$P_1(0,0,1) \quad P_2(0,1,0) \quad P_3(1,0,0)$

$\vec{A} = \overrightarrow{P_1P_2} = \langle 0, 1, -1 \rangle$

$\vec{B} = \overrightarrow{P_1P_3} = \langle 1, 0, -1 \rangle$

$\vec{C} = \overrightarrow{P_2P_3} = \langle 1, -1, 0 \rangle$

$$\vec{N} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = -\hat{\imath} - \hat{\jmath} - \hat{k}$$

$$\overrightarrow{P_1P} \cdot \vec{N} = 0 \Rightarrow (\vec{P} - \vec{P_1}) \cdot \vec{N} = 0 \Rightarrow \vec{N} \cdot \vec{P} = \vec{N} \cdot \vec{P_1}$$

$$N_x x + N_y y + N_z z = N_x P_{1x} + N_y P_{1y} + N_z P_{1z}$$

$$\Pi: \ -x - y - z = -1$$

$U_1(-1,1,0) \quad U_2(0,0,0) \quad U_3(1,2,0.5)$

$\vec{D} = \overrightarrow{U_1U_2} = \langle 1, -1, 0 \rangle, \quad D: \ \vec{P} = \vec{U_1} + (\vec{U_2} - \vec{U_1})t = \vec{U_1} + \vec{D}t$

$\vec{E} = \overrightarrow{U_1U_3} = \langle 2, 1, 0.5 \rangle \quad E: \ \vec{P} = \vec{U_1} + \vec{E}t$

$\vec{F} = \overrightarrow{U_2U_3} = \langle 1, 2, 0.5 \rangle \quad F: \ \vec{P} = \vec{U_2} + \vec{F}t$

$\vec{D}$ w/ Π: expect no plane intersection, $t = \mathbb{R} \neq 0 / 0$

plug in line equation: $P_x = x = U_{1x} + (U_{2x} - U_{1x})t$

$$N_x(U_{1x} + D_x t) + N_y(U_{1y} + D_y t) + N_z(U_{1z} + D_z t) = N_x P_{1x} + N_y P_{1y} + N_z P_{1z}$$

solve for t

$$t = \frac{N_x(P_{1x} - U_{1x}) + N_y(P_{1y} - U_{1y}) + N_z(P_{1z} - U_{1z})}{N_x D_x + N_y D_y + N_z D_z} = \frac{\vec{N} \cdot (\overrightarrow{U_1P_1})}{\vec{N} \cdot \vec{D}}$$

calculate

$$t = \frac{-(0-1) - (0+1) - (1-0)}{-(1) - (-1) - (0)} = \frac{-1}{0} = \frac{\mathbb{R} \neq 0}{0} \Rightarrow \text{no planar intersection}$$

$\vec{N} \cdot \vec{D} = 0 \Rightarrow \vec{N} \perp \vec{D}$

$\vec{N} \cdot \overrightarrow{U_1P_1} \neq 0 \Rightarrow \vec{N} \not\perp \overrightarrow{U_1P_1}$

$\vec{E}$ w/ Π: expect planar intersection, in range ($t \in [0,1]$), out of bounds

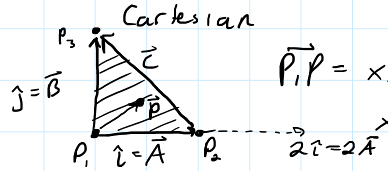
$$t = \vec{N} \cdot (\overrightarrow{U_1P_1}) / \vec{N} \cdot \vec{E} = \frac{-1}{-2-1-0.5} = \frac{1}{3.5}$$

$t \approx 0.28 \in [0,1] \Rightarrow$ intersects w/ plane btwn U_1 & U_3

line $\vec{E}$: $\vec{P}(0) = \vec{U_1}, \quad \vec{P}(1) = \vec{U_1} + \vec{E} = \vec{U_1} + \vec{U_2} - \vec{U_1} = \vec{U_2}$

determine if in $\Delta P_1P_2P_3$

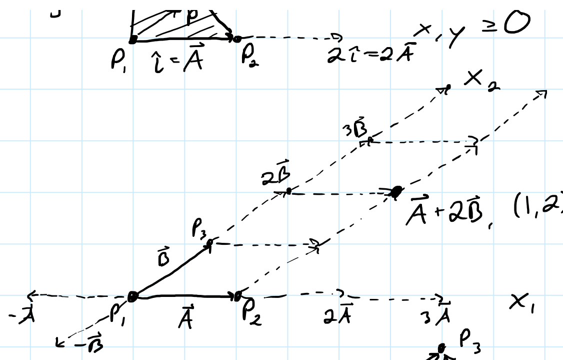
Cartesian



$\overrightarrow{P_1P} = x\vec{A} + y\vec{B} = x\hat{\imath} + y\hat{\jmath} = (x,y)$

