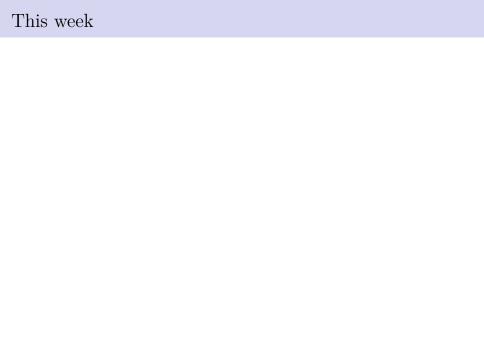
## Week 8: Differences among more than two samples Session 1

#### iClicker Question 1

- A two-sample t-test
- ${f B}$  Mann-Whitney paried test
- C Mann-Whitney U test
- ${\bf D}$  paired t-test

## ixlicker.

# Announcements



Having just analyzed some fish counts data in 16 lakes in Massachusetts, Thorsten found a significant 'lake' effect using an ANOVA, i.e., the mean number of fish was not the same in all lakes.

- 1. What would Thorsten do to find out which lakes were different from eachother?
- A) A series of t-tests
- B) A Tukey Honest Significant Difference test
- C) A Kruskal-Wallis test

I am interested in testing whether there is a significant difference between the population sizes of fish in 30 low salinity lakes and 30 high salinity lakes:

- 1. Which statistical test should I use?
- A) A t-test
- B) A One-Way ANOVA
- C) A Chi-square test
- D) A Two-Way ANOVA

I am interested in testing whether there is a significant difference between the population sizes of fish in 30 low salinity lakes and 30 high salinity lakes:

- 2. Which is the test statistic for the test?
- A) t
- B) F
- C) r
- D)  $\chi^2$

I am interested in testing whether there is a significant difference between the population sizes of fish in 30 low salinity lakes and 30 high salinity lakes. In fact, I actually sampled 10 large, 10 medium, and 10 small lakes in each of the high and low salinity lakes. I want to explore whether there are differences in population size based on lake salinity and lake size.

- 3. Now which statistical test should I use?
- A) A t-test
- B) A One-Way ANOVA
- C) A Chi-square test
- D) A Two-Way ANOVA

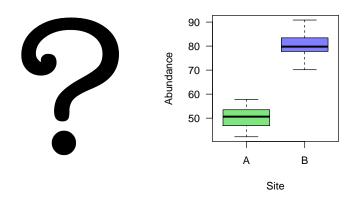
I am interested in testing whether there is a significant difference between the population sizes of fish in 30 low salinity lakes and 30 high salinity lakes. In fact, I actually sampled 10 large, 10 medium, and 10 small lakes in each of the high and low salinity lakes. I want to explore whether there are differences in population size based on lake salinity and lake size.

- 4. Now Which is the test statistic for the test?
- A) t
- B) F
- C) r
- D)  $\chi^2$

#### Comparing differences - two samples

#### Two samples:

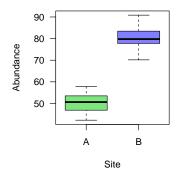
▶ Which test do we use?



#### Comparing differences - two samples

#### Two samples:

- ▶ the t-test?
- ▶ test whether group means differ significantly
- $\blacktriangleright$   $H_0$ : there is no significant difference between the means
- $\blacktriangleright$   $H_1$ : there is a significant difference between the means



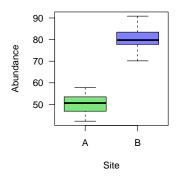
### Comparing differences - two smaples

#### Two samples:

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- $\blacktriangleright$   $H_0$ : there is no significant difference between the means
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#### Significance based on:

- degrees of freedom
- ▶ p-value



## Comparing differences - more than two samples

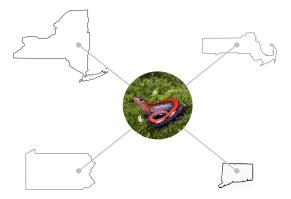
What about if there are more than 2 samples?

► can you think of any examples?



