NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2021-2022

EE2010 / IM2004 – SIGNALS AND SYSTEMS

April / May 2022 Time Allowed: 2 ½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 6 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of useful formulae is given in the Appendix on page 6.

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1. (a) Consider a discrete-time signal given by

$$x[n] = \{2n+1\} \times \text{rect}\left[\frac{n}{4}\right]$$

where

$$\text{rect}\left[\frac{n}{K}\right] = \begin{cases} 1, & \text{if } -K/2 \le n \le K/2, \\ 0, & \text{otherwise.} \end{cases}$$

(i) Sketch the waveform of the signal $x[n]$ and determine its energy level.
(ii) Sketch the waveforms of the even and odd components of the signal $x[n]$.
(iii) Determine the energy levels of the even and odd components of the signal $x[n]$. Is the energy level of $x[n]$ equal to the sum of the energy levels of its even and odd components? Justify your answer.
(iv) Sketch the waveform of the signal $y[n]$ given by

$$y[n] = -2x[-n-2].$$

(15 Marks)

Note: Question No. 1 continues on page 2.

1. (a) Consider a discrete-time signal given by

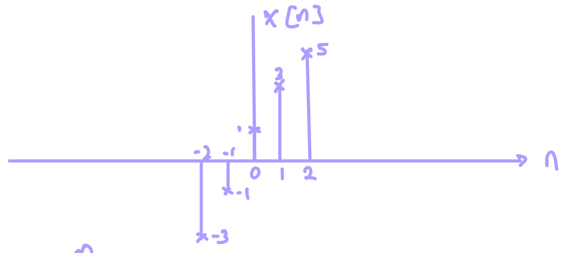
$$x[n] = \{2n+1\} \times \text{rect}\left[\frac{n}{4}\right]$$

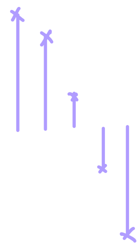
where

$$\text{rect}\left[\frac{n}{K}\right] = \begin{cases} 1, & \text{if } -K/2 \le n \le K/2, \\ 0, & \text{otherwise.} \end{cases}$$

(i) Sketch the waveform of the signal $x[n]$ and determine its energy level.

x[n]

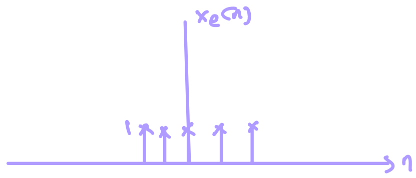




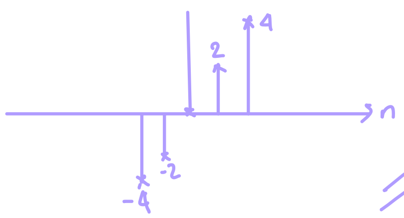
$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 = 45$

(ii) Sketch the waveforms of the even and odd components of the signal $x[n]$.

$x_e[n]$:



$x_o(n)$ =



(iii) Determine the energy levels of the even and odd components of the signal $x[n]$. Is the energy level of $x[n]$ equal to the sum of the energy levels of its even and odd components? Justify your answer.

$E_{x_e} = 5$

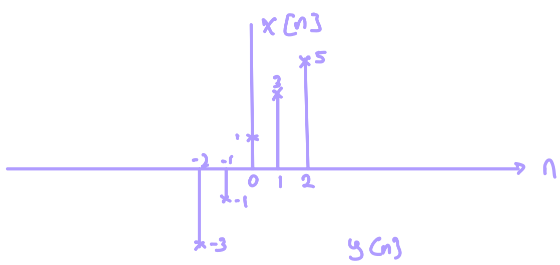
$E_{x_o} = 40$

Yes !!

