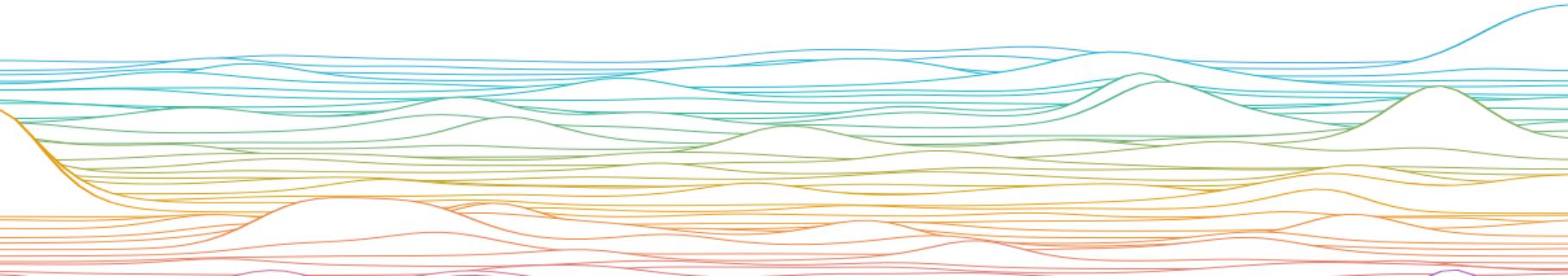


Self-Healing Neural Codes

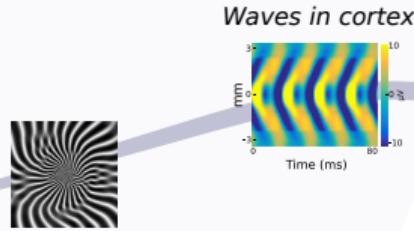
Changing to stay the same

M. Rule

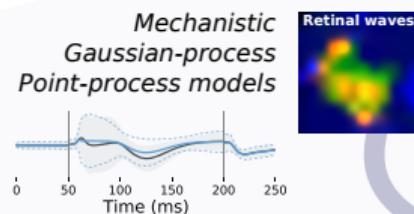
A job talk for lecturer in machine learning at the University of Edinburgh School of Informatics
20th June, 2022



Carnegie-Mellon University
B.S. Computer Science
Neural field theory



*Flicker-phosphene
hallucinations*



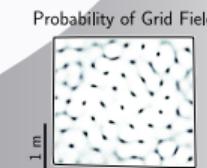
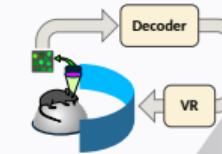
Brown University
Ph.D. Neuroscience
Statistical Modelling



Control Group
University of Cambridge
Homeostasis, Learning: Closed-loop

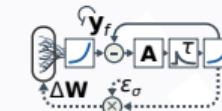
Past

Present



**Learning in
closed-loop**

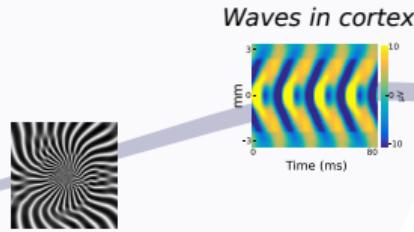
**Point-process
methods**



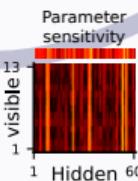
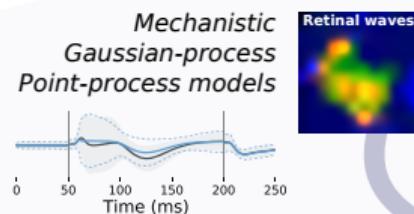
**Biological
Learning**

**University of
Edinburgh (II)**
Future

Carnegie-Mellon University
B.S. Computer Science
Neural field theory

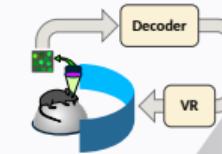


*Flicker-phosphene
hallucinations*

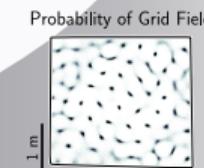


*Statistics of and
representations in
Boltzmann machines*

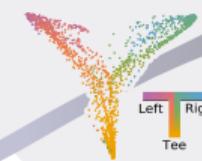
Brown University
Ph.D. Neuroscience
Statistical Modelling



**Learning in
closed-loop**



**Point-process
methods**



*Drift, homeostasis,
and learning*



**Biological
Learning**

Control Group
University of Cambridge
Homeostasis, Learning: Closed-loop

Present

University of Edinburgh (I)
Statistical methods

Past

**University of
Edinburgh (II)**
Future

How do real neurons learn?

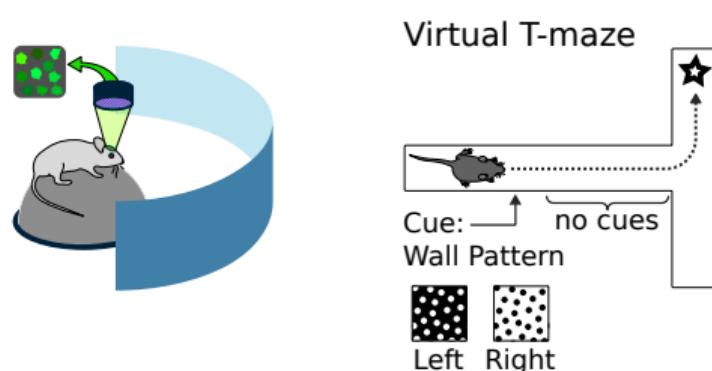
The brain is plastic.

The brain remembers.

How is the brain changing?

Driscoll & al. 2017 experiments:

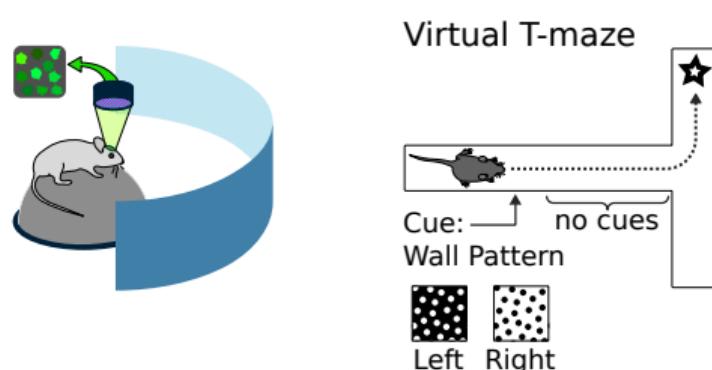
- ▶ Image neural population activity over time in a fixed virtual-reality task



How is the brain changing?

Driscoll & al. 2017 experiments:

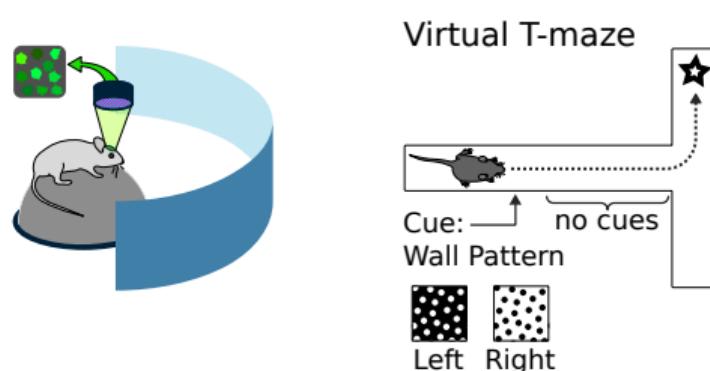
- ▶ Image neural population activity over time in a fixed virtual-reality task
- ▶ Posterior Parietal Cortex (PPC): Association area required for task



How is the brain changing?

Driscoll & al. 2017 experiments:

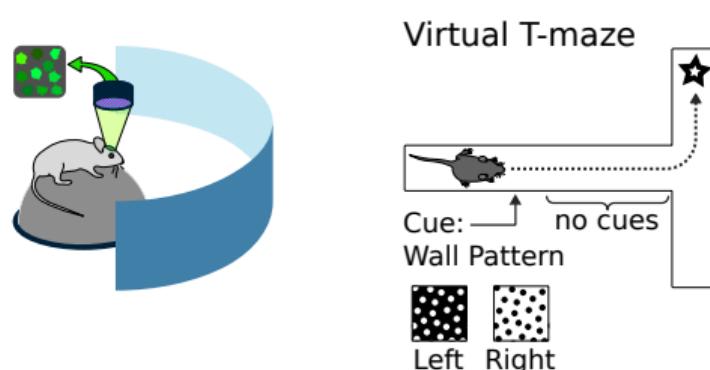
- ▶ Image neural population activity over time in a fixed virtual-reality task
- ▶ Posterior Parietal Cortex (PPC): Association area required for task
- ▶ Long-term recording at steady state: task performed at expert level



How is the brain changing?

Driscoll & al. 2017 experiments:

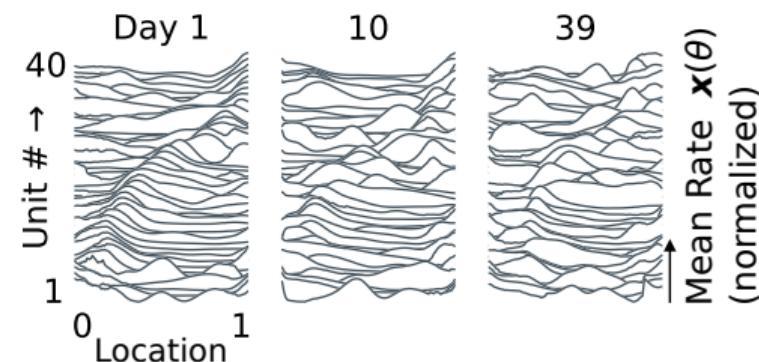
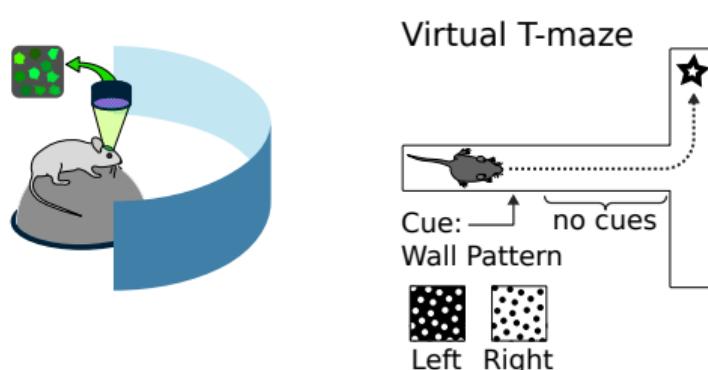
- ▶ Image neural population activity over time in a fixed virtual-reality task
- ▶ Posterior Parietal Cortex (PPC): Association area required for task
- ▶ Long-term recording at steady state: task performed at expert level
- ▶ No change in performance or behavior



How is the brain changing?

Driscoll & al. 2017 experiments:

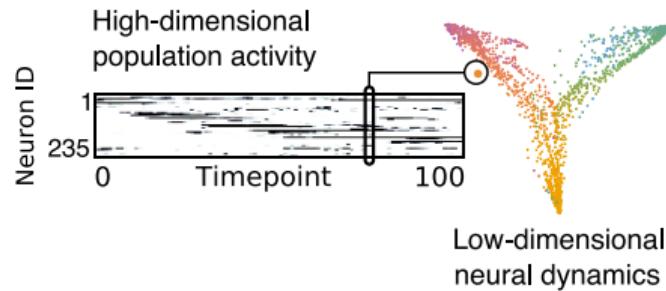
- ▶ Image neural population activity over time in a fixed virtual-reality task
- ▶ Posterior Parietal Cortex (PPC): Association area required for task
- ▶ Long-term recording at steady state: task performed at expert level
- ▶ No change in performance or behavior
- ▶ Neural population code “drifts”



... Neurons' roles change over time, even in representations supporting fixed, habitual tasks.