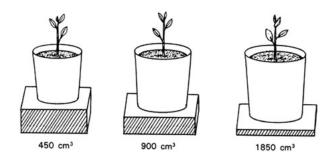
Regional differences in salamander abundance:

- ► comparing multiple populations
- ▶ quantify the differences between populations



Plant growth related to available resources (pot size):

- comparing multiple treatments
- quantify the effects of resource availability



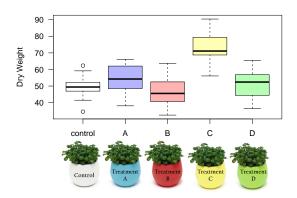
Plants productivity (dry mass in grams) related to fertilizer treatment

- ▶ do our treatments influence biomass production?
- ▶ is there a positive effect relative to a control?



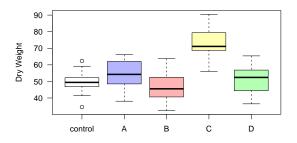
#### When there are more than 2 groups

- ▶ t-test doesn't help
  - need to do all possible pairs
  - time consuming
  - get spurious differences just by chance



#### Analysis of Variance (ANOVA):

- ▶ statistical test for testing for differences among >2 groups
- ▶ ANOVA and t-test are identical when there are 2 groups
- ▶ one factor/group/category (*One-way ANOVA*)

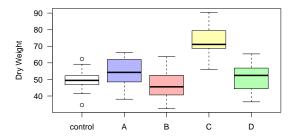


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#### Assumption:

▶ data are normally distributed



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#### Analysis of Variance (ANOVA):

- ▶ statistical test for testing for differences among >2 groups
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#### Assumption:

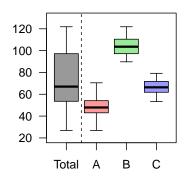
▶ data are normally distributed

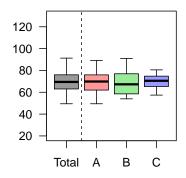
#### Hypotheses:

- $\blacktriangleright$   $H_0$ : there are no significant differences between the means
  - lack all means are equal
- $\blacktriangleright$   $H_1$ : there are significant differences between the means
  - lack all means are not equal

#### ANOVA explained

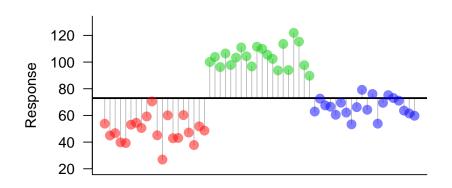
The ANOVA partitions the *total* variation into *within* sample variation with *between* sample variation to determine whether samples come from a single distribution or not.





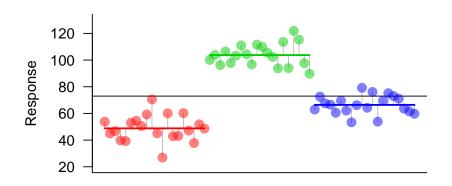
ightharpoonup Total sums of squares  $(SS_T)$ 

$$SS_T = \sum (x - \bar{x})^2$$



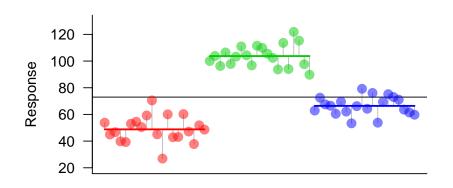
- ightharpoonup Within-sample sums of squares  $(SS_T)$
- $\blacktriangleright$  add up the within sample SS

$$SS_W = \sum (x_1 - \bar{x}_1)^2 + \sum (x_2 - \bar{x}_2)^2 + \sum (x_3 - \bar{x}_3)^2$$



