Week 7: Tests for Differences

Session 1

Spring 2020

iClicker Question 1

Scenario: You want to know whether the growth rates of oak trees differ in two habitats. You have two clones each of 37 different genotypes. One clone of each genotype is planted in a mesic prairie, the other in an old agricultural field.

- Which test should you use?

A `two-sample t-test`

B `Mann-Whitney paried test`

C `Mann-Whitney U test`

D `paired t-test`

iclicker

iClicker Question 2

Scenario: You are testing whether Pennsylvania sedge (C. pennsylvanica) cover is greater under hardwoods or conifers. You have collected percent cover data from 50 hardwood and 50 conifer patches at Mount Toby.

- Which test should you use?

A `two-sample t-test`

B `Mann-Whitney paried test`

C `Mann-Whitney U test`

D `paired t-test`

iClicker Question 3

Scenario: You want to know whether the growth rates of oak trees differ in two habitats. You think they will grow more quickly in a old agricultural field than in a mesic prairie.

- Which is the best alternative hypothesis?

A `growth is faster in the prairie`

B `there is no difference in growth between habitats`

C `growth is faster in the field`

D `there is a difference in growth between habitats`

E `growth rates are the same`

iClicker Question 4

Scenario: You want to know whether the growth rates of oak trees differ in two habitats: an old agricultural field and mesic prairie.

- Which is the best alternative hypothesis?

A `growth is faster in the prairie`

B `there is no difference in growth between habitats`

C `growth is faster in the field`

D `there is a difference in growth between habitats`

E `growth rates are the same`

iClicker Question 5

Scenario: You calculate a t-value, and an associated p-value, for individual length in two populations of Daphnia. Which critical significance category for the t-value would give you the best evidence that the two populations are different:

A 1%

B 10%

C 5%

D 95%

E 0%

If you're feeling stuck, remember my office hours are Tuesday/Thursday 1:00 - 2:00.

For Today

- Toward statistics
- Tests for differences

Follow-up questions from the Chapter 6 homework

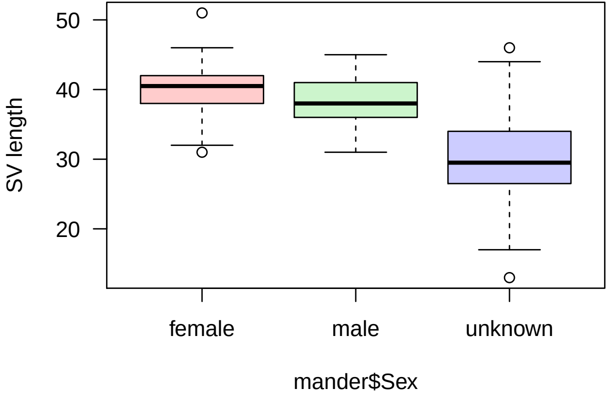
- What questions do you have?

Beyond graphs, Towards statistics

- Graphs are powerful tools that provide insight and understanding of the patterns and relationships in the data.
- Graphs alone don't give us the complete answer. We need to **quantify** the relationships we see in our plots.

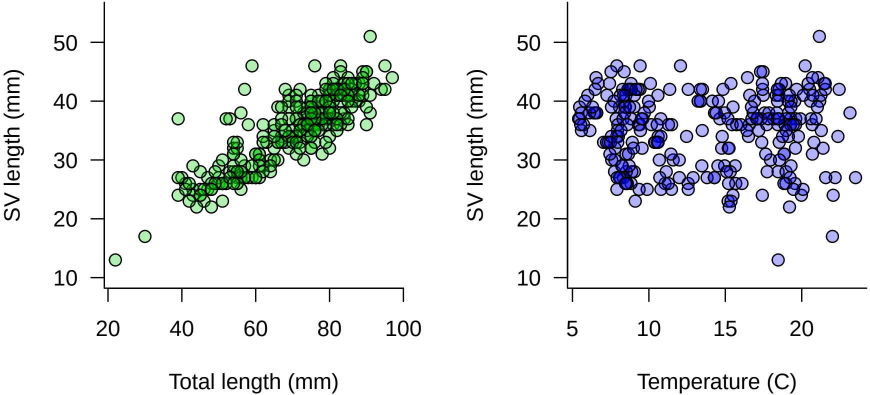
Beyond graphs, Towards statistics

- How can we **quantify** our evidence for relationships?
  - Are differences between groups *significant*?
  - Are differences between groups *meaningful*?