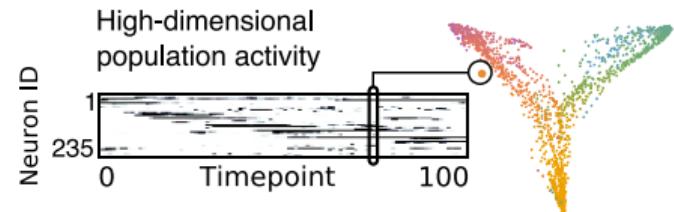
Underlying geometry is stable

Tim O'Leary & Chris Harvey



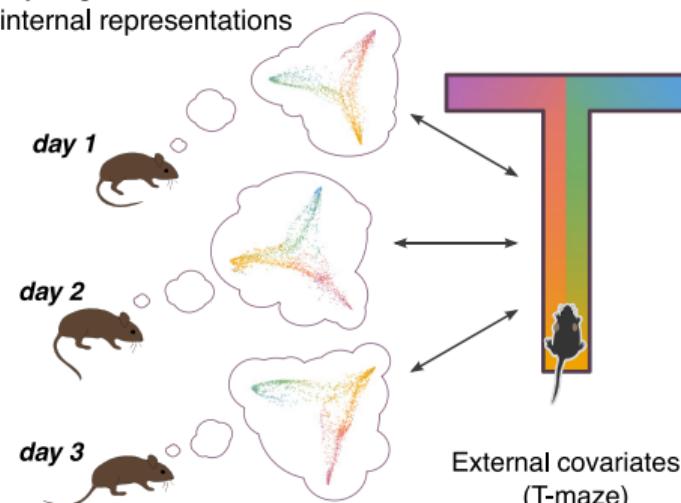
Underlying geometry is stable

Tim O'Leary & Chris Harvey



Low-dimensional neural dynamics

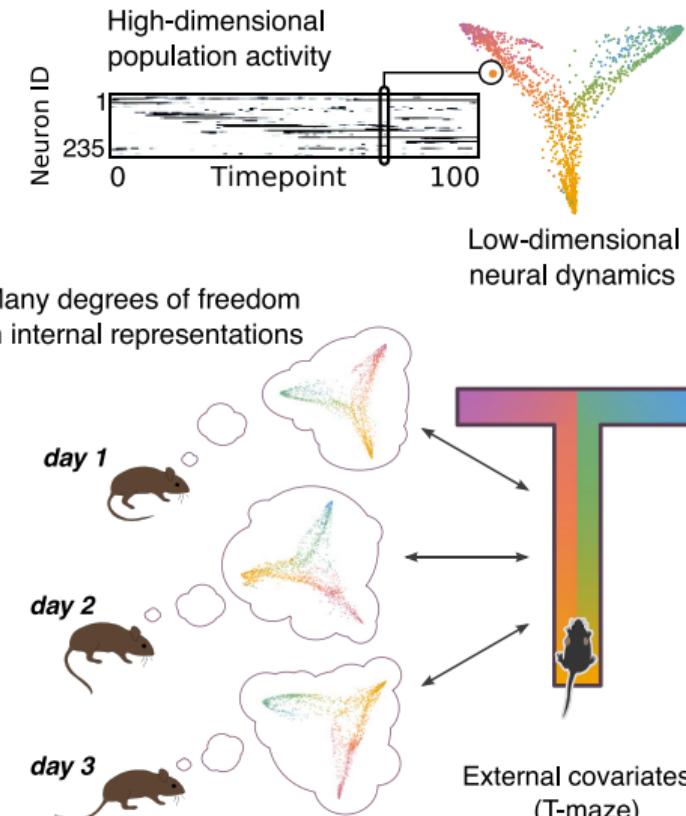
Many degrees of freedom
in internal representations



Rule, M. E., O'Leary, T., and Harvey, C. D. (2019). Causes and consequences of representational drift. *Current opinion in neurobiology*, 58:141–147

Underlying geometry is stable

Tim O'Leary & Chris Harvey



Ubiquitous, not uniform:

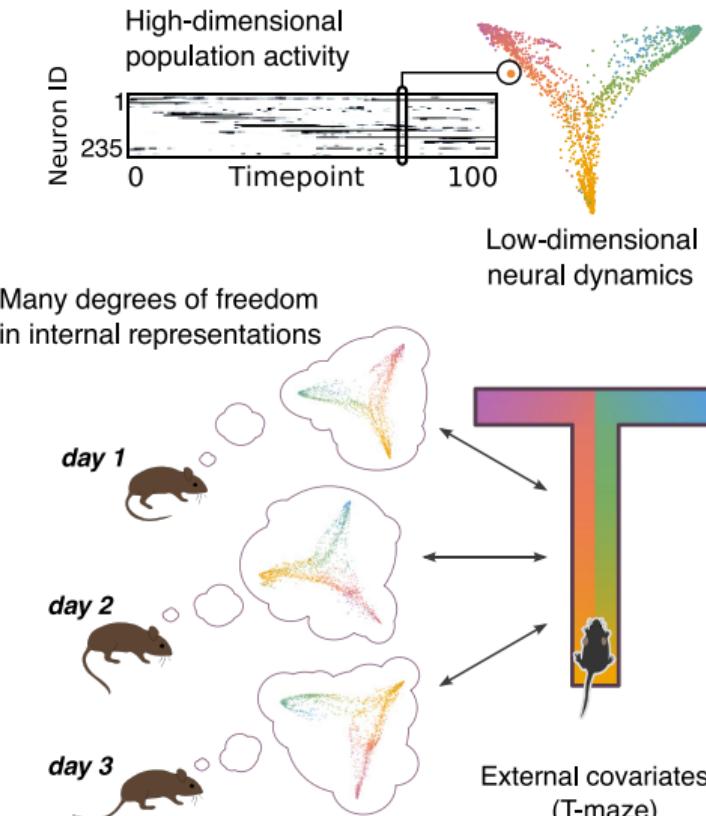
- ▶ **Hippocampus:**^{1,2} Fast turnover; Episodic memory?
- ▶ **Olfactory:**³ Experience-dependent: A code for novelty?
- ▶ **Visual⁴, Prefrontal⁵:** More stable near periphery?

¹Ziv & al. (2013); ²Levy & al. (2021); ³Schoonover, Fink & al. (2020);
⁴Deitch & al. (2020); ⁵Singh & al. (2019);

Rule, M. E., O'Leary, T., and Harvey, C. D. (2019). Causes and consequences of representational drift. *Current opinion in neurobiology*, 58:141–147

Underlying geometry is stable

Tim O'Leary & Chris Harvey



Ubiquitous, not uniform:

- ▶ **Hippocampus:**^{1,2} Fast turnover; Episodic memory?
- ▶ **Olfactory:**³ Experience-dependent: A code for novelty?
- ▶ **Visual⁴, Prefrontal⁵:** More stable near periphery?

¹Ziv & al. (2013); ²Levy & al. (2021); ³Schoonover, Fink & al. (2020);
⁴Deitch & al. (2020); ⁵Singh & al. (2019);

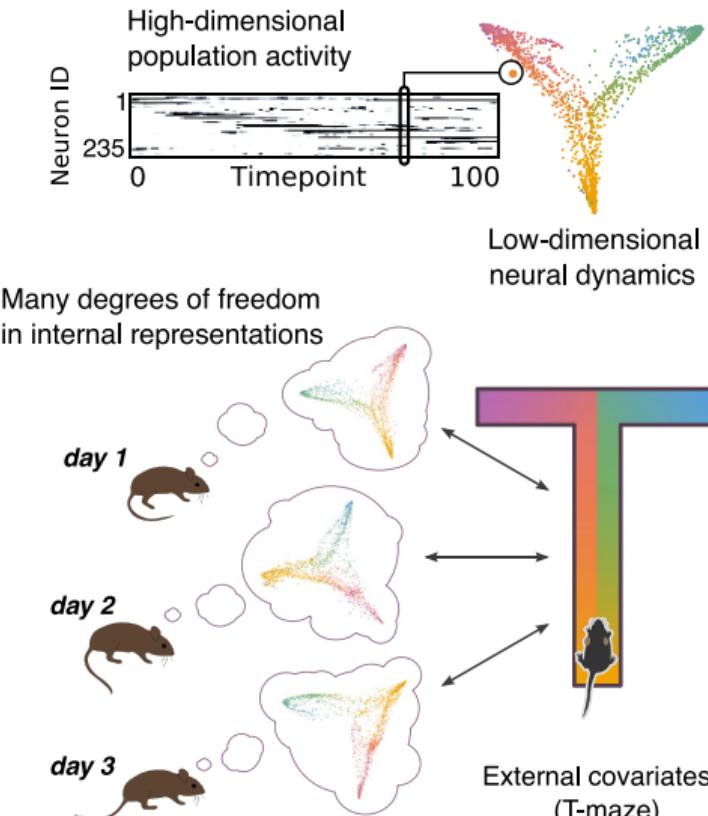
What can drift tell us about learning?

- ▶ Learning in over-parameterized networks?
- ▶ Strategies for rapid learning?
- ▶ Representations in large generative models?
- ▶ Avoiding catastrophic forgetting in continual learning?

Rule, M. E., O'Leary, T., and Harvey, C. D. (2019). Causes and consequences of representational drift. *Current opinion in neurobiology*, 58:141–147

Underlying geometry is stable

Tim O'Leary & Chris Harvey



Ubiquitous, not uniform:

- ▶ **Hippocampus:**^{1,2} Fast turnover; Episodic memory?
- ▶ **Olfactory:**³ Experience-dependent: A code for novelty?
- ▶ **Visual⁴, Prefrontal⁵:** More stable near periphery?

¹Ziv & al. (2013); ²Levy & al. (2021); ³Schoonover, Fink & al. (2020);
⁴Deitch & al. (2020); ⁵Singh & al. (2019);

What can drift tell us about learning?

- ▶ Learning in over-parameterized networks?
- ▶ Strategies for rapid learning?
- ▶ Representations in large generative models?
- ▶ Avoiding catastrophic forgetting in continual learning?

Rule, M. E., O'Leary, T., and Harvey, C. D. (2019). Causes and consequences of representational drift. *Current opinion in neurobiology*, 58:141–147

How does the brain achieve stable behavior despite internal change?

Could other neurons read a code like this?

How does the brain achieve stable behavior despite internal change?

Could other neurons read a code like this?

Proposition:

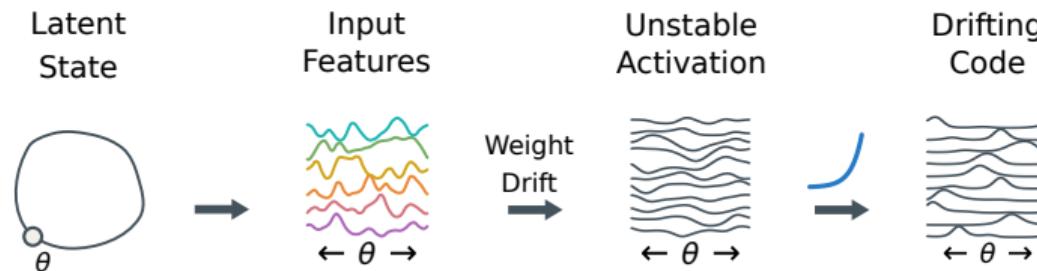
- ▶ Homeostasis stabilizes single neuron properties **and learned associations**
- ▶ Hebbian plasticity maintains learned representation **by re-enforcing existing correlation structure**

Supported by

LEVERHULME
TRUST
early-career fellowship

Unstable code, stable latent structure: A problem for long-term stability?