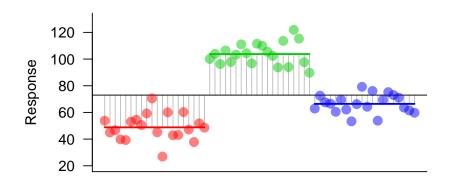
- $\blacktriangleright$  Within-sample sums of squares  $(SS_T)$
- ightharpoonup more generally (g is the number of groups)

$$SS_W = \sum_g \sum_i (x_{ig} - \bar{x}_g)^2$$



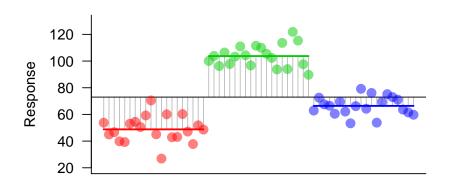
- ightharpoonup Between-sample sums of squares  $(SS_T)$
- ▶ add up the differences in the means

$$SS_B = n_1(\bar{x}_1 - \bar{x})^2 + n_2(\bar{x}_2 - \bar{x})^2 + n_3(\bar{x}_3 - \bar{x})^2$$

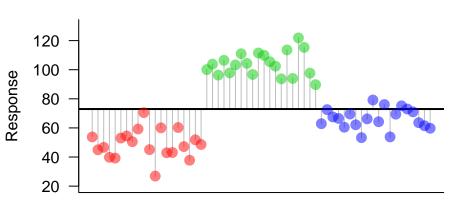


- ightharpoonup Between-sample sum of squares  $(SS_T)$
- ightharpoonup more generally (g is the number of groups)

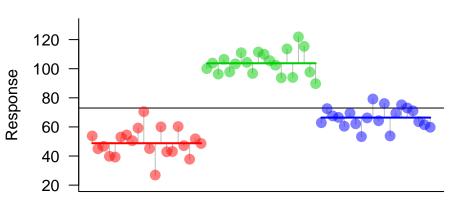
$$SS_B = \sum_g n_g (\bar{x}_g - \bar{x})^2$$



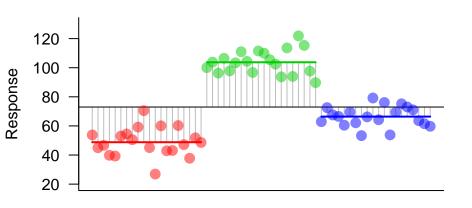
Total:



Within group:



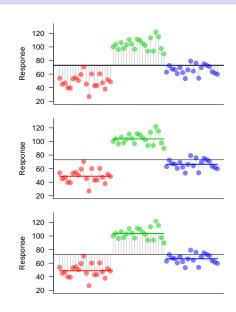
Between group:



$$SS_T = \sum (x - \bar{x})^2$$

$$SS_W = \sum_g \sum_i (x_{ig} - \bar{x}_g)^2$$

$$SS_B = \sum_g n_g (\bar{x}_g - \bar{x})^2$$



## ANOVA degrees of freedom

If we define the following:

- ightharpoonup n is the total sample size (number of observations)
- ightharpoonup g is the number of groups/samples

## ANOVA degrees of freedom

If we define the following:

- $\triangleright$  n is the total sample size (number of observations)
- $\triangleright$  g is the number of groups/samples

Then the degrees of freedom (df) are:

- ightharpoonup Total:  $df_T = n 1$
- ▶ Within:  $df_W = g 1$
- ▶ Between:  $df_B = n g$

## ANOVA the mean square

The mean square (MS) is the sum of squares divided by the degrees of freedom:

$$MS = SS/df$$

So:

► Total:  $MS_T = SS_T/df_T$ 

▶ Within:  $MS_W = SS_W/df_W$ 

▶ Between:  $MS_B = SS_B/df_B$ 

## ANOVA all the ingredients

	SS	df	MS
Total	$\sum (x - \bar{x})^2$	n-1	$SS_T/df_T$
Within	$\sum_{g} \sum_{i} (x_{ig} - \bar{x}_{j})^{2}$	n-g	$SS_W/df_W$
Between	$\sum_g n_g (\bar{x}_g - \bar{x})^2$	g-1	$SS_B/df_B$

#### ANOVA the statistical test

ANOVA results are usually presented in an ANOVA table \begin{center}

Source of variation	SS	df	MS	F	p
Between	$SS_B$	$df_B$	$MS_B$		
Within	$SS_W$	$df_W$	$MS_W$		
Total	$SS_T$	$df_T$	_		

 $\ensuremath{\operatorname{lend}}$ 

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Within	$SS_W$	$df_W$	$MS_W$		
Total	$SS_T$	$df_T$	_		

