

# Week 8: Differences among more than two samples

## Session 1

Spring 2020

## iClicker Question 1

A

B

C

D

# Announcements

This week's material builds on ideas from pairwise group comparisons.

The ANOVA material is more *dense* than what we've covered up until this point.

We're going to have to work on our *statistical intuition* to master these *inferential statistics* concepts.

# This week

Tuesday: Differences between more than two samples:

- ▶ Analysis of Variance (ANOVA) concepts
  - ▶ One-way ANOVA
  - ▶ Two-way ANOVA
  - ▶ Multiple testing

Thursday

- ▶ Continue ANOVA concepts
- ▶ Statistical analysis of salamanders

# Moving beyond two groups

T-tests are great, but what if we need to analyze more complicated scenarios?

Let's walk through some sampling and experimental scenarios to build intuition.

Scenario context: We're interested in bluegill population densities in Massachusetts lakes.



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<sup>1</sup>Image credit: New York Fish and Game Commission

# Scenario 1

Having just analyzed some fish counts data in 16 lakes in Massachusetts, Thorsten found a significant ‘lake’ effect using an ANOVA, i.e., the mean number of fish was not the same in all lakes.

1. Thorsten wants to know which *which lakes are different from each other*.
  - ▶ Think carefully: what does this actually mean?
  - ▶ What is the sampling unit?
  - ▶ What did he measure?
  - ▶ What would he need to compare?

# Scenario 1

Having just analyzed some fish counts data in 16 lakes in Massachusetts, Thorsten found a significant 'lake' effect using an ANOVA, i.e., the mean number of fish was not the same in all lakes.

2. What would Thorsten do to find out *which lakes were different from each other*?

- A) A series of  $t$ -tests
- B) A Tukey Honest Significant Difference test
- C) A Kruskal-Wallis test

## Scenario 2

I am interested in testing whether there is a significant difference between the population sizes of fish in 30 **low salinity** lakes and 30 **high salinity** lakes:

1. What is different from the last scenario?
2. What is the sampling unit?
3. What specific question(s) should I ask?
4. What, specifically, do I want to compare?



## Scenario 2

I am interested in testing whether there is a significant difference between the population sizes of fish in 30 **low salinity** lakes and 30 **high salinity** lakes:

5. Which statistical test could I use?

- A) A  $t$ -test
- B) A One-Way ANOVA
- C) A Chi-square test
- D) A Two-Way ANOVA

6. Which is the test statistic for the test I chose?

## Scenario 3

I am interested in testing whether there is a significant difference between the population density of fish in 30 **low salinity** lakes and 30 **high salinity** lakes.

In fact, I actually sampled 10 **large**, 10 **medium**, and 10 **small** lakes in each of the high and low salinity lakes.

I want to explore whether there are differences in population size based on lake **salinity** and lake **size**.

1. How has our sampling scheme changed?
2. What is the sampling unit?
3. How has our question changed?

## Scenario 3

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4. Now which statistical test should I use?

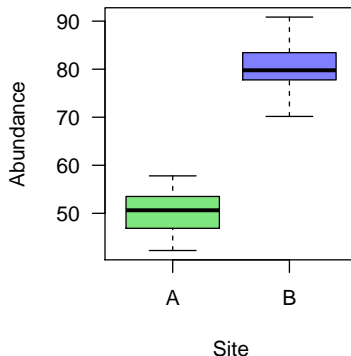
- A) A  $t$ -test
- B) A One-Way ANOVA
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5. Now which is the test statistic for the test?

# Comparing differences - two samples

Two samples:

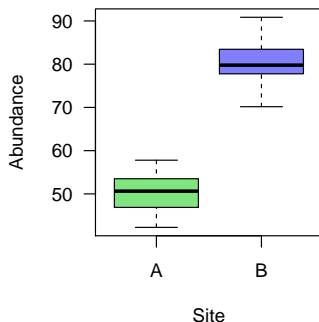
- Which test do we use?



# Comparing differences - two samples

Two samples:

- ▶ the t-test?
- ▶ test whether group means differ significantly
- ▶  $H_0$ : there is no significant difference between the means
- ▶  $H_1$ : there is a significant difference between the means



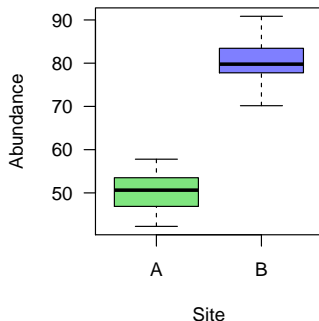
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Significance based on:

- ▶ t-statistic:  $t = \frac{|\bar{x}_a - \bar{x}_b|}{\sqrt{\frac{s_a^2}{n_a} + \frac{s_b^2}{n_b}}}$
- ▶ degrees of freedom
- ▶  $p$ -value



# Comparing differences - more than two samples

What about if there are more than 2 samples?

- ▶ can you think of any examples?



# Comparing multiple groups - examples

Regional differences in salamander abundance:

- ▶ comparing multiple populations
- ▶ quantify the differences between populations

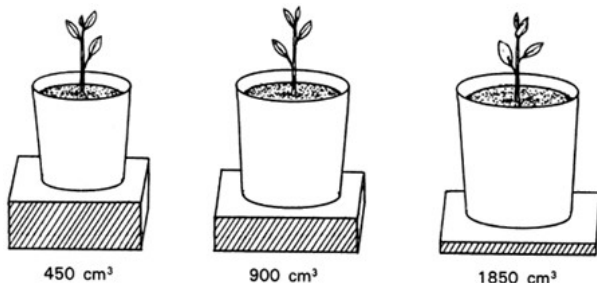




# Comparing multiple groups - examples

Plant growth related to available resources (pot size):

- ▶ comparing multiple treatments
- ▶ quantify the effects of resource availability



# Comparing multiple groups - examples

Plants productivity (dry mass in grams) related to fertilizer treatment

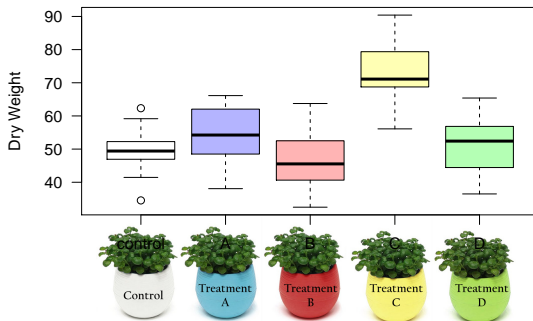
- ▶ do our treatments influence biomass production?
- ▶ is there a positive effect relative to a control?



