Lecture 06: Column Vectors

aka matrix with only one column

~~R~~ $\mathbb{R}^2$ is the set of all vectors with two real entries

ordered pairs

$\mathbf{u} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ or $(3, -4)$

$v = \begin{bmatrix} 0 \\ \pi \end{bmatrix}$ or $(0, \pi)$

bold face lower case

or

$\vec{u}$ or $\vec{v}$ *writing

*typical notation for vecs

Vector operations

Add

$$\begin{bmatrix} -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} -1+5 \\ 2+0 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

Multiply

$$\underbrace{3}_{\text{scalar}} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \times -1 \\ 3 \times 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \end{bmatrix}$$

components of the vector

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$+$$

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \end{bmatrix}$$

$$c\vec{u} = c \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} c \cdot u_1 \\ c \cdot u_2 \end{bmatrix}$$

operations are "componentwise"

The Zero Vector

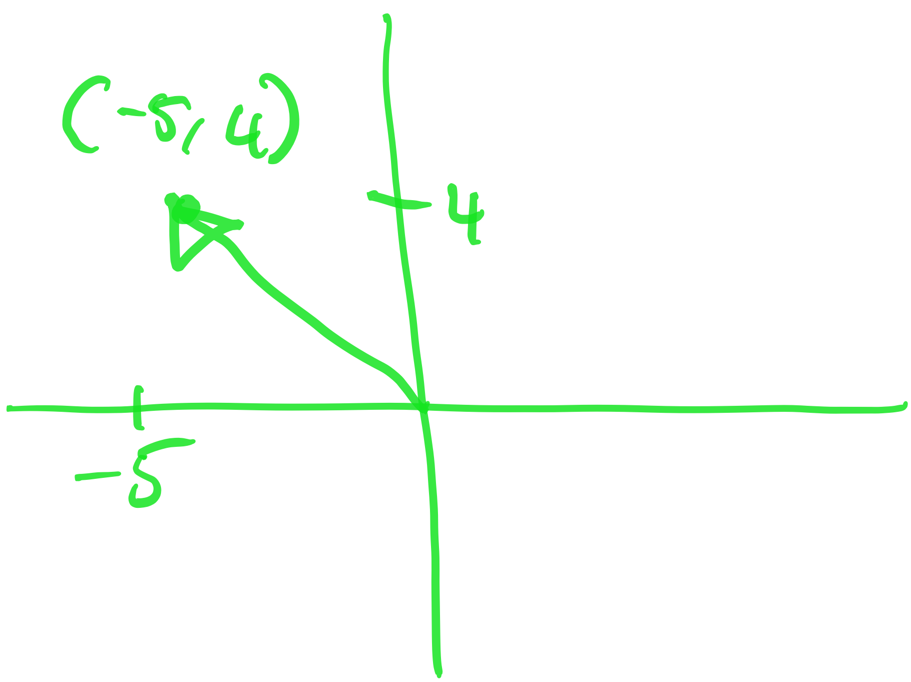
$$\vec{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\vec{0} + \vec{u} = \vec{u}$$