$E_{x_e} + E_{x_o} = E_x$

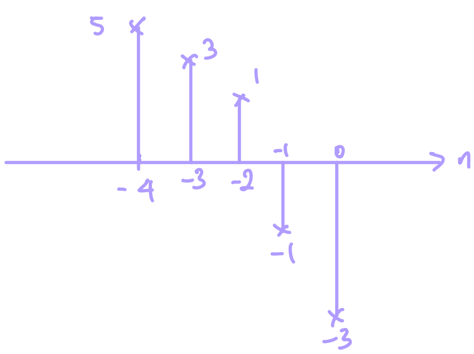
(iv) Sketch the waveform of the signal $y[n]$ given by

$$y[n] = -2x[-n-2].$$

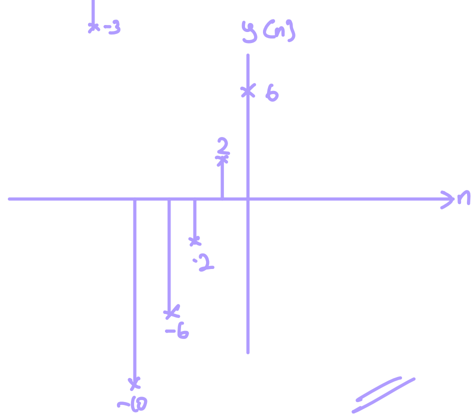


$y(n) = -2x[-(n+2)]$

$x[-(n+2)]$



$y(n)$:



(b) An input signal $x(t) = -2 + 2\cos(20\pi t)$ is passed through a full-wave rectifier system to produce an output signal given by $y(t) = |x(t)|$.

(i) Sketch the waveforms of the signals $x(t)$ and $y(t)$, respectively.
(ii) Determine the fundamental periods of the signals $x(t)$ and $y(t)$, respectively.
(iii) Determine the power level of the signal $x(t)$.
(iv) Determine whether this system is linear.

(10 Marks)

2. (a) The impulse response of a continuous-time system is given by

$$h(t) = 2 \times \mathrm{rect}\left(\frac{t-1}{2}\right)$$

where

$$\mathrm{rect}(t) = \begin{cases} 1, & \text{if } -1/2 \le t \le 1/2, \\ 0, & \text{otherwise.} \end{cases}$$

(i) Determine whether the system is memory-less, causal, and stable.
(ii) An input signal $x_1(t) = 2 \times \{u(t) - u(t-3)\}$ is applied to the system to produce the output signal $y_1(t)$. Sketch the waveforms of the signals $x_1(t)$ and $y_1(t)$, respectively.
(iii) Another signal $x_2(t)$ is applied to the system to produce the output $y_2(t)$, where

$$x_2(t) = \sum_{n=0}^{1} \left(-\frac{1}{2}\right)^n x_1(t-5n).$$

Based on the result that you have obtained in part (a)(ii), express the output $y_2(t)$ as a function of $y_1(t)$. Sketch the waveforms of the signals $x_2(t)$ and $y_2(t)$, respectively.

(13 Marks)

(b) Assuming that the conventional amplitude modulation (AM) signal with frequency-division multiplexing (FDM) is given by

$$x(t) = 2 \times \{1 + m_1(t)\}\cos\left(2\pi f_{c_1} t\right) + 2 \times \{1 + m_2(t)\}\cos\left(2\pi f_{c_2} t\right).$$

The amplitude spectra of the two baseband signals $m_1(t)$ and $m_2(t)$ are given by

$$M_1(f) = \begin{cases} 1, & \text{for } -150\text{Hz} \le f \le 150\text{Hz}, \\ 0, & \text{otherwise,} \end{cases}$$

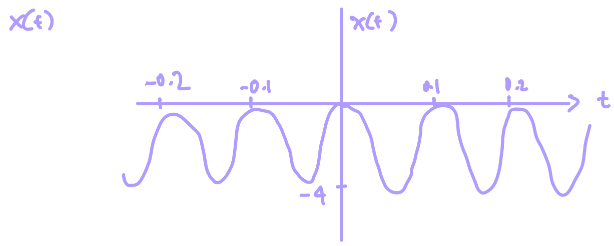
and

$$M_2(f) = \begin{cases} 1 - |0.01f|, & \text{for } -100\text{Hz} \le f \le 100\text{Hz}, \\ 0, & \text{otherwise.} \end{cases}$$

Note: Question No. 2 continues on page 3.