# Comparing multiple groups - examples

When there are more than 2 groups

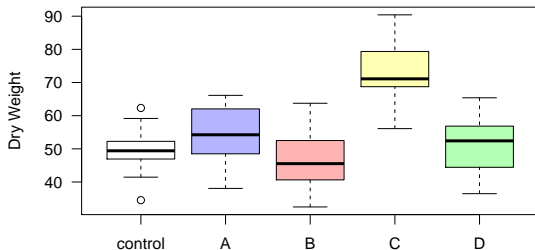
- ▶ t-test is probably not optimal:
  - ▶ We would need to do all possible pairs.
  - ▶ We might get spurious differences just by chance. Why?



# Comparing multiple groups - ANOVA

## Analysis of Variance (ANOVA):

- ▶ statistical test for testing for differences among  $>2$  groups
- ▶ ANOVA and t-test are identical when there are 2 groups
- ▶ one factor/group/category (*One-way ANOVA*)



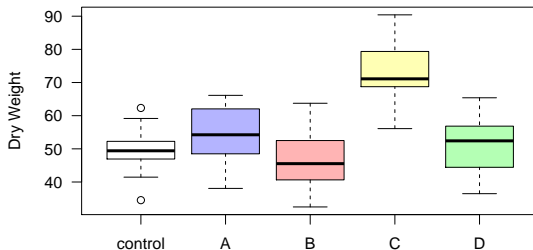
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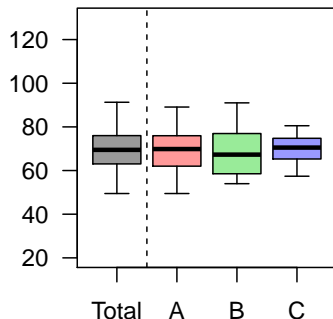
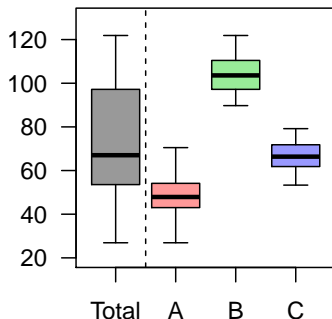
- ▶ data are normally distributed

Hypotheses:

- ▶  $H_0$ : there are no significant differences between the means
  - ▶ all means are equal
- ▶  $H_1$ : there are significant differences between the means
  - ▶ all means are not equal

# ANOVA explained

The ANOVA partitions the *total* variation into *within* sample (group) variation with *between* sample (group) variation to determine whether samples come from a single distribution or not.

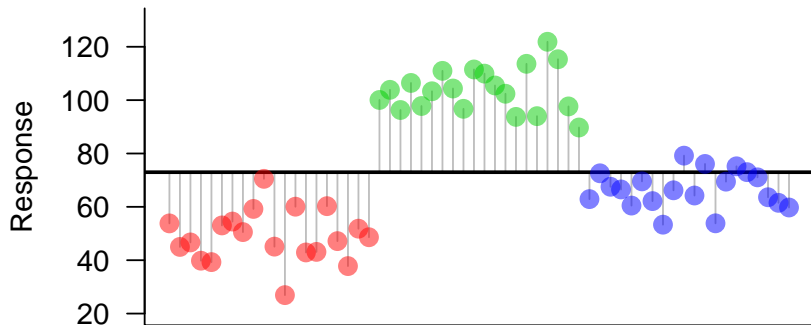




# ANOVA and the Sums of Squares

- *Total* sums of squares ( $SS_T$ )

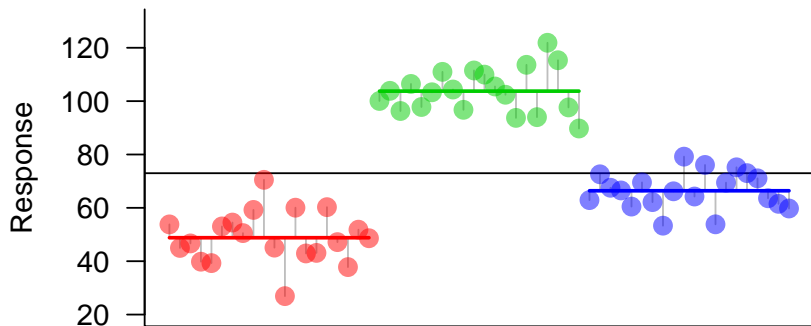
$$SS_T = \sum (x - \bar{x})^2$$



## ANOVA and the Sums of Squares

- *Within-sample* sums of squares ( $SS_W$ )
- add up the within sample SS

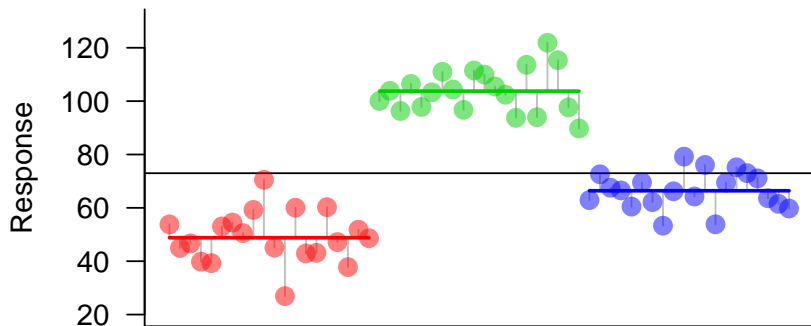
$$SS_W = \sum (x_1 - \bar{x}_1)^2 + \sum (x_2 - \bar{x}_2)^2 + \sum (x_3 - \bar{x}_3)^2$$



# ANOVA and the Sums of Squares

- ▶ *Within-sample* sums of squares ( $SS_W$ )
- ▶ more generally ( $g$  is the number of groups)

