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OPTIMAL CONTROL OF BATTERY ELECTRIC VEHICLE IN ENERGY EFFICIENCY COMPETITIONS

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Resumo. *Em competições de eficiência energética, como a Shell Eco-marathon (SEM), os veículos devem consumir a menor quantidade de energia em um trajeto fixo com tempo máximo determinado. Dois dos principais fatores que influenciam o consumo de energia de um veículo são a estratégia de direção e a relação de transmissão. O objeto deste estudo é defini-los de forma a minimizar o consumo de um protótipo elétrico para a SEM Americas 2019. A partir do modelo matemático desenvolvido para a dinâmica do veículo elétrico e das restrições da SEM foi proposto um problema de controle ótimo. Este problema foi solucionado computacionalmente. O consumo simulado para a estratégia ótima foi 3 vezes menor que o consumo na SEM Americas 2019. Sugerimos que não haja acionamento dos freios, que o tempo gasto seja o máximo permitido e que o motor seja ligado nos apenas nos trechos de aclive.*

Palavras chave: *Shell Eco-marathon, problema de controle ótimo, estrategia de direção, relação de transmissão.*

Abstract. *In energy efficiency competitions, such as the Shell Eco-marathon (SEM), vehicles must consume the least amount of energy on a track which must be completed under a given time. Two significant factors influencing a vehicle's energy consumption are the drive strategy and the gear ratio. This study aims to define these to minimize the consumption of a battery electric prototype in SEM Americas 2019. Based on the mathematical model developed for the electric vehicle dynamics and the restrictions of SEM, an optimal control problem was proposed. This problem was solved numerically. The simulated consumption for the optimal strategy was found to be three times lower than the consumption in SEM Americas 2019. Based on the results, we suggest no brake activation, that the time spent is the maximum allowed, and that the engine is connected only in uphill.*

Keywords: *Shell eco-marathon, optimal control problem, driving strategy, gear ratio.*

1. INTRODUCTION

Energy efficiency is the ratio of useful work performed over the total energy supplied to a conversion process or device. Increasing energy efficiency is a current need of society because it reduces energy production demand to perform the same work (IEA, 2019). This reduction has economic and environmental benefits, such as reducing vehicles' cost of use and emissions of pollutants. The transport sector is one of the main energy consumers, accounting for 20% of the world's energy consumption in 2018 (IEA, 2018).

The Shell company promotes the Shell Eco-marathon (SEM) to encourage the development of energy efficiency in the transportation sector. The SEM officially began in 1985 in France and is one of the largest student competitions in the world. It is currently held in 9 locations with more than 10,000 participating students from 52 countries (Shell, 2020?).

Several factors influence energy consumption, one of which is driving strategy. During the assessment of a vehicle's energy efficiency, it must follow restrictions, such as maximum instantaneous speed or minimum average speed. However, these restrictions allow numerous driving strategies. There is a need to find the strategy that maximizes energy efficiency to obtain the best results possible with a given prototype at the SEM.

To implement a driving strategy, two approaches are used — closed loop and open loop. In closed loop approaches (Liu *et al.*, 2018; Sawulski and Lawrynczuk, 2019; Briguiet *et al.*, 2020), the controller must be implemented in the vehicle. In open loop approaches (Guzzella, 2007; Targosz *et al.*, 2018; Gechev and Punov, 2020), the driving strategy is calculated on a computer and the pilot is oriented to follow it.

The gear ratio is another determining factor in energy consumption. It is relatively easy to change this in a competition vehicle that is already built. Spanoudakis *et al.* (2020), used simulations in Carmaker software to compare different gear ratios for an urban concept vehicle and found a possible 2.6% reduction in consumption.

This study focuses on determining, through an open mesh analysis, the steering strategy and the optimal transmission ratio for the DT1 prototype in the seven laps of the SEM Americas 2019 track. The DT1, Fig. 1, is an battery electric vehicle built at the Federal University of Minas Gerais (UFMG). This vehicle participated in SEM Americas in 2018 and 2019 and obtained the respective results $266.5\,\mathrm{km/kW\,h}$ (6th place) and $226.9\,\mathrm{km/kW\,h}$ (2nd place).

