

# Sequential Games

ECON 420: Game Theory

Spring 2018

## **Centipede game**

1. 2 players play for 10 rounds
2. Each round another unit of payoff is added to the pot (starting with one unit)
3. Players alternate turns, choose to either:
  - ▶ Stop, and collect the entire pot for themselves
  - ▶ Continue, one is added to the pot and next player chooses

## **Sequential games**

- ▶ Games where there is a strict order of play
- ▶ Games where players take turns moving are sequential
- ▶ Real-world games are generally combinations of sequential and simultaneous games

## Game trees

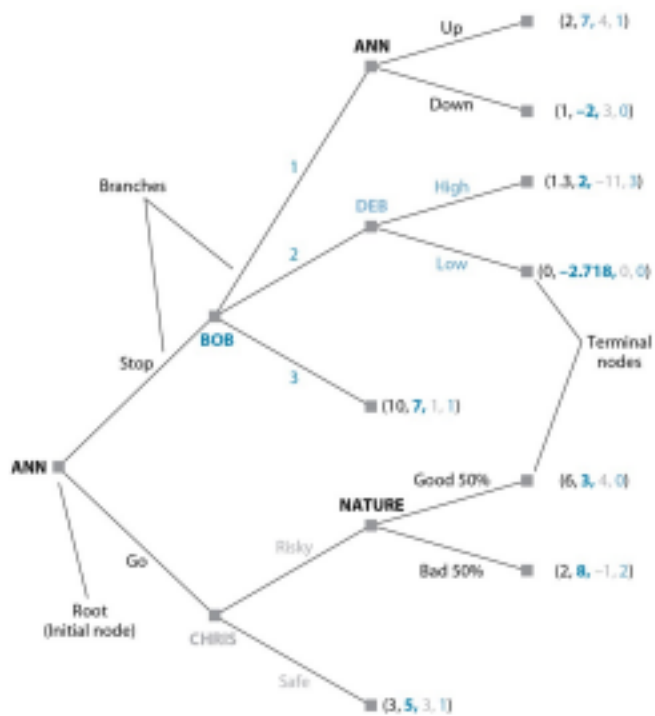
- ▶ We will visualize games using **game trees**
- ▶ Representing a game as a tree is known as the "extensive form" of a game
- ▶ The tree shows all components of a game: players, actions and strategies, payoffs

## Nodes and branches

- ▶ *Nodes* are points on the tree where choices are made
  - ▶ The first node is called the *root node*
  - ▶ The last nodes (without branches) are *terminal nodes*
- ▶ *Branches* show the actions available for the player to choose among at any node
- ▶ A node (and its branches) represent a "turn" for a player
- ▶ Payoffs are listed at the terminal nodes
  - ▶ Each player in the game gets a payoff at each node
  - ▶ Remember: Higher numbers are always better

## **External uncertainty**

- ▶ With external uncertainty, we introduce nature as a "player"
- ▶ Nature gets its own node, branches are possible outcomes
- ▶ Players calculate expected payoffs across the possible outcomes of nature's "choice"



## Moves vs strategy

- ▶ A choice of action at a node is called a *move*
- ▶ A strategy is a *complete plan of action*
  - ▶ A set of moves that will be performed if a certain situation arises
  - ▶ Strategies are collections of statements like "if  $X$  then  $Y$ " for *any possible*  $X$



### **Example**

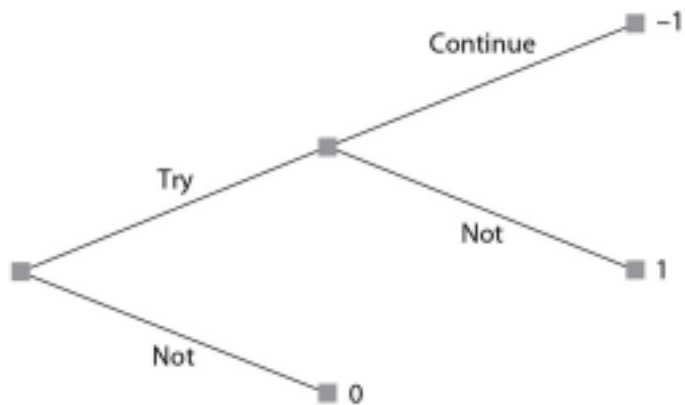
- ▶ How many strategies does Ann have?
- ▶ What are they?