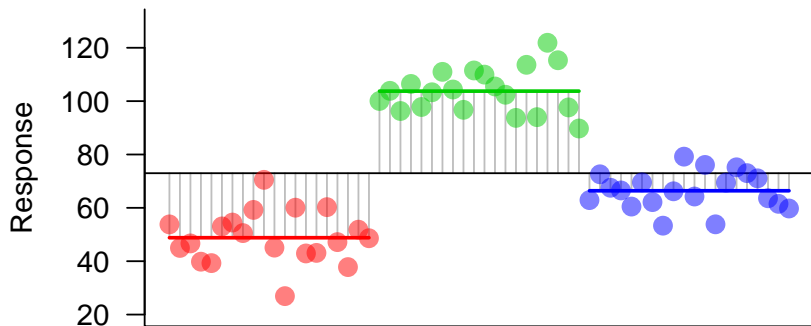
$$SS_W = \sum_g \sum_i (x_{ig} - \bar{x}_g)^2$$



# ANOVA and the Sums of Squares

- ▶ *Between-sample* sums of squares ( $SS_B$ )
- ▶ add up the differences in the means

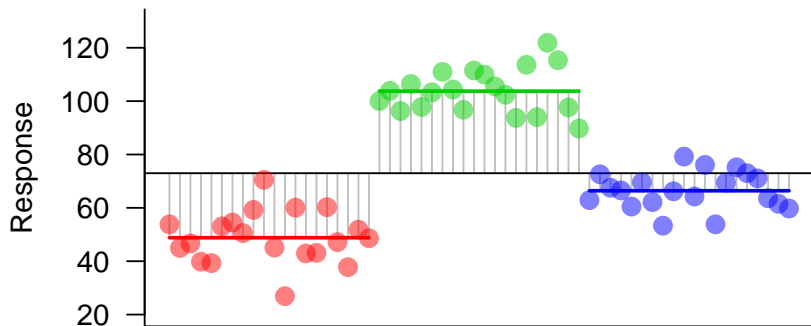
$$SS_B = n_1(\bar{x}_1 - \bar{x})^2 + n_2(\bar{x}_2 - \bar{x})^2 + n_3(\bar{x}_3 - \bar{x})^2$$



# ANOVA and the Sums of Squares

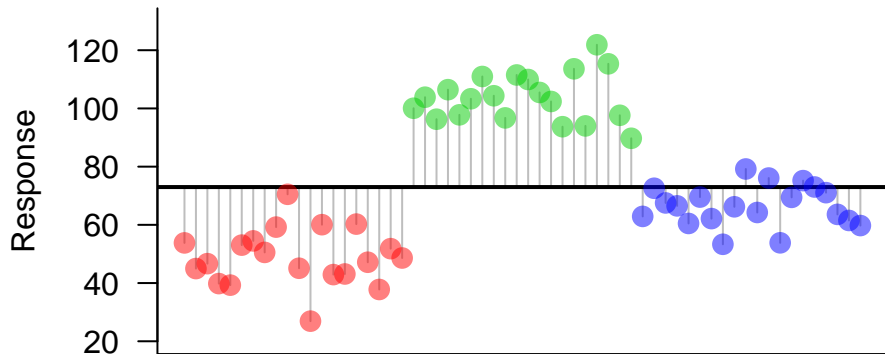
- ▶ *Between-sample* sum of squares ( $SS_B$ )
- ▶ more generally ( $g$  is the number of groups)

$$SS_B = \sum_g n_g (\bar{x}_g - \bar{x})^2$$



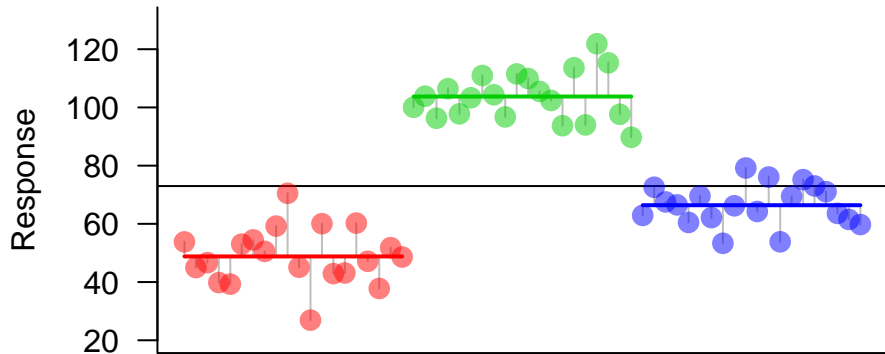
# ANOVA and the Sums of Squares

Total:



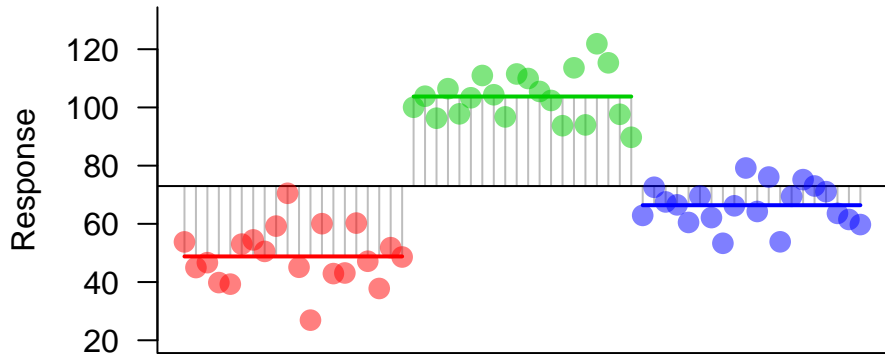
## ANOVA and the Sums of Squares

Within group:



# ANOVA and the Sums of Squares

Between group:



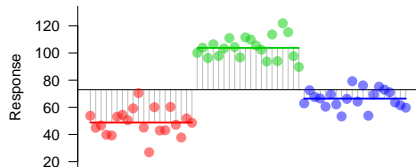
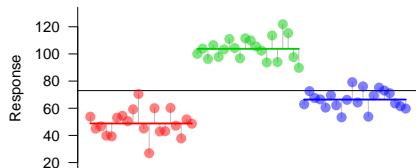
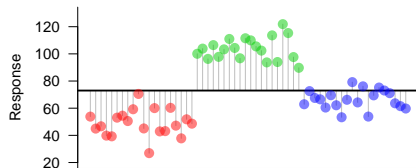