Figure 1. Battery electric prototype vehicle DT1

2. METHODOLOGY

The first step was to define mathematical models of the prototype vehicle DT1 and the track. A formulation was then proposed for the optimal control problem (OCP) to find the optimal driving strategy and gear ratio. Accordingly, this proposed OPC was solved utilizing MATLAB and the FALCON.m library.

2.1 Modeling of vehicle dynamics

A simple model of the vehicle's dynamics was used. Only the longitudinal dynamics was modeled. Aerodynamic drag did not consider wind effect. Tire rolling resistance was independent of temperature, pressure, or speed. Bearing friction was zero. The battery and power converter were treated as ideal. The dynamics of the inductor in the motor were disregarded. A white-box model was derived by using the fundamental principles that describe the system's behaviour. The following equation that describes the longitudinal dynamics was derived from the application of Newton's second law in the vehicle, as represented in Fig. 2.

$$
\begin{aligned}
(m_v + m_p + m_r)\,\ddot{x}(t) &= F_t - (F_a + F_g + F_r) \quad , \\
m_r &= \frac{N_r \cdot J_r + J_m \cdot \varphi^2}{r_r^2} \quad , \\
F_t &= K_t \cdot i(t) \cdot \eta \cdot \frac{\varphi}{r_r} \quad , \\
F_a &= \frac{\rho \cdot a_f \cdot c_d \cdot \dot{x}(t)^2}{2} \quad , \\
F_g &= (m_v + m_p) \cdot g \cdot \sin(\theta(x)) \quad , \\
F_r &= c_r \cdot (m_v + m_p) \cdot g \cdot \cos(\theta(x)) \quad ,
\end{aligned}
\tag{1}
$$

where m_v is the mass of the vehicle, m_p is the mass of the pilot, m_r is mass equivalent to the moment of inertia of the rotating parts (i.e., wheels and engine axle), x is the position, φ is the gear ratio, F_t is the propulsive force, i is the electric current in the motor, F_a is the aerodynamic drag, F_g is the component of the weight that is the direction of speed, F_r is the rolling resistance of tires on the track, and θ is the slope of the track. The model constants are shown in Tab. 1. The experimental validation of this model has not been performed.

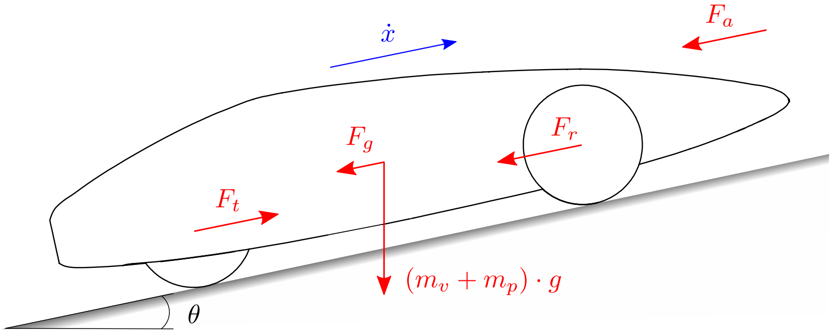

Figure 2. Diagram of forces of a moving vehicle

Table 1. Constants used in the vehicle model

Constant	Symbol	Value	Unit
Vehicle mass	m_v	36	kg
Pilot mass	m_p	50	kg
Number of wheels	N_r	3	-
Moment of inertia of the wheel	J_r	0.015	$\mathrm{kg\,m^2}$
Moment of inertia of the motor	J_m	0.0625×10^{-3}	$\mathrm{kg\,m^2}$
Wheel radius	r_r	0.254	m
Torque constant	K_t	0.119	$\mathrm{N\,m/A}$
Transmission efficiency	η	0.95	-
Air density	ρ	1.22	$\mathrm{kg/m^3}$
Frontal area of the vehicle	a_f	0.26	$\mathrm{m^2}$
Aerodynamic drag coefficient	c_d	0.164	-
Gravity acceleration	g	9.81	$\mathrm{m/s^2}$
Rolling resistance coefficient	c_r	0.0024	-

2.2 Track modeling

In 2018 and 2019, SEM Americas was held at the Sonoma Raceway. It is expected that the track will remain the same for the next editions of the competition. DEV (2018) made available GPS data collected on the Sonoma Raceway track in 2018. This data were used for track modeling. The layout of one lap in Sonoma Raceway is shown in Fig. 3, peaks of elevations are indicated with a triangle, valleys with a square.

