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CFRM 420
Homework 8

1.

(a)

For the ARMA process given by (1), $X_t+0.5X_{t-1} = \varepsilon_t+\varepsilon_{t-1}-\varepsilon_{t-2}$, or $\phi(z) = 1+0.5z$ we implement in this R. We have $\phi_1 = -0.5$, and solving gives us the root $z = 2$, which is not a zero within the unit circle on the complex plane, so there exists a causal stationary solution to this ARMA process.

(b)

For the ARMA process given by (2), $X_t - \frac{3}{2}X_{t-1} + \frac{1}{2}X_{t-2} = \varepsilon_t + \varepsilon_{t-1} - \varepsilon_{t-3}$, we implement in this R. We have that $\phi_1 = \frac{3}{2}, \phi_2 = -\frac{1}{2}$; solving this gives us the roots $z = 0.5615528, 3.5615528$ in the complex plane, so one of the zeros is within the unit circle on the complex plane, so there does not exist a causal stationary solution to this ARMA process.

For the ARMA process given by (3), $X_t - 5X_{t-1} + 8X_{t-2} - 2X_{t-5} = 2\varepsilon_t$, we implement in this R. We have that $\phi_1 = 5, \phi_2 = -8, \phi_5 = 2$; solving this gives us the zeros $z = 0.1593374, 1.7094081, 1.0000000, 1.0738921$, and 1.7094081 in the complex plane, which does not a causal stationary solution since there exists a zero within the unit circle on the complex plane.

2.

(a)

We assume values of h above $\max\{p, q\}$. We take the covariance of X_{t-h} and the right-hand side of (1) to get a covariance of 0. We then take the covariance of X_{t-h} and the left-hand side of (1) to get $\gamma(h) + 0.5\gamma(h-1)$. We equate these two covariances to get

$\gamma(h) + 0.5\gamma(h-1) = 0.$

which is exactly what we sought to find.

(b)

We continue for the value of $h = 0$. We take the covariance of X_{t-h} and the right-hand side of (1) in this case to get

$\mathrm{Cov}(X_t, \varepsilon_t + \varepsilon_{t-1} - \varepsilon_{t-2})$

$= \mathrm{Cov}(\varepsilon_t + \varepsilon_{t-1} - \varepsilon_{t-2} - 0.5X_{t-1}, \varepsilon_t + \varepsilon_{t-1} - \varepsilon_{t-2})$

$= -0.5\mathrm{Cov}(\varepsilon_{t-1} + \varepsilon_{t-2} - \varepsilon_{t-3} - 0.5X_{t-2}, \varepsilon_t + \varepsilon_{t-1} - \varepsilon_{t-2}) + \sigma^2 + \sigma^2 + \sigma^2$

$= 0.25\mathrm{Cov}(\varepsilon_{t-2} + \varepsilon_{t-3} - \varepsilon_{t-4} - 0.5X_{t-3}, \varepsilon_t + \varepsilon_{t-1} - \varepsilon_{t-2}) + 3 + \sigma^2 - \sigma^2$

$= -0.25\sigma^2 + 3$

$= -0.25 + 3$

$= 2.75.$

We then take the covariance of X_{t-h} and the left-hand side of (1) to get $\gamma(h) + 0.5\gamma(h-1)$ yet again. We equate these expressions to get

$\gamma(h) + 0.5\gamma(h-1) = 2.75.$

which is exactly what we sought to find.

(c)

We continue for the value of $h = 1$. We take the covariance of X_{t-h} and the right-hand side of (1) in this case to get

$\mathrm{Cov}(X_{t-1}, \varepsilon_t + \varepsilon_{t-1} - \varepsilon_{t-2})$

$= \mathrm{Cov}(X_{t-1}, \varepsilon_{t-1})$

$= \text{Cov}(\varepsilon_{t-1} + \varepsilon_{t-2} - \varepsilon_{t-3} - 0.5X_{t-2}, \varepsilon_{t-1})$

$= \text{Cov}(\varepsilon_{t-1} + \varepsilon_{t-2} - \varepsilon_{t-3} - 0.5X_{t-2}, \varepsilon_t + \varepsilon_{t-1} - \varepsilon_{t-2})$

$= 0.5\text{Cov}(X_{t-2}, \varepsilon_{t-2}) + \sigma^2 - \sigma^2$

$= 0.5\text{Cov}(\varepsilon_{t-2} + \varepsilon_{t-3} - \varepsilon_{t-4} - 0.5X_{t-3}, \varepsilon_{t-2})$

$= 0.5\sigma^2$

$= 0.5.$

We then take the covariance of X_{t-h} and the left-hand side of (1) to get $\gamma(h) + 0.5\gamma(h-1)$ yet again. We equate these expressions to get

$\gamma(h) + 0.5\gamma(h-1) = 0.5.$

which is exactly what we sought to find.

We continue for the value of $h = 2$. We take the covariance of X_{t-h} and the right-hand side of (1) in this case to get

$\text{Cov}(X_{t-2}, \varepsilon_t + \varepsilon_{t-1} - \varepsilon_{t-2})$

$= \text{Cov}(X_{t-2}, -\varepsilon_{t-2})$

$= -\text{Cov}(\varepsilon_{t-2} + \varepsilon_{t-3} - \varepsilon_{t-4} - 0.5X_{t-3}, \varepsilon_{t-2})$

$= -\sigma^2$

$= -1.$

We then take the covariance of X_{t-h} and the left-hand side of (1) to get $\gamma(h) + 0.5\gamma(h-1)$ yet again. We equate these expressions to get

$\gamma(h) + 0.5\gamma(h-1) = -1.$

which is exactly what we sought to find.

