Thinking about Games

ECON 420: Game Theory

Spring 2018

Reading

- This week: Chapters 1 and 2
- Next week: Chapter 3

Nim

- Today we'll play an alternative version of nim
- One row of 20 lines
- Each turn, the player must choose to remove 1, 2, or 3 lines
- The last person to remove a line wins

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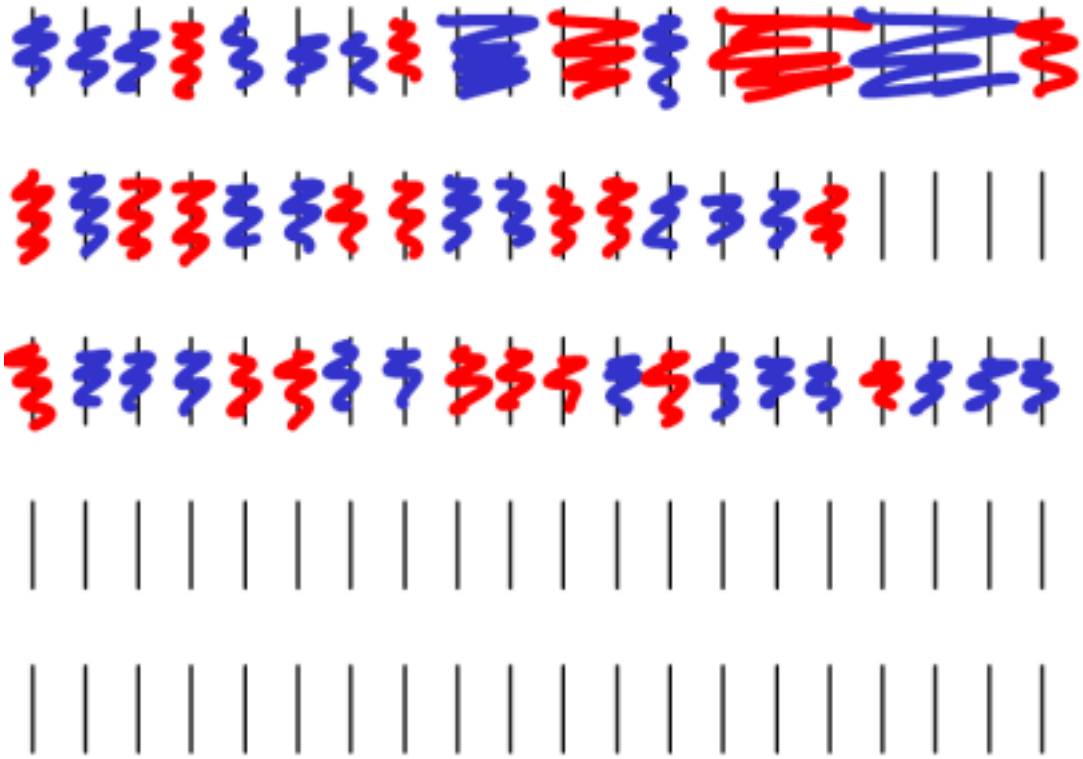

Decisions vs games

- *Decisions* are choices that can be made "without concern for reaction or response from others"
- *Strategic games* (or just *games*) are choices that occur among "mutually aware players"
  - Players in strategic games take into account the cross-effects of their actions and the actions of other players

Classifying games

- Games can be:
  - Sequential or simultaneous
  - Zero sum or non-zero sum
  - Single stage or repeated
    - ⋆ Infinite or finite repetition
  - Perfect or imperfect information
  - Fixed or manipulable rules

Sequential vs simultaneous games

- Sequential games
  - Players take turns (one after another)
  - Players look ahead at what might happen in the future to make choices
- Simultaneous games
  - Players make choices at the same time
  - Must predict what other players will do contemporaneously

Nim

- Sequential or simultaneous?
- *sequential*

Rock, paper, scissors

- Sequential or simultaneous?
- *simultaneous*

Chess

- Sequential or simultaneous?
- *sequential*

A single play in American football

- Sequential or simultaneous?
- *simultaneous*

A soccer penalty kick

- Sequential or simultaneous?
- *simultaneous*

Registering for classes

- Sequential or simultaneous?
- ?

