

OPTIMAL CONTROL OF BATTERY ELECTRIC VEHICLE IN ENERGY EFFICIENCY COMPETITIONS

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Resumo. Em competições de eficiência energética, como a Shell Eco-marathon (SEM), os veículos devem consumir a menor quantidade de energia em um trajeto fixo com tempo máximo determinado. Dois dos principais fatores que influenciam o consumo de energia de um veículo são a estratégia de direção e a relação de transmissão. O objeto deste estudo é defini-los de forma a minimizar o consumo de um protótipo elétrico na SEM Americas 2019. A partir do modelo matemático desenvolvido para a dinâmica do veículo elétrico e das restrições da SEM foi proposto um problema de controle ótimo. Este problema foi solucionado computacionalmente. O consumo simulado para a estratégia ótima 3 vezes menor que o consumo na SEM Americas 2019. Sugerimos que não haja acionamento dos freios, que o tempo gasto seja o máximo permitido e que o motor seja ligado nos apenas nos trechos de aclave.

Palavras chave: Shell Eco-marathon, problema de controle ótimo, estratégia de direção, relação de transmissão.

Abstract. In energy efficiency competitions, such as the Shell Eco-marathon (SEM), vehicles must consume the least amount of energy on a fixed track with a maximum set time. Two significant factors influencing a vehicle's energy consumption are the drive strategy and the gear ratio. This study aims to define these to minimize the consumption of a battery electric prototype, in SEM Americas 2019. Based on the mathematical model developed for the electric vehicle dynamics and the restrictions of SEM, an optimal control problem was proposed. This issue was computationally addressed. The simulated consumption for the optimal strategy was found to be three times lower than the consumption in SEM Americas 2019. Based on the results, we suggest no brake activation, that the time spent is the maximum allowed, and that the engine is connected only in uphill.

Keywords: Shell eco-marathon, optimal control problem, driving strategy, gear ratio.

1. INTRODUCTION

Energy efficiency is the relation between the energy supplied and the useful work performed by a conversion process or device. Increasing energy efficiency is a current need of society because it reduces energy production demand to perform the same work (IEA, 2019). This reduction has economic and environmental benefits, such as reducing vehicles' cost of use and emissions of pollutants. The transport sector is one of the main energy consumers, accounting for 20% of the world's energy consumption in 2018 (IEA, 2018).

The Shell company promotes the Shell Eco-marathon (SEM) to encourage the development of energy efficiency in the transportation sector. The SEM officially began in 1985 in France and is one of the largest student competitions in the world. It is currently held in 9 locations with more than 10,000 participating students from 52 countries (Shell, 2020?).

Several factors influence energy consumption, one of which is driving strategy. During the assessment of a vehicle's energy efficiency, it must follow the following restrictions, such as maximum instantaneous speed or minimum average speed, and these restrictions allow numerous management driving strategies. There is a need to find the strategy that maximizes energy efficiency.

Two approaches to driving strategy are used — closed loop and open loop. In closed loop approaches, the controller must be implemented in the vehicle. In open loop approaches, the driving strategy is calculated on a computer and the pilot is oriented to follow. Guzzella (2007), Targosz *et al.* (2018), Gechev and Punov (2020) presented open loop studies of optimal driving strategy. Liu *et al.* (2018), Sawulski and Lawrynczuk (2019), and Briguier *et al.* (2020) designed closed loop controllers to follow a optimal driving strategy.

The gear ratio is another determining factor in energy consumption. It is relatively easy to change this in a competition vehicle that is already built. Spanoudakis *et al.* (2020), used simulations in Carmaker software to compare different gear ratios for an urban concept vehicle and found a possible 2.6% reduction in consumption.

This study focuses on determining, through an open mesh analysis, the steering strategy and the optimal transmission ratio for the DT1 prototype in the seven laps of the SEM Americas 2019 track. The DT1, Fig. 1, is an electric battery vehicle built at the Federal University of Minas Gerais (UFMG).



Figure 1. Battery electric prototype vehicle DT1

2. METHODOLOGY

The first step was to define mathematical models of the prototype vehicle DT1 and the track. A formulation was then proposed for the optimal control problem (OCP) to find the optimal driving strategy and gear ratio. Accordingly, this proposed OPC was solved by finalizing MATLAB and the FALCON.m library.