Beyond graphs, Towards statistics

- How can we **quantify** our evidence for relationships?
  - Are associations between 2 variables *significatnt*?
  - Are associations between 2 variables *meaningful*?



Beyond graphs, Towards statistics

- Statistics is the tool we use to formally answer these questions!
  - Ahe differences *are/are not* significant!
  - Are associations *are/are not* significant!

Wait a second... what do we mean when we say **significant**?

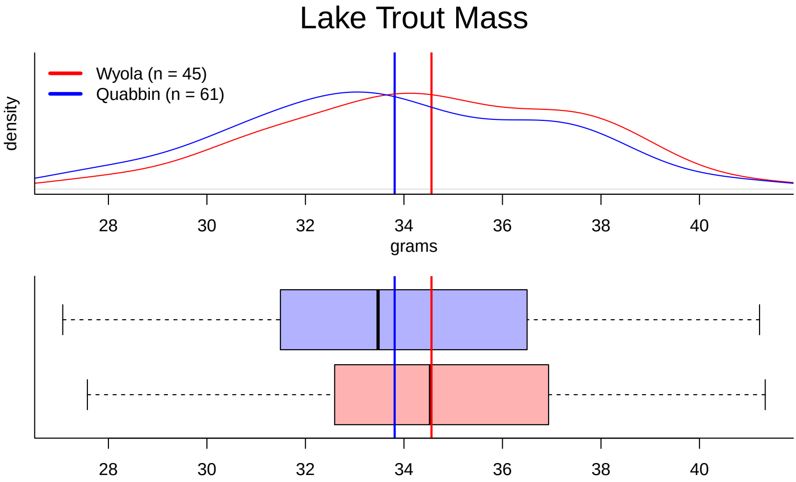
Let's examine some plots to gain intuition:

- Scenario: We want to know whether the size of 3-year-old bluegill (*Lepomis macrochirus*) are larger in some Massachusetts lakes than others.
- We have collected data for bluegill from Wyola Lake and the Quabbin Reservoir in Western Mass.

[1] Image credit: New York Fish and Game Commission

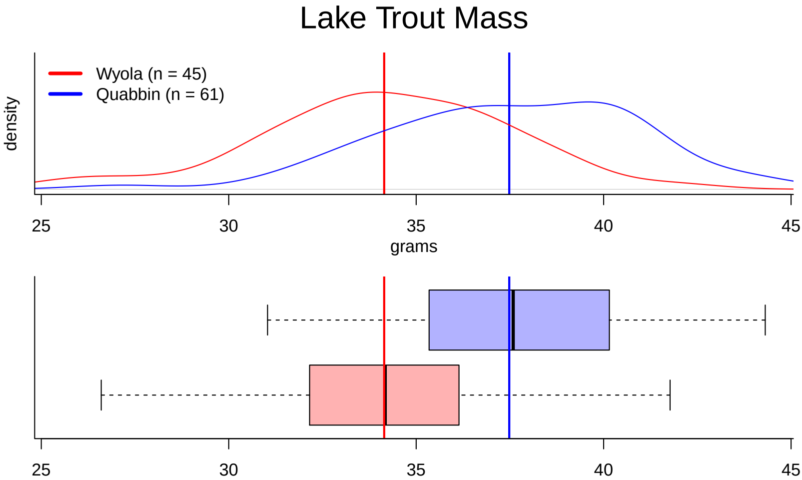
Bluegill Data I

- Are differences between lakes *significant*?
- Are differences between lakes *meaningful*?



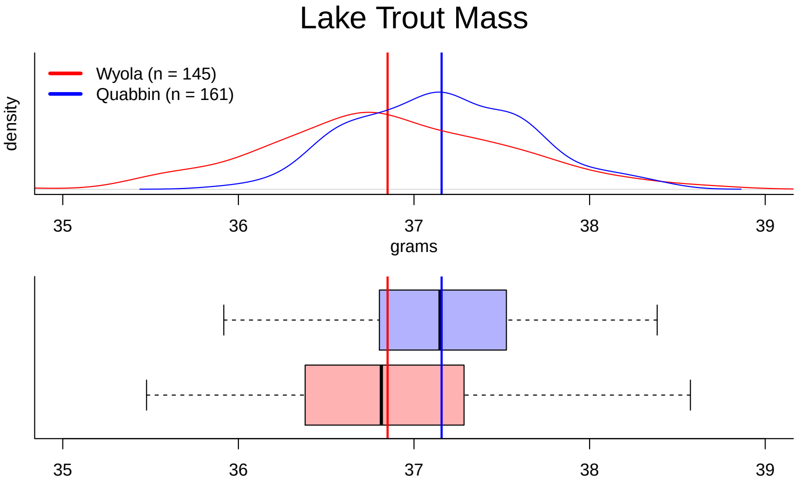
Bluegill Data II

- Are differences between lakes *significant*?
- Are differences between lakes *meaningful*?



