of the servo. Then simulate the controller in SIMULINK as well as hardware.

Theoretical Background

The system to be implemented can be modeled by the following block diagram:



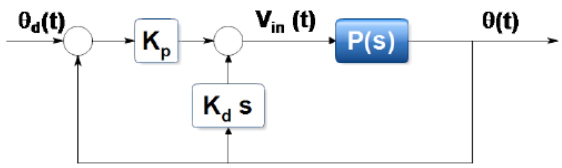
$$P(s) = \frac{K}{s(\tau s + 1)}, \qquad P_{bb}(s) = \frac{K_{bb}}{s^2}$$

The parameters of the systems defined below :

$$K = 1.53, \quad \tau = 0.0248, \quad K_{bb} = 0.419, \quad \omega_f = 2\pi, \quad V_{max} = 10V, \quad \theta_{max} = 0.9774rad$$

Task 1

Design and simulate a PD-DOO position controller for the servo motor by first finding K_p and K_d to satisfy a percent overshoot < 1% and a time to peak <= 0.2s. The block diagram below models the system:



The closed-loop equation for the system is:

$$\frac{x_d(t)}{x(t)} = \frac{(\frac{K_p K}{\tau})}{s^2 + (\frac{1 + KK_d}{\tau})s + \frac{K_p K}{\tau}} \tag{1}$$

Eqn. (1) resembles that of the equation for a general second order system:

$$2nd\ Order\ System = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{2}$$

The gains are able to be calculated by comparing the coefficients of 's' in the denominators of eqns. (1) and (2).

$$K_p = \frac{w_n^2 * \tau}{K} \tag{3}$$

$$K_d = \frac{\tau * 2\zeta w_n - 1}{K} \tag{4}$$

ζ can be solved for using the PO specification of 1%:

$$\zeta = \frac{\left|\ln\left(\frac{PO}{100}\right)\right|}{\sqrt{\pi^2 + \ln^2\left(\frac{PO}{100}\right)}} \tag{5}$$

$$\zeta = 0.8261$$

ω_n can be solved for using the time to peak specification:

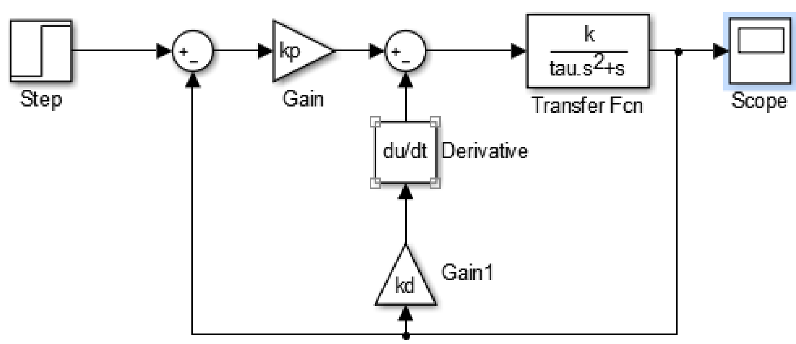
$$\omega_n = \frac{\pi}{t_{peak}\sqrt{1-\zeta^2}} \tag{6}$$

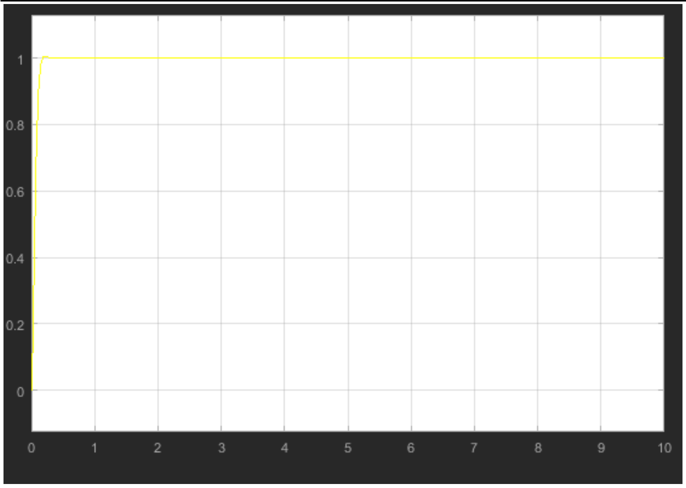
$$\omega_n = 27.8735$$

With ζ and ω_n solved for, the gains K_p and K_d can be calculated:

$$K_p = 12.5934$$
$$K_d = 0.0929$$

Knowing the gains, the system can be simulated in SIMULINK. Below is the model used for the simulation and the results of the simulation:





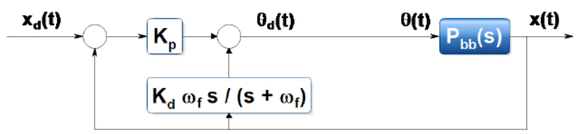

```
ans =

         RiseTime: 0.0922
     SettlingTime: 0.1425
      SettlingMin: 0.9050
      SettlingMax: 1.0100
        Overshoot: 1.0000
       Undershoot: 0
             Peak: 1.0100
         PeakTime: 0.2000
```

The system meets specifications.

Task 2

Design and simulate a PD-DOO position controller for the ball balancing model (assuming $\omega_f \ll s$) by first finding K_p and K_d for multiple percent overshoot and settling time specifications:



The closed-loop equation for the system is:

$$\frac{x_d(t)}{x(t)} = \frac{K_{bb}K_p}{s^2 + k_{bb}k_d s + k_p K_{bb}} \tag{7}$$

Eqn. (7) resembles that of the equation for a general second order system:

$$2nd\ Order\ System = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{8}$$

The gains are able to be calculated by comparing the coefficients of 's' in the denominators of eqns. (7) and (2).

$$K_p = \frac{w_n^2}{K_{bb}} \tag{9}$$

$$K_d = \frac{2\zeta w_n}{K_{bb}} \tag{10}$$

ζ can be solved for using the PO specification:

$$\zeta = \frac{\left|\ln\left(\frac{PO}{100}\right)\right|}{\sqrt{\pi^2 + \ln^2\left(\frac{PO}{100}\right)}} \tag{11}$$

ω_n can be solved for using the time to peak specification:

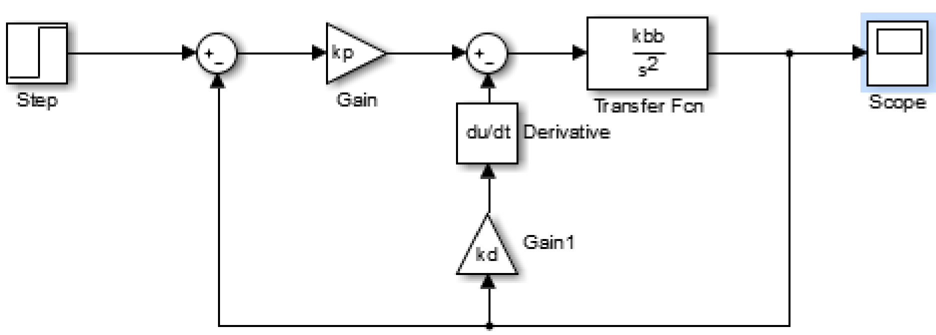
$$\omega_n = \frac{\pi}{t_{peak}\sqrt{1-\zeta^2}} \tag{12}$$

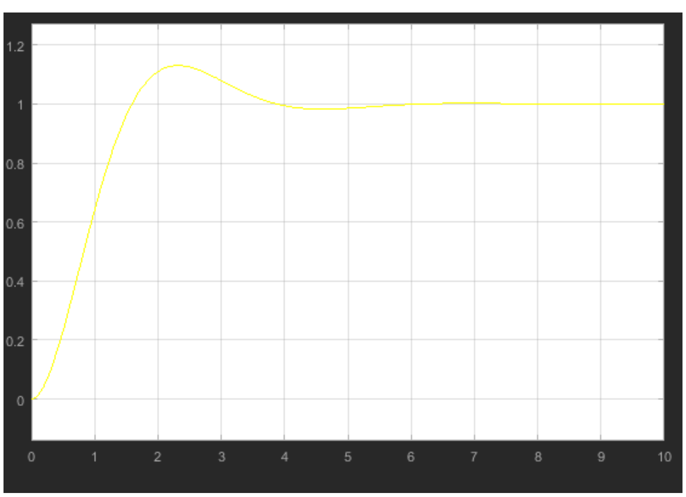
PO = 15% ts = 5s			
zeta	wn	kp	kd
0.8261	1.614	6.2175	6.3644

PO = 1% ts = 3s			
zeta	wn	kp	kd
0.8261	1.614	6.2175	6.3644

Knowing the gains, the systems can be simulated in SIMULINK.

