ECO 602 Analysis of Environmental Data

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Probability Distributions 1

Today’s Agenda

1. Stochasticity and models
2. Stochastic processes
3. Probability theory basics
4. Probability distributions
5. Discrete distributions

Dual-model paradigm: stochasticity

1. Deterministic: model of means
2. Stochastic: model of variability
   1. Error, noise, variability in the sample and population

Sources of error*

Your reading discusses 3 sources:

1. Measurement error
2. Process error
3. Model error

I want to add the concept of **sampling error** to the list.

* I don’t like the term ‘error’ but it is well established.

Measurement Error

We don’t have perfect accuracy or precision in our measurements.

Process error

There is randomness inherent in natural systems.

Model error

We won't always choose the right model! We might:

1. Choose an inappropriate deterministic model.
2. Infer wrong population parameter values for model.

Sampling error is intimately related to model error.

Sampling error

Sampling error is due to imperfect representation of the population in the sample.

Samples are just a subset of the population.

Different realizations of sampling procedure (usually) contain different observations.

Sampling error is greatest in small samples.

Larger samples allow better inference about the population.

Stochastic processes

A procedure whose outcome is **uncertain,** or **random**.

When we carry out a **realization** of a stochastic process, we produce a specific **event**.

We can often characterize the **probabilities** of observing different possible events.

Stochastic process examples

- Flipping a coin
- Drawing a hand of cards
- Random walk
- Brownian motion
- Lottery draw
- Different color balls in urns
- Examples in natural systems?

Probability theory and distributions: some essential terms*

Event, observation, realization, independence

Sample, sample space

Random variable

Probability mass, probability density

Cumulative probability

Empirical distribution

*Disclaimer: not an exhaustive list.

Probability theory

What is a probability?

Probability theory

Probability theory concerns the **likelihood**, of **events**.

Distributions are tools for describing the likelihood of observing **specific events** from the set of **all possible events.**

There are many named distributions with well-understood properties.

Probability key concepts

The sum of the probabilities of **all possible events** is 1.0

The probability of a **specific event** is usually less than 1.0

Independent events: the value of one observation gives us **no information** about the value of another observation.

Probability key concepts

Independent events: the probability of observing a **specific series** of events is equal to the **product** of the **individual events**.

The set of all possible events of a stochastic process is the **sample space**.

What is the sample space of a single coin flip?

What is the sample space of two independent coin flips?

Probability calculations can be difficult…

Probabilities can feel very unintuitive.

For example Bayes’ rule, which we’ll look at later, can have some very weird-seeming implications.

Combinatorics is the study of **combinations** and **permutations**.

The number of permutations and combinations can grow very quickly, even for small samples.

Sampling and sample spaces

- The Sample spaces can be discrete or continuous.
- A sample space can be finite or infinite
  - Finite, discrete example?
  - Infinite, discrete example?
  - Finite, continuous example?

Probability Distributions

A **distribution** associates a probability with every possible **event** in the **sample space**.

Theoretical distributions have well-defined functions.

Empirical distributions are calculated from data.

We usually want to **infer** a **theoretical distribution** of a **population** using an **empirical distribution** calculated from data.

Discrete distributions

Sample space is discrete – events cannot take on intermediate values.

For example, in a series of tosses of a coin, it is never possible to observe 1.34 heads.

But unintuitively, the sample space can still be **infinite**!

A simple distribution: Bernoulli

Binary outcome.

Each realization of the process is called a **trial**.

One parameter: the probability of success.

Let's sketch some distribution functions:

1. Probability mass
2. Cumulative mass

Binomial distribution

Describes a set of n independent Bernoulli trials.

Each trial has the same success probability.

Two parameters:

- p = probability of success
- n = number of trials

Repeated coin flips

Binomial distribution

Let's describe a series of three tosses of a fair coin as a binomial distribution:

- What is the sample space?
- How large is the sample space?
- How many heads would you expect to get?
- What are the values of n and p?

That was a lot!

It can be difficult, but not impossible, to gain intuition about distributions and distribution functions.

We'll take a break to work on assignment 2 before more complicated binomial examples.

Binomial examples from reading

Parameters:

n = number of trials

p = probability of success

Probability functions:

mass, cumulative mass

empirical, quantile

Binomial examples from reading

Brown creeper experiment:

- 10 sites
- Success: observing a bird at the site
- Failure: not observing a bird at the site
- Binary outcome, multiple trials.
- Binomial is a good candidate model.
- What could go wrong?