The track's relative altitude h was approximated by a sum of sines with 8 terms [1]. The approximation with trigonometric functions was chosen because of the periodicity of these functions. In this way, it would be possible to represent all track laps in a continuous function. Using continuous functions instead of interpolating values in tables reduces the computational cost. The slope model θ was obtained by deriving the altitude model with respect to x:

$$\theta(x) = arctg\left(\frac{\mathrm{d}h(x)}{\mathrm{d}x}\right)$$

$$\theta(x) = \arctan\left(\sum_{n=1}^{8} a_n \cdot \cos(b_n \cdot x + c_n)\right) \quad , \tag{2}$$

where the coefficients a_n, b_n and c_n are presented in Tab. 2. The fitted curve and the data are displays in Fig. 4

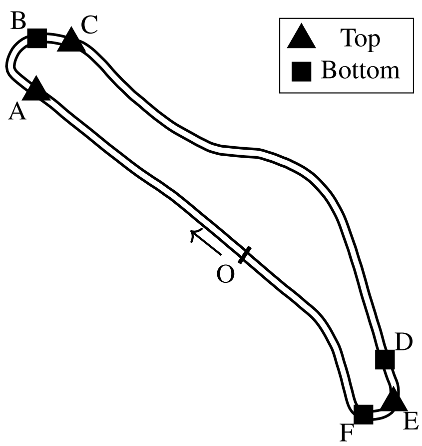

Figure 3. Layout of the racing track in Sonoma

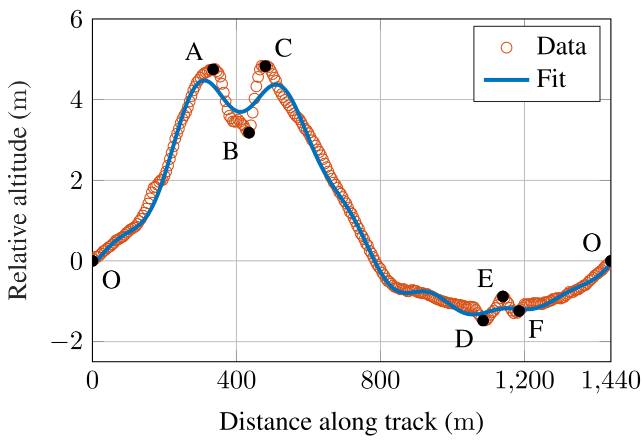

Figure 4. Curve adjusted to represent track altitude

[1]The fit was implemented in MATLAB using the command `fit(x, h, 'sin8')`

Table 2. Track's slope model coefficients

Coef.	Value	Coef.	Value	Coef.	Value	Coef.	Value
a_1	0.012293	a_3	0.005818	a_5	0.003958	a_7	0.004361
b_1	0.004363	b_3	0.008726	b_5	0.017449	b_7	0.021814
c_1	-0.228758	c_3	-2.193381	c_5	-3.058604	c_7	1.627015
a_2	0.003041	a_4	0.002926	a_6	0.005365	a_8	0.005539
b_2	0.000236	b_4	0.000246	b_6	0.026184	b_8	0.030552
c_2	0.399758	c_4	3.490690	c_6	0.579933	c_8	-1.400496

2.3 Formulation and solution of the optimal control problem

An optimal control problem aims to determine the parameters and control sequences of a system that will minimize a defined cost. Using the vehicle model in Eq. (1), the track model in Eq. (2), and the SEM competition rules, the following formulation was used for the optimal control problem.

