Lecture 07: Linear Combinations and Vector Equations

$c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_p\vec{v}_p$] linear combination

$c_1, c_2, \dots, c_p$: scalars

$\vec{u} = \begin{bmatrix} 0 \\ -3 \\ 5 \end{bmatrix} \in \mathbb{R}^3$ and $\vec{v} = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} \in \mathbb{R}^3$

$2\vec{u} + 3\vec{v} = 2\begin{bmatrix} 0 \\ -3 \\ 5 \end{bmatrix} + 3\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 31 \end{bmatrix}$

e.g. of linear combination of $\vec{u}$ and $\vec{v}$

Vector Equations

given vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p \in \mathbb{R}^n$
and another vector $\vec{b} \in \mathbb{R}^n$
is b a linear combo of the $\vec{v}$s?
/ does the vector equation
have a solution?

$$x_1 v_1 + x_2 v_2 + \ldots + x_p v_p = b$$

let

$$\vec{v}_1 = \begin{bmatrix} -\frac{1}{2} \\ 5 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} \frac{2}{5} \\ 6 \end{bmatrix}, b = \begin{bmatrix} 7 \\ 4 \\ 5 \end{bmatrix}$$

does the vector equation
have a solution?

$$x_1 \vec{v}_1 + x_2 \vec{v}_2 = \vec{b} \quad ?$$

$$x_1 \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix}$$

($\vec{v}_1$, $\vec{v}_2$, $\vec{b}$)

$$\begin{bmatrix} x_1 \cdot 1 \\ x_1 \cdot -2 \\ x_1 \cdot -5 \end{bmatrix} + \begin{bmatrix} x_2 \cdot 2 \\ x_2 \cdot 5 \\ x_2 \cdot 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + 2x_2 \\ -2x_1 + 5x_2 \\ -5x_1 + 6x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix}$$

$$x_1 + 2x_2 = 7$$
$$-2x_1 + 5x_2 = 4$$
$$-5x_1 + 6x_2 = -3$$

augmented matrix

$$\begin{bmatrix} 1 & 2 & 7 \\ -2 & 5 & 4 \\ -5 & 6 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 7 \\ 0 & 9 & 18 \\ 0 & 16 & 32 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 7 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 7 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

no pivot in
consistent
/has a
solution

$x_1 + 2x_2 = 7$

$x_2 = 2$

$x_1 = 3$