Probability Distributions... discrete

Example: *Binomial distribution*

Probability mass function (pmf):

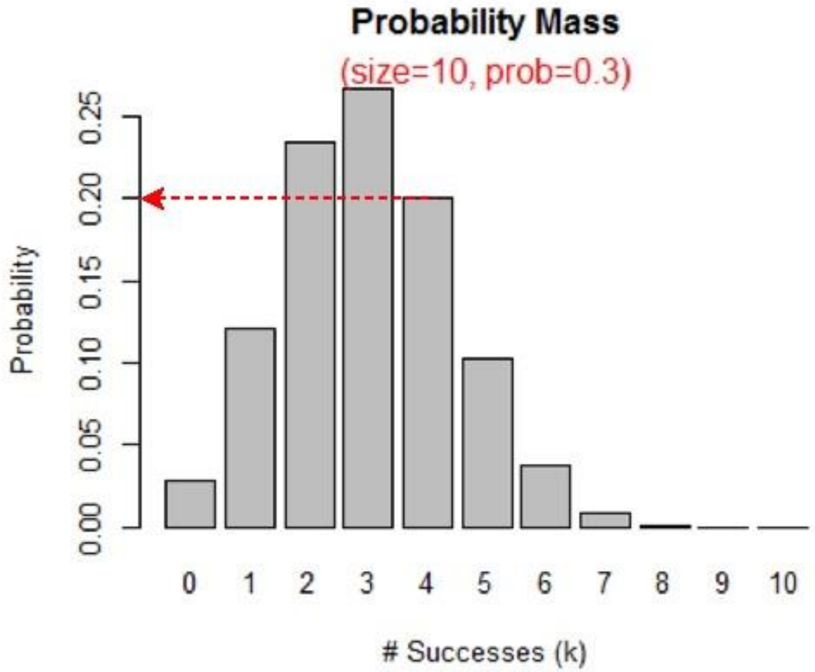
f(x) = Prob(X=x)

Binomial pmf:

$$\binom{N}{x} p^x (1-p)^{N-x}$$

N = trial size
p = per trial prob(success)
x = #successes (k)

dbinom(x=4,size=10,prob=0.3)
= 0.2



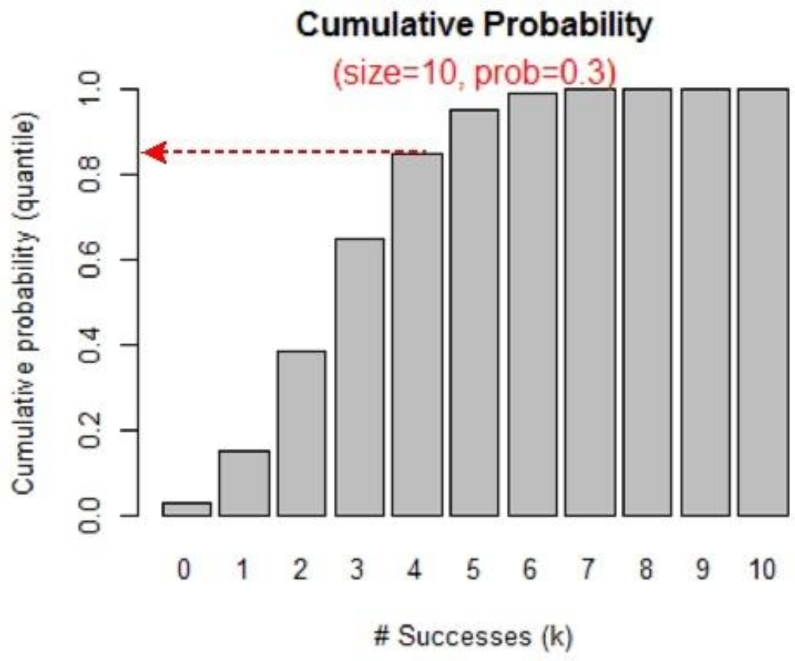
Probability Distributions… discrete

Example: *Binomial distribution*

Cumulative probability distribution:

$$f(x) = \text{Prob}(X \leq x)$$

- Denotes probability of x being less than or equal to any particular value (basis for *p*-values)