$$\min_{i(t),\varphi,T} \quad \int_0^T i(t)\cdot\left(i(t)\cdot R_a + \frac{K_v\cdot\dot{x}(t)\cdot\varphi}{r_r}\right)\,\mathrm{d}t \tag{3a}$$

$$\text{s.t.} \quad x(0)=\dot{x}(0)=0, \tag{3b}$$

$$\dot{v}(t)-\frac{F_t-(F_a+F_g+F_r)}{(m_v+m_p+m_r)}=0, \qquad t\in[0,T], \tag{3c}$$

$$i(t)-\frac{u_{bat}-K_v\cdot(\dot{x}(t)\cdot\varphi/r_r)}{R_a}\le 0, \qquad t\in[0,T], \tag{3d}$$

$$0\le i(t)\le i_{max}, \quad t\in[0,T], \tag{3e}$$

$$0\le \dot{x}(t)\le v_{max}, \quad t\in[0,T], \tag{3f}$$

$$\varphi_{min}\le\varphi\le\varphi_{max}, \tag{3g}$$

$$x(T)=x_f, \tag{3h}$$

$$T\le t_{max}, \tag{3i}$$

where

Eq. (3a) defines the electric energy consumed as the cost to minimize,

Eq. (3b) is a constraint for the initial states of distance traveled and velocity null,

Eq. (3c) is a constraint that ensure that the reset states respect the dynamics of the vehicle model,

Eq. (3d) is a path constraint to current due to the maximum battery voltage,

Eq. (3e) is a constraint for the maximum current,

Eq. (3f) is a constraint to the maximum car speed value set by the competition organization,

Eq. (3g) is the possible range for the gear ratio,

Eq. (3h) is a fixed end value constraint for the distance traveled, and

Eq. (3i) is a constraint to the maximum end time.

The constants in Eq. (3) are DT1 specifications and SEM rules. The constants values are shown in Tab. 3. The maximum time constant t_{max} was determined utilizing a safety margin of $3.68\,\%$ ($51.52\,\mathrm{s}$) to satisfy the $25.0\,\mathrm{km/h}$ of minimal average speed required on SEM rules. This value of t_{max} is equivalent to $25.92\,\mathrm{km/h}$ of average speed. The total distance constant x_f was determined considering that the distance covered in a lap on the track is 1440 meters.

FALCON.m is a free-use[2] software library for MATLAB, developed at the Institute of Flight System Dynamics of the Technische Universität München to solve and analyze optimal control problems (Rieck *et al.*, 2020). The OCP of Eq. (3) was solved using FALCON.m. The source code developed in this step was made using available open-source software.[3]

[2] The license can be requested at https://www.fsd.lrg.tum.de/software/falcon-m
[3] Available for download at https://github.com/michaelfsb/ocp_dt1

Table 3. Constants of the proposed OCP

	Constant	**Symbol**	**Value**	**Unit**
DT1 specs	Battery voltage	u_{bat}	42	V
	Maximum current	i_{max}	30	A
	Maximum gear ratio	φ_{max}	12.7	-
	Minimum gear ratio	φ_{min}	5.08	-
	Motor resistance	R_a	0.07	Ω
	Induced voltage constant	K_v	0.119	$\mathrm{V/(rad/s)}$
SEM rules	Maximum speed	v_{max}	12.5	$\mathrm{m/s}$
	Maximum time	t_{max}	1400	s
	Total distance	x_f	10080	m

In a direct current motor, the electric current is controlled through the applied voltage. The electric current was chosen as the control variable instead of the voltage because the motor is decoupled from the transmission when it is off. Decoupling the motor can only be represented by a discontinuous function, and this causes an increase in computational complexity in the OCP solution. The voltage required to produce the electrical current in the optimal case was calculated from the following equation

$$u(t) = \begin{cases} R_a \cdot i(t) + K_v \cdot \dfrac{x(t) \cdot \varphi}{r_r}, & \text{if } i(t) > 0, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

3. RESULTS

The solution found respected all the proposed restrictions and improve the autonomy to $905.68\,\mathrm{km/kWh}$. Optimized gear ratio φ and end time parameters T are indicated in Tab. 4 along with the media speed, and final speed metrics. The optimal driving strategy found was very similar to the strategy start-stop in which the engine is turned off when the speed is maximum and reconnected only when it is minimal. The world's most fuel-efficient vehicle used a start-stop-like driving strategy when it set the world record at SEM Americas 2018 (Grady *et al.*, 2019). However, in the optimal case, these values differed in the first, last, and other turns. The time spent on each lap and the speed range are shown in Tab. 5. The last lap is $\approx 23\,\mathrm{s}$ slower than the other laps because it is not started at the end of it.