(b) An input signal $x(t) = -2 + 2\cos(20\pi t)$ is passed through a full-wave rectifier system to produce an output signal given by $y(t) = |x(t)|$.

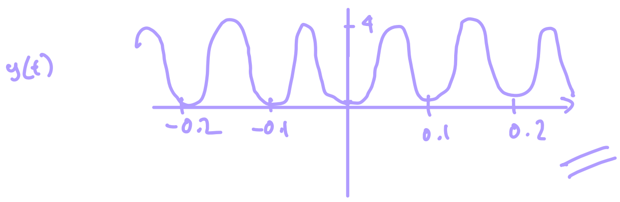
(i) Sketch the waveforms of the signals $x(t)$ and $y(t)$, respectively.



$2\pi f_0 t$

$f_0 : 10$

$\frac{1}{T} : \frac{1}{10}$



(ii) Determine the fundamental periods of the signals $x(t)$ and $y(t)$, respectively.

$f_0 \; x = 10Hz$

$f_0 \; y = 20Hz$

(iii) Determine the power level of the signal $x(t)$.

$\cos 2\theta = 2\cos^2\theta - 1$

$2\cos^2\theta = \cos 2\theta + 1$

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} 4 - 8\cos(20\pi t) + 4\cos^2(20\pi t) \, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} 4 - 8\cos(20\pi t) + 2\cos(40\pi t) + 2 \quad dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\left[6t - \frac{8}{20\pi}\sin(20\pi t) + \frac{2}{40}\sin(40\pi t)\right]_{-T/2}^{T/2}$$

$$= 6$$

(iv) Determine whether this system is linear.

(10 Marks)

$x(t) \longrightarrow y(t) = |x(t)|$

not linear

2. (a) The impulse response of a continuous-time system is given by

$$h(t) = 2 \times \text{rect}\left(\frac{t-1}{2}\right)$$

0-2

where

$$\text{rect}(t) = \begin{cases} 1, & \text{if } -1/2 \le t \le 1/2, \\ 0, & \text{otherwise.} \end{cases}$$

(i) Determine whether the system is memory-less, causal, and stable.

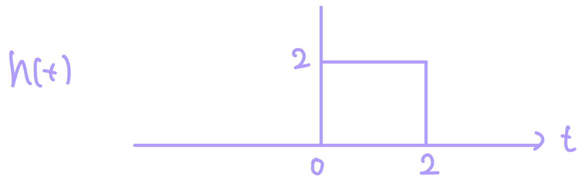
causal → yes, $t<0$ $h(t)=0$

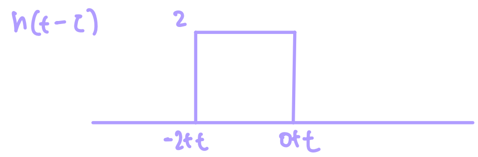
memoryless → no $h(t) \neq C\delta(t)$

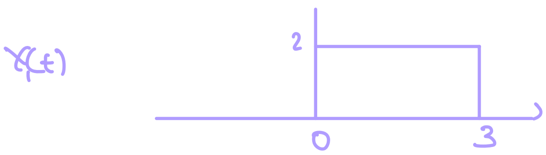
Stable → yes

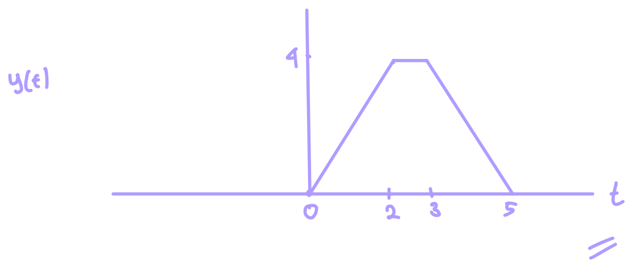
(ii) An input signal $x_1(t) = 2 \times \{u(t) - u(t-3)\}$ is applied to the system to produce the output signal $y_1(t)$. Sketch the waveforms of the signals $x_1(t)$ and $y_1(t)$, respectively.

