George Mason University

Compensator Design and Evaluation For Depth Rate Control

Report from laboratory experiment 6 conducted on 14 March 2016
As part of ECE 429 Control Systems Lab
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Introduction

This lab models the dynamics of the relationship between the rate of change of applied force to the rate of change of the vertical position for an underwater vehicle that only ascends and descends. The input to the plant is the rate at which water flows in and out of the depth control tanks and the output is the vertical velocity. This lab will design a lead compensator to meet frequency-domain specifications. The compensator will then be implemented as a Proportional-Derivative (PD) controller and a Proportional-Derivative-on-output-only (PD-DOO) controller.

Objective

The objective of this lab is two-fold. The first part is to design a compensator for the given system so as to satisfy frequency-domain specifications. The second objective is to observe the effects of the compensator as it is implemented in different configurations.

Experiment

The system plant was defined as:

$$G_p(s) = \frac{5 * 10^{-7}}{s^2(s + 0.75)}$$

and required the following frequency-domain specifications:

$$Phase\ Margin: \quad 45\ degrees \leq PM \leq\ 50\ degrees$$

$$Gain\ Crossover\ Frequency\ Range: \quad 0.045\ rad/sec \leq \omega_{gc} \leq 0.055\ rad/s$$

The lead compensator to be designed was of the form:

$$G_{cLead} = k_c * \frac{s + z}{s + p}$$

Where z is:

$$z = \omega_{gc} * \sqrt{\alpha}$$

Where α is:

$$\alpha = \frac{1 - \sin(\Phi_{comp.})}{1 + \sin(\Phi_{comp.})}$$

And where p is:

$$p = \omega_{gc} * \alpha$$

And where k_c is the gain necessary to achieve unity gain at ω_{gc}

The first step in designing the lead compensator was to evaluate the plant at the desired crossover frequency and determine the phase compensation necessary to achieve a phase margin of 47.5 degrees.

```
%System Specifications
wgc = 0.050; %(radians)
pm = 47.5; %(degrees)

%Define the transfer function
s = tf('s');
gp = (5e-7)/(s^2*(s+0.75));

%Calculate the necessary phase compensation
[magGp,phiGp] = bode(gp,wgc)
phic = (-180+pm) - (phiGp);
```

With the necessary phase compensation, the poles and zeros were calculated

```
%Calculate poles and zeros
alpha = (1-sind(phic))/(1+sind(phic));
z = wgc*sqrt(alpha);
p = z/alpha;
```

This yielded the following lead compensator:

$$G_{cLead} = k_c * \frac{(s + 0.0176)}{(s + 0.1424)}$$

The gain was calculated knowing that the gain of the compensated system must equal 1 at the crossover frequency:

$$1 = |G(\omega_{gc})|$$
$$1 = |G_{cLead}(\omega_{gc}) * G_p(\omega_{gc})|$$
$$1 = \left|k_c * \frac{(s+0.0176)}{(s+0.1424)} * \frac{5*10^{-7}}{s^2(s+0.75)}\right| \text{ Evaluated at } \omega_{gc}$$

Thus solving for k_c:

$$k_c = 10706$$

The calculation was done in MATLAB with the following code:

```
%Determine the gain of the lead compensator
gclead = (s+z)/(s+p);
g = gclead*gp;
[mag_g,phi_g] = bode(g,wgc);
k = 1/mag_g;
```

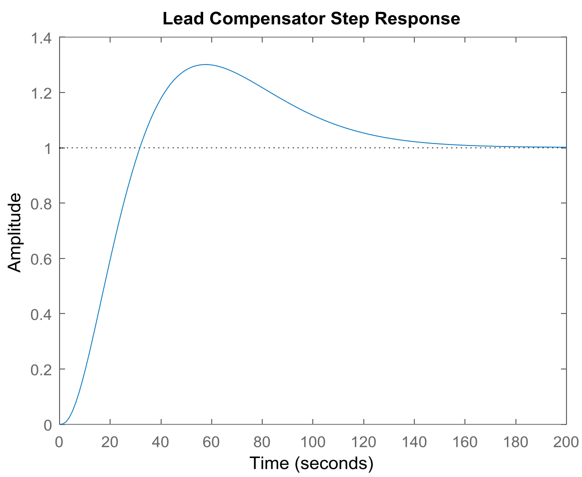
Resulting in the following compensated system:

```
%Compensated System
g = gclead*gp;
g_cl = feedback(g,1)
```

$$G(s) = \left(10706 * \frac{(s + 0.0176)}{(s + 0.1424)}\right)\left(\frac{5 * 10^{-7}}{s^2(s + 0.75)}\right)$$