Table 4. Optimized parameters and metrics for the optimal case

Description	**Value**	**Unit**
Gear ratio	9.2156	-
Total time	1400.0	s
Average speed	25.92	$\mathrm{km/h}$
Final speed	17.09	$\mathrm{km/h}$

Table 5. Time spent and speed limits on each lap for the optimal case

Lap	**Time** (s)	**Min. speed** (km/h)	**Max. speed** (km/h)
1	203.0	17.3	26.8
2,3,5,6	194.6	17.3	35.0
4	196.0	17.3	35.0
7	222.6	11.9	33.2

Unlike common sense, the speed at the final instant was greater than zero, i.e., the vehicle has an amount of energy that will not be used. This is due to the slow deceleration of the vehicle. More time would be spent on the last lap so that the final speed would be zero, and then the maximum time restriction would be violated.

The graph in Fig. 5 presents the voltage that must be applied to the motor to produce the optimal control sequence's electrical current. The motor drives are smooth, the maximum voltage applied is below the battery voltage, and the electric current achieves $30\,\mathrm{A}$ only in the first part. The power applied to the motor does not exceed the value of $194\,\mathrm{W}$.

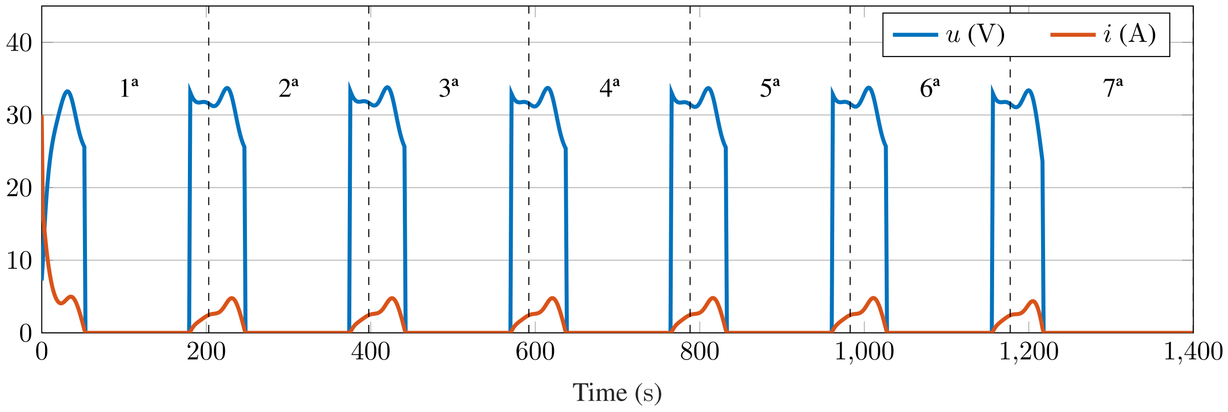

Figure 5. Voltage u and electric current i in optimal case

Looking at Fig. 6, it is apparent that the motor is started only once per lap during the uphill stretch (point F to point A in Fig. 3) and remains off in the curves and downhill stretch in the optimal driving strategy. The periodic behavior of the altitude model can also be observed in this graph.

