# Lecture 2: Solving Systems by Elimination

## Substitution

$x + 2y = 10$

$0{,}5x - 3y = -11$

$x = 10 - 2y$

$0{,}5(10 - 2y) - 3y = -11$

$5 - y - 3y = -11$

$5 - 4y = -11$

$-4y = -16$

$y = 4$

$x + 2(4) = 10$

$x = 10 - 8$

$x = 2$

# Elimination

down side - every step creates a new equation

so we replace an equation by the result of doing an operation

each operation has to be reversible so that we do not destroy any info e.g. don't multiply by $\varnothing$

Operations :

- multiply both sides by a non-zero const
- replace an equation by the sum of itself and a multiple of another equation
- swap positions of two equations

row = equation

after converting to matrix

# Solving a System: Elimination

form triangle

$$
\begin{aligned}
\_\,x_1 + \_\,x_2 + \_\,x_3 &= \_ \\
\_\,x_2 + \_\,x_3 &= \_ \\
\_\,x_3 &= \_
\end{aligned}
$$

Triangular Form

## Matrix Notation ← For convenience

$$
\begin{aligned}
x_1 - 3x_2 + 4x_3 &= -4 \\
3x_1 - 7x_2 + 7x_3 &= -8 \\
-4x_1 + 6x_2 - x_3 &= 7
\end{aligned}
$$

normal

↓ take consts

$$
\begin{bmatrix}
1 & -3 & 4 & -4 \\
3 & -7 & 7 & -8 \\
-4 & 6 & -1 & 7
\end{bmatrix}
$$

matrix

$$\xrightarrow{R_2+(-3)R_1} \begin{bmatrix} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ -4 & 6 & -1 & 7 \end{bmatrix}$$

$$\xrightarrow{R_3+(4)R_1} \begin{bmatrix} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & -6 & 15 & -9 \end{bmatrix}$$

$$\xrightarrow{R_3+(3)R_2} \begin{bmatrix} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$x_1 - 3x_2 + 4x_3 = -4$$

$$2x_2 - 5x_3 = 4$$

$$0 = 3 \text{ ?} \quad \times$$

no solution

# Example

$$
\begin{bmatrix} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix}
\xrightarrow{R_2 + R_1}
\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & -2 & 5 & 7 \\ 0 & 1 & 1 & 0 \end{bmatrix}
\xrightarrow{R_2 \leftrightarrow R_3}
\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 5 & 7 \end{bmatrix}
\xrightarrow{R_3 + (2)R_2}
\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 7 & 7 \end{bmatrix}
$$

$\therefore$

$$x_1 - 3x_2 + 0x_3 = 5$$

$$x_2 + x_3 = 0$$

$$7x_3 = 7$$

$$x_3 = 1$$

$$x_2 + (1) = 0$$

$$x_2 = -1$$

$$x_1 - 3(-1) = 5$$

$$x_1 + 4 = 5 \quad \therefore x_1 = 1$$