# Lecture 08: Span

span of a set of vectors, is all possible linear combos for these vectors

Span $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p\}$

contains all

$c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_p\vec{v}_p$

$c_1, c_2, \ldots, c_p \in \mathbb{R}$

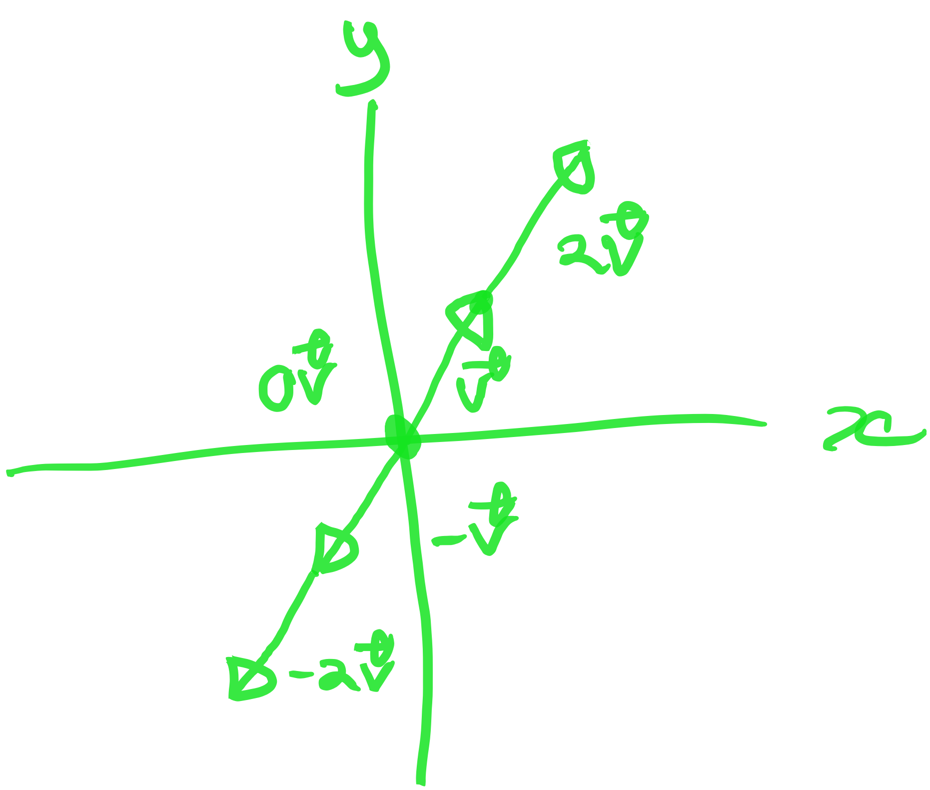
aka real numbers

when the set only has 1 vector

Span$\{\vec{v}\}$ contains all vectors in the for $c\vec{v}$ $c \in \mathbb{R}$

if $\vec{v}$ is nz, we can visualise it as a straight line through the origin

e.g. $\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}\begin{matrix} x \\ y \end{matrix}$