2.1 Modeling of vehicle dynamics

Simplifications were made for the modeling of the vehicle's dynamics. Only the longitudinal dynamic was modeled. Aerodynamic drag did not consider atmospheric effects. Tire rolling resistance was independent of temperature, pressure, or speed. Bearing friction was zero. The battery and power converter were treated as ideal. The dynamics of the inductor in the motor were disregarded. White box modeling, which consists of fundamental principles that describe the behavior of the system, was used. The following equation that describes the longitudinal dynamics was derived from the application of Newton's second law in the vehicle, as represented in Fig. 2.

$$\begin{aligned}
 (m_v + m_p + m_r) \ddot{x} &= F_t - (F_a + F_g + F_r) , \\
 m_r &= \frac{N_r \cdot J_r + J_m \cdot \varphi^2}{r_r^2} , \\
 F_t &= K_t \cdot i_a \cdot \eta \cdot \frac{\varphi}{r_r} , \\
 F_a &= \frac{\rho \cdot a_f \cdot c_d \cdot \dot{x}^2}{2} , \\
 F_g &= (m_v + m_p) \cdot g \cdot \sin(\theta(x)) , \\
 F_r &= c_r \cdot (m_v + m_p) \cdot g \cdot \cos(\theta(x)) ,
 \end{aligned} \tag{1}$$

where m_v is the mass of the vehicle, m_p is the mass of the pilot, m_r is mass equivalent to the moment of inertia of the rotating parts (i.e., wheels and engine axle), x is the position, φ is the gear ratio, F_t is the propulsion, i_a is the electric current in the motor, F_a is the aerodynamic drag, F_g is the component of the weight that is the direction of speed, F_r is the rolling resistance of tires on the track, and θ is the slope of the track. The model constants are shown in Tab. 1. The experimental validation of this model has not been performed.

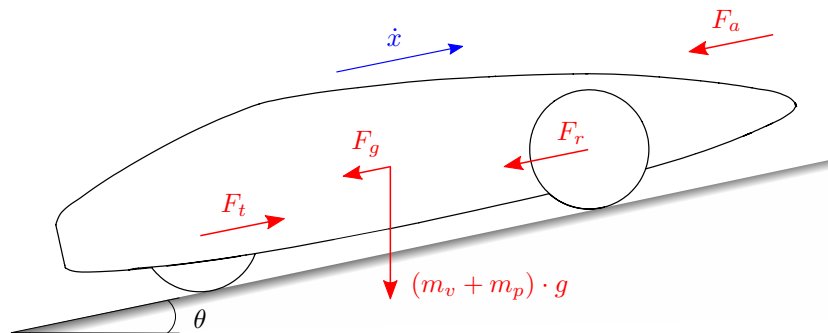


Figure 2. Diagram of forces of a moving vehicle

Table 1. Constants used in the vehicle model

Constant	Symbol	Unit	Value
Vehicle mass	m_v	[kg]	36
Pilot mass	m_p	[kg]	50
Armature resistance	R_a	[Ω]	0.07
Induced voltage constant	K_v	[V/(rad/s)]	0.119
Number of wheels	N_r	[]	3
Moment of inertia of the wheel	J_r	[kg.m ²]	0.015
Moment of inertia of the motor	J_m	[kg.m ²]	0.0625×10^{-3}
Wheel radius	r_r	[m]	0.254
Torque constant	K_t	[Nm/A]	0.119
Transmission efficiency	η	[]	0.95
Air density	ρ	[kg/m ³]	1.22
Frontal area of the vehicle	a_f	[m ²]	0.26
Aerodynamic drag coefficient	c_d	[]	0.164
Gravity acceleration	g	[m/s ²]	9.81
Rolling resistance coefficient	c_r	[]	0.0024

2.2 Track modeling

In 2018 and 2019, SEM Americas was held at the Sonoma Raceway. It is expected that the track will remain the same for the next editions of the competition. DEV (2018) made available GPS data collected on the Sonoma Raceway track in 2018. These data were used for track modeling. The track's relative altitude was approximated by a sum of sines with the `fit(x, h, 'sin8')` command of MATLAB. The slope model $\theta(x)$ was obtained by deriving $h(x)$ with respect to x :

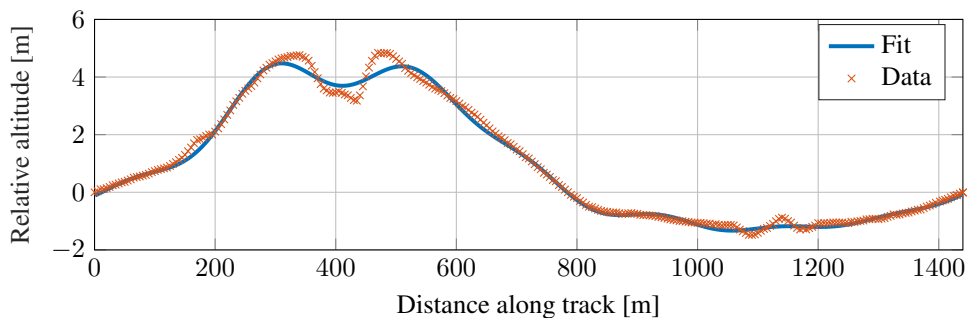
$$\theta(x) = \arctg \left(\sum_{n=1}^8 +a_n \cdot \cos(b_n \cdot x + c_n) \right) , \quad (2)$$

where the coefficients a_n , b_n and c_n are presented in Tab. 2. The fit curve and the data are shown in Fig. 3

