Attempted

1 Question 1

$a \in \mathbb{R}, b \in \mathbb{R}$ then by the mapping g , $g(a) \in \mathbb{R}$ and $g(b) \in \mathbb{R}$. Suppose a, b $\in \mathbb{N} and \mathbb{N} \subset \mathbb{R}$, then by the principle of recursive definition there exists a function ϕ such that $\phi(a+b) = \phi(a) + \phi(b) and \phi(ab) = \phi(a)\phi(b)$. If $a = b$, then $g(a)^2 \equiv g(a)g(a) \equiv g(a^2) and g(a^2 + a + g(a)) \equiv g(a^2) + g(a) + g(g(a))$. Then proceeding with contradiction taking from the problem we have

$$\begin{gathered}
g(a^2 + b + g(b)) = 2b + g(a)^2 \text{ taking } b = a, \forall a, b \in \mathbb{N} \subset \mathbb{R} \\
\text{by the principle of recursive definition} \\
g(a)g(a) + g(a) + g(g(a)) = 2a + g(a)g(a) \\
g(a) + g(g(a)) = 2a
\end{gathered}$$

but since $a \in \mathbb{N}$ and $\mathbb{N} \subset \mathbb{R}$ implies $a \in \mathbb{R}$,
but $g(a) \in \mathbb{R}$ and the function compositon of $g(g(a)) \in \mathbb{R} implies g(a) + g(g(a)) \neq 2a$
since the scaling a by 2 where 2 , $a \in \mathbb{N}$ does not imply equality despite $\mathbb{N} \subset \mathbb{R}$
this contradiction it makes the function$g : \mathbb{R} \longrightarrow \mathbb{R}$
undefined in the ordered field $\mathbb{R}$ completes the proof. ■

2 Question 2

3 Question 3