# ANOVA and the Sums of Squares

$$SS_T = \sum (x - \bar{x})^2$$

$$SS_W = \sum_g \sum_i (x_{ig} - \bar{x}_g)^2$$

$$SS_B = \sum_g n_g (\bar{x}_g - \bar{x})^2$$



# ANOVA degrees of freedom

If we define the following:

- ▶  $n$  is the total sample size (number of observations)
- ▶  $g$  is the number of groups/samples

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If we define the following:

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Then the degrees of freedom ( $df$ ) are:

- ▶ Total:  $df_T = n - 1$
- ▶ Within:  $df_W = g - 1$
- ▶ Between:  $df_B = n - g$

## ANOVA the *mean square*

The mean square ( $MS$ ) is the sum of squares divided by the degrees of freedom:

$$MS = SS/df$$

So:

- ▶ Total:  $MS_T = SS_T/df_T$
- ▶ Within:  $MS_W = SS_W/df_W$
- ▶ Between:  $MS_B = SS_B/df_B$

# ANOVA all the ingredients

|         | $SS$                                   | $df$    | $MS$        |
|---------|----------------------------------------|---------|-------------|
| Total   | $\sum (x - \bar{x})^2$                 | $n - 1$ | $SS_T/df_T$ |
| Within  | $\sum_g \sum_i (x_{ig} - \bar{x}_j)^2$ | $n - g$ | $SS_W/df_W$ |
| Between | $\sum_g n_g (\bar{x}_g - \bar{x})^2$   | $g - 1$ | $SS_B/df_B$ |

# ANOVA the statistical test

ANOVA results are usually presented in an ANOVA table

| Source of variation | $SS$   | $df$   | $MS$   | $F$ | $p$ |
|---------------------|--------|--------|--------|-----|-----|
| Between             | $SS_B$ | $df_B$ | $MS_B$ |     |     |
| Within              | $SS_W$ | $df_W$ | $MS_W$ |     |     |
| Total               | $SS_T$ | $df_T$ | –      |     |     |

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►  $F$  is the test statistic for the ANOVA

$$F = \frac{MS_B}{MS_W}$$

# ANOVA the statistical test

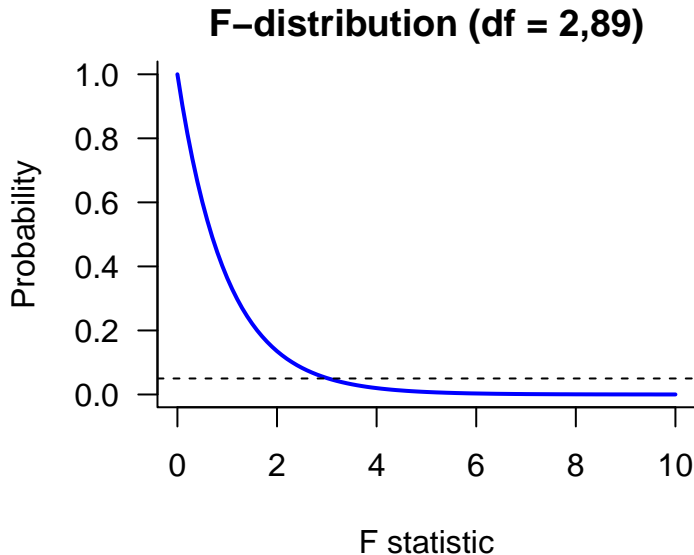
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| Total               | $SS_T$ | $df_T$ | –      |     |     |

- ▶  $p$  is the probability of observing the  $F$  statistic with a given degrees of freedom if the null hypothesis is true:
  - ▶ null hypothesis is ‘no difference between the means’
  - ▶ based on the  $F$ -distribution

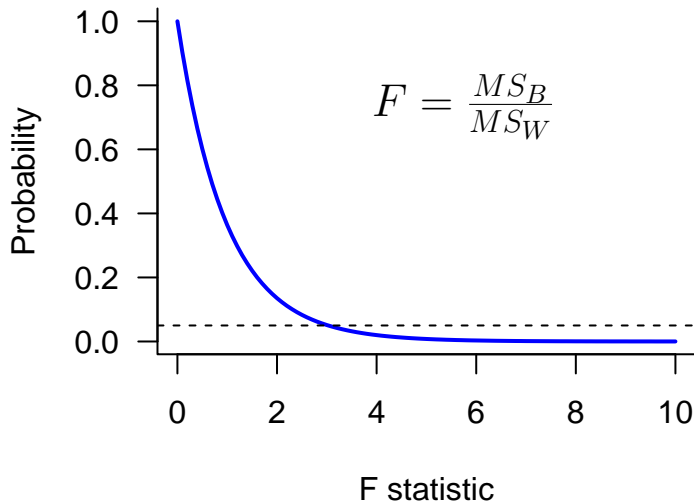


# ANOVA the $F$ distribution

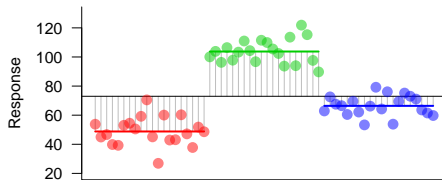


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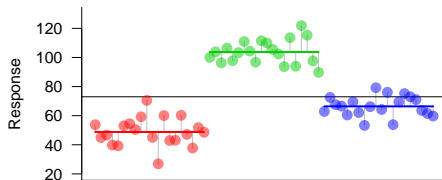
## F-distribution (df = 2,89)



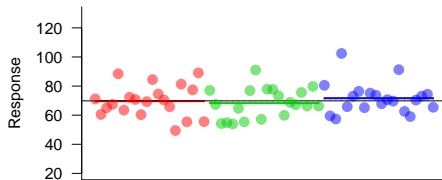
# ANOVA and the Sums of Squares



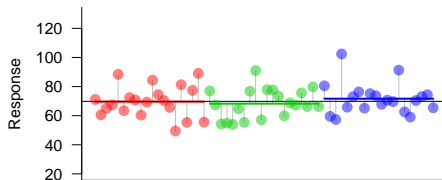
$$F = \frac{MS_B}{MS_W}$$



# ANOVA and the Sums of Squares



$$F = \frac{MS_B}{MS_W}$$



# ANOVA the $p$ value

Hypotheses:

- ▶  $H_0$ : there are no significant differences between the means
  - ▶ all means are equal
- ▶  $H_1$ : there are significant differences between the means
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When do we reject or fail to reject the null hypothesis?