$h(t-\tau) = h(-(\tau-t))$









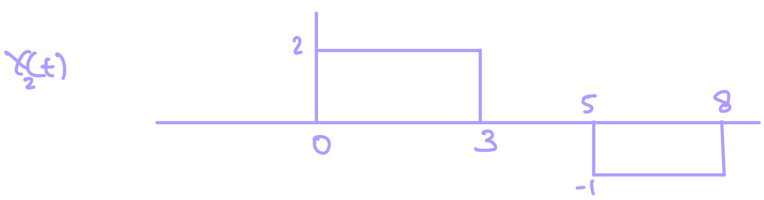
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

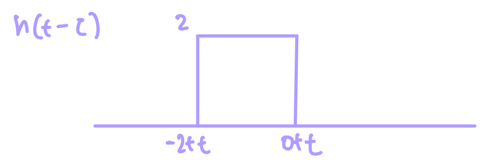
(iii) Another signal $x_2(t)$ is applied to the system to produce the output $y_2(t)$, where

$$x_2(t) = \sum_{n=0}^{1} \left(-\frac{1}{2}\right)^n x_1(t-5n).$$

Based on the result that you have obtained in part (a)(ii), express the output $y_2(t)$ as a function of $y_1(t)$. Sketch the waveforms of the signals $x_2(t)$ and $y_2(t)$, respectively.

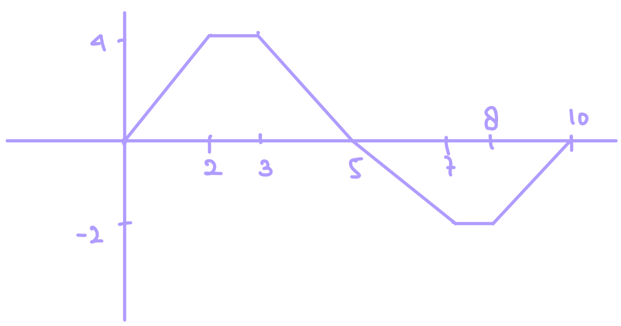
(13 Marks)





$$y_2(t) = \int_{-\infty}^{\infty} x_2(\tau) \cdot h(t-\tau)\, d\tau$$

$$y_2(t) = y_1(t) + -\frac{1}{2} y_1(t-5)$$



(b) Assuming that the conventional amplitude modulation (AM) signal with frequency-division multiplexing (FDM) is given by

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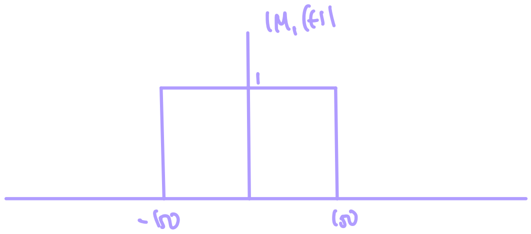
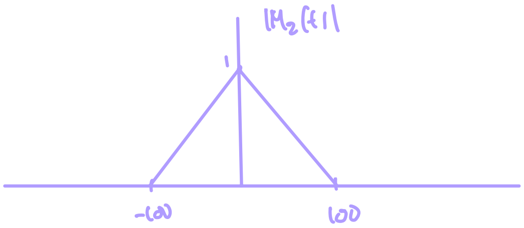
and

$$M_2(f) = \begin{cases} 1 - |0.01f|, & \text{for } -100\text{Hz} \le f \le 100\text{Hz}, \\ 0, & \text{otherwise.} \end{cases}$$

(i) Sketch the amplitude spectra of the signals $m_1(t)$ and $m_2(t)$, respectively. Determine the bandwidths of $m_1(t)$ and $m_2(t)$, respectively.