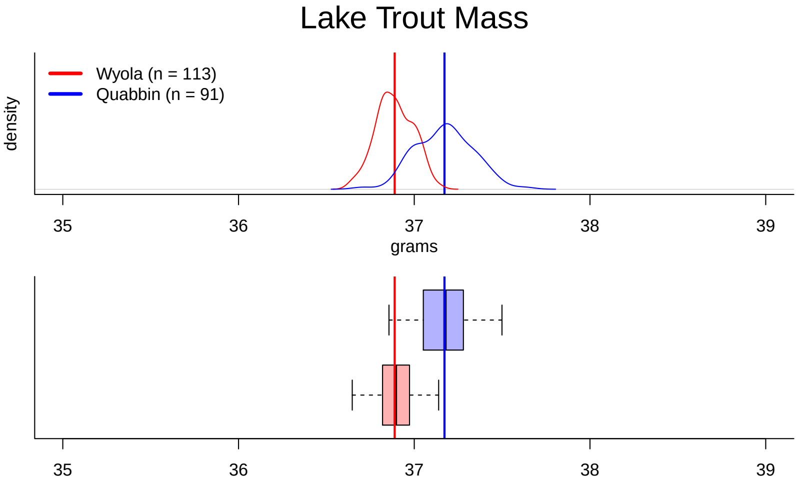
Bluegill Data III

- Are differences between lakes *significant*?
- Are differences between lakes *meaningful*?



Bluegill Data IV

- Are differences between lakes *significant*?
- Are differences between lakes *meaningful*?



Tests for differences

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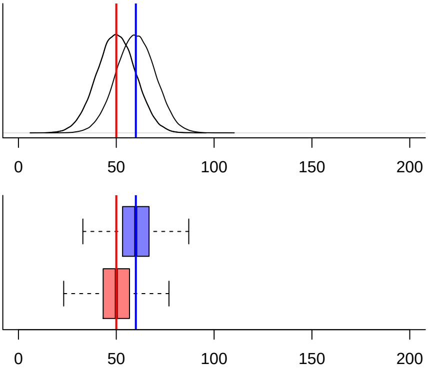
Tests for differences

Often we want to know if two of more samples are different

- are the sample *means* different?
- are the sample *medians* different?
- are the differences *statistically significant*?



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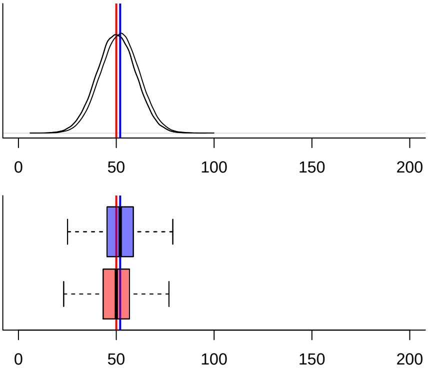

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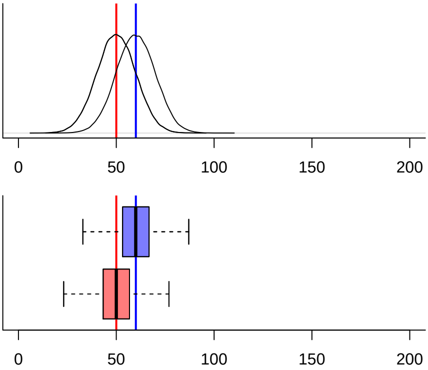

Tests for differences

Often we want to know if two of more samples are different

- are the sample *means* different?
- are the sample *medians* different?
- are the differences *statistically significant*?

To determine the significance of differences between **two**, we need a statistical test

- *t-test*
- *U-test*

Differences: t-test

Purpose:

- compare the means of two samples (say a and b)

Assumptions:

- both samples normally distributed
- both samples have equal variances

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$$t = \frac{|\bar{x}_a - \bar{x}_b|}{\sqrt{\frac{s_a^2}{n_a} + \frac{s_b^2}{n_b}}}$$

- t: the t-statistic
- $\bar{x}$: sample mean
- s: sample standard deviation
- n: sample size

Differences: t-test

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- if $|\bar{x}_a - \bar{x}_b|$ is large, then t is ????