 $\ensuremath{\operatorname{lend}}$ 

ightharpoonup F is the test statistic for the ANOVA

$$F = \frac{MS_B}{MS_W}$$

#### ANOVA the statistical test

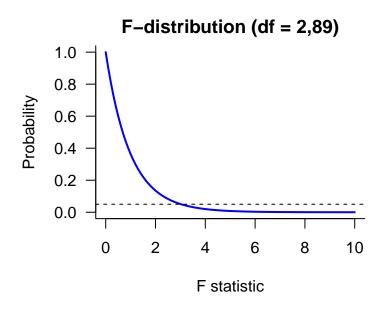
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Total	$SS_T$	$df_T$	_		

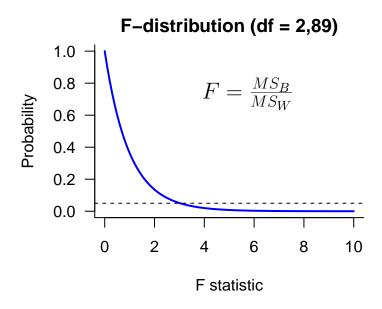
#### \end{center}

- $\triangleright$  p is the probability of observing the F statistic with a given degrees of freedom if the null hypothesis is true:
  - null hypothesis is 'no difference between the means'
  - $\triangleright$  based on the F-distribution

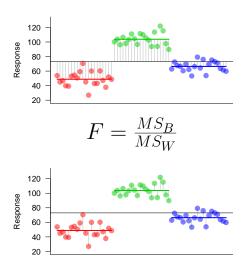
#### ANOVA the F distribution



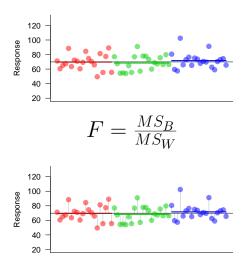
#### ANOVA the F distribution



### ANOVA and the Sums of Squares



### ANOVA and the Sums of Squares



# ANOVA the p value

#### Hypotheses:

- $\blacktriangleright$   $H_0$ : there are no significant differences between the means
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- $\blacktriangleright$   $H_1$ : there are significant differences between the means
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When do we reject or fail to reject the null hypothesis?

# ANOVA the p value

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When do we reject or fail to reject the null hypothesis?

- $\blacktriangleright$  if F is large, then p is small
- ▶ if p < 0.05 we reject the null hypothesis
- if p > 0.05 we fail to reject the null hypothesis

- ► Cannot use t-tests to make pairwise comparisons
  - ▶ multiple t-tests will lead to significant results by chance

- ► Instead we conduct *Post-hoc* testing
  - Tukey Honest Significant Difference test (Tukey HSD)
  - accounts for multiple tests being conducted
  - calculation of a t-statistic
  - a pair, so degrees of freedom is 1
  - ▶ 5% critical value for df = 1 is 4.303
    - if t > 4.303 then p < 0.05

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$$t_{a,b} = \frac{|\bar{x}_a - \bar{x}_b|}{\sqrt{\frac{MS_W\left(\frac{1}{n_a} + \frac{1}{n_b}\right)}{2}}}$$

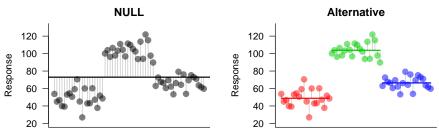
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    - if t > 4.303 then p < 0.05

	A	В	С
A	-	$t_{A,B}$	$t_{A,C}$
В	-	-	$t_{B,C}$
С	-	-	-

### ANOVA Recap

# Comparing differences between >2 samples (groups) using ANOVA

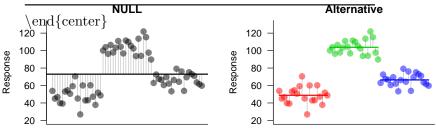
- ▶ null hypothesis:
  - no difference between the samples
  - data are from the same population
- ▶ alternative hypothesis:
  - sample means are different
  - data from the different populations



