Attempted

1 Question 1

$a \in \mathbb{R}, b \in \mathbb{R}$ then by the mapping g , $g(a) \in \mathbb{R}$ and $g(b) \in \mathbb{R}$. Suppose $a, b \in \mathbb{N}$ and $\mathbb{N} \subset \mathbb{R}$, then by the principle of recursive definition there exists a function ϕ such that $\phi(a+b) = \phi(a) + \phi(b)$ and $\phi(ab) = \phi(a)\phi(b)$. If $a = b$, then $g(a)^2 \equiv g(a)g(a) \equiv g(a^2)$ and $g(a^2 + a + g(a)) \equiv g(a^2) + g(a) + g(g(a))$. Then proceeding with contradiction taking from the problem we have

$$\begin{gathered} g(a^2 + b + g(b)) = 2b + g(a)^2 \\ \text{taking } b = a, \forall a, b \in \mathbb{N} \subset \mathbb{R} \\ \text{by the principle of recursive definition} \\ g(a)g(a) + g(a) + g(g(a)) = 2a + g(a)g(a) \\ g(a) + g(g(a)) = 2a \end{gathered}$$

but since $a \in \mathbb{N}$ and $\mathbb{N} \subset \mathbb{R}$ implies $a \in \mathbb{R}$, but $g(a) \in \mathbb{R}$ and the function compositon of $g(g(a)) \in \mathbb{R}$ implies $g(a) + g(g(a)) \neq 2a$
since the scaling a by 2 where 2 , $a \in \mathbb{N}$ does not imply equality despite $\mathbb{N} \subset \mathbb{R}$
this contradiction it makes the function$g : \mathbb{R} \longrightarrow \mathbb{R}$
undefined in the ordered field $\mathbb{R}$ and this completes the proof. ■

2 Question 2

(A)

let $A, B, C \in \mathbb{E}^n$ such that the $\triangle ABC$ is defined considering point A as the origin. Lines AB ,AC and BC are collinear thats is $AB, AC, BC \in \mathbb{R}^n$. Point Q is an interior point of $\triangle ABC$ iff there exists an open ball completely contained in set $A, B, C \in \mathbb{E}^n$ (inscribed circle in $\triangle ABC$) For equality of $\angle QCB$ and $\angle QAC$, $\angle QBC$ and $\angle QAB$ there should exist a line of symmetry that run from any vertix point on the $\triangle ABC$ for a metirc $d \in \mathbb{R}^n$, d being the euclidian distance function on $\mathbb{R}^n$ in $\mathbb{E}^n$ such that under the symmetry $d(AC) = d(AB) = d(BC)$ and the symmetry line is collinear with point Q and the verticies A, B, C of $\triangle ABC$ are othogonal to interior point $Q \in \triangle ABC$ subset of ball centered at Q with A, B, C as its boundary points With that you release that

$$\cos(\angle QCB) = \cos(\angle QAC)$$

$\frac{(Q-C)\cdot(B-C)}{|Q-C||B-C|} = \frac{(Q-A)\cdot(C-A)}{|Q-A||C-A|}$ Because of symmetry $d(QC) = |Q - C| = |Q - A|$ and $d(CA) = |C - A| = |B - C|$ then expressing $\cos\angle QAC$ in form of $\cos\angle QCB$

$$\cos\angle QCB = \frac{(Q-A)\cdot(C-A)}{|Q-A||C-A|}$$

given that $d(QA)$ =$d(QC)$ and $d(BC)$ =$d(CA)$ by symmetry. The same applies to the $\angle QBC$ and $\angle QAB$ since symmetry is an isometric transformation that preserve distances. and this completes the proof $\blacksquare$

3 Question 3