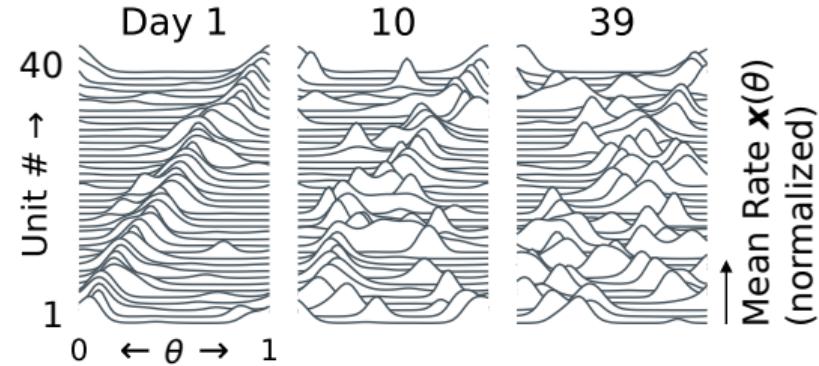
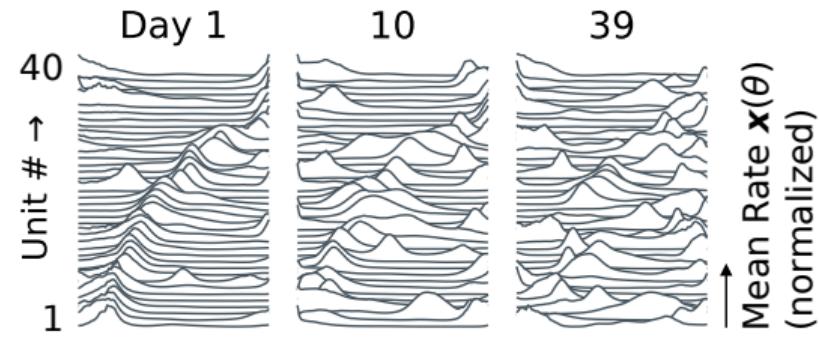
Model drift: Random features + drifting weights + homeostatic normalization



*Sensitivity & threshold homeostasis
preserve localized tuning[†]*

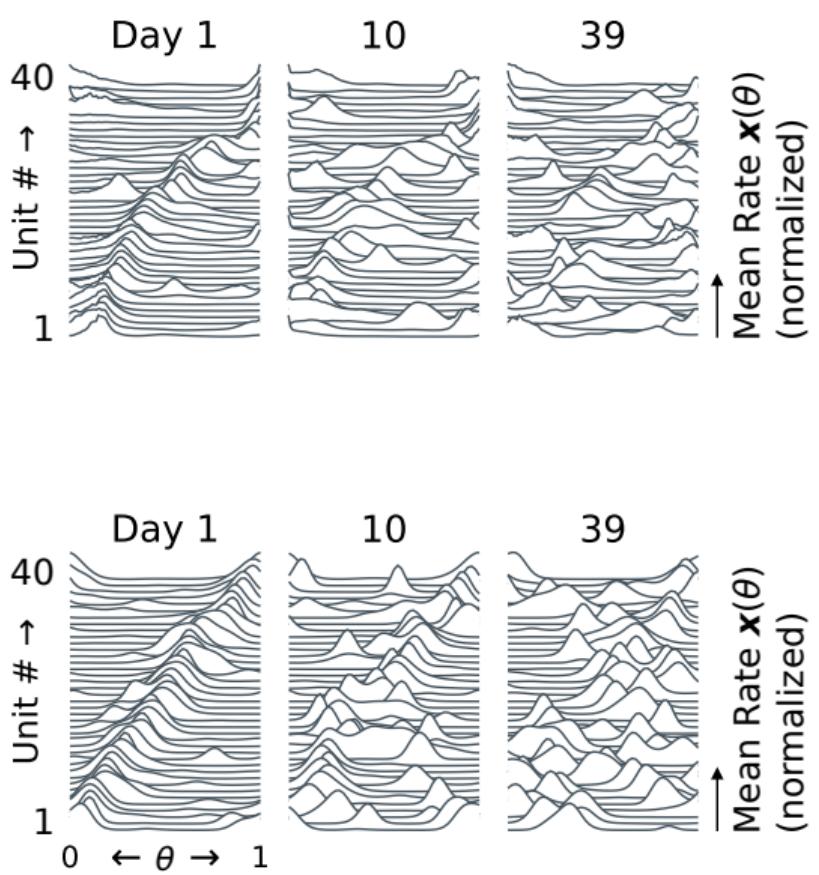


[†]**Homeostasis:** Cannon & Miller (2017, 19),

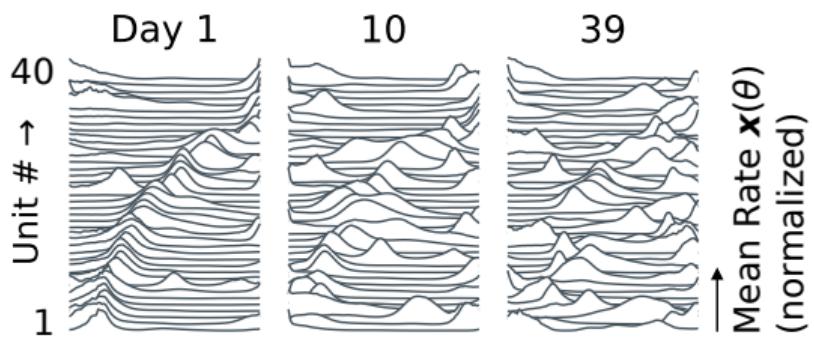


Simulated drift: which is data, which is model?

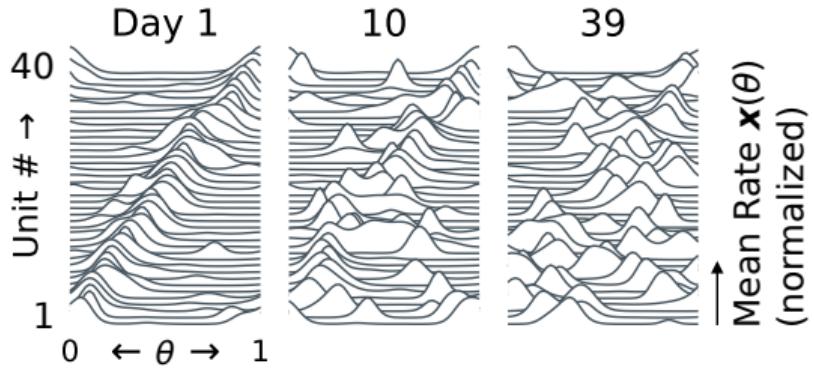
Simulated drift: which is data, which is model?



Simulated drift: which is data, which is model?



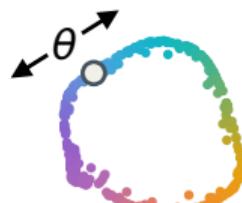
This one is data



This one is model

Is a stable readout possible?

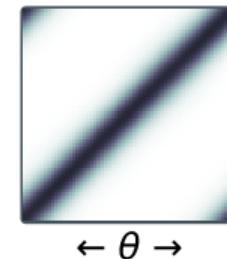
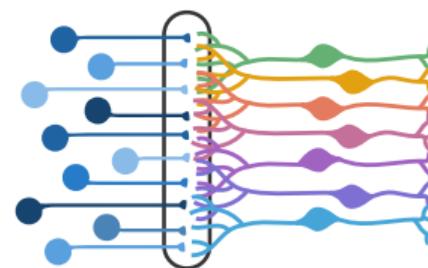
Latent State



Drifting Code



Stable Readout?



Cell #

0 Rate 1

A readout population learns to decode location from an encoding population that drifts.
Can it preserve this readout as the encoding changes?

Yes, with some ongoing learning



T. O'Leary



A. Loback



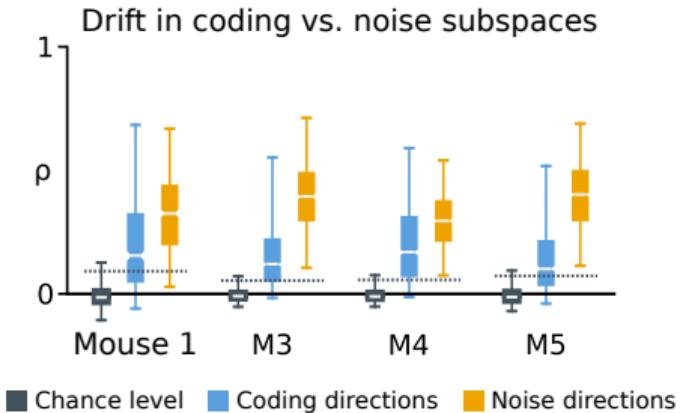
D. Raman



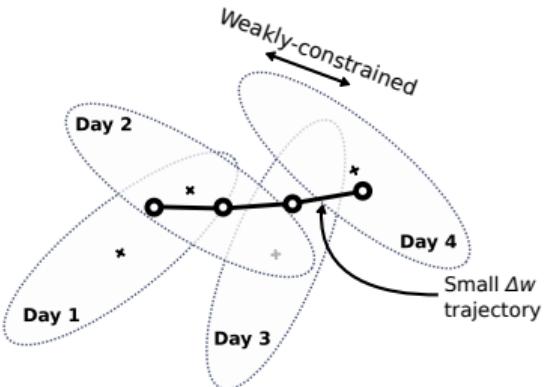
L. Driscoll



C. D. Harvey

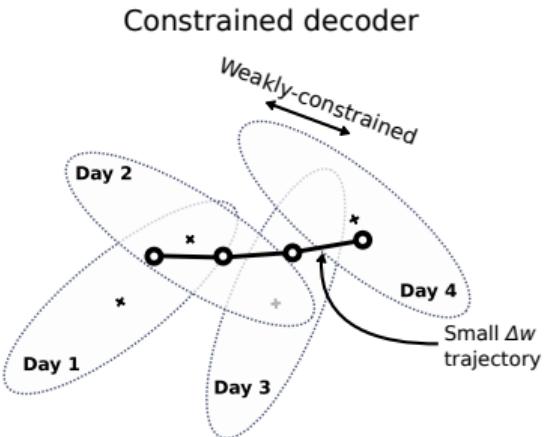
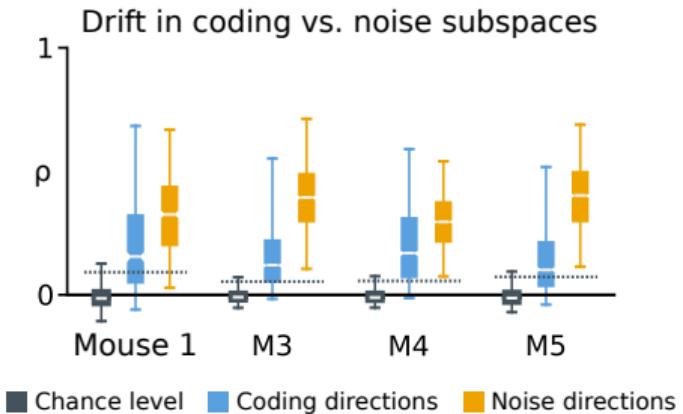


Constrained decoder



Rule, M. E., Loback, A. R., Raman, D. V., Driscoll, L. N., Harvey, C. D., and O'Leary, T. (2020). Stable task information from an unstable neural population. *Elife*, 9:e51121

Yes, with some ongoing learning



Rule, M. E., Loback, A. R., Raman, D. V., Driscoll, L. N., Harvey, C. D., and O'Leary, T. (2020). Stable task information from an unstable neural population. *Elife*, 9:e51121