# ANOVA Recap

Comparing differences between >2 groups using ANOVA \begin{center}

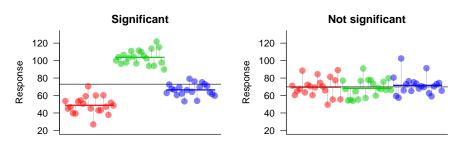
Source of variation	SS	df	MS	F	p
Between	$SS_B$	$df_B$	$MS_B$	$\frac{MS_B}{MS_W}$	
Within	$SS_W$	$df_W$	$MS_W$		
Total	$SS_T$	$df_T$	_		



### ANOVA Recap

#### Comparing differences between >2 groups using ANOVA

- ► Essentially comes down to:
  - $\triangleright$  a model with one mean or a model with a mean per group
  - which model best explains the data
  - which model significantly reduces the sums of squares



#### More than one factor with ANOVA

So far we have looked at multiple levels within a single factor

- ▶ factor: a single categorical predictor variable
- ▶ level: the categories within a factor

In some cases, we may be interested in >1 factor

- ▶ 2 factors: two-way ANOVA
- ▶ 3 factors: three-way ANOVA
- ► · · · multi-way ANOVA

Let's use a grazing example:

\begin{center}

\end{center}

	Site		
Grazing Treatment	Top	Lower	
Lo	9	7	
Lo	11	6	
Lo	6	5	
Mid	14	14	
Mid	17	17	
Mid	19	15	
Hi	28	44	
Hi	31	38	
Hi	32	37	

Lets use the example from the book (in R looks like this):

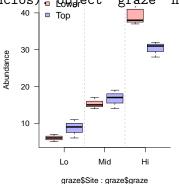
graze

## Error in eval(expr, envir, enclos): object 'graze' not :

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graze

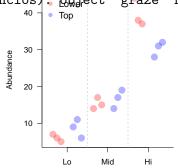
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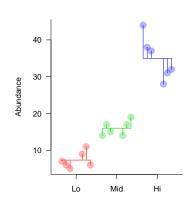
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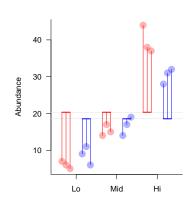
#### Step one:

- ► SS for each factor
  - graze
  - site
- $SS_{graze} = \sum (x_{i,graze} \bar{x}_{graze})^2$
- ► Ignore site grouping



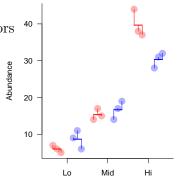
#### Step one:

- ► SS for each factor
  - graze
  - site
- $ightharpoonup SS_{site} = \sum (x_{i,site} \bar{x}_{site})^2$
- ► Ignore graze grouping



#### Step two:

- ▶ SS for each combinations of factors
- ► Treat all groupings as unique
- $\triangleright$   $SS_{within} = (x_{i,g} \bar{x}_g)^2$

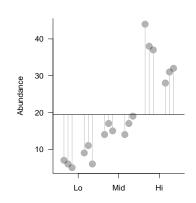


#### Step three:

- ► Sums of squares of both factors

#### Step four:

- ► Total sums of squares
- $ightharpoonup SS_{total} = \sum (x_i \bar{x})^2$
- ightharpoonup the null model
- ► Ignore all group structure



# Conducting the ANOVA - sums of squares $\,$

### $\verb|\begin{center}|$

	SS	df	MS	F	p
Graze	$SS_{graze}$				
Site	$SS_{site}$				
Both factors(interaction)	$SS_{both}$				
Within group	$SS_{within}$				
Total	$SS_{total}$				

 $\ensuremath{\operatorname{lend}}$ 

# Degrees of freedom

#### In general:

- ► Factor 1 (F1): number of levels 1
- ► Factor 2 (F2): number of levels 1
- ▶ Within: n (levels in F1 × levels in F2)
- ► Total: *n* 1

# Degrees of freedom

#### In general:

- ► Factor 1 (F1): number of levels 1
- ► Factor 2 (F2): number of levels 1
- ▶ Within: n (levels in F1 × levels in F2)
- ► Total: *n* 1