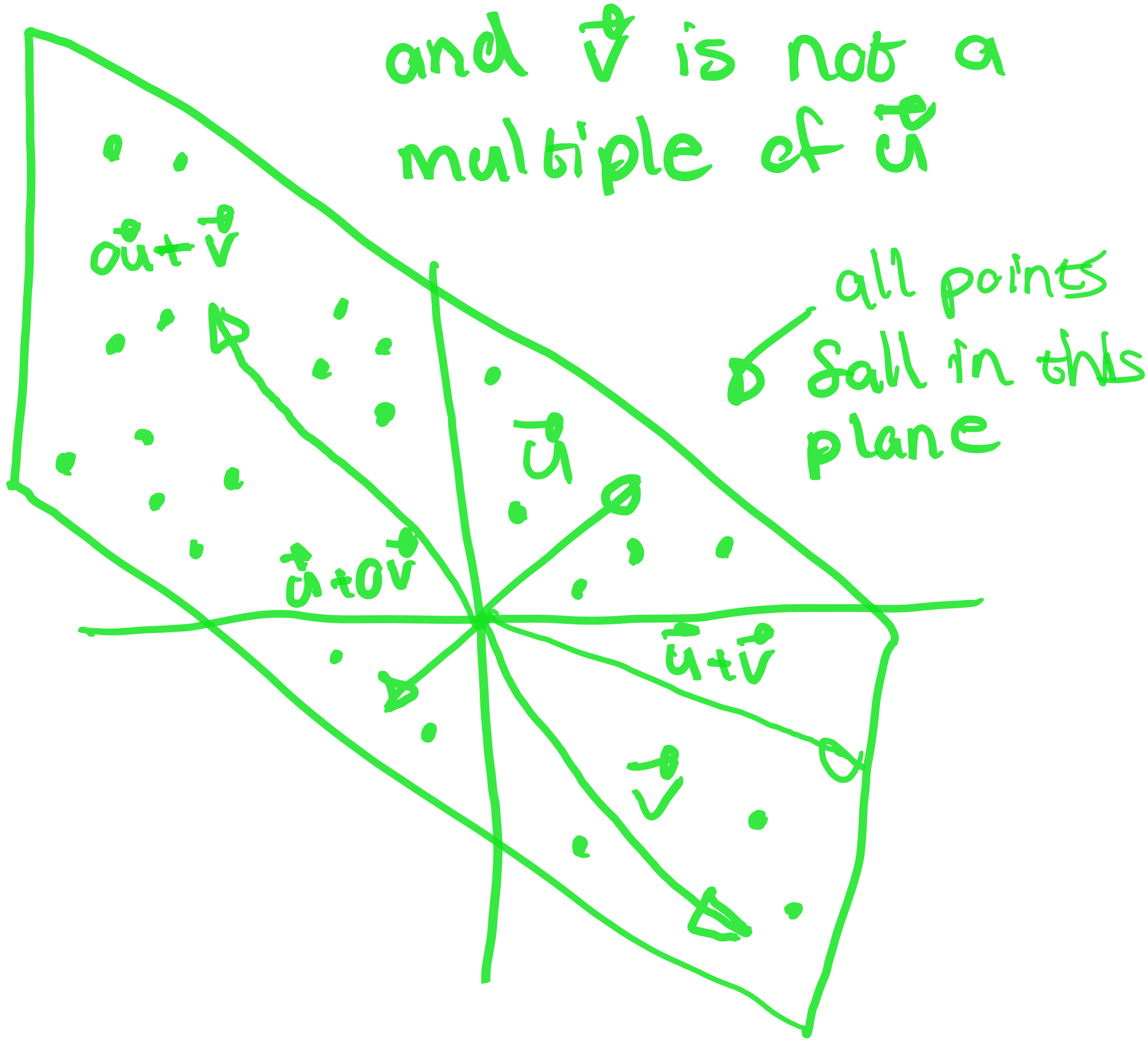
Span $\{\vec{u}, \vec{v}\}$

contains all vectors in form

$a\vec{u} + b\vec{v}$

$a, b \in \mathbb{R}$

visualise using plane through the origin if not when $\vec{u} = \vec{0}$ and $\vec{v}$ is not a multiple of $\vec{u}$



# Spans and linear Combos

is b a linear combo of ....

same as

is b is Span of ....

"Span" as a verb

sometimes every vector $\mathbb{R}^n$ is in Span$\{\vec{v}_1 \ldots \vec{v}_n\}$

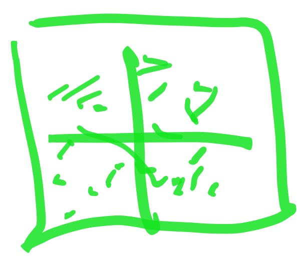
in this case $\{\vec{v}_1 \ldots \vec{v}_n\}$ span $\mathbb{R}^n$

e.g $\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\vec{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

↳ basically with these two vectors we can get any combo of numbers we want

$\{\vec{u}, \vec{v}\}$ spans $\mathbb{R}^2$



# Two Common Span Q's

## Is $\vec{b}$ in Span$\{\vec{v}_1, \ldots, \vec{v}_p\}$ 1

equivalent ~~qs~~ q's

is $\vec{b}$ a linear combo of $\vec{v}_1, \ldots, \vec{v}_p$

does the vec equation

$x_1\vec{v}_1 + x_p\vec{v}_p = \vec{b}$

have a solution

is the following vec equation consistent?