- ▶ Most drift looks like trial-to-trial variability on slow time-scales

Yes, with some ongoing learning



T. O'Leary



A. Loback



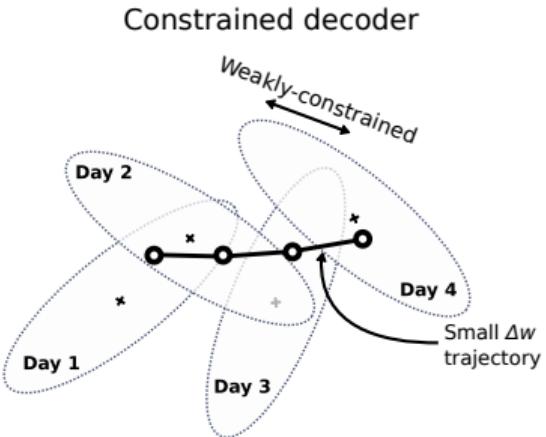
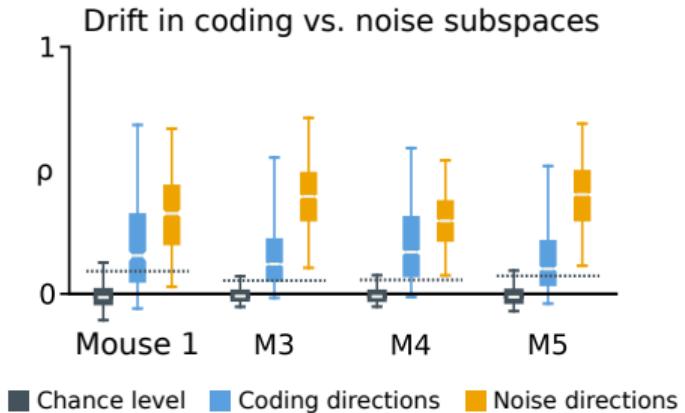
D. Raman



L. Driscoll



C. D. Harvey

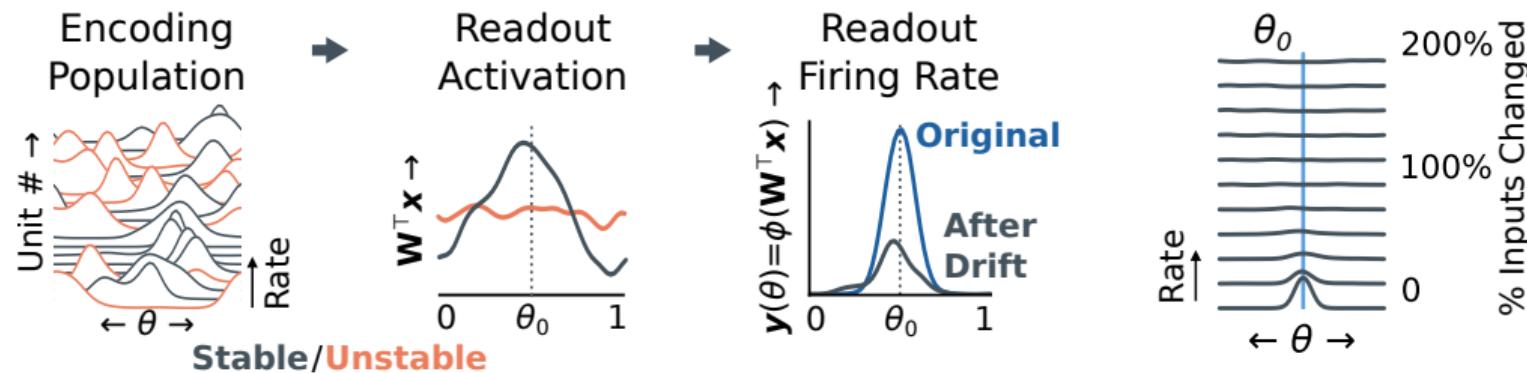


Rule, M. E., Loback, A. R., Raman, D. V., Driscoll, L. N., Harvey, C. D., and O'Leary, T. (2020). Stable task information from an unstable neural population. *Elife*, 9:e51121

- ▶ Most drift looks like trial-to-trial variability on slow time-scales
- ▶ Change in the underlying representation is gradual
 - Only small weight changes needed to stabilize decoding

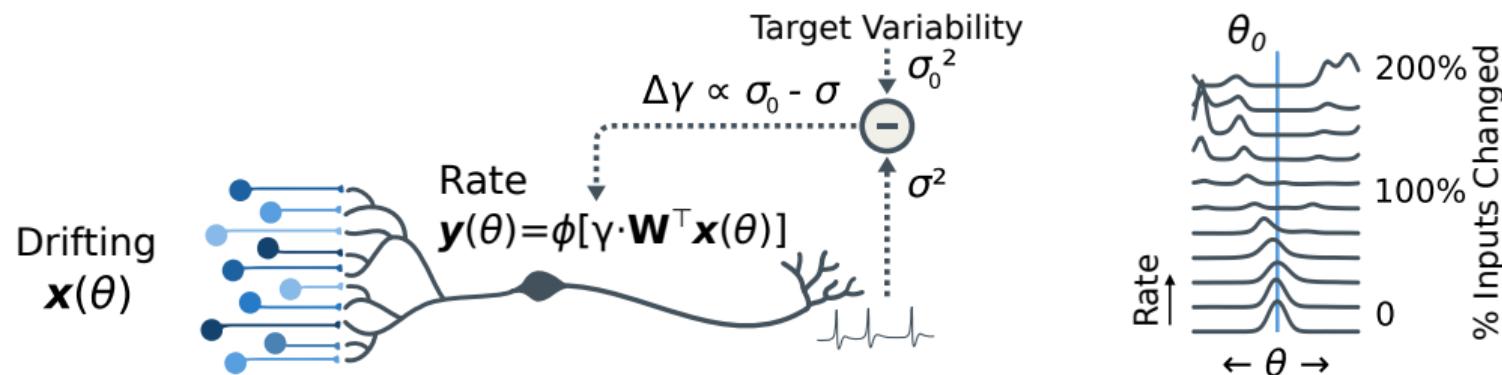
*Can we track a drifting code for continuous variables
without an external reference?*

Fixed weights: Drift attenuates excitatory drive



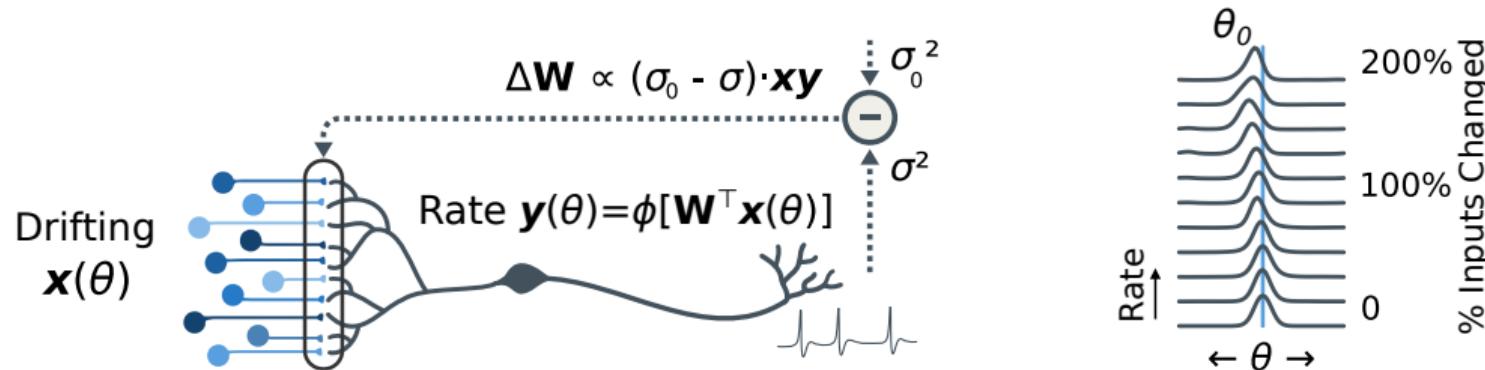
Readout neurons require a specific conjunction of inputs to fire.
Random drift destroys this excitatory drift, but doesn't change tuning.

Sensitivity homeostasis: Tuning is robust to modest change



Homeostatically scaling up weights stabilizes activity against small amounts of drift, since the input is somewhat redundant.

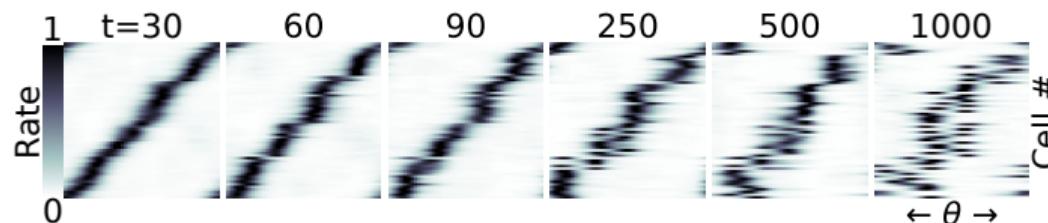
Hebbian homeostasis: Use the neuron's own output as a training signal



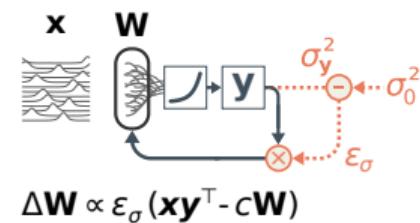
Using the neuron's own output to adjust weights stabilizes localized, bump-like tuning. This uses unsupervised Hebbian learning to homeostatically restore activity as drift destroys excitatory drive.

Use recurrent predictions as a learning signal

Hebbian Homeostasis



Schematic

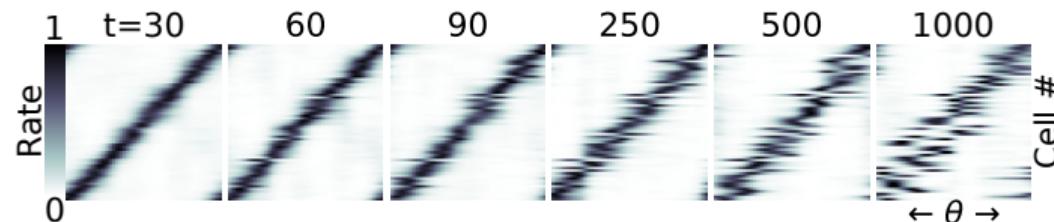


Hebbian Homeostasis provides stability, but tuning decays toward high-variance components[†]

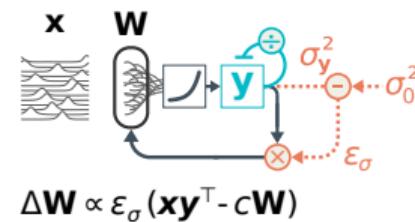
[†]c.f. Oja (1982)

Use recurrent predictions as a learning signal

Population Competition



Schematic



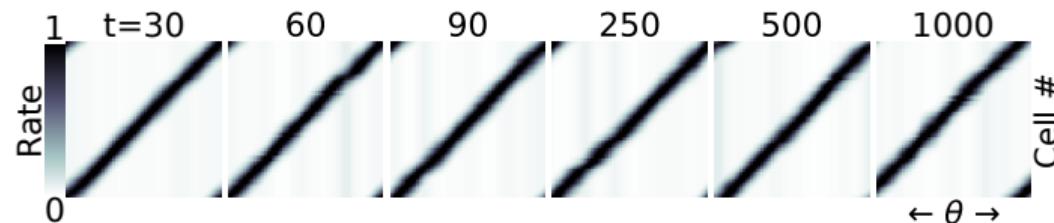
Hebbian Homeostasis provides stability, but tuning decays toward high-variance components[†]
Competition between readout cells stabilizes further[‡]

[†]c.f. Oja (1982)

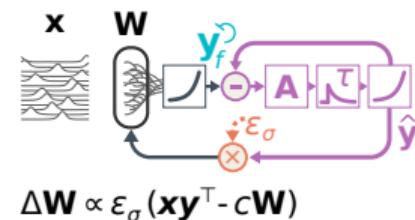
[‡]e.g. Földiák & Fdilr, Barlow & Földiák (1989); Pehlevan & al. (2015, 17, 19, 20)

