Homework 1

625.433

1. (15 points). Verify the following extension of the addition rule (a) by an appropriate Venn diagram and (b) by a formal argument using the axioms of probability.

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

2. (10 points). A player throws darts at a target. On each trial, independently of the other trials, he hits the bull's-eye with probability .05. How many times should he throw it so that his probability of hitting the bull's-eye at least once is .5?

3. (15 points). Two boys play basketball in the following way: They take turns shooting and stop when a basket is made. Player A goes first and has probability p_1 of making a basket on any throw. Player B, who shoots second, has probability p_2 of making a basket. The outcomes of the successive trials are assumed to be independent. Find the probability mass function for the total numer of attempts.

4. (15 points). A line segment of length 1 is cut once at random. What is the probability that the longer piece is more than twice the length of the shorter piece?

5. (15 points). Prove that the expected value of a binomial random variable is np, and that its variance is $np(1-p)$.

6. (15 points). Consider a random experiment where we draw uniformly and indepnedently n numbers, $X_1, X_2, \ldots, X_n$, from the interval $[0, 1]$. Let $X_{(1)}$ be the smallest of the n numbers. Determine the pdf of $X_{(1)}$.

7. (15 points). Let $Y = \exp\{X\}$, where $X \sim N(0, 1)$.

   (a) Determine the pdf of Y.

   (b) Determine the expected value of Y.