#### Grazing example:

- ▶ Graze: 3 1 = 2
- ▶ Site: 2 1 = 1
- ▶ Within:  $18 (3 \times 2) = 12$
- ▶ Total: 18 1 = 17

# Degrees of freedom

#### In general:

- ► Factor 1 (F1): number of levels 1
- ► Factor 2 (F2): number of levels 1
- ▶ Within: n (levels in F1 × levels in F2)
- ► Total: *n* 1

#### \begin{center}

$egin{array}{lll} { m Graze} & SS_{graze} & df_{graze} \ & Site & SS_{site} & df_{site} \ & Both \ { m factors(interaction)} & SS_{both} & df_{both} \ & Within \ { m group} & SS_{within} & df_{within} \ & Total & SS_{total} & df_{total} \ & \end{array}$		SS	df	MS	F	p
Both factors (interaction) $SS_{both}$ $df_{both}$ Within group $SS_{within}$ $df_{within}$	Graze	$SS_{graze}$	$df_{graze}$			
Within group $SS_{within}$ $df_{within}$	Site	$SS_{site}$	$df_{site}$			
	Both factors(interaction)	$SS_{both}$	$df_{both}$			
Total $SS_{total}$ $df_{total}$	Within group	$SS_{within}$	$df_{within}$			
	Total	$SS_{total}$	$df_{total}$			

\end{center}

### Mean squares

▶ the mean squares are calculated by dividing the sums of squares by the degrees of freedom for each element

### $\left\langle \operatorname{begin}\left\{ \operatorname{center}\right\} \right\rangle$

	SS	df	MS	F	p
Graze	$SS_{graze}$	$df_{graze}$	$MS_{graze} = \frac{SS_{graze}}{df_{graze}}$		
Site	$SS_{site}$	$df_{site}$	$MS_{site} = \frac{SS_{site}}{df_{site}}$		
Both factors	$SS_{both}$	$df_{both}$	$MS_{both} = \frac{SS_{both}}{df_{both}}$		
Within group	$SS_{within}$	$df_{within}$	$MS_{within} = \frac{SS_{within}}{df_{within}}$		
Total	$SS_{total}$	$df_{total}$			

 $\ensuremath{\operatorname{lend}}$ 

#### F statistic

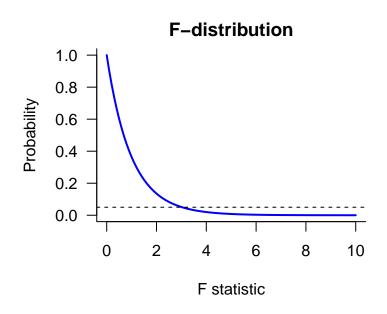
ightharpoonup the F-statistic is calculated by taking the element of interest divided by the within group MS (the error term)

#### \begin{center}

	SS	df	MS	F	p
Graze	$SS_{graze}$	$df_{graze}$	$MS_{graze} = \frac{SS_{graze}}{df_{graze}}$	$\frac{MS_{graze}}{MS_{within}}$	
Site	$SS_{site}$	$df_{site}$	$MS_{site} = \frac{SS_{site}}{df_{site}}$	$\frac{MS_{site}}{MS_{within}}$	
Both factors	$SS_{both}$	$df_{both}$	$MS_{both} = \frac{SS_{both}}{df_{both}}$	$\frac{MS_{both}}{MS_{within}}$	
Within group	$SS_{within}$	$df_{within}$	$MS_{within} = \frac{SS_{within}}{df_{within}}$		
Total	$SS_{total}$	$df_{total}$			

 $\ensuremath{\operatorname{lend}}$ 

#### ANOVA the F distribution



▶ Read in the data as a data frame

graze

## Error in eval(expr, envir, enclos): object 'graze' not :

```
oneway.site <- aov(Abundance ~ Site, data = graze)</pre>
```

```
## Error in terms.formula(formula, "Error", data = data):
```

```
summary(oneway.site)
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## Error in summary(oneway.site): object 'oneway.site' not
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```