Use recurrent predictions as a learning signal

Recurrent Dynamics



Schematic



Hebbian Homeostasis provides stability, but tuning decays toward high-variance components[†]

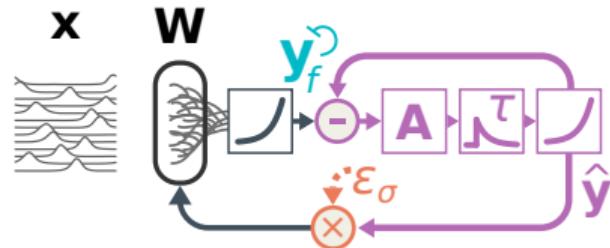
Competition between readout cells stabilizes further[‡]

Recurrent predictions improve error-correction → long-term stability

[†]c.f. Oja (1982)

[‡]e.g. Földiák & Fdilr, Barlow & Földiák (1989); Pehlevan & al. (2015, 17, 19, 20)

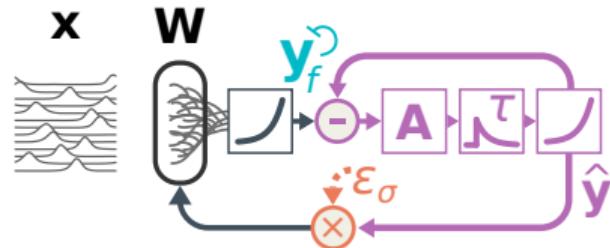
Repair readout using a predictive model stored in recurrent weights



Generative model of spikes $\mathbf{y} = \{y_1, \dots, y_n\}$

- ▶ Latent variables $\mathbf{z} = \{z_1, \dots, z_n\} \sim P_z$
- ▶ Observation model $P_{\mathbf{y}|z}$
- ▶ $P_{\mathbf{y}} = \int_{dz} P_{\mathbf{y}|z} P_z$

Repair readout using a predictive model stored in recurrent weights

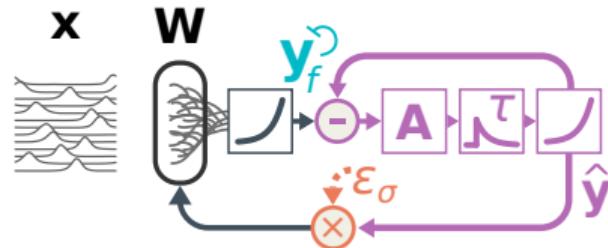


Generative model of spikes $\mathbf{y} = \{y_1, \dots, y_n\}$

- ▶ Latent variables $\mathbf{z} = \{z_1, \dots, z_n\} \sim P_z$
- ▶ Observation model $P_{\mathbf{y}|z}$
- ▶ $P_{\mathbf{y}} = \int_{dz} P_{\mathbf{y}|z} P_z$

Learn a prior on likely states in latent variables \mathbf{z} , and how to map $\mathbf{z} \leftrightarrow \mathbf{y}$. Save this prior, and use it to repair readouts in the face of drift.

Repair readout using a predictive model stored in recurrent weights



Specifically, a Log-Gaussian-Poisson model:

$$P_z = \mathcal{N}[0, \Sigma] \propto \exp(-\frac{1}{2} z^\top \Sigma^{-1} z)$$

$$P_{y_i|z_i} = \text{Poisson}[\lambda_i = e^{z_i}] \propto \lambda_i^{y_i} e^{-\lambda_i}$$

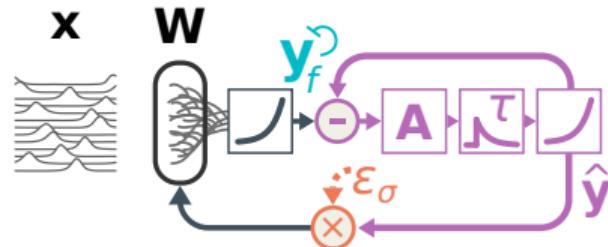
$$\ln P_{z|\mathbf{y}} = \mathbf{y}^\top z - \mathbb{1}^\top \lambda(z) - \frac{1}{2} z^\top \Sigma^{-1} z$$

Generative model of spikes $\mathbf{y} = \{y_1, \dots, y_n\}$

- ▶ Latent variables $z = \{z_1, \dots, z_n\} \sim P_z$
- ▶ Observation model $P_{\mathbf{y}|z}$
- ▶ $P_{\mathbf{y}} = \int dz P_{\mathbf{y}|z} P_z$

Learn a prior on likely states in latent variables z , and how to map $z \leftrightarrow \mathbf{y}$. Save this prior, and use it to repair readouts in the face of drift.

Repair readout using a predictive model stored in recurrent weights



Generative model of spikes $\mathbf{y} = \{y_1, \dots, y_n\}$

- ▶ Latent variables $\mathbf{z} = \{z_1, \dots, z_n\} \sim P_z$
- ▶ Observation model $P_{\mathbf{y}|z}$
- ▶ $P_{\mathbf{y}} = \int_{dz} P_{\mathbf{y}|z} P_z$

Learn a prior on likely states in latent variables \mathbf{z} , and how to map $\mathbf{z} \leftrightarrow \mathbf{y}$. Save this prior, and use it to repair readouts in the face of drift.

Specifically, a Log-Gaussian-Poisson model:

$$P_z = \mathcal{N}[0, \Sigma] \propto \exp(-\frac{1}{2}\mathbf{z}^\top \Sigma^{-1} \mathbf{z})$$

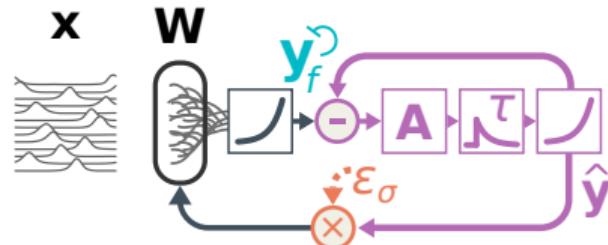
$$P_{y_i|z_i} = \text{Poisson}[\lambda_i = e^{z_i}] \propto \lambda_i^{y_i} e^{-\lambda_i}$$

$$\ln P_{z|\mathbf{y}} = \mathbf{y}^\top \mathbf{z} - \mathbb{1}^\top \lambda(\mathbf{z}) - \frac{1}{2}\mathbf{z}^\top \Sigma^{-1} \mathbf{z}$$

Error correct: get \mathbf{z} most likely to explain \mathbf{y}

$$\dot{\mathbf{z}} \propto \nabla_{\mathbf{z}} \ln P_{z|\mathbf{y}} = \mathbf{y} - \lambda(\mathbf{z}) - \Sigma^{-1} \mathbf{z}$$

Repair readout using a predictive model stored in recurrent weights



Generative model of spikes $\mathbf{y} = \{y_1, \dots, y_n\}$

- ▶ Latent variables $\mathbf{z} = \{z_1, \dots, z_n\} \sim P_z$
- ▶ Observation model $P_{\mathbf{y}|z}$
- ▶ $P_{\mathbf{y}} = \int_{dz} P_{\mathbf{y}|z} P_z$

Learn a prior on likely states in latent variables \mathbf{z} , and how to map $\mathbf{z} \leftrightarrow \mathbf{y}$. Save this prior, and use it to repair readouts in the face of drift.

Specifically, a Log-Gaussian-Poisson model:

$$P_z = \mathcal{N}[0, \Sigma] \propto \exp(-\frac{1}{2}\mathbf{z}^\top \Sigma^{-1} \mathbf{z})$$

$$P_{y_i|z_i} = \text{Poisson}[\lambda_i = e^{z_i}] \propto \lambda_i^{y_i} e^{-\lambda_i}$$

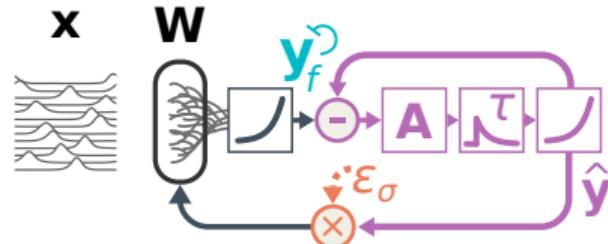
$$\ln P_{\mathbf{z}|\mathbf{y}} = \mathbf{y}^\top \mathbf{z} - \mathbb{1}^\top \lambda(\mathbf{z}) - \frac{1}{2}\mathbf{z}^\top \Sigma^{-1} \mathbf{z}$$

Error correct: get \mathbf{z} most likely to explain \mathbf{y}

$$\dot{\mathbf{z}} \propto \nabla_{\mathbf{z}} \ln P_{\mathbf{z}|\mathbf{y}} = \mathbf{y} - \lambda(\mathbf{z}) - \Sigma^{-1} \mathbf{z}$$

Equivalent: $\dot{\mathbf{z}} \propto -\mathbf{z} + \Sigma[\mathbf{y} - \lambda(\mathbf{z})]$

Repair readout using a predictive model stored in recurrent weights



Generative model of spikes $\mathbf{y} = \{y_1, \dots, y_n\}$

- ▶ Latent variables $\mathbf{z} = \{z_1, \dots, z_n\} \sim P_z$
- ▶ Observation model $P_{\mathbf{y}|z}$
- ▶ $P_{\mathbf{y}} = \int dz P_{\mathbf{y}|z} P_z$

Learn a prior on likely states in latent variables \mathbf{z} , and how to map $\mathbf{z} \leftrightarrow \mathbf{y}$. Save this prior, and use it to repair readouts in the face of drift.

Specifically, a Log-Gaussian-Poisson model:

$$P_z = \mathcal{N}[0, \Sigma] \propto \exp(-\frac{1}{2}\mathbf{z}^\top \Sigma^{-1} \mathbf{z})$$

$$P_{y_i|z_i} = \text{Poisson}[\lambda_i = e^{z_i}] \propto \lambda_i^{y_i} e^{-\lambda_i}$$

$$\ln P_{\mathbf{z}|\mathbf{y}} = \mathbf{y}^\top \mathbf{z} - \mathbb{1}^\top \lambda(\mathbf{z}) - \frac{1}{2} \mathbf{z}^\top \Sigma^{-1} \mathbf{z}$$

Error correct: get \mathbf{z} most likely to explain \mathbf{y}

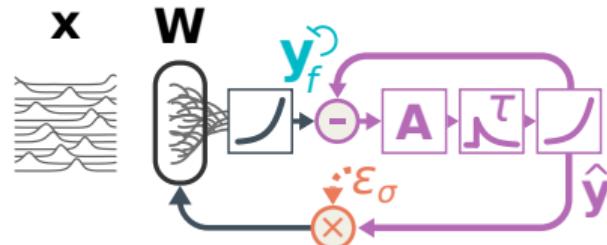
$$\dot{\mathbf{z}} \propto \nabla_{\mathbf{z}} \ln P_{\mathbf{z}|\mathbf{y}} = \mathbf{y} - \lambda(\mathbf{z}) - \Sigma^{-1} \mathbf{z}$$

Equivalent: $\dot{\mathbf{z}} \propto -\mathbf{z} + \Sigma[\mathbf{y} - \lambda(\mathbf{z})]$

Predictions $\lambda(\mathbf{z})$ cancel inputs \mathbf{y} via inhibition, through recurrent weights Σ : A rate network, but not quite physiological¹.

¹compare to Masset & al. (2022), which shares some assumptions;

Repair readout using a predictive model stored in recurrent weights



Generative model of spikes $\mathbf{y} = \{y_1, \dots, y_n\}$

- ▶ Latent variables $\mathbf{z} = \{z_1, \dots, z_n\} \sim P_z$
- ▶ Observation model $P_{\mathbf{y}|z}$
- ▶ $P_{\mathbf{y}} = \int dz P_{\mathbf{y}|z} P_z$

Learn a prior on likely states in latent variables \mathbf{z} , and how to map $\mathbf{z} \leftrightarrow \mathbf{y}$. Save this prior, and use it to repair readouts in the face of drift.

