Homework 1

Exercises

Partial Derivatives

1. $\frac{\partial}{\partial u} f(u,v) = \frac{\partial}{\partial u} 10u^2v^3 + 3v^2 + 4u$
   $= 20uv^3 + 4$
2. $\frac{\partial}{\partial v} f(u,v) = \frac{\partial}{\partial v} 10u^2v^3 + 3v^2 + 4u$
   $= 30u^2v^2 + 6v$
3. $\frac{\partial}{\partial u} g(u,v,w) = x\log(u) + yuvw^2 + 10x^2$
   $= \frac{x}{u} + yvw^2$
4. $\frac{\partial}{\partial v} g(u,v,w) = x\log(u) + yuvw^2 + 10x^2$
   $= yuw^2$
5. $\frac{\partial}{\partial w} g(u,v,w) = x\log(w) + yuvw^2 + 10x^2$
   $= 2yuvw$
6. $\frac{\partial}{\partial u} h(u,v) = \sum_{i=1}^{m} \frac{1}{2}(x^{(i)}u + y^{(i)}v)^2$
   $= mx(ux + vy)$
7. $\frac{\partial}{\partial v} h(u,v) = \sum_{i=1}^{m} \frac{1}{2}(x^{(i)}u + y^{(i)}v)^2$
   $= my(ux + vy)$

Partial Derivative Intuition

1) $\frac{\partial}{\partial u} f(-2,-2)$: Positive; moving in the positive x direction increases the level curve.
2) $\frac{\partial}{\partial v} f(-2,-2)$: Positive; moving in the positive y direction increases the level curve
3) $\frac{\partial}{\partial u} f(3,-3)$: Positive; moving in the positive x direction increases the level curve.
4) $\frac{\partial}{\partial v} f(3,-3)$: Negative; moving in the positive y direction decreases the level curve.
5) $(1,2)$

Problems

1) $J(\theta_1) = \frac{1}{2}(\theta_1 - 3)^2 + \frac{1}{2}(-\theta_1 - 2)^2 + \frac{1}{2}(2\theta_1 - 4)^2$
$\frac{\partial}{\partial \theta_1} J(\theta) = \theta_1 - 3 - \theta_1 + 2 + 2\theta_1 - 4$
$0 = 2\theta_1 - 5$
$\theta_1 = 2.5$

2) $J(\theta) = \frac{1}{2}(\theta_1 x^1 - y^1)^2 + \frac{1}{2}(\theta_1 x^2 - y^2)^2 + \frac{1}{2}(\theta_1 x^3 - y^3)^2$
$\frac{\partial}{\partial \theta_1} J(\theta) = \theta_1 x^1 - y^1 + \theta_1 x^2 - y^2 + \theta_1 x^3 - y^3$
$0 = \theta_1(x^1 + x^2 + x^3) - (y^1 + y^2 + y^3)$

$\frac{(y^1 + y^2 + y^3)}{(x^1 + x^2 + x^3)} = \theta_1$

$\frac{3 - 2 + 4}{1 - 1 + 2} = \frac{5}{2} = 2.5$