

Lecture 05: Solutions to Linear Systems

$$x_1 + x_2 - 6x_3 + x_4 = -4$$

$$x_1 - 9x_3 + x_4 = -3$$

$$2x_2 + 6x_3 + x_4 = -9$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -6 & 1 & -4 \\ 1 & 0 & -9 & 1 & -3 \\ 0 & 2 & 6 & 1 & 9 \end{array} \right]$$

augmented matrix

$$\left[\begin{array}{cccc|c} 1 & 0 & -9 & 0 & 4 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1 & -7 \\ x_1 & x_2 & x_3 & x_4 & b \end{array} \right]$$

row reduction

basic vars: x_1, x_2, x_4

free vars: x_3

↳ appears more than once, thus no leading entry

$$x_3 = 1 \quad x_3 = 2$$

$$x_1 = 9x_3 + 4 \quad x_1 = 13 \quad x_1 = 22$$

$$x_2 = -3x_3 - 1 \quad x_2 = -4 \quad x_2 = -7$$

x_3 is free

$$x_3 = 1 \quad x_3 = 2$$

$$x_4 = -7$$

$$x_4 = -7 \quad x_4 = -7$$

different values of $\uparrow x_3$ give different solutions

Steps to solve a system of linear equations.

1. Create Augmented Matrix
2. Row-Reduce Matrix
3. Rewrite reduced matrix in Equation Form
4. Solve each equation

Describing Solutions

does the system have any solutions? aka. consistent

does the system have one unique or many solutions?

echelon form is enough to answer

Example:

$$2x_1 + 6x_3 + 7x_4 = 7$$

$$6x_1 + 18x_3 + 24x_4 = 6$$

$$2x_1 - x_2 + 5x_4 = 14$$

$$2x_2 + 12x_3 + 4x_4 = -10 \quad \text{! Aug Matrix}$$

$$\left[\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & b \\ 2 & 0 & 6 & 7 & 7 \\ 6 & 0 & 18 & 24 & 6 \\ 2 & -1 & 0 & 5 & 14 \\ 0 & 2 & 12 & 4 & -10 \end{array} \right]$$

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad b$

2	-1	0	5	10
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0	1	6	2	-7
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0	0	0	1	-5
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0	0	0	0	4
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Echelon
Form

$$0x_1 + 0x_2 + 0x_3 + 0x_4 = 4$$

? $L \neq R \therefore$ no solution

pivot in last col = no solution

consistent = unique or many sol

inconsistent = no solutions \emptyset

free vars