Below is the model used for the simulation and the results of the PO = 15% and ts = 5s simulation:



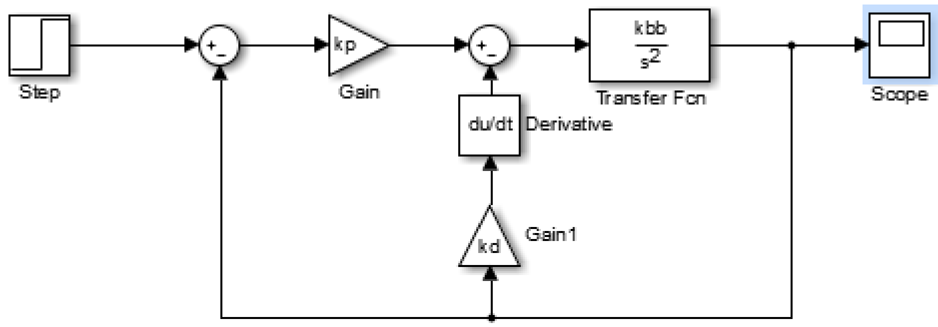


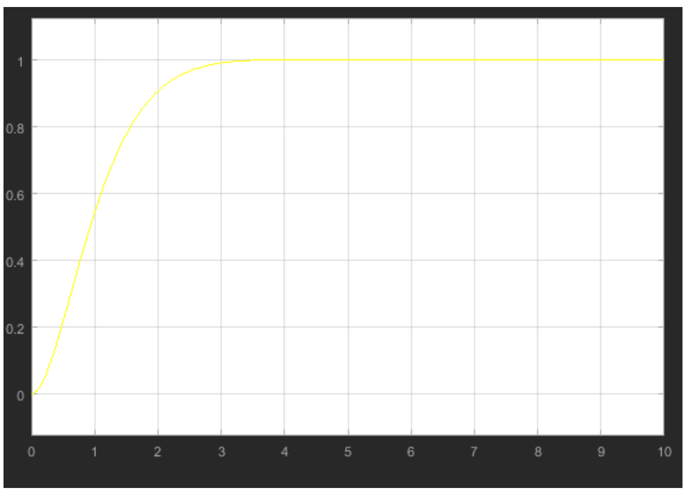

```
ans =

      RiseTime: 1.0810
  SettlingTime: 5.0000
   SettlingMin: 0.9239
   SettlingMax: 1.1500
     Overshoot: 14.9977
    Undershoot: 0
          Peak: 1.1500
      PeakTime: 2.3601
```

The system meets specifications.

Below is the model used for the simulation and the results of the PO = 1% and ts = 3s simulation:





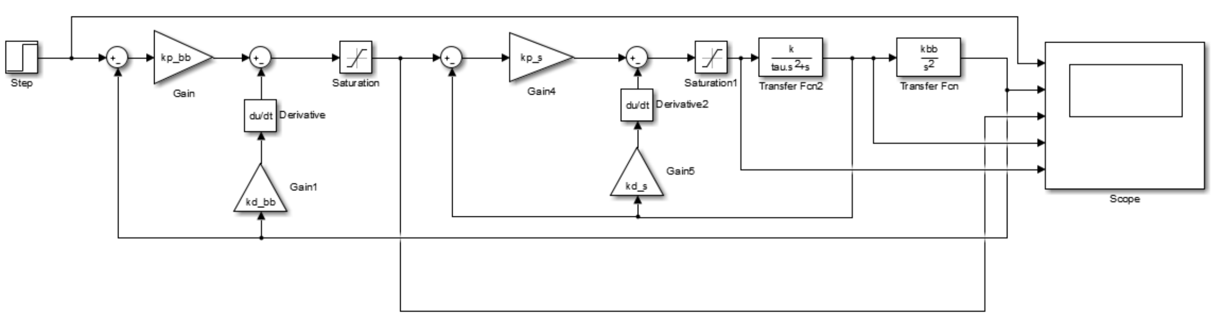

```
ans =

       RiseTime: 1.5920
   SettlingTime: 2.4611
    SettlingMin: 0.9050
    SettlingMax: 1.0100
      Overshoot: 1.0000
     Undershoot: 0
           Peak: 1.0100
       PeakTime: 3.4539
```