```
## Error in summary(oneway.graze): object 'oneway.graze' no
```

```
oneway.graze <- aov(Abundance ~ graze, data = graze)</pre>
```

```
## Error in terms.formula(formula, "Error", data = data):
```

```
summary(oneway.graze)
```

```
## Error in summary(oneway.graze): object 'oneway.graze' no
```

► Conduct *any* test using formula syntax

```
twoway.additive <- aov(Abundance ~ Site + graze, data = gra
## Error in terms.formula(formula, "Error", data = data): of
## Error</pre>
```

```
summary(twoway.additive)
```

## Error in summary(twoway.additive): object 'twoway.addit;

► Conduct *any* test using formula syntax

```
twoway.additive <- aov(Abundance ~ Site + graze, data = gra
## Error in terms.formula(formula, "Error", data = data): of
## Error</pre>
```

```
summary(twoway.additive)
```

## Error in summary(twoway.additive): object 'twoway.addit;

```
twoway.interaction <- aov(Abundance ~ Site * graze, data =</pre>
```

```
## Error in terms.formula(formula, "Error", data = data):
```

```
summary(twoway.interaction)
```

```
## Error in summary(twoway.interaction): object 'twoway.in
```

```
twoway.interaction <- aov(Abundance ~ Site * graze, data =</pre>
```

```
## Error in terms.formula(formula, "Error", data = data):
```

```
summary(twoway.interaction)
```

```
## Error in summary(twoway.interaction): object 'twoway.in
```

We will conduct three analyses using the *salamANOVA*. We are interested in whether salamander snout-to-vent length (SVL) varies by sex and/or site. The data look like this:

```
str(sals)
```

## Error in str(sals): object 'sals' not found

- ► Site: there are four sites (P1A, P1B, P2A, P2B)
- ► Sex: M (male) and F (female)
- ► SVL: the snout-to-vent length in mm

#### Analysis 1: Does SVL vary by sex?

- ▶ What is the null hypothesis?
- ▶ Make a plot to visualize the hypothesis.
- ▶ What statistical test will you use to test  $H_0$ ?
- ▶ What is the:
  - $\triangleright$  test statistic for this particular test (e.g., t, F, etc)
  - degrees of freedom (calculate this)
  - significance level
- ► Conduct the analysis:
  - what is the value of the test statistic
  - $\triangleright$  what the *p*-value
- ▶ Write a short paragraph reporting the conclusion, use values from the statistical test to suppo, supported by the results from the test.

#### Analysis 2: Does SVL vary by site?

- ▶ What is the null hypothesis?
- ► Make a plot to visualize the hypothesis.
- $\blacktriangleright$  What statistical test will you use to test  $H_0$ ?
- ▶ What is the:
  - $\triangleright$  test statistic for this particular test (e.g., t, F, etc)
  - degrees of freedom (calculate this)
  - significance level
- ► Conduct the analysis:
  - what is the value of the test statistic
  - $\triangleright$  what the *p*-value
- ▶ Write a short paragraph reporting the conclusion, use values from the statistical test to suppo, supported by the results from the test.

#### Analysis 3: Does SVL vary by sex and/or site?

- ▶ What is the null hypothesis?
- ▶ Make a plot to visualize the hypothesis.
- ▶ What statistical test will you use to test  $H_0$ ?
- ▶ What is the:
  - $\triangleright$  test statistic for this particular test (e.g., t, F, etc)
  - degrees of freedom (calculate this)
  - significance level
- ► Conduct the analysis:
  - what is the value of the test statistic
  - $\triangleright$  what the *p*-value
- ▶ Write a short paragraph reporting the conclusion, use values from the statistical test to suppo, supported by the results from the test.

Assignment: Statistical analysis of variation in salamnder SVL.

- ► Write a report with four sections:
  - 1. Analysis 1
  - 2. Analysis 2
  - 3. Analysis 3
  - 4. Reflection: how does analysis 3 compare to analyses 1 and 2?
- ► Sections 1 to 3 sould report on each of the prompts in the previous slides.
- ▶ Section 4 is an opportunity to demonstrate your undertanding of the material covered over the previous weeks.
- $\blacktriangleright$  Assignment due: 11.55pm Tuesday November  $20^th$