George Mason University

Position Control for a Servo Motor
System Identification

Report from laboratory experiment C.1 conducted on 29 March 2016
As part of ECE 429 Control Systems Lab
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Objective

There are two objectives to this lab. The first is to generate the closed loop step-response of the rigid body torsional system and observe the response overshoot and time-to-peak. The second objective is to calculate the moment of inertia and frictional damping coefficient of the system.

Theoretical Background

The system to be controlled is modeled by the equation:

$$T(t) = J\ddot{\theta}(t) + c\dot{\theta}(t) \tag{1}$$

Where

J – Moment of Inertia of the Disc
Θ – Angular Position of the Disc (Output)
T- Torque Applied by the Motor (Input)

The transfer function is defined as the output divided by the input:

$$G_p(s) = \frac{\theta(s)}{T(s)} \tag{2}$$

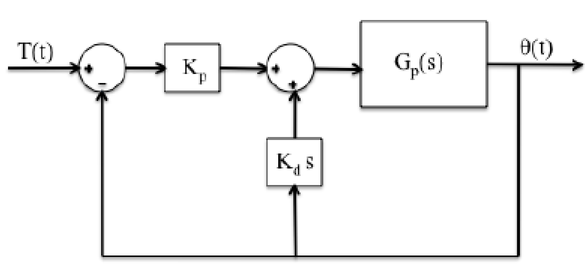
Applying the Laplace transform to eqn. (1):

$$T(s) = Js^2\theta(s) + cs\theta(s) \tag{3}$$

Solving for eqn. (2), the open-loop transfer function of the plant is obtained:

$$G_p(s) = \frac{\theta(s)}{T(s)} = \frac{1}{s(Js + c)} \tag{4}$$

The controller used on the system is a Proportional-Derivative, Derivative on the Output Only (PD-DOO) controller, which is modeled by the following block diagram:



For an open-loop plant function, $G_{ol}(s)$, with a feedback function $H(s)$, the closed loop equation can be found using the following equation:

$$G_{cl} = \frac{G_{ol}(s)}{1 + G_{ol}(s)H(s)} \tag{5}$$

The first step in solving for the closed-loop equation of the total system is to find the closed-loop equation of $G_p(s)$ with the feedback function $K_d s$ using eqn. (5):

$$G_{pCL1} = \frac{\left(\frac{1}{s(Js+c)}\right)}{1 + \left(\frac{1}{s(Js+c)}\right)K_d s}$$

$$G_{pCL1} = \frac{1}{Js^2 + (c + K_d)s} \tag{6}$$

To solve for the total closed-loop equation of the system, $G_{pCL1} * K_p$ will act as the open-loop function, and since unity feedback is applied, $H(s)$ is simply 1. Thus using eqn. (5) the total closed-loop equation is obtained:

$$G_{pCLtotal} = \frac{K_p\left(\frac{1}{Js^2 + (c + K_d)s}\right)}{1 + K_p\left(\frac{1}{Js^2 + (c + K_d)s}\right)}$$

$$G_{pCLtotal} = \frac{K_p}{Js^2 + (C + K_d)s + K_p}$$

$$G_{pCLtotal} = \frac{\left(\frac{K_p}{J}\right)}{s^2 + \left(\frac{C + K_d}{J}\right)s + \left(\frac{K_p}{J}\right)} \tag{7}$$

Eqn. (7) resembles that of the equation for a general second order system:

$$2nd\ Order\ System = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{8}$$

The torsional plant is able to be characterized in comparing eqn. (7) and (8), it can be deduced that :

$$\omega_n^2 = \frac{K_p}{J}$$

$$J = \frac{K_p}{\omega_n^2} \tag{9}$$

And:

$$2\zeta\omega_n = \frac{C + K_d}{J}$$

$$C = J(2\zeta\omega_n) - K_d \tag{10}$$

Thus knowing the gains of the controller, the damping factor and the natural frequency of the system, the moment of inertia of the disc (J) and the damping constant (C) can be obtained.