Specifically, a Log-Gaussian-Poisson model:

$$P_z = \mathcal{N}[0, \Sigma] \propto \exp(-\frac{1}{2}\mathbf{z}^\top \Sigma^{-1} \mathbf{z})$$

$$P_{y_i|z_i} = \text{Poisson}[\lambda_i = e^{z_i}] \propto \lambda_i^{y_i} e^{-\lambda_i}$$

$$\ln P_{\mathbf{z}|\mathbf{y}} = \mathbf{y}^\top \mathbf{z} - \mathbb{1}^\top \lambda(\mathbf{z}) - \frac{1}{2} \mathbf{z}^\top \Sigma^{-1} \mathbf{z}$$

Error correct: get \mathbf{z} most likely to explain \mathbf{y}

$$\dot{\mathbf{z}} \propto \nabla_{\mathbf{z}} \ln P_{\mathbf{z}|\mathbf{y}} = \mathbf{y} - \lambda(\mathbf{z}) - \Sigma^{-1} \mathbf{z}$$

Equivalent: $\dot{\mathbf{z}} \propto -\mathbf{z} + \Sigma[\mathbf{y} - \lambda(\mathbf{z})]$

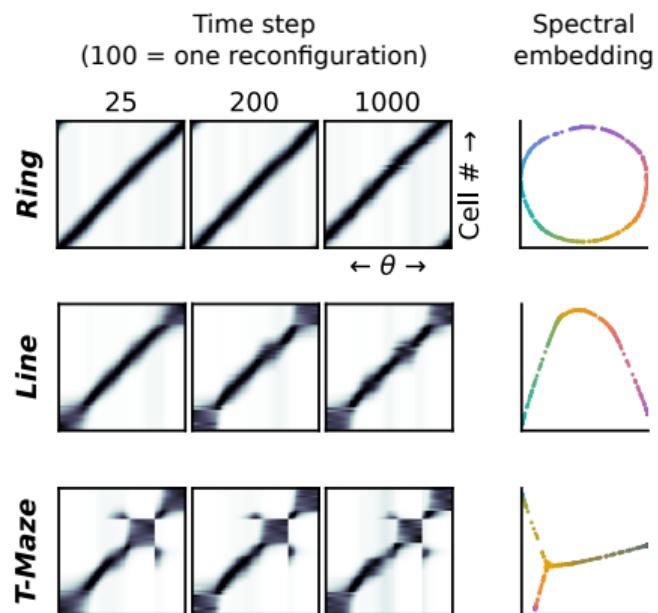
Predictions $\lambda(\mathbf{z})$ cancel inputs \mathbf{y} via inhibition, through recurrent weights Σ : A rate network, but not quite physiological¹.

These questions remain open: Future work.

¹compare to Masset & al. (2022), which shares some assumptions;

Drift in low-dimensional representations is benign

Homeostasis and internal predictions confer long-term stability to drift



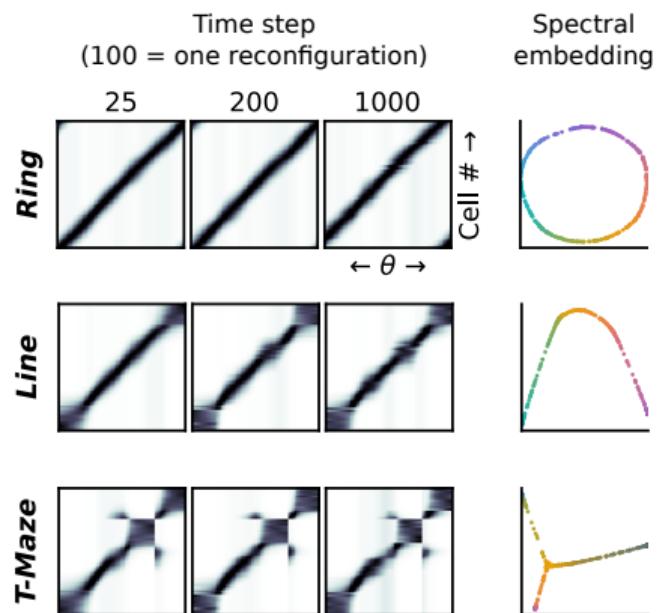
Rule, M. E. and O'Leary, T. (2022). Self-healing codes: How stable neural populations can track continually reconfiguring neural representations. *Proceedings of the National Academy of Sciences*, 119(7):e2106692119

Drift in low-dimensional representations is benign

Homeostasis and internal predictions confer long-term stability to drift

Drift: Unstable code, stable low-D structure

- ▶ Redundant codes have robust readout
- ▶ Monitor drift rate as deviation from homeostatic target



Rule, M. E. and O'Leary, T. (2022). Self-healing codes: How stable neural populations can track continually reconfiguring neural representations. *Proceedings of the National Academy of Sciences*, 119(7):e2106692119

Drift in low-dimensional representations is benign

Homeostasis and internal predictions confer long-term stability to drift

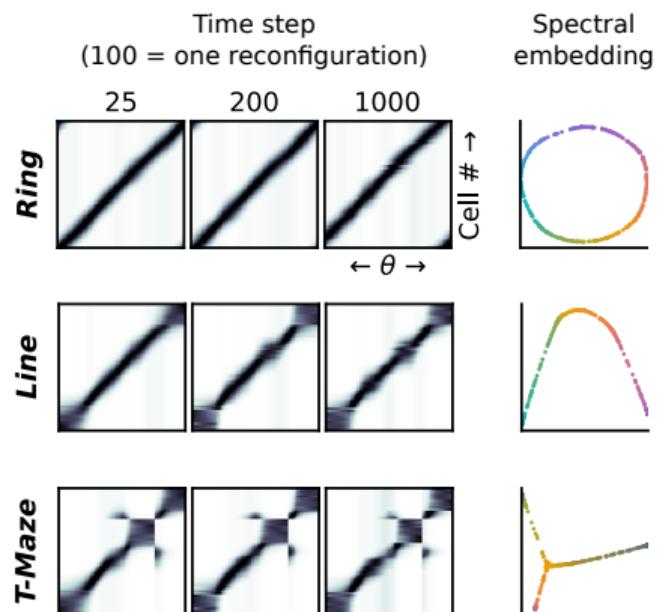
Drift: Unstable code, stable low-D structure

- ▶ Redundant codes have robust readout
- ▶ Monitor drift rate as deviation from homeostatic target

Hebbian homeostasis:

- ▶ Correct errors using an internal training signal
- ▶ Re-learn tuning as inputs change

Rule, M. E. and O'Leary, T. (2022). Self-healing codes: How stable neural populations can track continually reconfiguring neural representations. *Proceedings of the National Academy of Sciences*, 119(7):e2106692119



Drift in low-dimensional representations is benign

Homeostasis and internal predictions confer long-term stability to drift

Drift: Unstable code, stable low-D structure

- ▶ Redundant codes have robust readout
- ▶ Monitor drift rate as deviation from homeostatic target

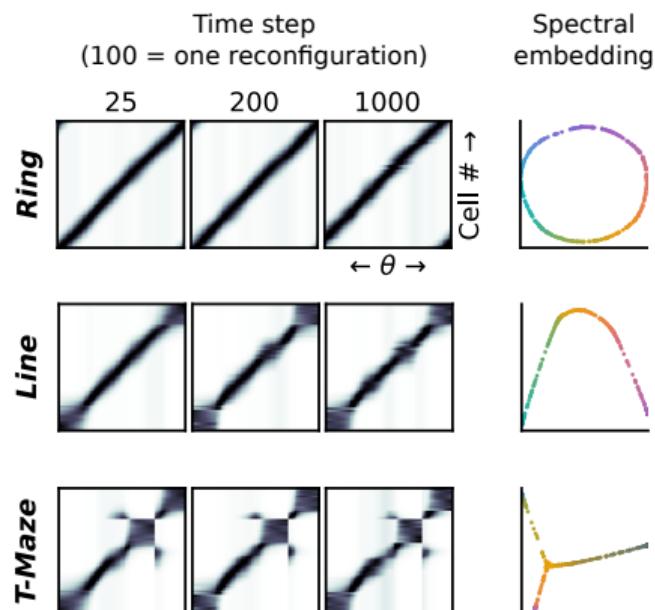
Hebbian homeostasis:

- ▶ Correct errors using an internal training signal
- ▶ Re-learn tuning as inputs change

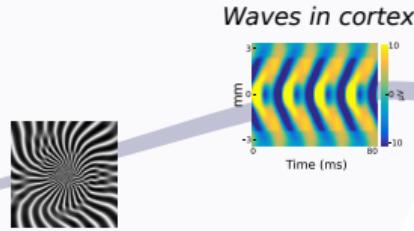
Recurrent population interactions

- ▶ Predictions correct errors, provide a stable readout

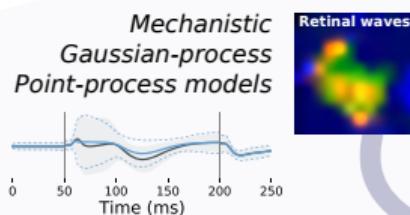
Rule, M. E. and O'Leary, T. (2022). Self-healing codes: How stable neural populations can track continually reconfiguring neural representations. *Proceedings of the National Academy of Sciences*, 119(7):e2106692119



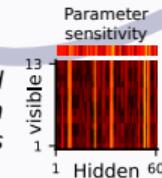
Carnegie-Mellon University
B.S. Computer Science
Neural field theory



Brown University
Ph.D. Neuroscience
Statistical Modelling



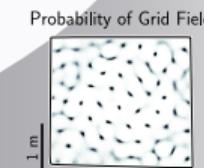
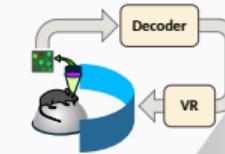
Statistics of and representations in Boltzmann machines



Control Group
University of Cambridge
Homeostasis, Learning: Closed-loop

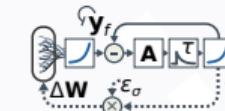
Past

Present



Learning in closed-loop

Point-process methods

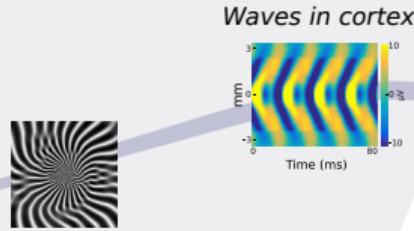


Biological Learning

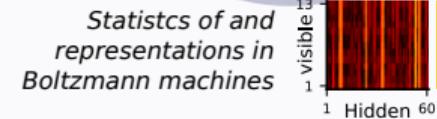
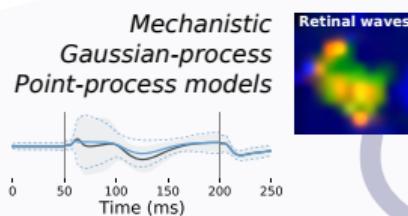
University of Edinburgh (I)
Statistical methods

Future

Carnegie-Mellon University
B.S. Computer Science
Neural field theory



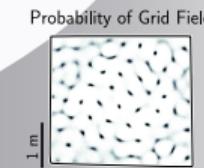
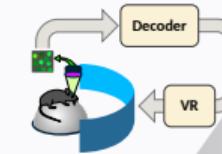
Brown University
Ph.D. Neuroscience
Statistical Modelling



Control Group
University of Cambridge
Homeostasis, Learning: Closed-loop

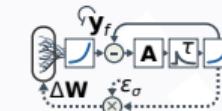
Past

Present



Learning in closed-loop

Point-process methods



Biological Learning

University of Edinburgh (I)
Statistical methods

Future

Background: models of the brain, dynamical and statistical



B. Ermentrout W. Truccolo

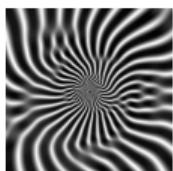
Undergraduate (Bard Ermentrout): Neural-field theory

(Rule et al., 2011) What do neural networks see?

