Lecture 10: Homogenous Systems

A system of linear equations is homogenous if it can be written in the form

$$A\vec{x} = 0$$

Example

$$x_1 + x_2 - 6x_3 + x_4 = 0$$

$$x_1 \quad - 9x_3 + x_4 = 0$$

$$2x_2 + 6x_3 + x_4 = 0$$

$A\vec{x} = 0$ where

$$A = \begin{bmatrix} 1 & 1 & -6 & 1 \\ 1 & 0 & -9 & 1 \\ 0 & 2 & 6 & 1 \end{bmatrix}$$

$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ is a solution for a homogenous system

thus always has atleast one solution

Lets discover more features of a homogenous syst!

form aug matrix

$$\begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & b \end{matrix} \\ \begin{bmatrix} 1 & 1 & -6 & 1 & 0 \\ 1 & 0 & -9 & 1 & 0 \\ 0 & 2 & 6 & 1 & 0 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 1 & 0 & -9 & 0 & 0 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$x_1 - 9x_3 = 0$

$x_2 + 3x_3 = 0$

x_3 is free

$x_4 = 0$

we want to know which vectors are solutions to this system.

rewrite in vector form

$$\begin{aligned} x_1 &= 9x_3 \\ x_2 &= -3x_3 \\ x_3 &= x_3 \\ x_4 &= 0 \end{aligned} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \underbrace{\begin{bmatrix} 9 \\ -3 \\ 1 \\ 0 \end{bmatrix}}$$

solution set is Span$\left\{ \begin{bmatrix} 9 \\ -3 \\ 1 \\ 0 \end{bmatrix} \right\}$

we will always be able to write the solution set of a homogenous system as the span of some number of vectors

there will be a vector foreach free variable

if there are no free vars only solution is $\text{Span}\{\vec{0}\}$

Equations of Planes

plane: $x - 3y + 4z = 0$

aug matrix: $[1 \;\; -3 \;\; 4 \;\; 0]$ → already in reduced row ech

$x - 3y + 4z = 0$

y is free

z is free

$$\begin{aligned} x &= 3y - 4z \\ y &= y \\ z &= z \end{aligned} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = y\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ solution set $= \text{Span}\left\{\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}\begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}\right\}$