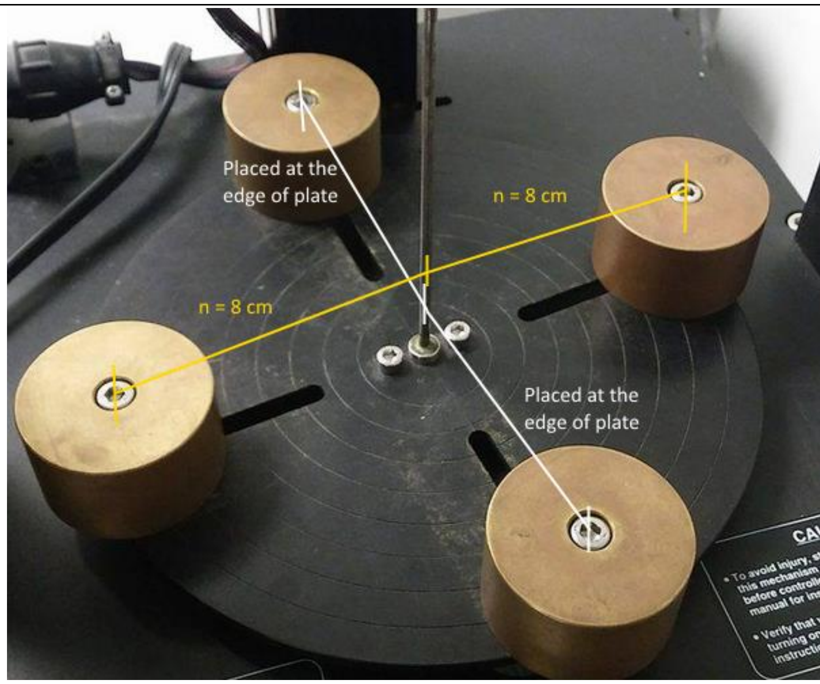
Implementation

Task 1

The Torsional plant was setup according to the instructions as outlined in the Unit C pre-lab. The first step was turning on the computer and then the hardware (plant). *Degrees* was selected under USER UNITS. To setup the control algorithm, I navigated to SETUP CONTROL ALGORITHM and selected *Continuous Time.* Then selected *PI with Velocity Feedback* as the Algorithm. Then *Setup Algorithm* was clicked and upon where the gains for the controller were all set to zero. Next *Implement Algorithm* was clicked followed by *Ok.* Then I navigated to COMMAND and then TRAJECTORY CONFIGURATION, where *Step* was selected and then *Setup. Closed Loop Step* was selected with a *Step Size* of 25 degrees. *Dwell Time* was set to 5000 ms with 1 as the *Number of Steps.*

Task 2

Arranged the weights on the rotational plate accordingly:



Note that the outer edge of the plate is 18 cm in diameter, or 9 cm in radius.

Task 3

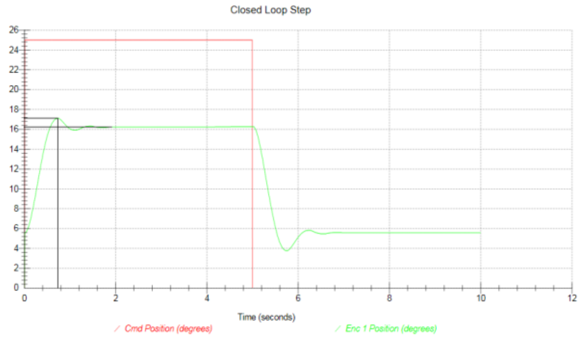
The initial values of $K_p$and $K_d$of the PI with velocity feedback controller were set to zero. K_p was adjusted to obtain a stable step response curve from a 25 degree step input. Two additional K_p values that yielded a stable step response were also obtained.

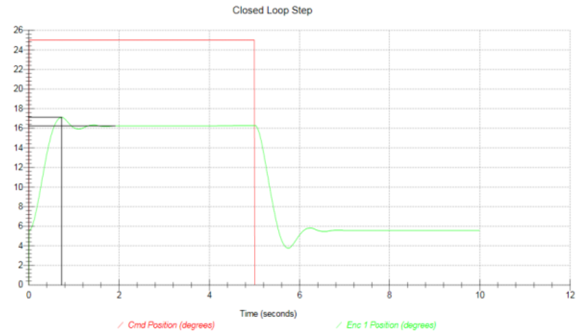
The three values of $K_p$were 0.01, 0.02, 0.035.