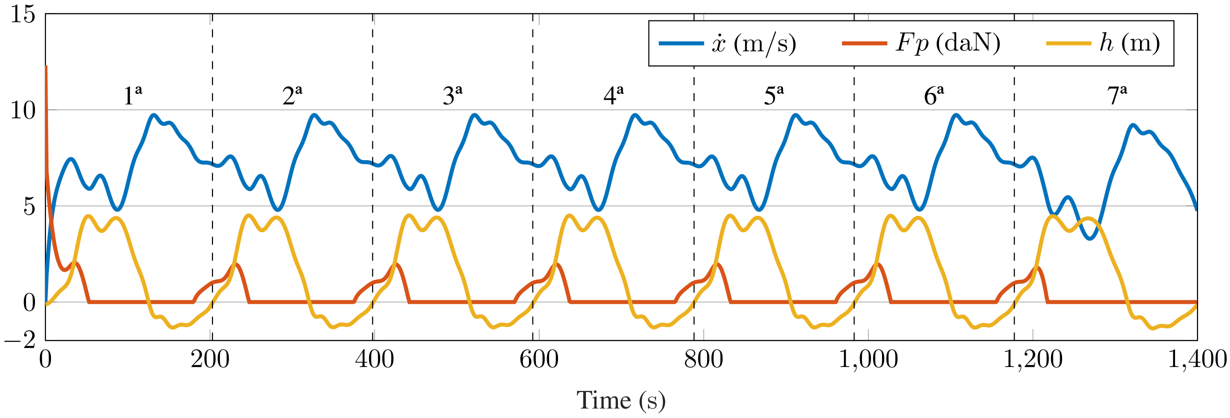

Figure 6. Speed $\dot{x}$, propulsive force F_p and, relative altitude h in optimal case

As Fig. 7 shows, the greatest resistive force that acted on the vehicle was the weight parallel component to the track F_g, ranging from $-18.45\,\mathrm{N}$ to $25.84\,\mathrm{N}$. Although F_g is conservative and performs zero work in a closed circuit. It strongly influences, given its magnitude compared to the other resistive forces, the prototype's accelerations, and, consequently, the motor's working region. The F_a aerodynamic drag reached a maximum of $2.46\,\mathrm{N}$, and the F_r rolling resistance had a constant value of $2.02\,\mathrm{N}$.

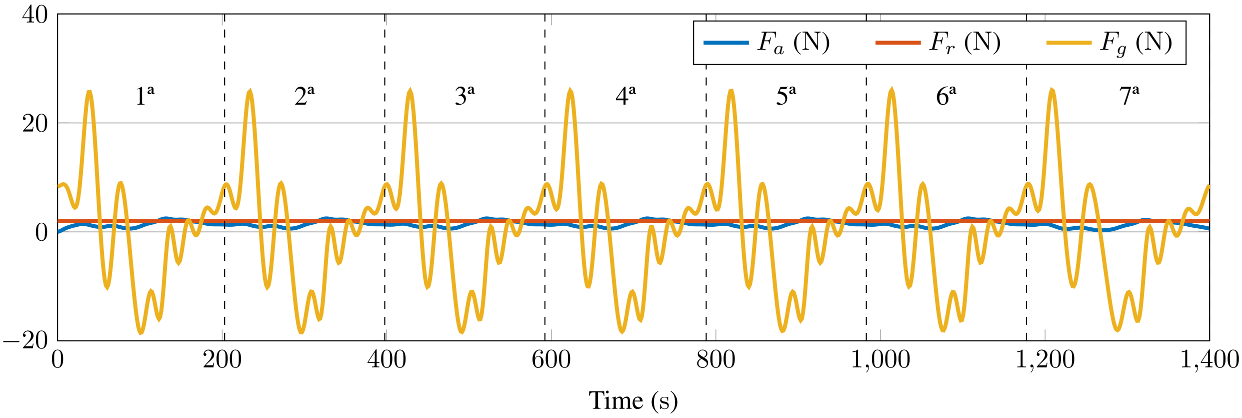

Figure 7. Aerodynamic drag F_a, rolling resistance F_r and, weight component parallel to speed F_g in optimal case

4. CONCLUSION

The current study's main goal was to determine the optimal driving strategy and transmission ratio for an electric battery vehicle in an energy efficiency competition. This problem was addressed using optimal control theory. Accordingly, a computational solution was proposed. The optimal driving strategy and the gear ratio would bring the prototype's autonomy to $905.7\,\mathrm{km/kWh}$. This autonomy is at least three times greater than the autonomy marks achieved.

The vehicle's model used has limitations due to the simplifications performed. The curve speed limit is one of these limitations because the lateral dynamics have not been modeled. The studied vehicle has good stability in curves when compared to other vehicles of energy efficiency. However, it is necessary to verify that it is possible to perform the curves with speed calculated in the optimal solution.