The following code was used to observe the system response to a step input:

```
%Step-Response
figure(1)
step(g_cl);
stepinfo(g_cl);
```

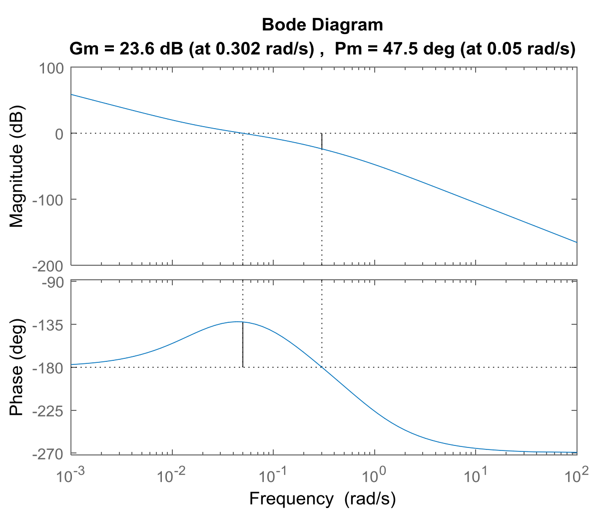


ans =

RiseTime: 21.1238
SettlingTime: 142.2798
SettlingMin: 0.9026
SettlingMax: 1.3010
Overshoot: 30.1029
Undershoot: 0
Peak: 1.3010
PeakTime: 58.0176

The following code was used to verify the frequency-domain specifications:

```
%Margin Specifications
figure(2)
margin(g)
```



Thus, the system achieves a phase margin of 47.5 degrees with a crossover frequency of 0.05 rad/s which satisfies the specifications.

A lead compensator with the form:

$$G_{lead}(s) = \frac{k_c(s - z_c)}{(s - p_c)}$$

Can be converted to a PD controller in the form:

$$G_{PD}(s) = \frac{k_d s + k_p}{\tau s + 1}$$

Using the following equations to transform G_{lead} into G_{PD}:

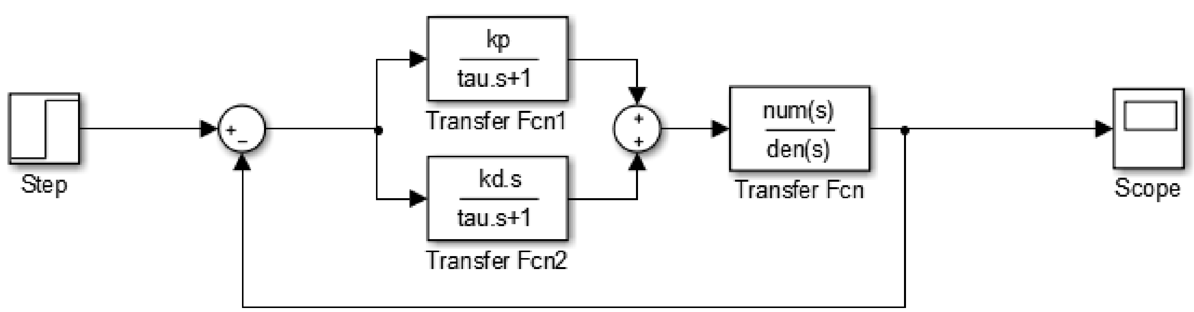
$$\tau = -\frac{1}{p_C} \quad k_p = \frac{k_c z_c}{p_c} \quad k_d = k_c \tau$$

```
%Lead Compensator <--> PD Controller Conversion factors
tau = -1/(-p);
kp = k*(-z)/(-p);
kd = k*tau;
```