# ANOVA the $p$ value

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When do we reject or fail to reject the null hypothesis?

- ▶ if  $F$  is large, then  $p$  is small
- ▶ if  $p < 0.05$  we reject the null hypothesis
- ▶ if  $p > 0.05$  we *fail to* reject the null hypothesis

# Pairwise comparisons with ANOVA

The  $F$  statistic tells us whether there are differences, but *not* what the differences are:

- ▶ *Cannot* use  $t$ -tests to make pairwise comparisons
  - ▶ multiple  $t$ -tests will lead to significant results by chance

# Pairwise comparisons with ANOVA

The  $F$  statistic tells us whether there are differences, but *not* what the differences are:

- ▶ Instead we conduct *Post-hoc* testing
  - ▶ Tukey Honest Significant Difference test (Tukey HSD)
  - ▶ accounts for multiple tests being conducted
  - ▶ calculation of a  $t$ -statistic
  - ▶ a pair, so degrees of freedom is 1
  - ▶ 5% critical value for  $df = 1$  is 4.303
    - ▶ if  $t > 4.303$  then  $p < 0.05$



# Pairwise comparisons with ANOVA

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$$t_{a,b} = \frac{|\bar{x}_a - \bar{x}_b|}{\sqrt{\frac{MS_W \left( \frac{1}{n_a} + \frac{1}{n_b} \right)}{2}}}$$

# Pairwise comparisons with ANOVA

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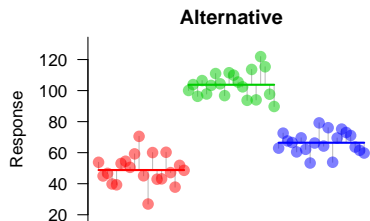
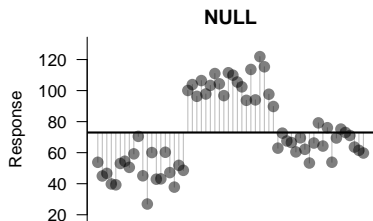
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|   | A | B         | C         |
|---|---|-----------|-----------|
| A | - | $t_{A,B}$ | $t_{A,C}$ |
| B | - | -         | $t_{B,C}$ |
| C | - | -         | -         |

# ANOVA Recap

Comparing differences between  $>2$  samples (groups) using ANOVA

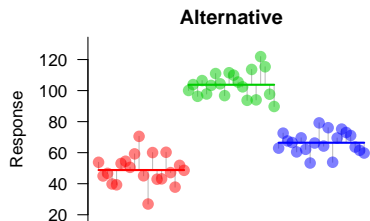
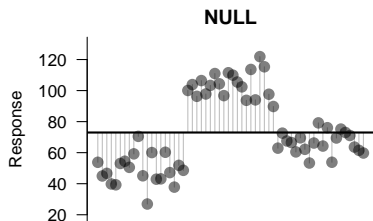
- ▶ null hypothesis:
  - ▶ no difference between the samples
  - ▶ data are from the same population
- ▶ alternative hypothesis:
  - ▶ sample means are different
  - ▶ data from the different populations



# ANOVA Recap

Comparing differences between  $>2$  groups using ANOVA

| Source of variation | $SS$   | $df$   | $MS$   | $F$                 | $p$ |
|---------------------|--------|--------|--------|---------------------|-----|
| Between             | $SS_B$ | $df_B$ | $MS_B$ | $\frac{MS_B}{MS_W}$ |     |
| Within              | $SS_W$ | $df_W$ | $MS_W$ |                     |     |
| Total               | $SS_T$ | $df_T$ | —      |                     |     |

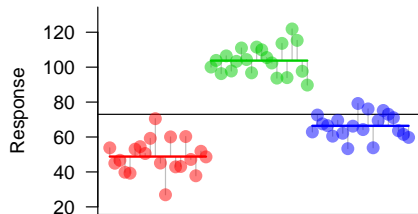


# ANOVA Recap

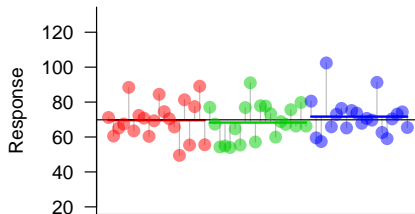
Comparing differences between  $>2$  groups using ANOVA

- ▶ Essentially comes down to:
  - ▶ a model with one mean *or* a model with a mean per group
  - ▶ which model best explains the data
  - ▶ which model significantly reduces the sums of squares

**Significant**



**Not significant**



# More than one factor with ANOVA

So far we have looked at multiple levels within a single factor

- ▶ factor: a single categorical predictor variable
- ▶ level: the categories within a factor

In some cases, we may be interested in  $>1$  factor

- ▶ 2 factors: *two-way* ANOVA
- ▶ 3 factors: *three-way* ANOVA
- ▶  $\dots$  multi-way ANOVA

# Two-way ANOVA

Let's use a grazing example:

| Grazing Treatment | Site |       |
|-------------------|------|-------|
|                   | Top  | Lower |
| Lo                | 9    | 7     |
| Lo                | 11   | 6     |
| Lo                | 6    | 5     |
| Mid               | 14   | 14    |
| Mid               | 17   | 17    |
| Mid               | 19   | 15    |
| Hi                | 28   | 44    |
| Hi                | 31   | 38    |
| Hi                | 32   | 37    |

