Row Operations:

Scaling: R x constant

Replacement: sub sum of itself + multiple of another row

Swapping: $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 0 & 1 & 2 & 0 & -1 \\ -2 & 2 & 0 & 3 & 2 \\ 2 & 8 & 20 & 0 & 6 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2}$$

$$\begin{bmatrix} -2 & 2 & 0 & 3 & 2 \\ 0 & 1 & 2 & 0 & -1 \\ 2 & 8 & 20 & 0 & 6 \end{bmatrix} \xrightarrow{R_3 + R_1}$$

$$\begin{bmatrix} -2 & 2 & 0 & 3 & 2 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 10 & 20 & 3 & 8 \end{bmatrix} \xrightarrow{R_3 - 10R_2}$$

$$\begin{bmatrix} -2 & 2 & 0 & 3 & 2 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 3 & 18 \end{bmatrix}$$

Pivots and Pivot Colums

when a matrix is in echelon form, the leading entry of each row is called a **pivot**

matrices can have different echelon forms, but the pivots will always be in the same locations

a column with a pivot is called a pivot column

Row-Reducing a Matrix

1. Begin with the first nz col
   This is a pivot col and the pivot is at the top

2. if nescecary, swap nz rows to get a nz entry into the pivot position

3. Use "replacement" row ops to create zeros in all positions below the pivot

4. Move to next pivot col and repeat

for reduced use scaling to get 1

$$\begin{bmatrix} 2 & 0 & -2 & 1 & -1 \\ -1 & 1 & 3 & 0 & 1 \\ 0 & -2 & -4 & -3 & 0 \\ 1 & 0 & -1 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & 2 & 1 \\ -1 & 1 & 3 & 0 & 1 \\ 0 & -2 & -4 & -3 & 0 \\ 2 & 0 & -2 & 1 & -1 \end{bmatrix} \begin{matrix} \\ R_2 + R_1 \\ \\ R_4 - (2)R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & 2 & 1 \\ 0 & 1 & 2 & 2 & 2 \\ 0 & -2 & -4 & -5 & 0 \\ 0 & 0 & -2 & -7 & -2 \end{bmatrix} \quad R_3 + 2(R_1)$$

$$\begin{bmatrix} 1 & 0 & -1 & 2 & 1 \\ 0 & 1 & 2 & 2 & 2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & -2 & -7 & -2 \end{bmatrix} \quad \text{swap}$$

$$\begin{bmatrix} 1 & 0 & -1 & 2 & 1 \\ 0 & 1 & 2 & 2 & 2 \\ 0 & 0 & -2 & -7 & -2 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

$R_1 + R_3(-\frac{1}{2})$

~~$R_3 \div$~~

$R_2 + R_3$

$\frac{7}{1} \times \frac{1}{2} = \frac{7}{2}$

$+2$

$-2 \times -\frac{1}{2} = 1$

$$\begin{bmatrix} 1 & 0 & 0 & 11/2 & 2 \\ 0 & 1 & 0 & -5 & 0 \\ 0 & 0 & -2 & -7 & -2 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

$R_3 \div -2$

$$\begin{bmatrix} 1 & 0 & 0 & 11/2 & 2 \\ 0 & 1 & 0 & -5 & 0 \\ 0 & 0 & 1 & 7/2 & 1 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$