$$g(z) = \frac{1}{1+e^{-z}}$$

1) $\frac{d}{dz} g(z) = g(z)(1-g(z))$

$\frac{d}{dz} g(z) = \frac{\frac{d}{dz}(e^{-x}+1)}{(e^{-x}+1)^2}$

$= \frac{e^{-x} \cdot \frac{d}{dz}(-z)}{(e^{-x}+1)^2}$

$= \frac{(-\frac{d}{dz}(z))e^{-z}}{(e^{-z}+1)^2}$

$= \frac{e^{-z}}{(e^{-z}+1)^2}$

$\frac{1}{1+e^{-z}}\left(1-\frac{1}{1+e^{-z}}\right)$

$\frac{1}{1+e^{-z}} - \frac{1}{1+e^{-2z}}$

$\frac{1 \cdot e^{-z}}{(1+e^{-z})^2} - \frac{1}{(1+e^{-z})^2}$

$\frac{e^{-z}}{(1+e^{-z})^2}$ ←

2) $1-g(z) = g(-z)$

$1+e^{-z} = e^0 \cdot e^{-z} = e^0 + \frac{1}{e^z}$

$1 - \frac{1}{1+e^{-z}} : 1 - \frac{1}{e^0 + \frac{1}{e^z}}$

$: 1 - \frac{e^z}{e^z+1}$

$= \frac{1(e^z+1) - e^z}{e^z+1}$

$= \frac{e^z + 1 - e^z}{e^z+1}$

$= \frac{1}{e^z+1} = g(-z)$

3) $J(\theta) = -y\log(h_\theta(x)) - (1-y)\log(1-h_\theta(x))$

$\frac{d}{d\theta_j}[-y\log(h_\theta(x))] = -y[\log(h_\theta(x))]$

$= -y\left(\frac{1}{h_\theta(x)}\right)\frac{d}{d\theta_j}[h_\theta(x)]$

$\frac{d}{d\theta_j}[-1(1-y)\log(1-h_\theta(x))] = (1-y)\frac{1}{1-h_\theta(x)} \cdot \frac{d(1-h_\theta(x))}{d\theta_j} = \frac{(1-y)x_j e^{\theta^Tx}}{(1-h_\theta(x))(1+e^{-\theta^Tx})}$

$\frac{d}{d\theta_j} - y\log(h_\theta(x)) = \frac{-y x_j e^{-\theta^Tx}}{h_\theta(x)(1+e^{\theta^Tx})}$

$\frac{-y x_j e^{-\theta^Tx}}{h_\theta(x)(1+e^{-\theta^Tx})} + \frac{(1-y)x_j e^{-\theta^Tx}}{(1-h_\theta(x))(1+e^{-\theta^Tx})} = \frac{h_\theta(x) - y}{h_\theta(x)(1-h_\theta(x))} \cdot x_j = (h_\theta(x)-y)x_j$

4) A couple ways to improve the algorithm's accuracy include regularization, feature engineering, and error analysis. Regularization could be used to limit the impact of irrelevant features, while placing a more appropriate weight on features that truly matter. This could mean a message like "congratulations!" Which is likely to be considered as spam, could be more appropriately weighted as ham. Feature engineering is another way to tackle this issue, by helping to provide more semantic meaning than simply the text. These new features could help differentiate between if a congratulations is just a wholesome message from a friend, or a spam message and phishing attempt. Error analysis is another great way to improve the algorithm, identifying common mistakes that can inspire future engineered features. Feedback mechanisms could be used to help the model cont