# Two-way ANOVA

Lets use the example from the book (in R looks like this):

```
graze
```

| ##    | graze | Site  | Abundance |
|-------|-------|-------|-----------|
| ## 1  | Lo    | Top   | 9         |
| ## 2  | Lo    | Top   | 11        |
| ## 3  | Lo    | Top   | 6         |
| ## 4  | Mid   | Top   | 14        |
| ## 5  | Mid   | Top   | 17        |
| ## 6  | Mid   | Top   | 19        |
| ## 7  | Hi    | Top   | 28        |
| ## 8  | Hi    | Top   | 31        |
| ## 9  | Hi    | Top   | 32        |
| ## 10 | Lo    | Lower | 7         |
| ## 11 | Lo    | Lower | 6         |
| ## 12 | Lo    | Lower | 5         |
| ## 13 | Mid   | Lower | 14        |

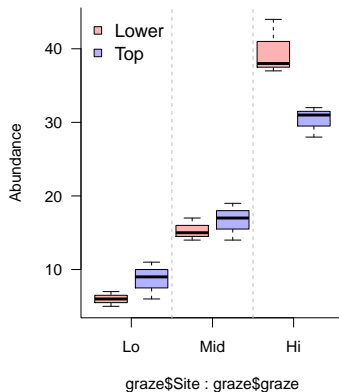


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| ## 2  | Lo    | Top   | 11        |
| ## 3  | Lo    | Top   | 6         |
| ## 4  | Mid   | Top   | 14        |
| ## 5  | Mid   | Top   | 17        |
| ## 6  | Mid   | Top   | 19        |
| ## 7  | Hi    | Top   | 28        |
| ## 8  | Hi    | Top   | 31        |
| ## 9  | Hi    | Top   | 32        |
| ## 10 | Lo    | Lower | 7         |
| ## 11 | Lo    | Lower | 6         |
| ## 12 | Lo    | Lower | 5         |
| ## 13 | Mid   | Lower | 14        |

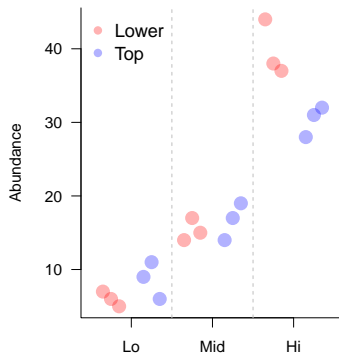


# Two-way ANOVA

Lets use the example from the book (in R looks like this):

```
head(graze, 9)
```

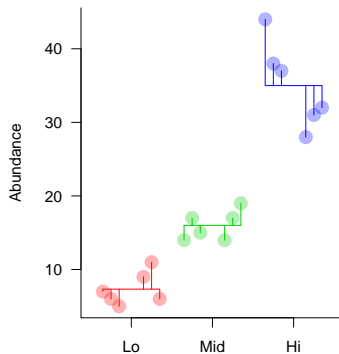
| ##   | graze | Site | Abundance |
|------|-------|------|-----------|
| ## 1 | Lo    | Top  | 9         |
| ## 2 | Lo    | Top  | 11        |
| ## 3 | Lo    | Top  | 6         |
| ## 4 | Mid   | Top  | 14        |
| ## 5 | Mid   | Top  | 17        |
| ## 6 | Mid   | Top  | 19        |
| ## 7 | Hi    | Top  | 28        |
| ## 8 | Hi    | Top  | 31        |
| ## 9 | Hi    | Top  | 32        |



# Conducting the ANOVA

Step one:

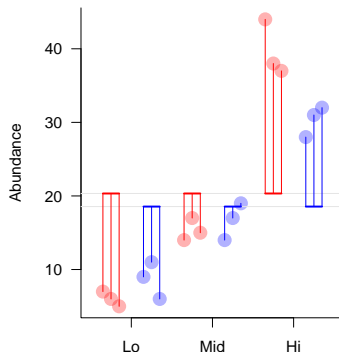
- ▶ SS for each factor
  - ▶ graze
  - ▶ site
- ▶  $SS_{graze} = \sum (x_{i,graze} - \bar{x}_{graze})^2$
- ▶ Ignore site grouping



# Conducting the ANOVA

Step one:

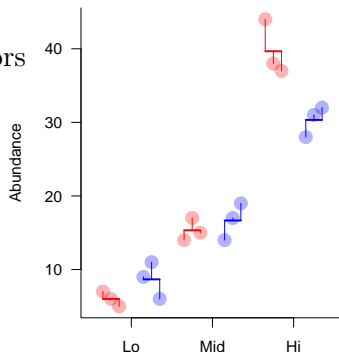
- ▶ SS for each factor
  - ▶ graze
  - ▶ site
- ▶  $SS_{site} = \sum (x_{i,site} - \bar{x}_{site})^2$
- ▶ Ignore graze grouping



# Conducting the ANOVA

Step two:

- ▶ SS for each combinations of factors
- ▶ Treat all groupings as unique
- ▶  $SS_{within} = (x_{i,g} - \bar{x}_g)^2$



# Conducting the ANOVA

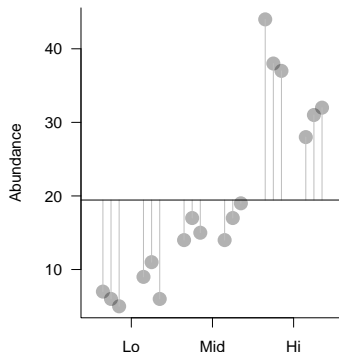
Step three:

- ▶ Sums of squares of both factors
- ▶  $SS_{both} = SS_{total} - SS_{graze} - SS_{site} - SS_{within}$

# Conducting the ANOVA

Step four:

- ▶ Total sums of squares
- ▶  $SS_{total} = \sum (x_i - \bar{x})^2$
- ▶ the *null* model
- ▶ Ignore all group structure



# Conducting the ANOVA - sums of squares

|                           | $SS$          | $df$ | $MS$ | $F$ | $p$ |
|---------------------------|---------------|------|------|-----|-----|
| Graze                     | $SS_{graze}$  |      |      |     |     |
| Site                      | $SS_{site}$   |      |      |     |     |
| Both factors(interaction) | $SS_{both}$   |      |      |     |     |
| Within group              | $SS_{within}$ |      |      |     |     |
| Total                     | $SS_{total}$  |      |      |     |     |



# Degrees of freedom

In general:

- ▶ Factor 1 (F1): number of levels - 1
- ▶ Factor 2 (F2): number of levels - 1
- ▶ Within:  $n - (\text{levels in F1} \times \text{levels in F2})$
- ▶ Total:  $n - 1$

# Degrees of freedom

In general:

- ▶ Factor 1 (F1): number of levels - 1
- ▶ Factor 2 (F2): number of levels - 1
- ▶ Within:  $n - (\text{levels in F1} \times \text{levels in F2})$
- ▶ Total:  $n - 1$

Grazing example:

- ▶ Graze:  $3 - 1 = 2$
- ▶ Site:  $2 - 1 = 1$
- ▶ Within:  $18 - (3 \times 2) = 12$
- ▶ Total:  $18 - 1 = 17$

# Degrees of freedom

In general:

- ▶ Factor 1 (F1): number of levels - 1
- ▶ Factor 2 (F2): number of levels - 1
- ▶ Within:  $n$  - (levels in F1  $\times$  levels in F2)
- ▶ Total:  $n$  - 1

|                           | $SS$          | $df$          | $MS$ | $F$ | $p$ |
|---------------------------|---------------|---------------|------|-----|-----|
| Graze                     | $SS_{graze}$  | $df_{graze}$  |      |     |     |
| Site                      | $SS_{site}$   | $df_{site}$   |      |     |     |
| Both factors(interaction) | $SS_{both}$   | $df_{both}$   |      |     |     |
| Within group              | $SS_{within}$ | $df_{within}$ |      |     |     |
| Total                     | $SS_{total}$  | $df_{total}$  |      |     |     |

# Mean squares

- the mean squares are calculated by dividing the sums of squares by the degrees of freedom for each element

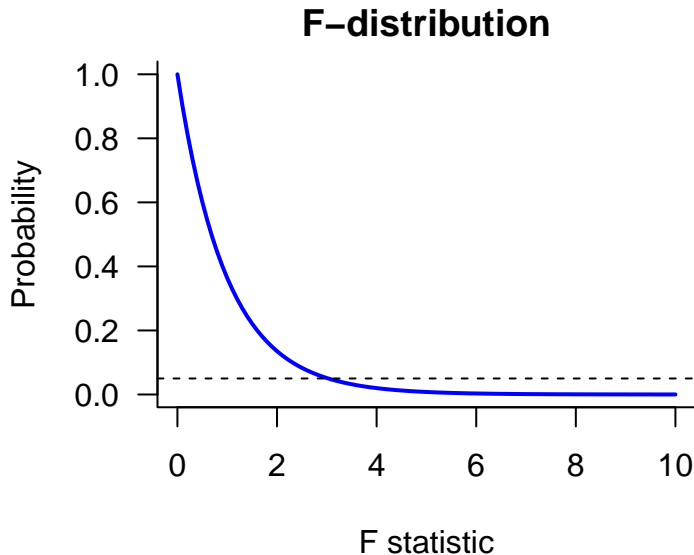
|              | $SS$          | $df$          | $MS$                                            | $F$ | $p$ |
|--------------|---------------|---------------|-------------------------------------------------|-----|-----|
| Graze        | $SS_{graze}$  | $df_{graze}$  | $MS_{graze} = \frac{SS_{graze}}{df_{graze}}$    |     |     |
| Site         | $SS_{site}$   | $df_{site}$   | $MS_{site} = \frac{SS_{site}}{df_{site}}$       |     |     |
| Both factors | $SS_{both}$   | $df_{both}$   | $MS_{both} = \frac{SS_{both}}{df_{both}}$       |     |     |
| Within group | $SS_{within}$ | $df_{within}$ | $MS_{within} = \frac{SS_{within}}{df_{within}}$ |     |     |
| Total        | $SS_{total}$  | $df_{total}$  |                                                 |     |     |

# F statistic

- ▶ the  $F$ -statistic is calculated by taking the element of interest divided by the within group MS (the *error* term)

|              | $SS$          | $df$          | $MS$                                            | $F$                              | $p$ |
|--------------|---------------|---------------|-------------------------------------------------|----------------------------------|-----|
| Graze        | $SS_{graze}$  | $df_{graze}$  | $MS_{graze} = \frac{SS_{graze}}{df_{graze}}$    | $\frac{MS_{graze}}{MS_{within}}$ |     |
| Site         | $SS_{site}$   | $df_{site}$   | $MS_{site} = \frac{SS_{site}}{df_{site}}$       | $\frac{MS_{site}}{MS_{within}}$  |     |
| Both factors | $SS_{both}$   | $df_{both}$   | $MS_{both} = \frac{SS_{both}}{df_{both}}$       | $\frac{MS_{both}}{MS_{within}}$  |     |
| Within group | $SS_{within}$ | $df_{within}$ | $MS_{within} = \frac{SS_{within}}{df_{within}}$ |                                  |     |
| Total        | $SS_{total}$  | $df_{total}$  |                                                 |                                  |     |

# ANOVA the $F$ distribution



# ANOVA in practice - R

- Read in the data as a data frame

```
graze
```

| ##    | graze | Site  | Abundance |
|-------|-------|-------|-----------|
| ## 1  | Lo    | Top   | 9         |
| ## 2  | Lo    | Top   | 11        |
| ## 3  | Lo    | Top   | 6         |
| ## 4  | Mid   | Top   | 14        |
| ## 5  | Mid   | Top   | 17        |
| ## 6  | Mid   | Top   | 19        |
| ## 7  | Hi    | Top   | 28        |
| ## 8  | Hi    | Top   | 31        |
| ## 9  | Hi    | Top   | 32        |
| ## 10 | Lo    | Lower | 7         |
| ## 11 | Lo    | Lower | 6         |
| ## 12 | Lo    | Lower | 5         |
| ## 13 | Mid   | Lower | 14        |

# Any ANOVA in practice - R

- Conduct *any* test using formula syntax

```
oneway.site <- aov(Abundance ~ Site, data = graze)  
summary(oneway.site)
```

| ##           | Df | Sum Sq | Mean Sq | F value | Pr(>F) |
|--------------|----|--------|---------|---------|--------|
| ## Site      | 1  | 14.2   | 14.22   | 0.087   | 0.771  |
| ## Residuals | 16 | 2602.2 | 162.64  |         |        |