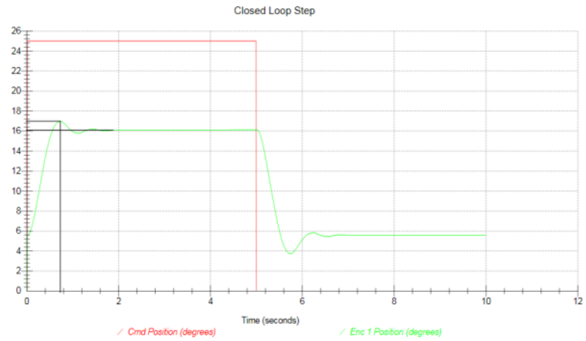
Task 4

Three system responses were recorded for each value of K_p. From the responses percent overshoot and time to peak were measured using pixel resolution for time and amplitude values. The following are the simulation results along with the tabularized data for each value of K_p:

---------------------------- $K_p = 0.01$ ----------------------------

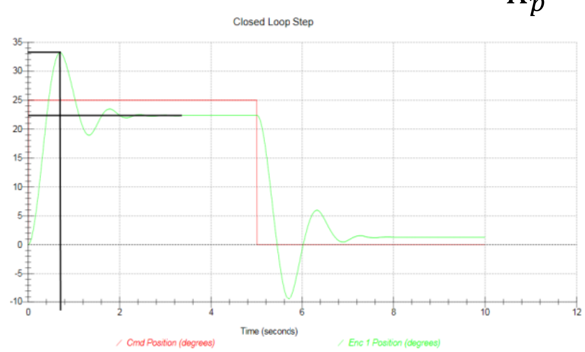


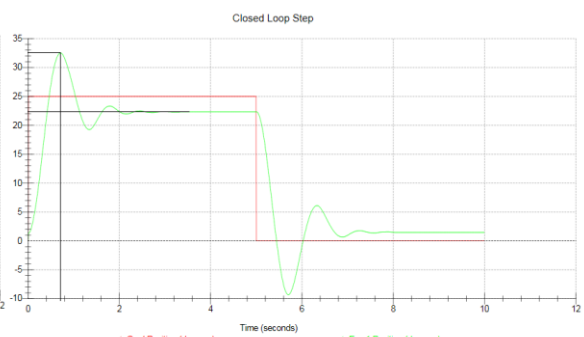


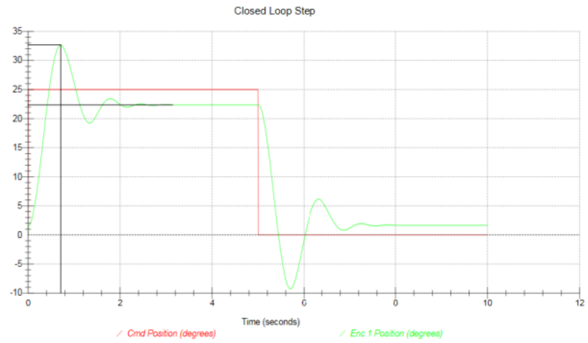


Kp	Trial	Final Value	Peak Value	PO	Peak Time (s)
0.01	1	16.2370	17.0670	5.1118	0.7540
	2	16.2670	17.1350	5.3360	0.7540
	3	16.1330	17.0010	5.3803	0.7540
	Average	16.2123	17.0677	5.2760	0.7540