- ▶ 5–40 Hz flickering light: “phosphene” hallucinations¹
- ▶ To detect edges, visual cortex inhibits nearby inputs.
- ▶ Inverting input when inhibition arrives excites edges.
- ▶ Rate network with recurrent convolutions

(Heitmann, Rule et al. 2017) Optogenetic stimulation switches cortex from travelling waves to oscillations.

¹Purkinje 1819

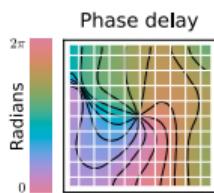


Neural-field theory simplifies collective activity for mathematical tractability.

Ph.D. (Wilson Truccolo): Point-process models

Collective Dynamics in Primate Motor Cortex

- ▶ (Rule et al., 2017; 2018) Before moving, rhythmic spiking encodes movement plans; Phase (re)alignments cause waves across cortex.
- ▶ (Rule et al., 2015) When moving, local electric fields relate to past/future movement; Past spiking predicts future variability.



Point-process models infer why neurons spike

Can we combine these? Simplify point-process models, infer neural fields from spikes?

Autoregressive point processes as state-space models



Autoregressive models predict the future from the past.

Coarse graining (“zoom out”):

- ▶ Reduce the dimension of a large model
- ▶ Linear models with Gaussian noise are simpler to analyze

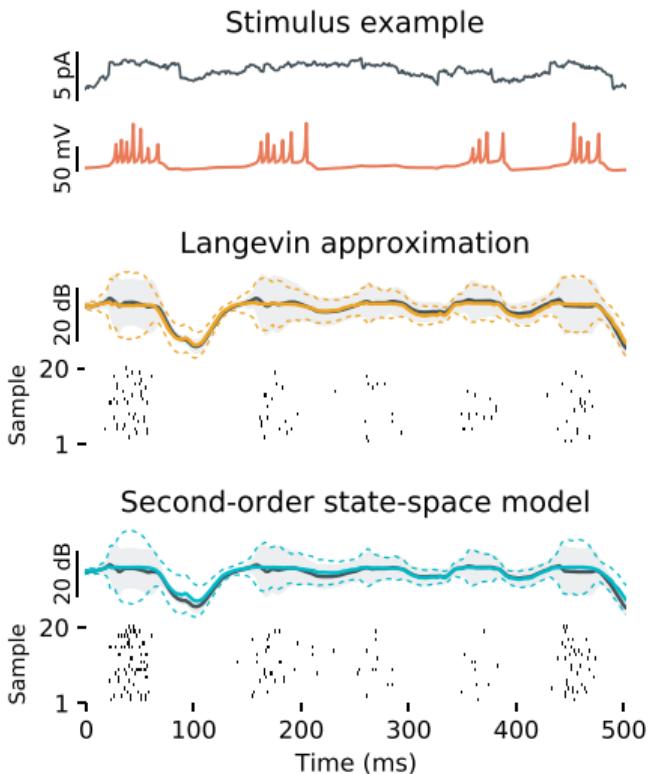
The probability of a neuron spiking can be predicted as a linear function of past variables, followed by a pointwise nonlinearity.

- ▶ Not Gaussian, but a Gaussian approximation is ok because predictions average over many spikes.

Process history \sim Gaussian process; Propagate in time using

- ▶ Moment-matching (“moment closure”), or
- ▶ Locally-quadratic approximation (improved stability)

Rule, M. and Sanguinetti, G. (2018). Autoregressive point processes as latent state-space models: A moment-closure approach to fluctuations and autocorrelations. *Neural computation*, 30(10):2757–2780



Neural field models for latent state inference



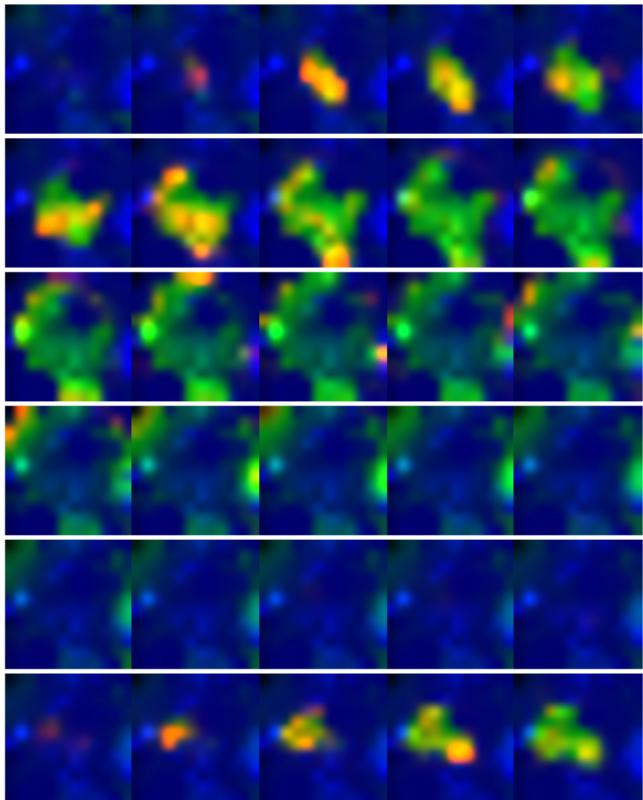
Neural field models are simple descriptions of spatiotemporal neural population dynamics.

- ▶ Partial Differential Equations (PDEs) over space and time.
- ▶ # neurons often → ∞ so that noise averages away.

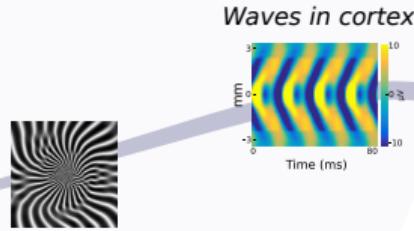
Model noise in finite populations → *stochastic* PDE.

- ▶ Use as a latent-variable model of waves in the retina.
- ▶ 3 states: Quiescent, Active, Refractory
(compare to Susceptible, Infected, Recovered)
- ▶ Approximate latent distribution as a Gaussian process
- ▶ Estimate from spiking observations via Bayesian filtering

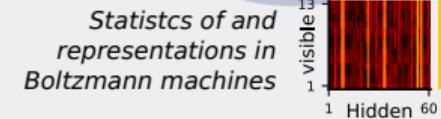
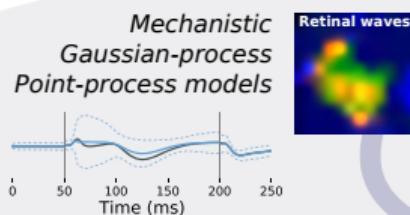
Rule, M. E., Schnoerr, D., Hennig, M. H., and Sanguinetti, G. (2019). Neural field models for latent state inference: Application to large-scale neuronal recordings. *PLoS computational biology*, 15(11):e1007442



Carnegie-Mellon University
B.S. Computer Science
Neural field theory

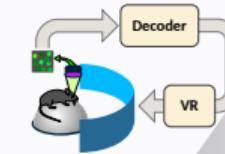


Brown University
Ph.D. Neuroscience
Statistical Modelling

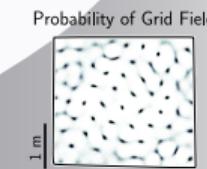


Control Group
University of Cambridge
Homeostasis, Learning: Closed-loop

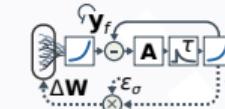
Present



Learning in closed-loop



Point-process methods



Biological Learning

University of Edinburgh (I)
Statistical methods

Past

University of Edinburgh (II)
Future

Are binary latent-variable models like the early sensory system?

Restricted Boltzmann Machines (RBMs) are latent-variable models where the observed/latent variables are Bernoulli-distributed conditioned on each other.

Martino Sorbaro was training RBMs to model population spiking activity, kept finding models close to “critical”.

Rare stimuli suppressed variability

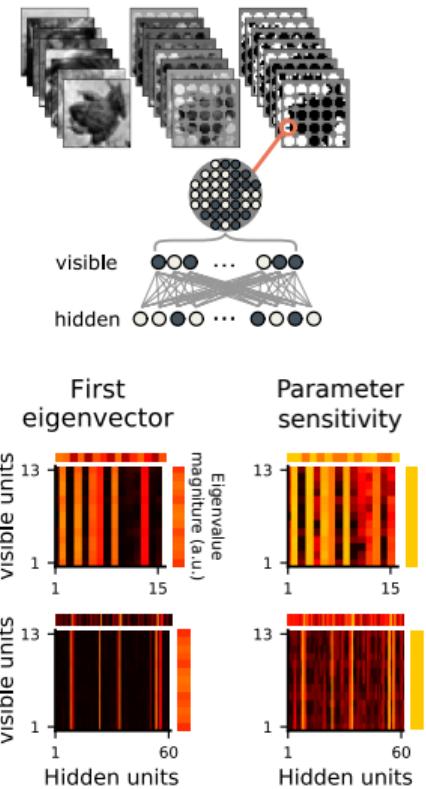
- ▶ Noise limits bandwidth; Rare stimuli need more bits to encode
- ▶ Incoming stimuli reduce neural variability¹
- ▶ Suppression of firing can be informative²

∞ -large models: 1/f statistics and rank ≈ 1 Fisher information matrix³

- ▶ Implicit prior on model’s statistics
- ▶ Finite models also approximate this, given sufficient capacity.
- ▶ \Rightarrow Easy to measure a weight’s importance from local activity

Rule, M. E., Sorbaro, M., and Hennig, M. H. (2020). Optimal encoding in stochastic latent-variable models. *Entropy*, 22(7):714

¹Churchland & al. (2010), Echeveste & al. (2020), ²Schneidman (2011), ³Schwab (2014)



Can we understand developmental pruning via information theory?



The brain removes neurons during maturation. How does it choose which ones?

Fisher Information Matrix (FIM): amount a model changes when we change its parameters \approx Importance

- ▶ Only the diagonal can be detected locally. But, if FIM \approx rank-1, the diagonal is ok.

In Boltzmann machines, weight FI depends on pre/post activity

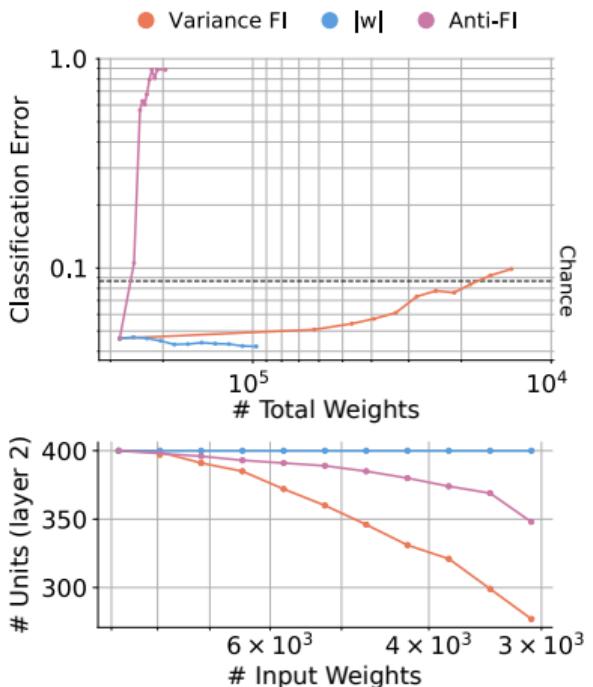
- ▶ More pre-post correlations \Rightarrow more important
- ▶ Prune small weights: **reduced** error, didn't shrink network
- ▶ FI pruning: good performance, shrinks network

Must re-train post-pruning¹

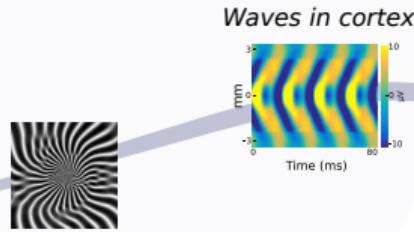
- ▶ Use homeostasis for “internal” transfer learning during pruning?