# Any ANOVA in practice - R

- Conduct *any* test using formula syntax

```
oneway.site <- aov(Abundance ~ Site, data = graze)  
summary(oneway.site)
```

| ## |           | Df | Sum Sq | Mean Sq | F value | Pr(>F) |
|----|-----------|----|--------|---------|---------|--------|
| ## | Site      | 1  | 14.2   | 14.22   | 0.087   | 0.771  |
| ## | Residuals | 16 | 2602.2 | 162.64  |         |        |

# Any ANOVA in practice - R

- Conduct *any* test using formula syntax

```
oneway.graze <- aov(Abundance ~ graze, data = graze)
summary(oneway.graze)
```

```
##              Df Sum Sq Mean Sq F value    Pr(>F)
## graze          2 2403.1   1201.6    84.48 6.84e-09 ***
## Residuals     15   213.3     14.2
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
```

# Any ANOVA in practice - R

- Conduct *any* test using formula syntax

```
oneway.graze <- aov(Abundance ~ graze, data = graze)
summary(oneway.graze)
```

```
##              Df Sum Sq Mean Sq F value    Pr(>F)
## graze          2 2403.1   1201.6    84.48 6.84e-09 ***
## Residuals     15   213.3     14.2
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
```

# Any ANOVA in practice - R

- Conduct *any* test using formula syntax

```
twoway.additive <- aov(Abundance ~ Site + graze, data = graze)
summary(twoway.additive)
```

```
##              Df Sum Sq Mean Sq F value    Pr(>F)
## Site           1   14.2    14.2      1.00    0.334
## graze          2 2403.1  1201.6    84.48 1.54e-08 ***
## Residuals     14   199.1    14.2
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
```

# Any ANOVA in practice - R

- Conduct *any* test using formula syntax

```
twoway.additive <- aov(Abundance ~ Site + graze, data = graze)
summary(twoway.additive)
```

```
##              Df Sum Sq Mean Sq F value    Pr(>F)
## Site           1   14.2    14.2      1.00    0.334
## graze          2 2403.1  1201.6    84.48 1.54e-08 ***
## Residuals     14   199.1    14.2
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
```

# Any ANOVA in practice - R

- Conduct *any* test using formula syntax

```
twoway.interaction <- aov(Abundance ~ Site * graze, data =  
summary(twoway.interaction))
```

```
##              Df Sum Sq Mean Sq F value    Pr(>F)  
## Site           1   14.2     14.2    2.462  0.14264  
## graze          2 2403.1  1201.6  207.962 4.86e-10 ***  
## Site:graze     2   129.8     64.9   11.231  0.00178 **  
## Residuals     12    69.3      5.8  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
```

# Any ANOVA in practice - R

- Conduct *any* test using formula syntax

```
twoway.interaction <- aov(Abundance ~ Site * graze, data =  
summary(twoway.interaction))
```

```
##              Df Sum Sq Mean Sq F value    Pr(>F)  
## Site           1   14.2     14.2    2.462  0.14264  
## graze          2 2403.1  1201.6  207.962 4.86e-10 ***  
## Site:graze     2   129.8     64.9   11.231  0.00178 **  
## Residuals     12    69.3      5.8  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
```

## Group Exercise - *salamANOVA*

We will conduct three analyses using the *salamANOVA*. We are interested in whether salamander snout-to-vent length (SVL) varies by sex and/or site. The data look like this:

```
str(sals)
```

```
## Error in str(sals): object 'sals' not found
```

- ▶ Site: there are four sites (P1A, P1B, P2A, P2B)
- ▶ Sex: M (male) and F (female)
- ▶ SVL: the snout-to-vent length in mm



# Group Exercise - *salamANOVA*

Analysis 1: Does SVL vary by sex?

- ▶ What is the null hypothesis?
- ▶ Make a plot to visualize the hypothesis.
- ▶ What statistical test will you use to test  $H_0$ ?
- ▶ What is the:
  - ▶ test statistic for this particular test (e.g.,  $t$ ,  $F$ , etc)
  - ▶ degrees of freedom (calculate this)
  - ▶ significance level
- ▶ Conduct the analysis:
  - ▶ what is the value of the test statistic
  - ▶ what the  $p$ -value
- ▶ Write a short paragraph reporting the conclusion, use values from the statistical test to support, supported by the results from the test.

# Group Exercise - *salamANOVA*

Analysis 2: Does SVL vary by site?

- ▶ What is the null hypothesis?
- ▶ Make a plot to visualize the hypothesis.
- ▶ What statistical test will you use to test  $H_0$ ?
- ▶ What is the:
  - ▶ test statistic for this particular test (e.g.,  $t$ ,  $F$ , etc)
  - ▶ degrees of freedom (calculate this)
  - ▶ significance level
- ▶ Conduct the analysis:
  - ▶ what is the value of the test statistic
  - ▶ what the  $p$ -value
- ▶ Write a short paragraph reporting the conclusion, use values from the statistical test to support, supported by the results from the test.

## Group Exercise - *salamANOVA*

Analysis 3: Does SVL vary by sex and/or site?

- ▶ What is the null hypothesis?
- ▶ Make a plot to visualize the hypothesis.
- ▶ What statistical test will you use to test  $H_0$ ?
- ▶ What is the:
  - ▶ test statistic for this particular test (e.g.,  $t$ ,  $F$ , etc)
  - ▶ degrees of freedom (calculate this)
  - ▶ significance level
- ▶ Conduct the analysis:
  - ▶ what is the value of the test statistic
  - ▶ what the  $p$ -value
- ▶ Write a short paragraph reporting the conclusion, use values from the statistical test to support, supported by the results from the test.

# Group Exercise - *salamANOVA*

Assignment: Statistical analysis of variation in salamnder SVL.

- ▶ Write a report with four sections:
  1. Analysis 1
  2. Analysis 2
  3. Analysis 3
  4. Reflection: how does analysis 3 compare to analyses 1 and 2?
- ▶ Sections 1 to 3 sould report on each of the prompts in the previous slides.
- ▶ Section 4 is an opportunity to demonstrate your undertanding of the material covered over the previous weeks.