# Attempted

## 1 Question 1

$a \in \mathbb{R}, b \in \mathbb{R}$ then by the mapping $g$ , $g(a) \in \mathbb{R}$ and $g(b) \in \mathbb{R}$. Suppose $a, b \in \mathbb{N}$ and $\mathbb{N} \subset \mathbb{R}$, then by the principle of recursive definition there exists a function $\phi$ such that $\phi(a+b) = \phi(a) + \phi(b)$ and $\phi(ab) = \phi(a)\phi(b)$. If $a = b$ , then $g(a)^2 \equiv g(a)g(a) \equiv g(a^2)$ and $g(a^2 + a + g(a)) \equiv g(a^2) + g(a) + g(g(a))$. Then proceeding with contradiction taking from the problem we have

$$\begin{gathered} g(a^2 + b + g(b)) = 2b + g(a)^2 \\ \text{taking } b = a, \forall a, b \in \mathbb{N} \subset \mathbb{R} \\ \text{by the principle of recursive definition} \\ g(a)g(a) + g(a) + g(g(a)) = 2a + g(a)g(a) \\ g(a) + g(g(a)) = 2a \end{gathered}$$

but since $a \in \mathbb{N}$ and $\mathbb{N} \subset \mathbb{R}$ implies $a \in \mathbb{R}$, but $g(a) \in \mathbb{R}$ and the function compositon of $g(g(a)) \in \mathbb{R}$ implies $g(a) + g(g(a)) \neq 2a$
since the scaling a by 2 where 2 , $a \in \mathbb{N}$ does not imply equality despite $\mathbb{N} \subset \mathbb{R}$
this contradiction it makes the function$g : \mathbb{R} \longrightarrow \mathbb{R}$
undefined in the ordered field $\mathbb{R}$ and this completes the proof. ■

## 2 Question 2

# 3 Question 3