Scholl, C., Rule, M. E., and Hennig, M. H. (2021). [The information theory of developmental pruning: Optimizing global network architectures using local synaptic rules](#). *PLoS computational biology*, 17(10):e1009458

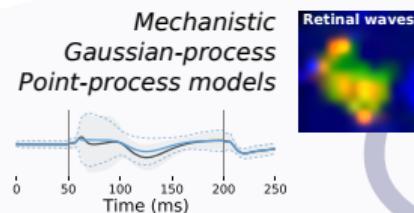
¹Not the best approach for industrial ML: Crowley & al. (2018, 2019)



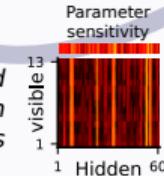
Carnegie-Mellon University
B.S. Computer Science
Neural field theory



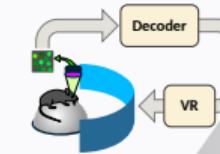
*Flicker-phosphene
hallucinations*



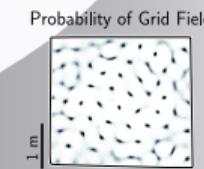
*Statistics of and
representations in
Boltzmann machines*



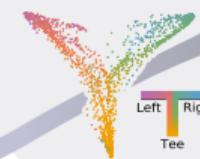
Brown University
Ph.D. Neuroscience
Statistical Modelling



**Learning in
closed-loop**



**Point-process
methods**



*Drift, homeostasis,
and learning*



**Biological
Learning**

Control Group
University of Cambridge
Homeostasis, Learning: Closed-loop

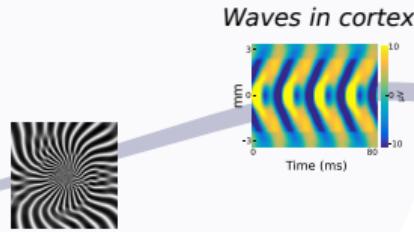
Present

University of Edinburgh (I)
Statistical methods

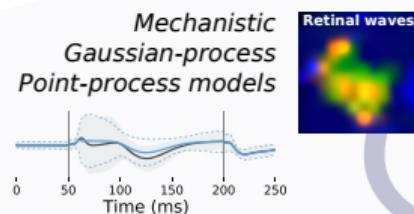
Past

**University of
Edinburgh (II)**
Future

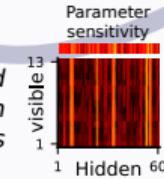
Carnegie-Mellon University
B.S. Computer Science
Neural field theory



Flicker-phosphene hallucinations



Statistics of and representations in Boltzmann machines



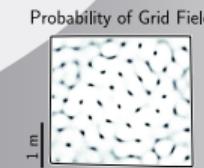
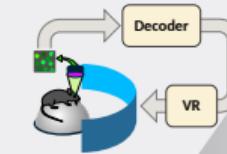
Brown University
Ph.D. Neuroscience
Statistical Modelling



Control Group
University of Cambridge
Homeostasis, Learning: Closed-loop

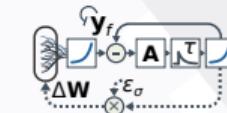
Past

Present



Learning in closed-loop

Point-process methods



Biological Learning

University of Edinburgh (I)
Statistical methods

Future

Gaussian processes for grid cells

“Grid cells” in hippocampal formation fire as a periodic (hexagonal) function of animal’s location “ x ”.

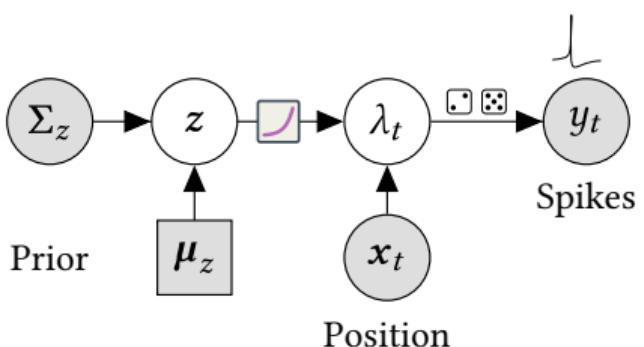
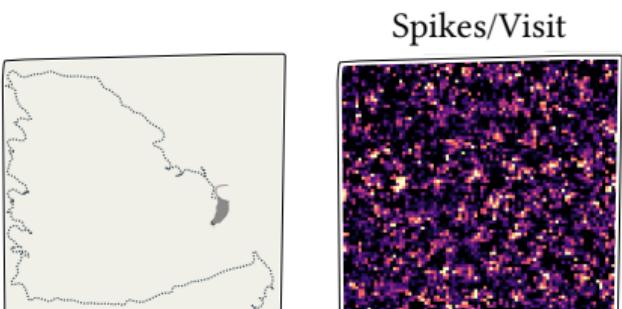
Can we model distribution of firing-rate maps $P_{\text{spike}}(x)$ from limited data?

- ▶ (Latent) log-rate function $z(x)$ with Gaussian-process prior
 $P_{z(x)} = \mathcal{GP}(\mu_0, \Sigma)$
- ▶ Point process observations:
 $P_{y(x)|z(x)} = \text{Poisson}[\lambda(x) = e^{z(x)} dx]$

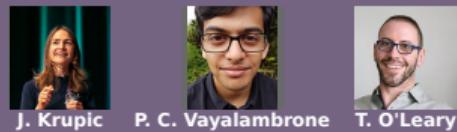
Covariance kernel for periodic grids

- ▶ Bessel function; 0th order, 1st kind
- ▶ Multiply by a local window

M. Rule, P. Chaudhuri-Vayalambrone, J. Krupic, T. O’Leary (2023). Variational log-Gaussian point-process modelling: Application to hippocampal grid cells. In Preparation.



Gaussian processes for grid cells: Make it faster



Pandemic lockdown: run everything on an old laptop,
 2^{16} dimensions, large datasets, how?

Variational Bayes:

- $P_{z(x)}|y(x) \approx Q_{z(x)} = \mathcal{GP}(\mu, \Sigma)$
- $\Sigma^{-1} = \text{prior} + \text{diagonal update}$

Fit iteratively

- $\hat{\mu} \leftarrow \underset{\mu}{\text{argmax}} \langle \log\text{-prior} + \log\text{-likelihood} \rangle_Q^*$
- $\Sigma \leftarrow \Sigma_0 + \text{diag}[\langle \lambda(x) \rangle]$

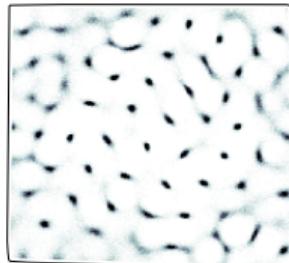
Evaluate on grid

- Bin data with interpolation to ‘pseudopoints’
- Σ_0 becomes a convolution

Low-rank spatial frequency subspace

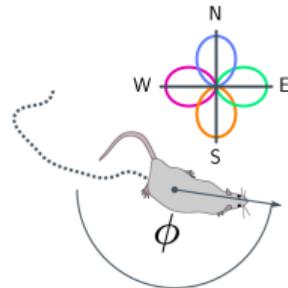
- Discard frequencies ≈ 0 in the prior
- (Fast) Hartley transform gives real-valued components compatible w. Krylov subspace solvers

Probability of Grid Field

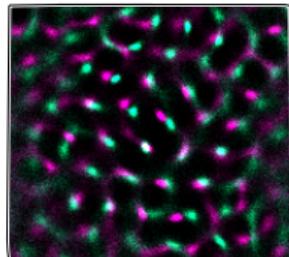


Heading-Dependent Weights

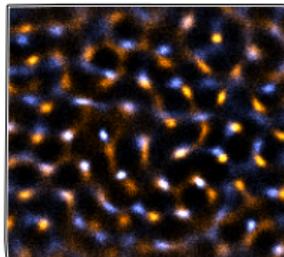
$$w = \cos(\phi - \phi_0)^2$$



Eastward (green) vs
Westward (red)



Northward (blue) vs
Southward (orange)



Calcium-imaging brain-machine interface



Closed-loop control in an adaptive system

- ▶ How does the brain adapt motor control to new circumstances?

Brain-Machine interface

- ▶ Force parietal cortex to act as a motor output
- ▶ We control the motor response to neuronal firing

Decode directly from images

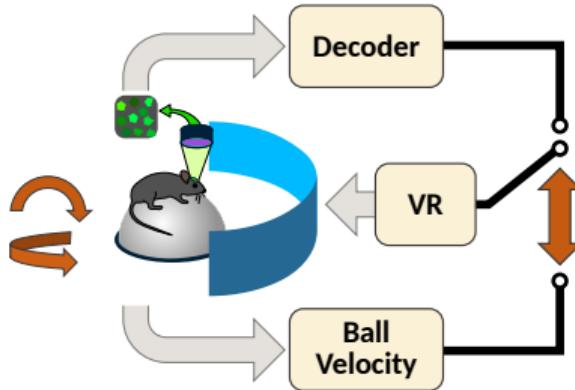
- ▶ Calibrate during behavior using stochastic gradient descent
- ▶ Test in closed loop

It works

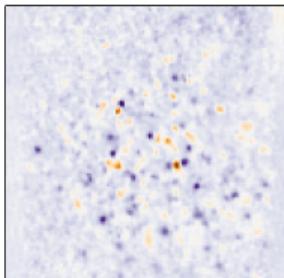
- ▶ No substantial tuning drift over five days
- ▶ Mean-rates of neurons become sensitive to BMI/control context

Sorrell, E., Rule, M. E., and O'Leary, T. (2021). Brain-machine interfaces: Closed-loop control in an adaptive system. *Annual Review of Control, Robotics, and Autonomous Systems*, 4

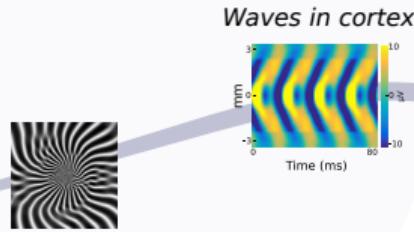
Sorrell, E., Wilson, D., Rule, M., Yang, H., Forni, F., Harvey, C., O'Leary, T. (2022) A Calcium Imaging Based Brain-Machine Interface for Virtual Navigation. In preparation.



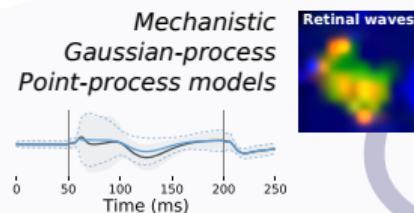
Normalized weights
(forward velocity)



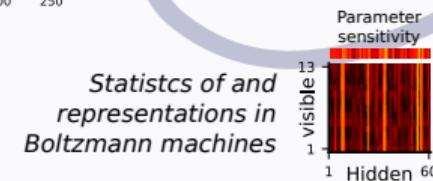
Carnegie-Mellon University
B.S. Computer Science
Neural field theory



*Flicker-phosphene
hallucinations*



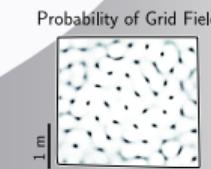
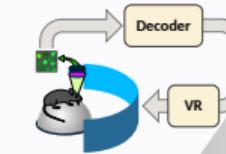
Brown University
Ph.D. Neuroscience
Statistical Modelling



Control Group
University of Cambridge
Homeostasis, Learning: Closed-loop

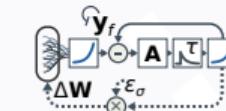
Past

Present



**Learning in
closed-loop**

**Point-process
methods**



**Biological
Learning**

**University of
Edinburgh (II)**
Future