Table 2. Track's slope model coefficients

Coef.	Value	Coef.	Value	Coef.	Value	Coef.	Value
a_1	0.012293	a_3	0.005818	a_5	0.003958	a_7	0.004361
b_1	0.004363	b_3	0.008726	b_5	0.017449	b_7	0.021814
c_1	-0.228758	c_3	-2.193381	c_5	-3.058604	c_7	1.627015
a_2	0.003041	a_4	0.002926	a_6	0.005365	a_8	0.005539
b_2	0.000236	b_4	0.000246	b_6	0.026184	b_8	0.030552
c_2	0.399758	c_4	3.490690	c_6	0.579933	c_8	-1.400496

Figure 3. Curve adjusted to represent track altitude



2.3 Formulation and solution of the optimal control problem

Using the vehicle model in Eq. (1), the track model in Eq. (2), and the SEM competition rules, the following formulation was used for the optimal control problem.

$$\min_{i(t), \varphi, T} \int_0^T i(t) \cdot \left(i(t) \cdot R_a + \frac{Kv \cdot v(t) \cdot \varphi}{r_r} \right) dt \quad (3a)$$

$$\text{s.t.} \quad x(0) = v(0) = 0, \quad (3b)$$

$$\dot{x}(t) - v(t) = 0, \quad t \in [0, T], \quad (3c)$$

$$\dot{v}(t) - \frac{F_t - (F_a + F_g + F_r)}{(m_v + m_p + m_r)} = 0, \quad t \in [0, T], \quad (3d)$$

$$i(t) - \frac{u_{bat} - Kv \cdot (v(t) \cdot \varphi / r_r)}{R_a} \leq 0, \quad t \in [0, T], \quad (3e)$$

$$0 \leq i(t) \leq i_{max}, \quad t \in [0, T], \quad (3f)$$

$$0 \leq v(t) \leq v_{max}, \quad t \in [0, T], \quad (3g)$$

$$\varphi_{min} \leq \varphi \leq \varphi_{max}, \quad (3h)$$

$$x(T) = x_f, \quad (3i)$$

$$T \leq t_{max}, \quad (3j)$$

where the integral term of Eq. (3a) defines the electricity consumed as the cost to minimize, Eq. (3b) is a constraint for the initial states of distance traveled and velocity null, Eq. (3c) and (3d) are constraints that ensure that the reset states respect the dynamics of the vehicle model, Eq. (3e) is a path-to-current constraint due to the maximum battery voltage, Eq. (3f) is a restriction for the maximum current value depending on the current limit delivered by the vehicle's power converter, Eq. (3g) is a restriction to the maximum car speed value set by the competition organization, Eq. (3h) is the possible range for the gear ratio, Eq. (3j) is a restriction to the maximum end time value due to the minimum average speed limit determined by the competition organization, and Eq. (3i) is a fixed end value constraint for the distance traveled equivalent to seven laps on the track. Table 3 presents the constants of Eq. (3).

Table 3. Constants of the proposed OCP

Constant	Symbol	Unit	Value
Battery voltage	u_{bat}	[V]	42
Maximum current	i_{max}	[A]	30
Maximum speed	v_{max}	[m/s]	12.5
Total distance	x_f	[m]	10080
Maximum time	t_{max}	[s]	1400
Minimum gear ratio	φ_{min}	[]	5.08
Maximum gear ratio	φ_{max}	[]	12.7

FALCON.m is a free-use software library for MATLAB, developed at the Institute of Flight System Dynamics of the Technische Universität München to solve and analyze optimal control problems (Rieck *et al.*, 2020). The OCP of Eq. (3) was solved using FALCON.m. The source code developed in this step was made using available open-source software.¹

3. RESULTS

The optimal track strategy found was very similar to the strategy start-stop in which the engine is turned off when the speed is maximum km/h and reconnected only when it is minimal. The world's most fuel-efficient vehicle used a start-stop-like driving strategy when it set the world record at SEM Americas 2018 (Grady *et al.*, 2019). However, in the optimal case, these values differed in the first, last, and other turns. The maximum and minimum speeds were 26.8 and 17.3 km/h, respectively, in the first lap. In last lap, the maximum and minimum values were 33.2 and 11.9 km/h, respectively. Meanwhile, in other laps, the maximum and minimum values were 35.0 and 17.3 km/h, respectively. The time spent on each lap was 194.6 s for the first, second, fourth, and fifth laps, 196.0 s for the third lap, and 222.6 s for the last lap.

