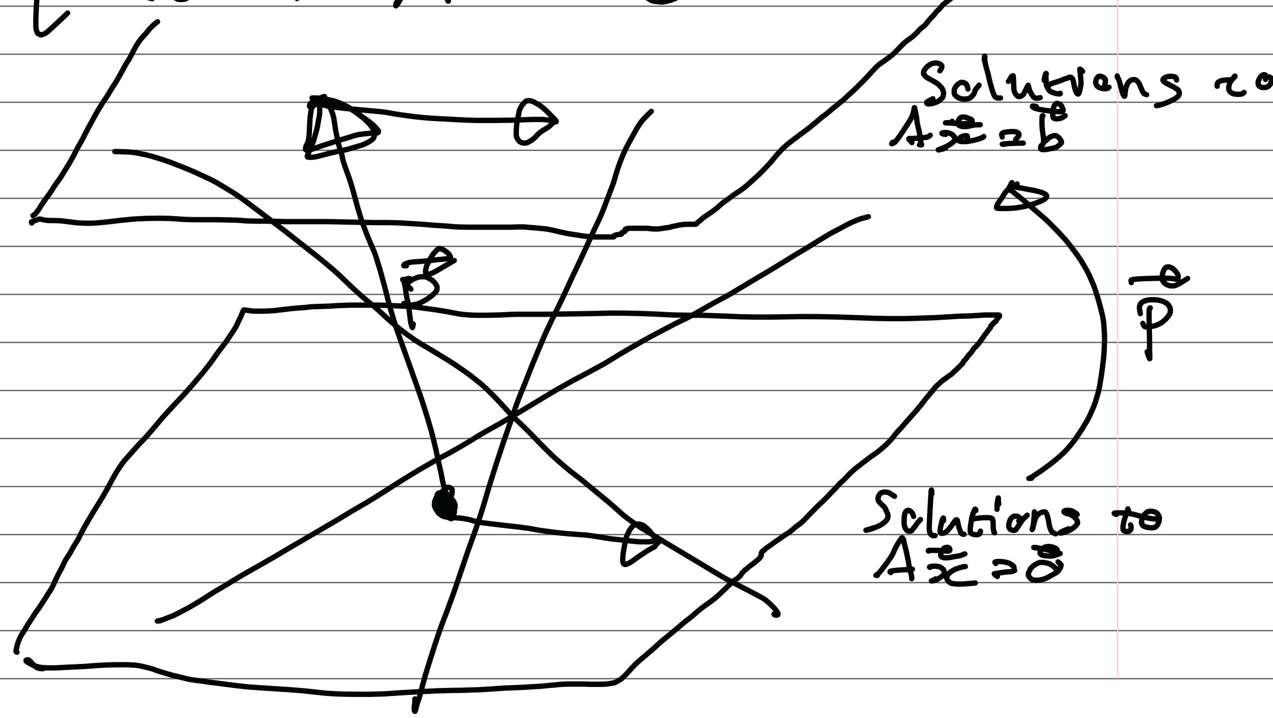
Lecture 11: Non-Homogenous Linear System

Theorom

Suppose the equation $A\vec{x} = \vec{b}$ is consistent for some given $\vec{b}$, and let $\vec{p}$ be a solution. Then the solution set of $A\vec{x} = \vec{b}$ is the set of all vectors of the form $\vec{w} = \vec{p} + \vec{v}_h$, where $\vec{v}_h$ is a solution of the homogenous equation $A\vec{x} = \vec{0}$



Proving the Theorom

$A\vec{x}=\vec{b}$ $\qquad$ $\vec{p}+\vec{v}_h$

any vector that is a solution to $A\vec{x}=\vec{0}$

a solution to $A\vec{x}=\vec{b}$

We have to prove two things

1. Any vector of the form $\vec{p}+\vec{v}_h$ is a solution to $A\vec{x}+\vec{b}$

2. Every solution to $A\vec{x}=\vec{b}$ has the form $\vec{p}+\vec{v}_h$

to do the ~~wh~~ we prove that each set contains the other

Proof of (1)

1 Any vector of the form $\vec{p} + \vec{v}_n$ is a solution to $A\vec{x} = \vec{b}$

plug $\vec{p} + \vec{v}_n$ in

Compute

$$A(\vec{p} + \vec{v}_n) = A\vec{p} + A\vec{v}_n = \vec{b} + \vec{0} = \vec{b}$$

$\vec{p}$ is a particular solution to $A\vec{x} = \vec{b}$

plug $\vec{p}$ into $\vec{x}$
results in $\vec{b}$

$\vec{v}_n$ any solution to $A\vec{x} = \vec{0}$

if i plug $\vec{v}_n$ into it
results in $\vec{0}$

Proof of (2)

2. Every solution of $A\vec{x}=\vec{b}$ has the form $\vec{p}+\vec{v}_h$

Let $\vec{w}$ be a solution to $A\vec{x}=\vec{b}$. Now $\vec{w}-\vec{p}$ is a solution to the homogenous equation $A\vec{x}=\vec{0}$ since

$$A(\vec{w}-\vec{p})=A(\vec{w})-A(\vec{p})=\vec{b}-\vec{b}=\vec{0}$$

$\vec{w}$ has the form

$\vec{p}+(\vec{w}-\vec{p})$ and $\vec{w}-\vec{p}$ is a solution to the homogenous equation $A\vec{x}=\vec{0}$

Notes

$A\tilde{x}=\tilde{0}$ always has atleat one solution

$A\tilde{x}=\tilde{b}$ might not be consistent

if $A\tilde{x}=\tilde{b}$ has any solutions then the theorom applies

important