Constant-sum vs non-constant-sum games

- Constant sum
  - The sum total payoffs are fixed
  - Playing the game only determines the allocation of payoffs, not the total amount
- Zero sum
  - A special case of constant sum where total payoffs are zero
  - Often used to refer to constant-sum games
- Non-constant sum
  - Total payoffs depend on choices of players

Nim

- Constant or non-constant sum?
- *constant (zero)*

Rock, paper, scissors

- Constant or non-constant sum?
- *constant (zero)*

Splitting the last piece of cake with someone

- Constant or non-constant sum?
- *constant (not zero)*

Chicken (stay straight or swerve)

- Constant or non-constant sum?
- *non-constant*

International trade

- Constant or non-constant sum?
- *non-constant*

Example

- All-pay auction
  - You will bid to receive a $5 bill
  - You have to pay me your highest bid *regardless if you win or lose the auction*
  - Everyone who bids has to pay, but only one person will win the $5

Constant-sum games

- Constant-sum games can be either:
  - Negative sum (war, household chores)
  - Zero sum (sports, games with a "winner" and "loser")
  - Positive sum (eating cake)
- Non-constant-sum games can be any of the above, too
  - Sometimes positive *and* negative sums are possible in the same game (all-pay auction)

Single-stage and repeated games

- Single-stage games are played (against some particular opponents) and never again
- Repeated games are played over and over
  - Can be finitely or infinitely repeated
  - Choices in one round (stage) might affect later rounds (and vice versa)

Golden Balls (split or steal)

- Single-stage or repeated?
- *single-stage*

"Battle of wits" (poison cups)

- Single-stage or repeated?
- *single-stage*

A baseball plate appearance (pitcher vs batter)

- Single-stage or repeated?
- *repeated*

OPEC oil production

- Single-stage or repeated?
- *repeated*

Perfect and imperfect information

- Perfect information
  - Players know exactly what choice are available to each player and what the payoffs will be (given choices)
- Imperfect information
  - Uncertainty over choices or payoffs (or both)
  - External uncertainty: "Nature" (the state of the world) changes choices or payoffs
  - Strategic uncertainty: Imperfect information about what other players are doing or have done in the past

Nim

- Perfect or imperfect information?
- *perfect information*

Chess

- Perfect or imperfect information?
- *perfect information*

Vacation planning

- Perfect of imperfect information?
- *imperfect information (external uncertainty)*

Applying for jobs

- Perfect of imperfect information?
- *imperfect information (strategic uncertainty)*

Poker

- Perfect of imperfect information?
- *imperfect information (strategic uncertainty)*

Fixed and manipulable rules

- Fixed rules can't be altered by players
- The choices available to each player are constant and known

Nim

- Fixed or manipulable rules?
- *fixed*

Political campaigns

- Fixed or manipulable rules?
- *manipulable*

Advertising

- Fixed or manipulable rules?
- *manipulable*

Defining a game

- A strategic game must contain three elements:
  1. Players
  2. Strategies
  3. Payoffs
- We make various assumptions about these elements

Players

- The participants in the game who make choices
- Humans, firms, "nature", etc
- We assume players are *rational*
  - They can calculate outcomes from different strategies and will choose the optimum
- We assume players have *common knowledge of the rules*
  - Rules are fixed *at some level*
    - ⋆ Example: Releasing tax returns when running for president
    - ⋆ Example: Battle of wits
  - Whether or not to follow rules is *itself* part of a larger game

Example Who are the players in a game of poker?

- People playing
- Nature - determines cards

Strategies

- The set of choices available to the player
- A complete strategy is a "map" (set of instructions) on how to play a game given any possible set of choices from the other players
- Strategies are collections of choices
- A strategy is complete if you could give your instructions to someone else (or a machine) to do

Payoffs

- The outcomes of the game
- Can be profits, utility, money, wins and losses, etc
- In this class we will assume that higher payoffs are more desirable
- We will often need to calculate *expected payoffs* if there is some randomness or uncertainty (imperfect information)
  - For any possible outcome i with payoff π_i, expected payoffs are $\sum_i p_i \pi_i$, where p_i is the probability of i occurring

Examples

1. Suppose I flip a coin
   - Heads: You get \$100
   - Tails: You lose \$50
2. Suppose I flip 2 coins:
   - Heads/heads: You get \$100
   - Heads/tails: You get \$20
   - Tails/tails: You lose \$40

H H
H T
T H
T T

1. Expected payoff ($E(\pi)$)

$$\frac{1}{2} \cdot 100 + \frac{1}{2}(-50)$$

$$\frac{1}{2}(100 - 50) = \frac{1}{2}(50)$$

$$= 25$$

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2. $\frac{1}{4}(100) + \frac{1}{2}(20) + \frac{1}{4}(-40)$

HH HT or TH

$$= 25 + 10 - 10$$

$$= 25$$

Equilibrium

- The outcome where nobody can do better by *unilaterally* changing their strategy
- At equilibrium, nobody can say "I wish I had done that differently"
- Equilibrium does *not* mean that the outcome is optimal
- We don't expect to always obtain an equilibrium
- There is *at least* one equilibrium in every game

One half the average game What is an equilibrium of this game?

- Everyone picks 0

Extra-credit game (choosing points) What is an equilibrium of this game?

- Everyone chooses 1 point
- Any other equilibria?

Guess the average

- Everyone write down a number between 0 and 100
- The winning number is the average number guessed
- Trade papers after writing down your number