The optimal control sequence of i and the state generated by it, as indicated by v , are presented in Fig. 4. Optimized

¹Available for download at https://github.com/michaelfsb/ocp_dt1

end time parameters T and φ transmission ratio are indicated in Tab. 4 along with the brand, media speed, and final speed metrics.

Table 4. Optimized parameters and metrics for the optimal driving strategy

Description	Unit	Value
Autonomy	[km/kWh]	905.68
Gear ratio	[]	9.2156
Total time	[s]	1400.0
Average speed	[km/h]	25.92
Final speed	[km/h]	17.09

The graphs in Fig. 5 show the voltage that must be applied to the motor to produce the electrical current of the optimal control sequence and the slope of the track at each instant of time. Notably, the engine is connected only at the highest positive peak of inclination, that is, the engine is only connected to the slope of the track.

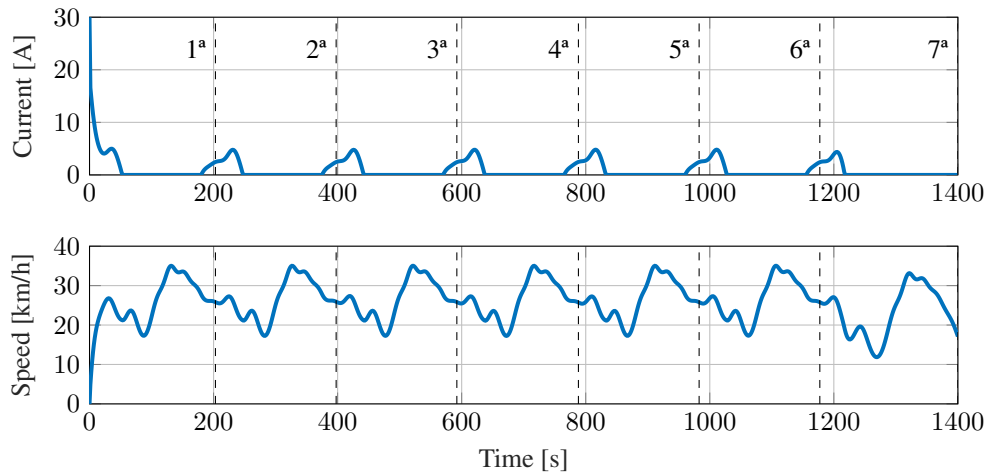


Figure 4. Optimal control sequence i and corresponding speed \dot{x}

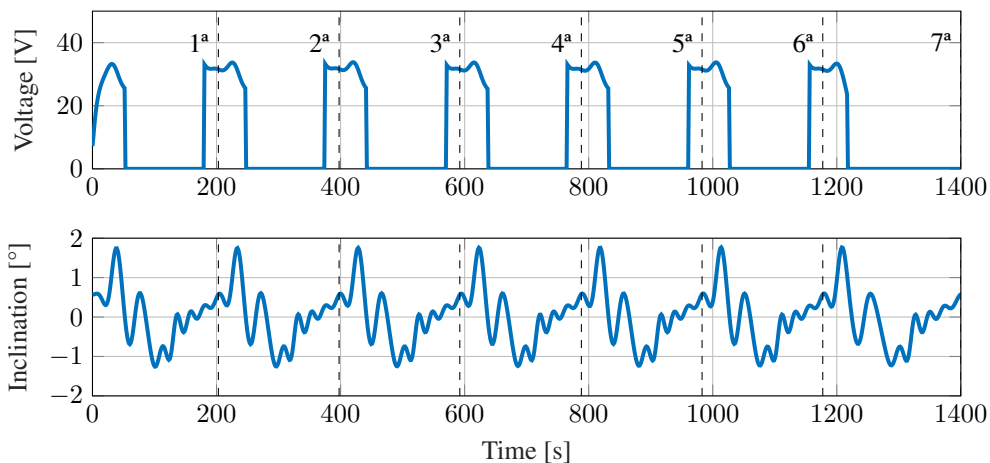


Figure 5. Voltage required for optimal control sequence and track inclination

4. CONCLUSION

This study addressed the problem of determining the optimal steering strategy and transmission ratio for an electric battery vehicle in an energy efficiency competition. This problem was addressed using optimal control theory. Accordingly, a computational solution was proposed. The source code developed can help competition teams and contribute to the study of vehicles that circulate on predefined paths.

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7. RESPONSIBILITY FOR INFORMATION

The authors are solely responsible for the information included in this study.