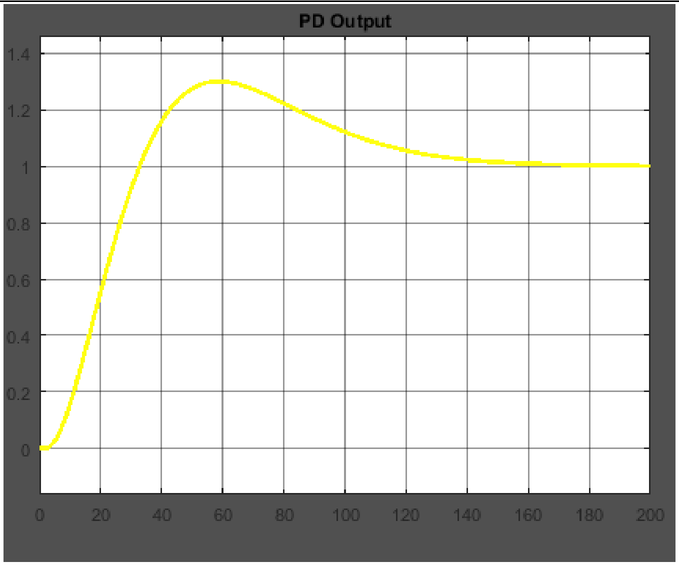
G_{PD} was obtained:

$$G_{PD}(s) = \frac{(75166)s + 1319.3}{(7.0207)s + 1}$$

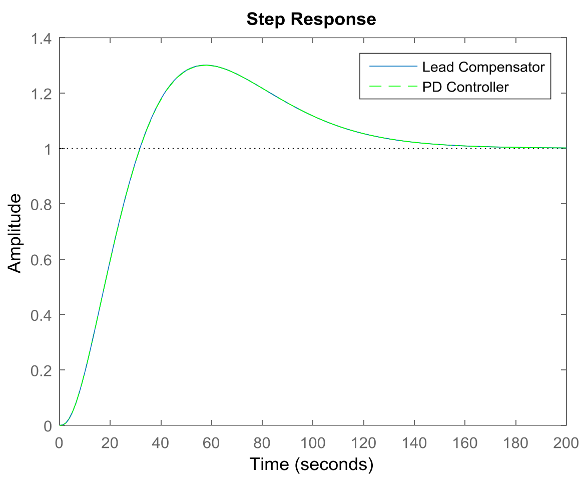
The following is the SIMULINK model used to simulate the step response of the system with the PD controller:



Which yields the following output:

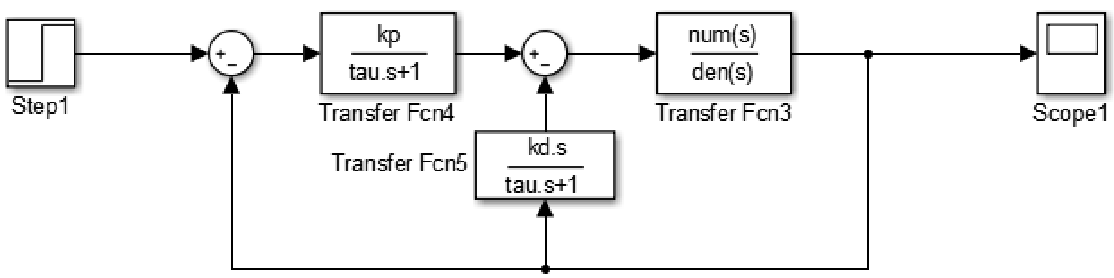


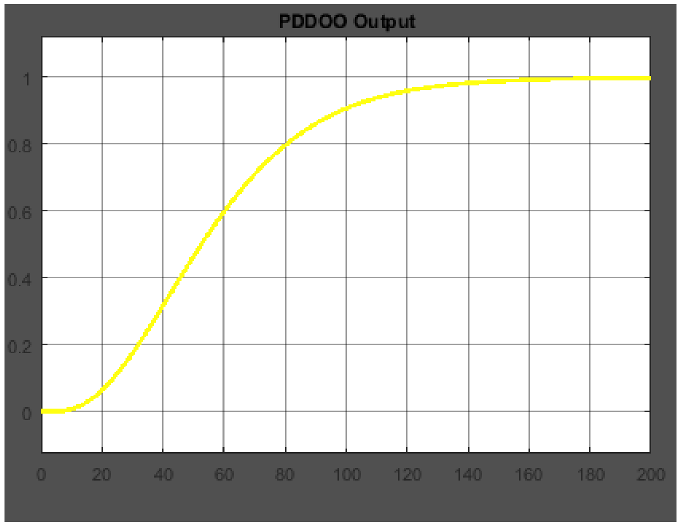
Comparing the output of the PD controller with the output of the lead compensator:



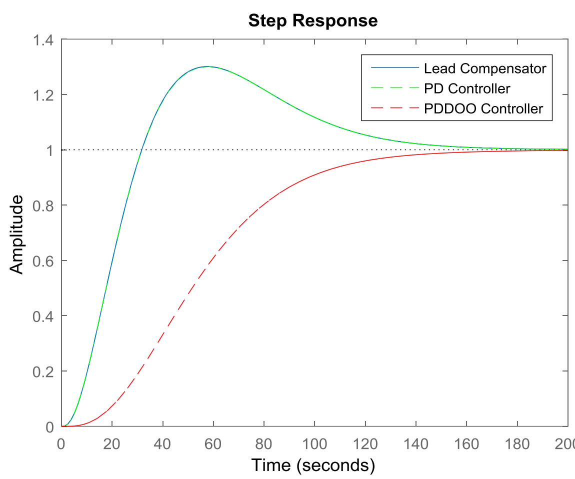
It is apparent that the PD controller and the lead compensator yield the same output for the given step input, and thus are equivalent in implementation as expected.

The following is the SIMULINK model for the PDDOO controller and system used to simulate the step response of the overall system:





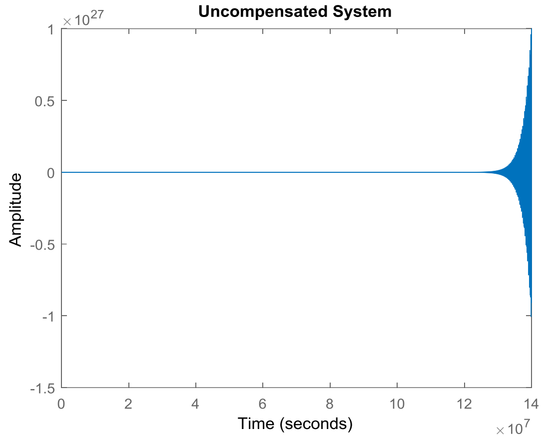
Comparing the output of the lead compensator, PD controller, and the PDDOO controller:

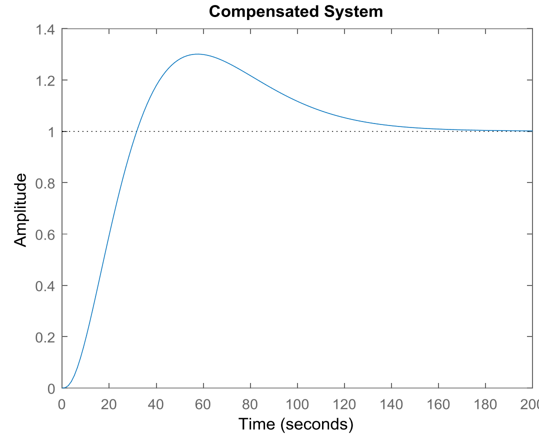


The system response with the PDDOO controller differs from the PD controller and the lead compensator in that it eliminates overshoot in system response. They all maintain approximately the same settling time.

Conclusion

The uncompensated closed loop system step-response proved to be unstable so a compensator was designed yield a stable response:





A stable response is critical for depth control. The uncompensated system exhibits no control since there are oscillations tend towards $\pm\infty$. In a physical system this would translate from oscillating from the surface of the water to the sea floor.

This lab demonstrated that a lead compensator can be transformed into a PD controller and output the same results. However the PDDOO controller seemed to be superior since it minimized the detrimental effects from the sharp change of the step input. This occurs because the derivative control action no longer saw the step input since it was placed on the output only. The resulting response of the PDDOO controlled system was the best since it produced no overshoot and a settling time virtually equivalent to the lead and PD compensator.