BW = 150

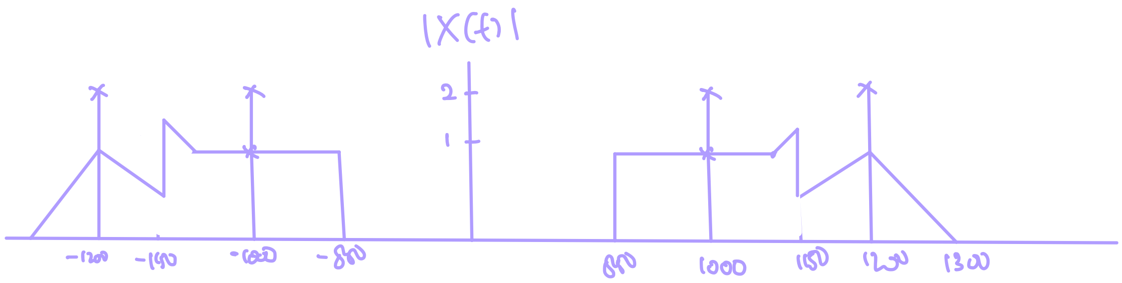


BW = 100

(ii) Assuming that $f_{c_1} = 1{,}000$ Hz and $f_{c_2} = 1{,}200$ Hz, sketch the amplitude spectrum of the signal $x(t)$ and determine its bandwidth.

Ac Ac

$$x(t) = 2 \times \{1 + m_1(t)\} \cos(2\pi f_{c_1} t) + 2 \times \{1 + m_2(t)\} \cos(2\pi f_{c_2} t).$$



BW = 450Hz

(iii) Determine whether the original signals $m_1(t)$ and $m_2(t)$ can be recovered from $x(t)$ without any distortion. Justify your answer.

(12 Marks)

no distortion 1000 > 150 and 1200 > 100

(i) Sketch the amplitude spectra of the signals $m_1(t)$ and $m_2(t)$, respectively. Determine the bandwidths of $m_1(t)$ and $m_2(t)$, respectively.

(ii) Assuming that $f_{c_1} = 1{,}000\,\text{Hz}$ and $f_{c_2} = 1{,}200\,\text{Hz}$, sketch the amplitude spectrum of the signal $x(t)$ and determine its bandwidth.

(iii) Determine whether the original signals $m_1(t)$ and $m_2(t)$ can be recovered from $x(t)$ without any distortion. Justify your answer.

(12 Marks)

3. (a) Consider two periodic signals $x(t)$ and $y(t)$ that have the following time-domain relationship:

$$y(t) = 8x(8t - 8) + 8.$$

Further denote the fundamental angular frequencies of the signals $x(t)$ and $y(t)$ as ω_x and ω_y, and their complex-exponential Fourier series coefficients as c_n and d_n with phase angles θ_n and $\emptyset_n$, respectively.

(i) For n not equal to zero, express d_n in terms of c_n. If $|c_4| = 1.25$, then what is the magnitude of d_4?

(ii) If the dc value of $x(t)$ is 0.5, then what is the dc value of $y(t)$?

(iii) Express $\emptyset_n$ in terms of θ_n. If $\theta_1 = 0.2$ rad, $\omega_x = 0.15$ rad/s, then what is the value of $\emptyset_1$?

(iv) If the 5th harmonic of $x(t)$ is 500 Hz, then what is the frequency of the 10th harmocnic of $y(t)$?

(20 Marks)

(b) Consider three periodic signals $x(t)$, $y(t)$ and $z(t)$ that have the following time-domain relationship: $z(t) = x(t) - 9y(t)$ where $y(t) = x(30t)$. Further denote the complex-exponential Fourier series coefficients of the signals $x(t)$, $y(t)$ and $z(t)$ as c_n, $d_n$and g_n, respectively. If $c_{39} = 9$, then what is the value of g_{39}?

(5 Marks)

3. (a) Consider two periodic signals $x(t)$ and $y(t)$ that have the following time-domain relationship:

$$y(t) = 8x(8t - 8) + 8.$$

Further denote the fundamental angular frequencies of the signals $x(t)$ and $y(t)$ as ω_x and ω_y, and their complex-exponential Fourier series coefficients as c_n and d_n with phase angles θ_n and ϕ_n, respectively.