- if $\sqrt{\frac{s_a^2}{n_a} + \frac{s_b^2}{n_b}}$ is large, then t is ????

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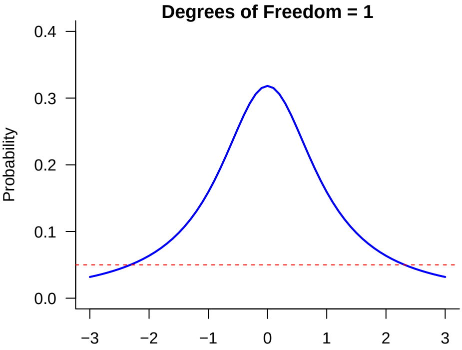
- if $|\bar{x}_a - \bar{x}_b|$ is large, then t is large
- if $\sqrt{\frac{s_a^2}{n_a} + \frac{s_b^2}{n_b}}$ is large, then t is small

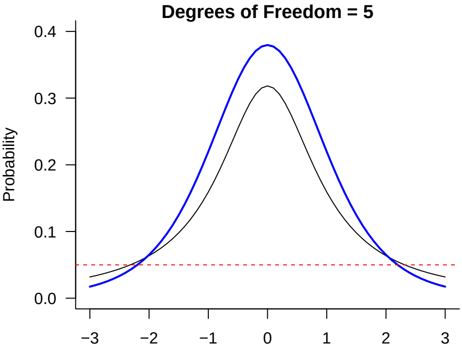
Differences: t-test

Understanding the *t-distribution*:



Differences: t-test

Understanding the *t-distribution*:



Differences: t-test

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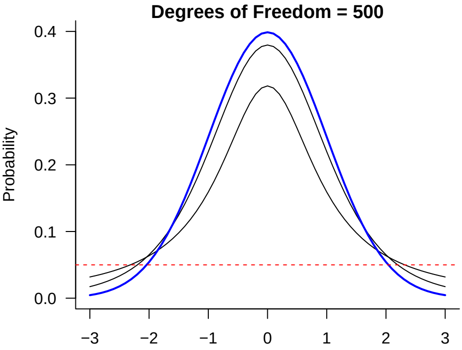

Differences: t-test

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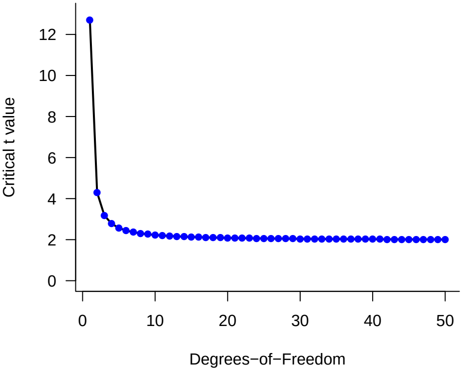
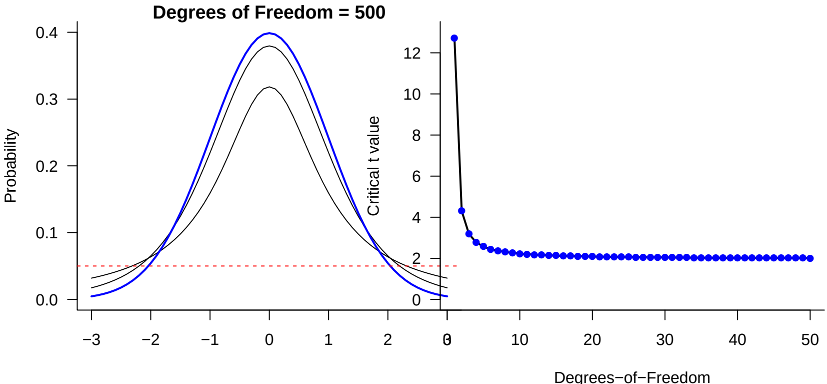


Differences: t-test

Understanding the *t-distribution*:

- whether a difference is significant depends on:
  - the *t-statistic*
  - degrees-of-freedom ($n_a - 1 + n_b - 1$)
- larger *t-statistics* more likely to be significant



Differences: t-test

Understanding the *p-value*:

- *p-value* is the probability of observing a *t-statistic* as high as we did by chance
- if *p-value* is lower than significance level (e.g. 5%):
  - difference is significant
  - reject the null hypothesis
  - accept the alternative hypothesis

Differences: t-test

Which *t-test*?