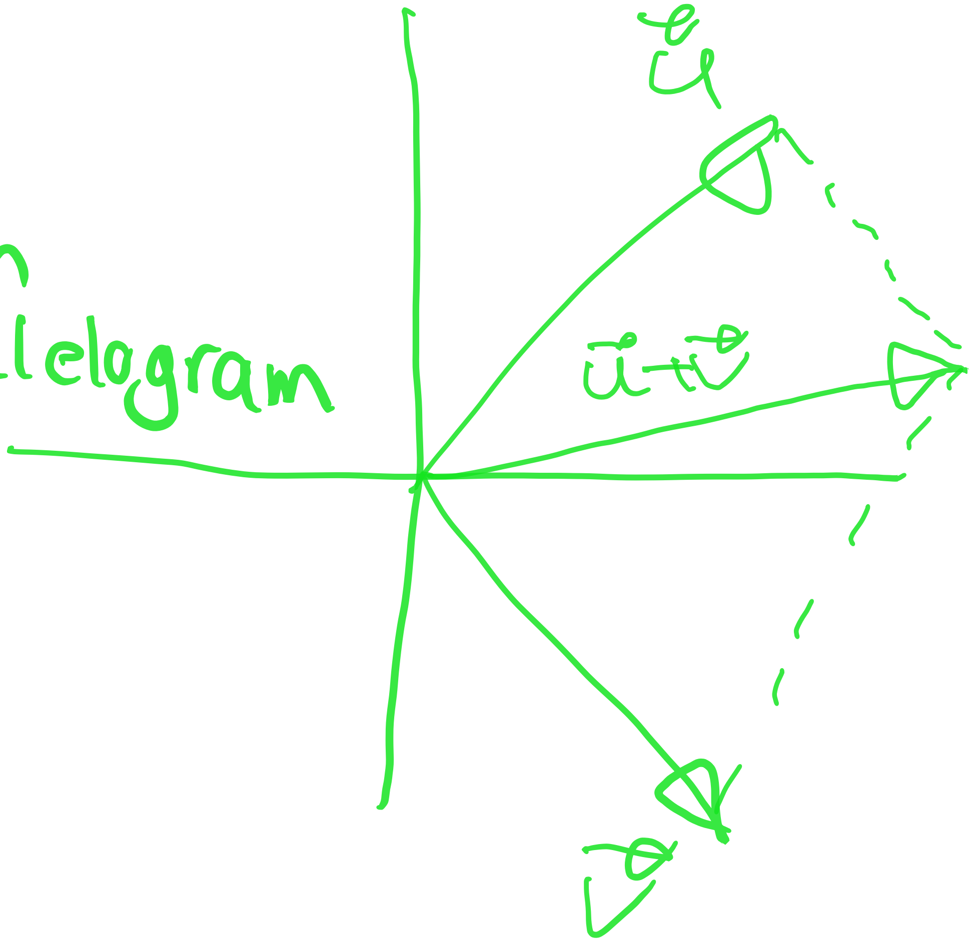
$$\vec{0} \times \vec{v} = \vec{0}$$

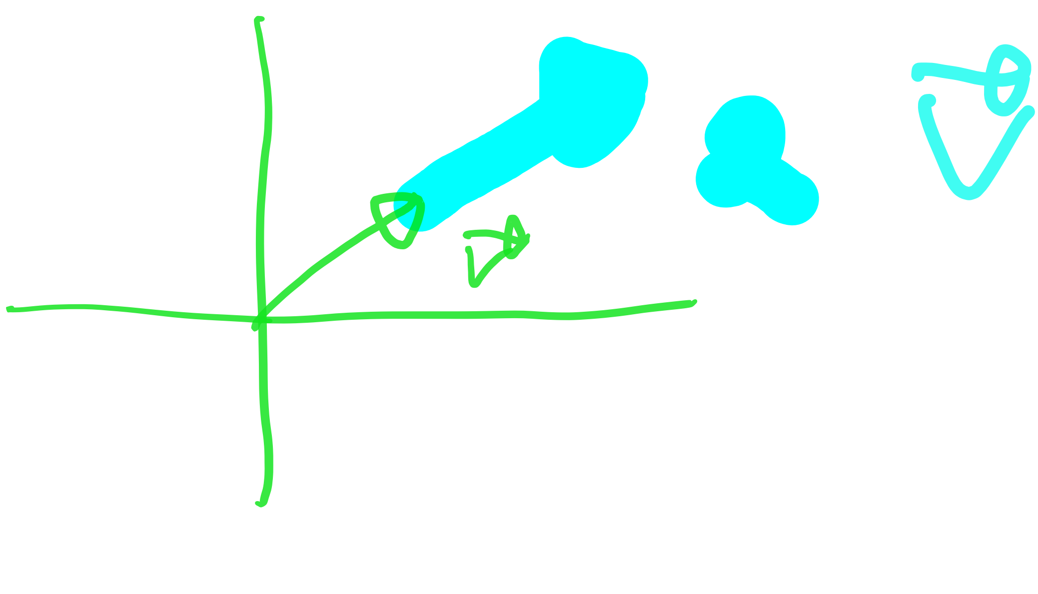
Geometry of vectors

$$\begin{array}{c c} \begin{bmatrix} -5 \\ 4 \end{bmatrix} & \begin{array}{c} x \\ y \end{array} \end{array}$$



form parrelelogram





Higher Dimensions

$\mathbb{R}^n$ contains all n-tuples of real numbers

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

Algebraic Properties

i) $\vec{u}+\vec{v}=\vec{v}+\vec{u}$ *communitative

ii) $(\vec{u}+\vec{v})+\vec{w}=\vec{u}+(\vec{v}+\vec{w})$ *associative

iii) $\vec{u}+\vec{0}=\vec{0}+\vec{u}=\vec{u}$ $\vec{0}$ is an additive identity

iv) $\vec{u}+(-\vec{u})=(-\vec{u})+\vec{u}=\vec{0}$

$-\vec{u}=(-1)\vec{u}$ $-\vec{u}$ is the additive inverse of $\vec{u}$

v) $c(\vec{u}+\vec{v})=c\vec{u}+c\vec{v}$

vi) $(c+d)\vec{u}=c\vec{u}+d\vec{u}$

} distributive props

vii) $c(d\vec{u})=(cd)\vec{u}$ "associative" sorta

viii) $1\vec{u}=\vec{u}$ multiplicative identity