We put all of these Yule-Walker equations together into a system of linear equations in R and solve. We have $\mathbf{c} = [2.75, 0.5, -1]$ and $A = \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0 \\ 0 & 0.5 & 1 \end{bmatrix}$

Solving in R, we can verify that our result γ is indeed $[3.3333333, -1.1666667, -0.4166667]$, which corresponds accordingly to $\gamma(0), \gamma(1), \gamma(2)$.

(d)

We implement this in R, using the fact that $\rho(h) = \gamma(h)/\gamma(0)$ and substituting as necessary using equations 4-6 and the equation in part (a):

$\rho(0) = 1$

$\rho(1) = -0.35$

$\rho(2) = -0.125$

$\rho(3) = 0.0625$

$\rho(4) = -0.03125$

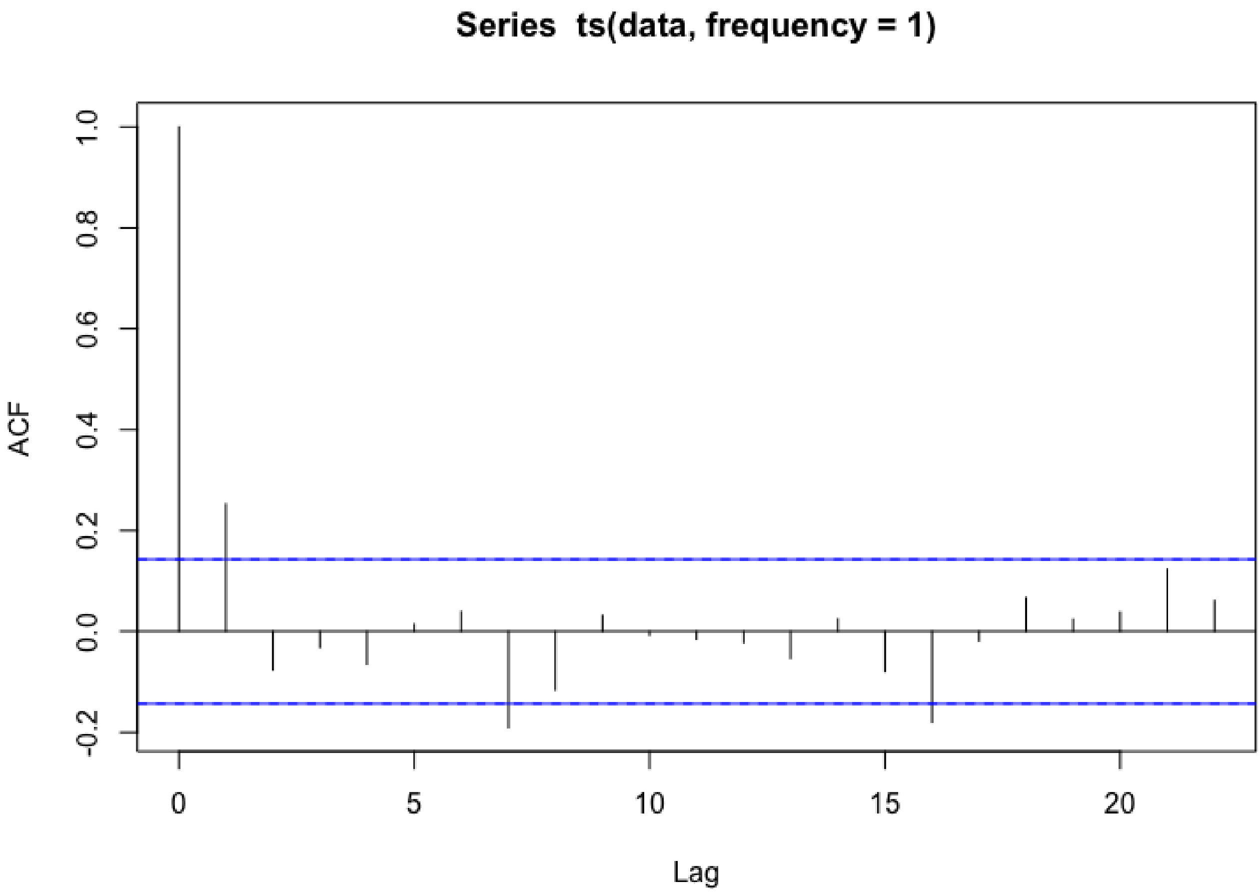
$\rho(5) = 0.015625$

Using the ARMAacf() function in R, we also verify these results.

3.

(a)

We implement this in R:



(b)

In regards to the hypothesis test at a 5% significance level for $\rho(1) = 0$, we see that the value at $\rho(1)$ is well above the confidence intervals, so we successfully rejected the null hypothesis and find that $\rho(1) \neq 0$.

(c)

Looking at the graphed sample autocorrelation function at the other lag values, we see that the null hypothesis of zero autocorrelation is rejected at 2 lags. We would expect to see that the null hypothesis of zero autocorrelation would never be rejected (i.e. there are 0 times it will be outside of the confidence intervals) at a 5% significance level if the data were independent white noise, since for independent white noise, $\hat{\rho}(h)$ should lie within the band; in the same vein, it is reasonable to assume the data follows MA(1) process since $\hat{\rho}(h)$ lies within the band for most $|h| > 1$, since for an MA(1) process, $\rho(h) = 0$ for all $|h| > 1$.