(i) For n not equal to zero, express d_n in terms of c_n. If $|c_4| = 1.25$, then what is the magnitude of d_4?

let fundamental angular freq x = $\omega_x = 2\pi f_{0x}$

———————— " ———————— y: $\omega_y = 8\omega_x$

$$x(t) = \sum_{-\infty}^{\infty} C_n e^{jn\omega_x t}$$

$$x(8t-8) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_x (8t-8)}$$

$$8x(8t-8) + 8 = 8 + \sum_{n=-\infty}^{\infty} 8C_n e^{jn\omega_x (8t-8)}$$

$$y(t) = 8 + \sum_{n=-\infty}^{\infty} 8C_n e^{-8jn\omega_x} e^{jn8\omega_x t}$$

$$d_n = 8C_n e^{-8jn\omega_x} \quad n \neq 0$$

$$|d_4| = 10$$

(ii) If the dc value of $x(t)$ is 0.5, then what is the dc value of $y(t)$?

$$C_0 = 0.5$$

$$d_0 = 8 + 8 \cdot 0.5 = 12$$

(iii) Express ϕ_n in terms of θ_n. If $\theta_1 = 0.2$ rad, $\omega_x = 0.15$ rad/s, then what is the value of ϕ_1?

$$d_n = 8C_n e^{-8jn\omega_x} \qquad \phi_n = \theta_n - 8 \cdot n \cdot \omega_x$$

$$\phi_1 = 0.2 + (-8 \cdot 0.15) = -1 \text{ rad}$$

(iv) If the 5th harmonic of $x(t)$ is 500 Hz, then what is the frequency of the 10th harmocnic of $y(t)$?

(20 Marks)

$$5f_{0x} = 500$$

$$f_{0x} = 100 \qquad f_{0y} = 8 \cdot 100 = 800$$

$$10 \cdot f_{0y} = 8000 \text{ Hz}$$

(b) Consider three periodic signals $x(t)$, $y(t)$ and $z(t)$ that have the following time-domain relationship: $z(t) = x(t) - 9y(t)$ where $y(t) = x(30t)$. Further denote the complex-exponential Fourier series coefficients of the signals $x(t)$, $y(t)$ and $z(t)$ as c_n, d_n and g_n, respectively. If $c_{39} = 9$, then what is the value of g_{39}?

(5 Marks)

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_x t} \qquad \omega_x \qquad d_{39} = 9$$

$$y(t) = \sum_{n=-\infty}^{\infty} d_n e^{jn\omega_y t} \qquad \omega_y = 30\,\omega_x$$

$$z(t) = \sum_{n=-\infty}^{\infty} g_n e^{jn\omega_z t} \qquad \omega_z = \omega_x$$

$$x(t) - 9y(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_x t} - 9 d_n e^{jn30\omega_x t} = \sum_{n=-\infty}^{\infty} g_n e^{jn\omega_x t}$$

$$g_{39} = c_{39} = 9$$

4. (a) $x(t)$ is a periodic signal formed by truncated segments of sinousiods as shown in Figure 1:

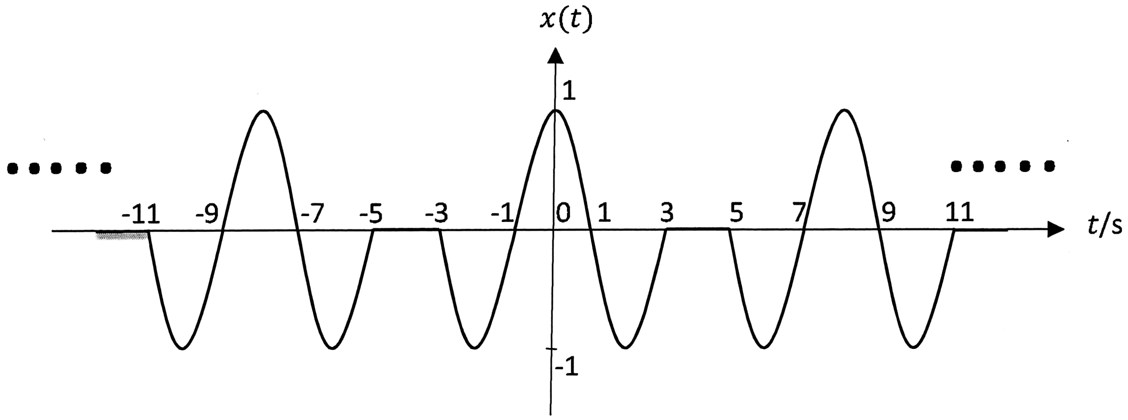


Figure 1

Given the Fourier analysis equation, $c_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}dt$, the expression used to find the complex-exponential Fourier series coefficients is determined to be

$$c_n = \frac{1}{A}\int_{-B}^{B} \cos\left(\frac{2\pi}{D}t\right)e^{-j2\pi nt/E}dt.$$

What are the values of A, B, D and E in the above expression for computing c_n? Select B such that it has the smallest positive value.

(8 Marks)

(b) If the signal $x(t)$ is given by the unit impulse function $\delta(t)$, that is, $x(t) = \delta(t)$, and the Fourier Transform of $x(t)$ is $X(\omega)$, find $X(0)$ and $X(\pi)$.

(4 Marks)

(c) An analog system H has the following impulse response:

$$h(t) = 1000 \times sinc(1000\pi t),$$

where t is time in second.

Draw the magnitude response of the system. What is the real bandwidth of the system in Hz? Note that $sinc$ function is defined here and in the Appendix, as:

$$sinc(x) \triangleq \frac{\sin(x)}{x}.$$

(6 Marks)

Note: Question No. 4 continues on page 5.

4. (a) $x(t)$ is a periodic signal formed by truncated segments of sinousiods as shown in Figure 1:

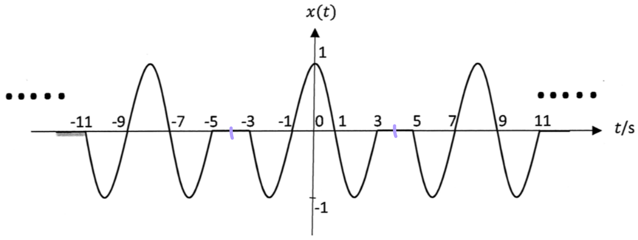


Figure 1

T_0: 8

$A = 8$

$B = 3$

$D: 4$

$E: 8$

$\frac{2\pi}{D} : \frac{\pi}{2}$ $D: 4$

Given the Fourier analysis equation, $c_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}dt$, the expression used to find the complex-exponential Fourier series coefficients is determined to be

$$c_n = \frac{1}{A}\int_{-B}^{B} \cos\left(\frac{2\pi}{D}t\right)e^{-j2\pi nt/E}dt.$$

What are the values of A, B, D and E in the above expression for computing c_n? Select B such that it has the smallest positive value.

(8 Marks)

(b) If the signal $x(t)$ is given by the unit impulse function $\delta(t)$, that is, $x(t) = \delta(t)$, and the Fourier Transform of $x(t)$ is $X(\omega)$, find $X(0)$ and $X(\pi)$.

(4 Marks)

$x(t) = \delta(t)$

$X(\omega) = 1$

$X(0) = 1$

$X(\pi) = 1$

(c) An analog system H has the following impulse response:

$$h(t) = 1000 \times sinc(1000\pi t),$$

where t is time in second.

Draw the magnitude response of the system. What is the real bandwidth of the system in Hz? Note that $sinc$ function is defined here and in the Appendix, as:

$$sinc(x) \triangleq \frac{\sin(x)}{x}.$$

(6 Marks)

$\frac{1000\pi}{\pi}$ $sinc(1000\pi t) \leftrightarrow rect\left(\frac{\omega}{2000\pi}\right)$

$2\pi f: 1000\pi$

$f: 500Hz$

BW: 500Hz

(d) Another analog system G has its impulse response given by

$$g(t) = h(t) \times \text{rect}\left(\frac{t}{0.1}\right),$$

where $h(t)$ is defined in part (c).