--------------------------- $K_p = 0.02$ ---------------------------

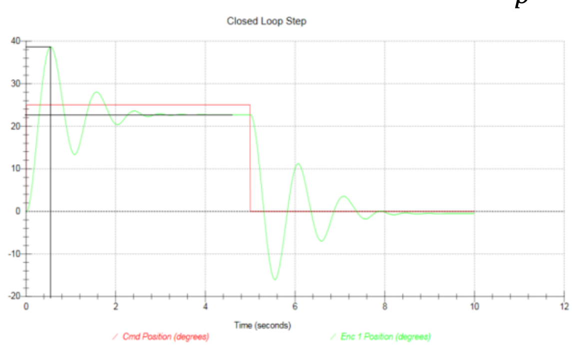


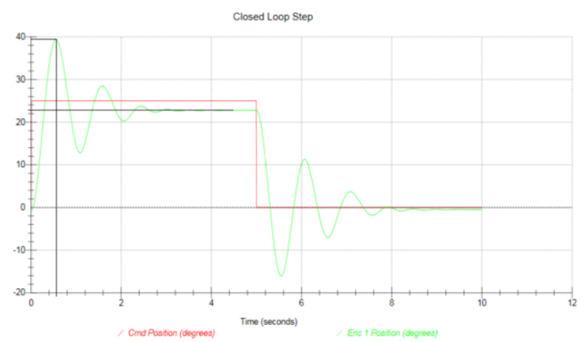


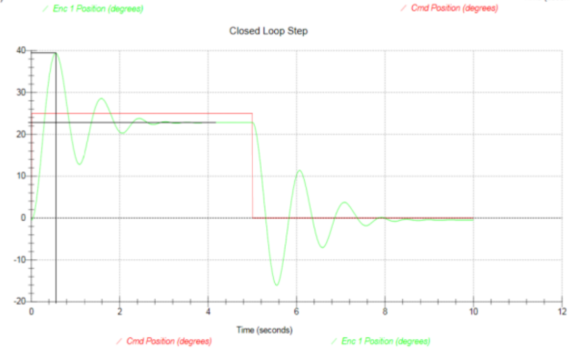


Kp	Trial	Final Value	Peak Value	PO	Peak Time (s)
0.02	1	22.3750	33.3750	49.1620	0.7110
	2	22.3750	32.6250	45.8101	0.7110
	3	22.3750	32.7500	46.3687	0.7110
	Average	22.3750	32.9167	47.1136	0.7110

--------------------------- $K_p = 0.035$ ---------------------------







Kp	Trial	Final Value	Peak Value	PO	Peak Time (s)
0.035	1	22.8330	38.8330	70.0740	0.5629
	2	23.0000	39.6670	72.4652	0.5629
	3	23.0000	39.6667	72.4639	0.5629
	Average	22.9443	39.3889	71.6677	0.5629

------------------------- Comparison of the $K_p$average values -------------------------

Kp	Final Value	Peak Value	PO	Peak Time (s)
0.010	16.2123	17.0677	5.2760	0.7540
0.020	22.3750	32.9167	47.1136	0.7110
0.035	22.9443	39.3889	71.6677	0.5629

Using the percent overshoot results, the damping factor (ζ) of the system can be determined by the following equation:

$$\zeta = \frac{\left|\ln\left(\frac{PO}{100}\right)\right|}{\sqrt{\pi^2 + \ln^2\left(\frac{PO}{100}\right)}} \tag{11}$$

Kp	PO	ζ
0.010	5.2760	0.6835
0.020	47.1136	0.2330
0.035	71.6677	0.1054

Knowing the damping factor and the time to peak, the natural frequency ω_n of the system can be determined by the following equation:

$$\omega_n = \frac{\pi}{t_{peak}\sqrt{1-\zeta^2}} \tag{12}$$

Kp	tp (s)	ζ	ωn (rad/s)
0.010	0.7540	0.6835	5.7083
0.020	0.7110	0.2330	4.5436
0.035	0.5629	0.1054	5.6124

Task 5

Knowing the gains of the controller, the natural frequency and damping factor of the system, eqns. (9) and (10) can be used to calculate the moment of inertia of the disc (J) and damping factor due to friction (C), respectively:

Kp	Kd	ζ	ωn (rad/s)	J	C
0.010	0.000	0.6835	5.708308864	0.0003069	0.002395
0.020	0.000	0.2330	4.543577458	0.0009688	0.002051
0.035	0.000	0.1054	5.612374447	0.0011112	0.001315

Note that for each simulation J and C change. According to eqn. (9), J is a directly proportional to K_p. Thus an increased $K_p$would yield an increased J. On the other hand, C is directly proportional to $J(2\zeta\omega_n)$. Since the magnitude of the damping factor (ζ) decrease is larger than the magnitude of the increase of K_p, we see a decrease in the value of C, despite the increase in K_p.

Conclusion

By observing the system response, specifically the time to peak and percent overshoot, we can model the system by comparing the denominator of the closed loop system to that of the general form of a second order system.