Another important point regarding the vehicle model is that it has not been validated experimentally. Therefore, it cannot state that the optimal response found is optimal for the real case. Despite its limitations, the study certainly contributes to a qualitative compression of the efficient driving strategy for low-speed battery electric vehicles. This work's natural progression is to validate the vehicle's model experimentally and implement the optimal case during the competition.

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6. REFERENCES

Briguiet, G., Caldas, J., Lima, L. and Santiago, V., 2020. "Eficiência energética de um veículo elétrico". *Revista Militar de Ciência e Tecnologia*, Vol. 37.

DEV, 2018. "Sonoma raceway". Duke Electric Vehicles. 22 Nov. 2020 <http://www.duke-ev.org/blog/2018/11/23/sonoma-raceway>.

Gechev, T. and Punov, P., 2020. "Driving strategy for minimal energy consumption of an ultra-energy-efficient vehicle in Shell Eco-marathon competition". *IOP Conference Series: Materials Science and Engineering*, Vol. 1002, No. 1, p. 012018. doi:10.1088/1757-899x/1002/1/012018.

Grady, P., Chen, G., Verma, S., Marellapudi, A. and Hotz, N., 2019. "A Study of Energy Losses in the World's Most Fuel Efficient Vehicle". *2019 IEEE Vehicle Power and Propulsion Conference (VPPC)*, Vol. 00, pp. 1–6. doi:10.1109/vppc46532.2019.8952212.

Guzzella, L., 2007. "Modeling and Control of Advanced Propulsion Systems". *Oil & Gas Science and Technology - Revue de l'IFP*, Vol. 62, No. 4, pp. 585–594. doi:10.2516/ogst:2007040.

IEA, 2018. "Data and statistics". International Energy Agency. 22 Nov. 2020 <https://www.iea.org/data-and-statistics>.

IEA, 2019. "Multiple benefits of energy efficiency". International Energy Agency. 22 Nov. 2020 <https://www.iea.org/reports/multiple-benefits-of-energy-efficiency>.

Liu, T., Feng, L., Hellgren, M. and Wikander, J., 2018. "Increasing Fuel Efficiency of a Hybrid Electric Competition Car by a Binary Equivalent Consumption Minimization Strategy". *2018 IEEE 14th International Conference on Automation Science and Engineering (CASE)*, Vol. 00, pp. 1–7. doi:10.1109/coase.2018.8560378.

Rieck, M., Bittner, M., Grüter, B., Diepolder, J. and Piprek, P., 2020. "Falcon.m user guide". Technische Universität München. 22 Nov. 2020 <https://www.fsd.lrg.tum.de/software/wp-content/uploads/UserGuideMain.pdf>.

Sawulski, J. and Lawrynczuk, M., 2019. "Real-Time Optimisation of Control Strategy for a Low Fuel Consumption Vehicle Engine Using STM32 Microcontroller: Preliminary Results". *2019 18th European Control Conference (ECC)*, pp. 4246–4251. doi:10.23919/ecc.2019.8796299.

Shell, 2020? "Shell eco-marathon". Shell. 22 Nov. 2020 <https://www.makethefuture.shell/en-gb/shell-eco-marathon>.

Spanoudakis, P., Moschopoulos, G., Stefanoulis, T., Sarantinoudis, N., Papadokokolakis, E., Ioannou, I., Piperidis, S., Doitsidis, L. and Tsourveloudis, N.C., 2020. "Efficient Gear Ratio Selection of a Single-Speed Drivetrain for Improved Electric Vehicle Energy Consumption". *Sustainability*, Vol. 12, No. 21, p. 9254. doi:10.3390/su12219254.

Targosz, M., Skarka, W. and Przystałka, P., 2018. "Model-Based Optimization of Velocity Strategy for Lightweight Electric Racing Cars". *Journal of Advanced Transportation*, Vol. 2018, pp. 1–20. doi:10.1155/2018/3614025.

7. RESPONSIBILITY FOR INFORMATION

The authors are solely responsible for the information included in this study.