Without any derivation and computation, comment on the key difference between the frequency responses of systems H and G.

(4 Marks)

$g(t) = h(t) \cdot rect\left(\frac{t}{0.1}\right)$

?

(e) If the signal $x(t)$ is given by $h(t)$ defined in part (c), that is, $x(t) = h(t)$, what is the Nyquist rate for sampling the signal $x(t)$?

(3 Marks)

f_N: 2·500Hz: 1kHz

(d) Another analog system G has its impulse response given by

$$g(t) = h(t) \times \text{rect}\left(\frac{t}{0.1}\right),$$

where $h(t)$ is defined in part (c).

Without any derivation and computation, comment on the key difference between the frequency responses of systems H and G.

(4 Marks)

(e) If the signal $x(t)$ is given by $h(t)$ defined in part (c), that is, $x(t) = h(t)$, what is the Nyquist rate for sampling the signal $x(t)$?

(3 Marks)

Appendix

Fourier Transform Pairs			Fourier Transform Operations	
$x(t)$	$X(\omega)$		$x(t)$	$X(\omega)$
$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a>0$	$kx(t)$	$kX(\omega)$
$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a>0$	$x_1(t)+x_2(t)$	$X_1(\omega)+X_2(\omega)$
$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a>0$	$x^*(t)$	$X^*(-\omega)$
$\delta(t)$	1		$X(t)$	$2\pi x(-\omega)$
1	$2\pi\delta(\omega)$		$x(at)$	$\dfrac{1}{\|a\|}X\left(\dfrac{\omega}{a}\right)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$		$x(t-t_0)$	$X(\omega)e^{-j\omega t_0}$
$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$		$x(t)e^{j\omega_0 t}$	$X(\omega-\omega_0)$
$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$		$x_1(t)*x_2(t)$	$X_1(\omega)X_2(\omega)$
$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$		$x_1(t)x_2(t)$	$\dfrac{1}{2\pi}X_1(\omega)*X_2(\omega)$
$\text{rect}\left(\dfrac{t}{\tau}\right)$	$\tau\,\text{sinc}\left(\dfrac{\omega\tau}{2}\right)$		$\dfrac{d^n x(t)}{dt^n}$	$(j\omega)^n X(\omega)$
$\dfrac{W}{\pi}\,\text{sinc}\,(Wt)$	$\text{rect}\left(\dfrac{\omega}{2W}\right)$		$\int_{-\infty}^{t} x(u)du$	$\dfrac{X(\omega)}{j\omega}+\pi X(0)\delta(\omega)$
$\sum_{n=-\infty}^{\infty}\delta(t-nT_0)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\dfrac{2\pi}{T_0}$		

Useful Trigonometric Identities

$\exp(\pm j\theta)=\cos(\theta)\pm j\sin(\theta)$	$\cos(A)+\cos(B)=2\cos\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$
$\cos(\theta)=\frac{1}{2}[\exp(j\theta)+\exp(-j\theta)]$	$\sin(\theta)=\frac{1}{2j}[\exp(j\theta)-\exp(-j\theta)]$
$2\cos(A)\cos(B)=\cos(A-B)+\cos(A+B)$	$\cos^2(A)=\frac{1}{2}[1+\cos(2A)]$
$2\sin(A)\sin(B)=\cos(A-B)-\cos(A+B)$	$\sin^2(A)=\frac{1}{2}[1-\cos(2A)]$
$2\cos(A)\sin(B)=\sin(A+B)-\sin(A-B)$	$\sin(2A)=2\cos(A)\sin(A)$
$\cos(A\pm B)=\cos(A)\cos(B)\mp\sin(A)\sin(B)$	$\sin(A\pm B)=\sin(A)\cos(B)\pm\cos(A)\sin(B)$

END OF PAPER

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1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.