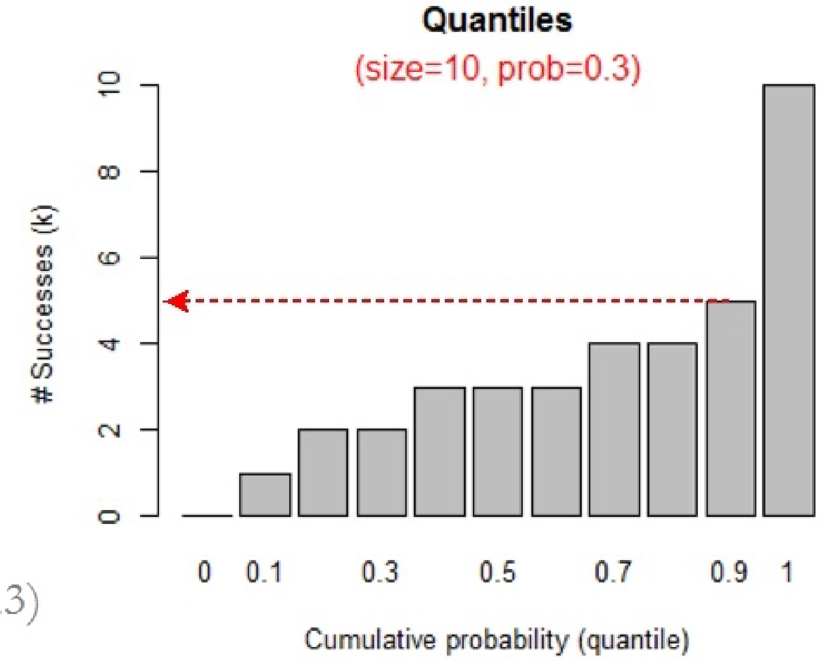
pbinom(x=4,size=10,prob=0.3) = 0.85

Probability Distributions… discrete

Example: *Binomial distribution*

Quantile distribution:

- Denotes value of x for any given quantile of the cumulative probability distribution; i.e., it is the opposite of the cumulative probability distribution



qbinom(p=.9,size=10,prob=0.3) = 5

Quantile functions are confusing*
*to me

I find the concept of quantile functions much more confusing than probability mass or cumulative probability functions!

We'll go over the concept several times.

Quantile functions

The reading says they are the ‘opposite’ of the cumulative mass function.

You can think of it as an inverse function to the cumulative mass function.

If you have a headache at this point, it’s ok.

Probability Distributions... discrete

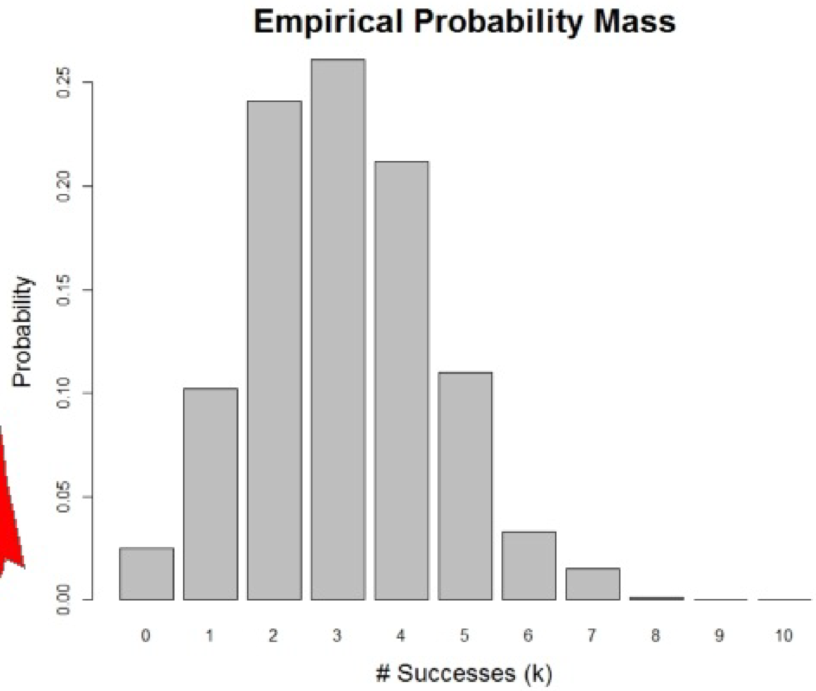
Example: *Binomial distribution*

<u>Example:</u>

Size(#trials) = 10
prob(present) = 0.3

Sample	Trial outcome	*k*
Sample 1	0 1 0 0 0 1 0 1 1 0	4
Sample 2	0 0 0 0 0 0 1 0 0 0	1
Sample 3	0 1 1 0 0 0 0 0 1 0	3
etc...		

Note, divide frequencies by total frequency to convert to a probability



Histograms and mass functions

1. Did you notice a similarity between histograms, probability mass functions, and empirical mass functions?
2. Hint: probability mass functions are just a type of normalized histogram.

For next time:

1. Finish discrete distributions
2. Continuous distributions