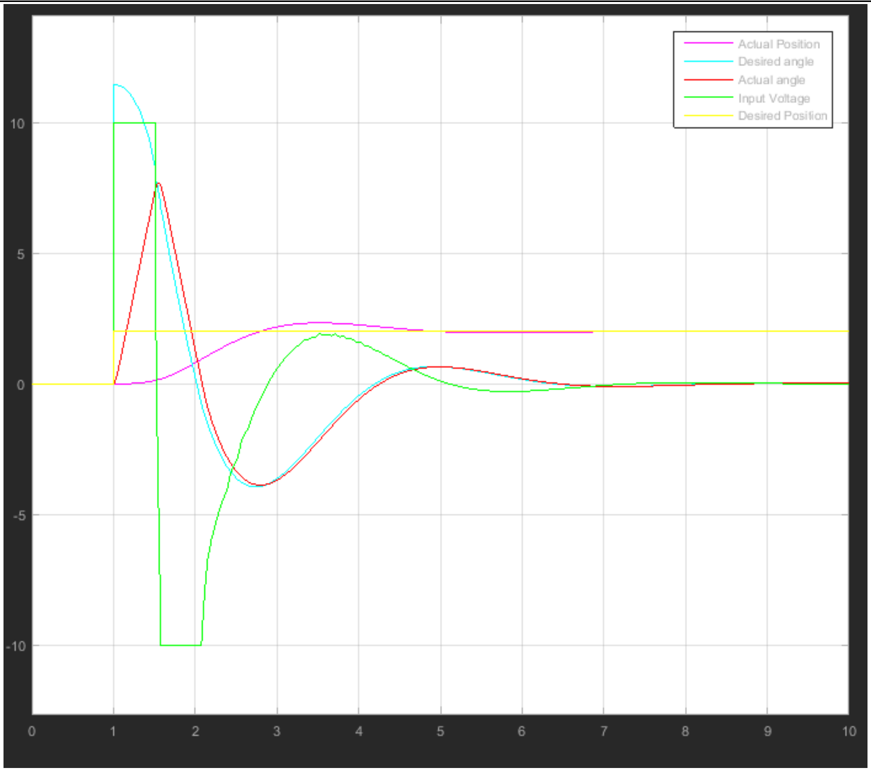
The system meets specifications.

Task 3

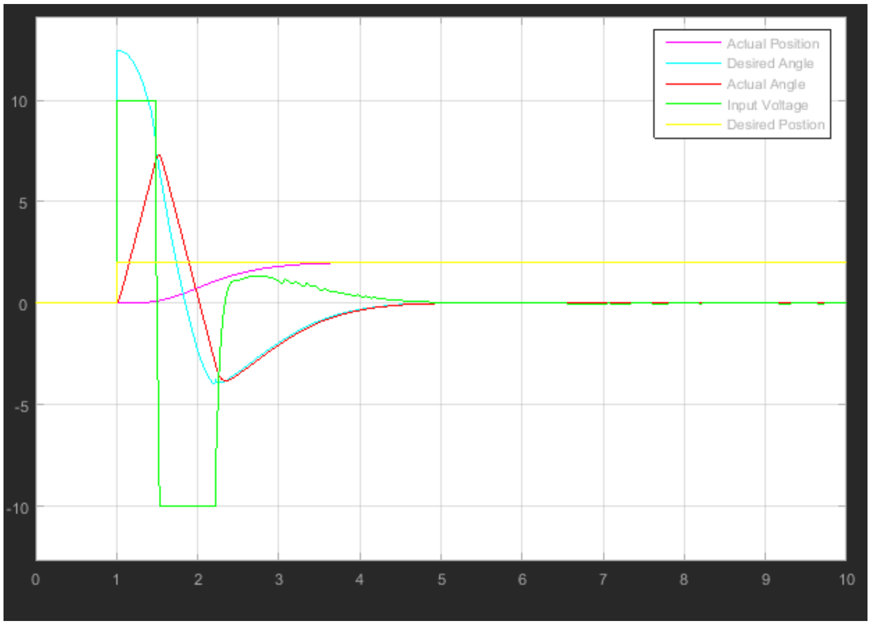
Implementing the whole system for both cases in SIMULINK:



Test Case 2: PO = 1% and ts = 3s.



Test Case 2: PO = 1% and ts = 3s.



The system simulations were as expected. They satisfied the specifications they were designed for.