- standard *t-test*
  - compare two independent samples
  - both normally distributed
  - equal (similar) variances
  - samples sizes can be the same or not

$$t = \frac{|\bar{x}_a - \bar{x}_b|}{\sqrt{\frac{s_a^2}{n_a} + \frac{s_b^2}{n_b}}}$$

- t: the t-statistic
- $\bar{x}$: sample mean
- s: sample standard deviation
- n: sample size

Differences: paired t-test

Sometimes samples are not independent

- compare pairs of samples
  - e.g., before-after
  - e.g., north-south
  - e.g., left-right
- both normally distributed
- equal (similar) variances
- samples sizes *must* be the

Differences: paired t-test

Which *t-test*?

- paired *t-test*
  - compare pairs of samples
  - both normally distributed
  - equal (similar) variances
  - samples sizes *must* be the

$$t = \frac{\bar{D}}{\sqrt{\frac{s_D^2}{n}}}$$

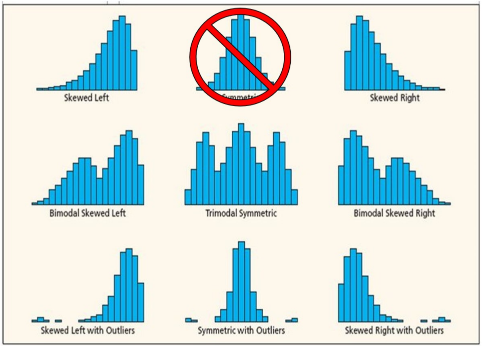
- t: the t-statistic
- $\bar{D}$: mean of the *differences*
- s: standard deviation of the *differences*
- n: number of *paired* samples

Differences: U-test

?

Differences: U-test

- compare two samples
- both *not* normally distributed
- based on *median*, *range*, and *ranks*
- rank all values as one sample, calculate group rank sums R
- calculate a U-value, a measure of overlap



Differences: U-test

- compare two samples
- both or differences *not* normally distributed
- based on *median*, *range*, and *ranks*
- rank all values as one sample, calculate group rank sums R
- calculate a U-value, a measure of overlap

$$U_a = n_a \times n_b + \frac{n_a(n_a + 1)}{2} - R_a$$

$$U_b = n_b \times n_a + \frac{n_b(n_b + 1)}{2} - R_b$$

- n_a: number of samples in sample a
- n_b: number of samples in sample b
- R_a: sum of the ranks of values in a
- R_b: sum of the ranks of values in b

Differences: U-test

- compare two samples
- both or differences *not* normally distributed
- based on *median*, *range*, and *ranks*
- rank all values as one sample, calculate group rank sums R
- calculate a U-value, a measure of overlap

$$U_a = n_a \times n_b + \frac{n_a(n_a + 1)}{2} - R_a$$

$$U_b = n_b \times n_a + \frac{n_b(n_b + 1)}{2} - R_b$$

- smallest is used to find the p-value
- unlike the t-statistic, lower U-values are more likely to be significant

Differences: Wilcoxon matched-pairs test

- both or differences *not* normally distributed
- based on ranked *differences*
  - first calculate the differences
  - second rank the differences
  - 0's not ranked
- sum and compare +ve and -ve ranks

$$W^+ = \sum R^+$$

$$W^- = \sum R^-$$

- W^+: the Wilcoxon test statistic for positive differences
- W^+: the Wilcoxon test statistic for negative differences
- R^+: the sum of the ranks of positive differences
- R^+: the sum of the ranks of negative differences

Differences: Wilcoxon matched-pairs test

- pairs or differences *not* normally distributed
- based on ranked *differences*
  - first calculate the differences
  - second rank the differences
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$$W^+ = \sum R^+$$

$$W^- = \sum R^-$$

- smallest is used to find the p-value
- lower W-values are more likely to be significant

Group Assignment

Using the whale count data, compare the differences between first and second abundance guesstimates using first `excel` and then `R`.

Submit a single written group report that outlines the following points:

1. state the null and alternative hypotheses being tested
2. the reason for choosing the statistical test you used
3. a summary of the results:
   - degrees-of-freedom, test statistic, p-values (at 5% level)?
   - did you accept or reject the null hypothesis?
   - is there a difference?
4. conduct the analysis in `R` and `excel` and submit:
   - a written report of points 1, 2 and 3 as **PDF**
   - an `excel` workbook showing your results
   - a saved `R` file showing results