$x_1\vec{v}_1 + \ldots + x_p\vec{v}_p = \vec{b}$

Does the set $\{\vec{v}_1 \ldots \vec{v}_p\}$ span $\mathbb{R}^n$ (2)

equivalent questions

- is every vector $b \in \mathbb{R}^n$ a linear combination of $\vec{v}_1, \ldots \vec{v}_p$

- does the vec equation have a solution/ consistent no matter what $\vec{b}$ is

$$x_1\vec{v}_1 \ldots + x_p\vec{v}_p = \vec{b}$$

# Example 1:

$\vec{v}_1 = \begin{bmatrix} -2 \\ 5 \\ 0 \\ -3 \end{bmatrix}$, $\vec{v}_2 \begin{bmatrix} 4 \\ -3 \\ -1 \\ 0 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 0 \\ 7 \\ 0 \\ -6 \end{bmatrix}$

is $\vec{b}$ is Span $\{\vec{v}_1, \vec{v}_2\}$

the question is asking whether the equation $x_1\vec{v}_1 + x_2\vec{v}_2 = \vec{b}$ has a solution / is consistent

last col has a pivot

$$\begin{array}{ccc} \vec{v}_1 & \vec{v}_2 & \vec{b} \end{array}$$
$$\begin{bmatrix} -2 & 4 & 0 \\ 5 & -3 & 7 \\ 0 & -1 & 0 \\ -3 & 0 & -6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rightarrow$ thus no solution

~~or~~ $0\vec{v}_1 + 0\vec{v}_2 \neq 1$

$\therefore \vec{b}$ is not in Span

Example 2

$\vec{u}_1 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 6 \\ -5 \\ -1 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} -8 \\ 5 \\ 3 \end{bmatrix}, \vec{u}_4 = \begin{bmatrix} -4 \\ -5 \\ -1 \end{bmatrix}$

do $\vec{u}_1, \vec{u}_2, \vec{u}_3$ and $\vec{u}_4$ span $\mathbb{R}^3$

using Spanning Column Theorom

1. vectors $\vec{v}_1 + \dots + \vec{v}_p$ span $\mathbb{R}^n$
2. matrix whose columns are $\vec{v}_1 \dots \vec{v}_p$ has a pivot in every row

if 2 is true so is 1

We will use the spanning column theorem to check whether there is a pivot in every row of the matrix whose colums are $\vec{u}_1 - \vec{u}_4$

$$\begin{bmatrix} -2 & 6 & -8 & -4 \\ 1 & -5 & 5 & -5 \\ 1 & -1 & 3 & -1 \end{bmatrix} \longrightarrow \begin{bmatrix} \boxed{1} & 0 & 5/2 & 0 \\ 0 & \boxed{1} & -1/2 & 0 \\ 0 & 0 & 0 & \boxed{1} \end{bmatrix}$$

*not an aug matrix

since there is a pivot in every row, yes the vectors $\vec{u}_1 \ldots \vec{u}_4$ span $\mathbb{R}^3$

Solving Type ② Q

is $\vec{b}$ is Span $\{\vec{v}_1, \dots, \vec{v}_p\}$

① Setup vec equation

② Construct and row reduce augmented matrix

③ if consitent then Yes else No

- check if pivot in last col

pivot = not consistent

# Solving Type 2 Eq

does the set $\{v_1 \dots v_p\}$ span $\mathbb{R}^n$

1. Using spanning col theorom

2. Construct and reduce matrix

3. if pivot in every row = Yes
   else = No

# General tips

- Explain what a matrix represents, don't just row reduce
- translate terminology into equations
- write a conclusion