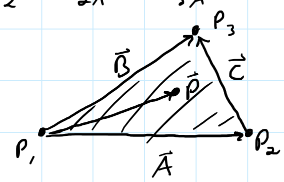
$x, y \geq 0 \qquad x + y \leq 1 \qquad y \leq -x + 1$

x_2



$x, y \geq 0 \qquad x+y \leq 1 \qquad y \leq -x+1$

$\vec{P} = x_1\vec{A} + x_2\vec{B}$



$\overrightarrow{P_1P} = x_1\vec{A} + x_2\vec{B}$

$x_1, x_2 \geq 0$

$x_1 + x_2 \leq 1$

Basis, $\mathcal{B} = \{\vec{A}, \vec{B}, \vec{N}\}$ of $\mathbb{R}^3$

put in coordinates of $\mathcal{B}$ using change-of-base matrix $M = \begin{bmatrix} | & | & | \\ \vec{A} & \vec{B} & \vec{N} \\ | & | & | \end{bmatrix}$

$$[\vec{P}]_{\mathcal{B}} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = M^{-1}\vec{P} \Rightarrow \vec{P} = x_1\vec{A} + x_2\vec{B} + x_3\vec{N}$$

we know that $x_3 = 0$ b/c $\vec{P}$ in plane

ie normal component is 0

$$\vec{E}(t) - \vec{P}_1 = \vec{P} = x_1\vec{A} + x_2\vec{B} \Rightarrow \text{3 eqns, 2 unknowns}$$

$$\Rightarrow \left[\begin{array}{cc|c} A_x & B_x & E(t)_x - P_{1x} \\ A_y & B_y & E(t)_y - P_{1y} \\ A_z & B_z & E(t)_z - P_{1z} \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cc|c} 1 & 0 & x_1 \\ 0 & 1 & x_2 \\ 0 & 0 & 0 \end{array}\right]$$ check $x_1, x_2 \geq 0$, $x_1 + x_2 \leq 1$

$\vec{E}(t=0.28) \approx (-.44, 1.28, .14) = \vec{U}_1 + \vec{E}t$

$\vec{E}(t=0.28) - \vec{P}_1 = (-.44, 1.28, -.86)$

$$\left[\begin{array}{cc|c} | & | & | \\ \vec{A} & \vec{B} & \overrightarrow{P_1E(t)} \\ | & | & | \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 0 & 1 & \sim -.44 \\ 1 & 0 & \sim 1.28 \\ -1 & -1 & \sim -.86 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cc|c} 1 & 0 & 1.2857 \\ 0 & 1 & -.4286 \\ 0 & 0 & 0 \end{array}\right]$$

$x_1 = 1.2857 \geq 0$ ✓

$x_2 = -.4286 < 0$ ✗ no collision

$\vec{F}$ w Π: expect planar intersection, in range, in bounds $t \in [0,1]$

$t = \vec{N}\cdot\overrightarrow{U_2P_1} / \vec{N}\cdot\vec{F} = -1/-3.5 \approx .28 \in [0,1] \Rightarrow \vec{F}$ intersects w/ Π btwn U_2 & U_3

$\vec{F}(t = 1/3.5) \approx (.28, .56, .14)$

$\vec{F}(t = 1/3.5) \approx (.28, .56, -.86)$

$$\left[\begin{array}{cc|c} \vec{A} & \vec{B} & \overrightarrow{P_1F(t)} \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 0 & 1 & \sim .28 \\ 1 & 0 & \sim .56 \\ -1 & -1 & \sim -.86 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cc|c} 1 & 0 & .5715 \\ 0 & 1 & .2857 \\ 0 & 0 & 0 \end{array}\right]$$

$x_1 = .5715 \geq 0$ ✓

$x_2 = .2857 \geq 0$ ✓

$x_1 + x_2 = .8572 \leq 1$ ✓

$x_1\vec{A} + x_2\vec{B} = \overrightarrow{P_1F(t)}$

$x_1A_x + x_2B_x = F(t)_x - P_{1x}$

$x_1A_y + x_2B_y = F(t)_y - P_{1y}$

$x_1A_z + x_2B_z = F(t)_z - P_{1z}$

on Π

$\vec{G} = \overrightarrow{V_1V_2}$: expect complete planar intersection $\quad t = \frac{0}{0}$

$$t = \vec{N} \cdot \overrightarrow{V_1P_1} / \vec{N} \cdot \vec{G} = 0/0$$

intersect $\vec{G}$ w/ each bounding line (A, B, C)

$\vec{G}$: $\quad x = V_{1x} + (V_{2x} - V_{1x})t \qquad A: \quad x = P_{1x} + (P_{2x} - P_{1x})s$

$\vdots \qquad \vdots$

$$G_x = A_x, \quad G_y = A_y, \quad G_z = A_z$$

$V_{1x} + (V_{2x} - V_{1x})t = P_{1x} + (P_{2x} - P_{1x})s \Rightarrow A_x s - G_x t = V_{1x} - P_{1x}$

$\vdots \qquad \vdots$

3 eqns, 2 unknowns

$$\left[\begin{array}{cc|c} A_x & -G_x & (\overrightarrow{P_1V_1})_x \\ A_y & -G_y & (\overrightarrow{P_1V_1})_y \\ A_z & -G_z & (\overrightarrow{P_1V_1})_z \end{array}\right] \xRightarrow{RREF} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

s & $t = 0$

$A_z(0) - G_z(0) = 1$

$0 - 0 \neq 1 \Rightarrow$ inconsistent $\Rightarrow$ no intersection

$$\left[\begin{array}{cc|c} 1 & 0 & s \\ 0 & 1 & t \\ 0 & 0 & 0 \end{array}\right]$$

$0 \leq t, s \leq 1$

$\Rightarrow$ for intersection to be within the points