## Strategies

- ▶ Strategies must include actions at *each node where a player can move*
- ▶ This includes the nodes that won't be reached if a player chooses a particular set of actions
- ▶ This is because hypothetical moves might help determine which moves should be chosen at earlier nodes
- ▶ Choices early in a game are affected by *expectations* about what will happen later in the game

## **Finding equilibria in game trees**

- ▶ Consider one person's decision tree (is this a game?)
- ▶ The player (Carmen) is considering whether or not to start smoking
- ▶ Carmen first decides whether to start, then decides whether to continue
- ▶ What should Carmen do?

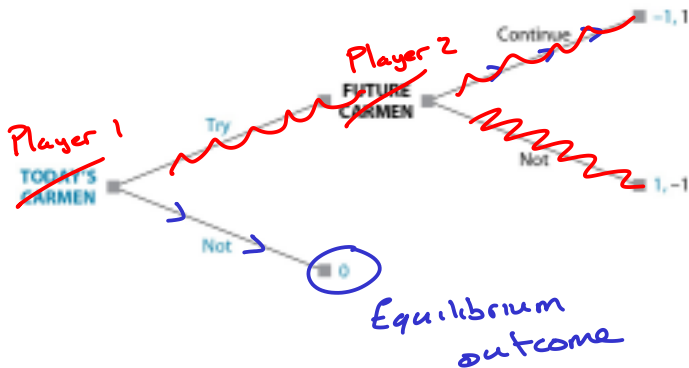


## **A decision tree as a game**

- ▶ Previous decision tree ignore that Carmen may become addicted if she starts smoking
  - ▶ Once addicted, quitting becomes worse (payoffs are lower)
- ▶ Carmen knows she may become addicted and that her payoffs might change if she starts smoking
- ▶ We can think of this as a game where the players are Carmen today and Carmen in the future (after the initial decision is made)
- ▶ Today's Carmen and future Carmen have different payoffs

Today's carmen:  $N$

Future carmen:  $C$



## Pruning

- ▶ Starting at the end, we can "prune" the branches that we know will not be chosen
- ▶ When there is one action remaining at the final nodes, this means that the "final" decision moves back to the previous node (rollback)
- ▶ Starting at the end and moving backward by pruning allows today's Carmen to choose the best option for herself
- ▶ When all players use rollback analysis, the result of the game is called a *rollback equilibrium*

## **Smoking game**

- ▶ What are the rollback equilibrium strategies?
- ▶ Can either player do better by changing their strategies?

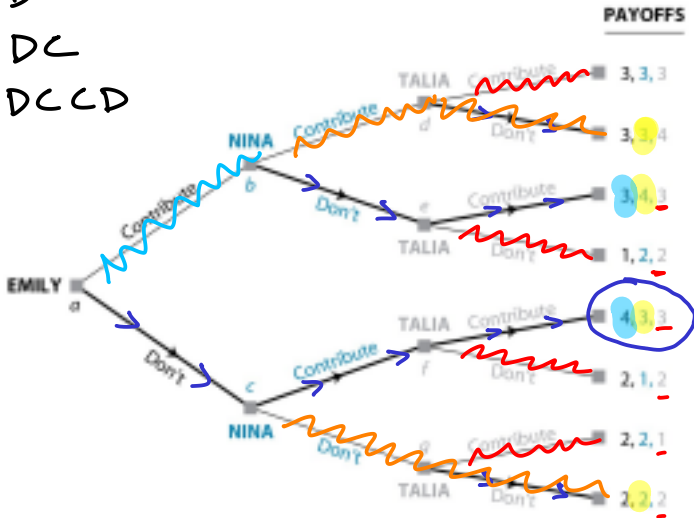


# Equilibrium strategies

Emily: D

Nina: DC

Talia: DCCD



Emily's strategies: (1) C  
(2) D

### Three-player game

- ▶ How many strategies does each player have?
- ▶ What is the rollback equilibrium?
- ▶ What are the rollback equilibrium strategies?

Nina: (1) Contribute always

(2) C if Emily C, D if Emily D

(3) D if Emily C, C if Emily D

(4) Always D

Nina: ① (C,C)

② C at b, D at C or (C,D) choice at C

③ (D,C)

④ (D,D)

choice at b

Talia's strategies in the form (d,e,f,g)

① (D,C,C,D)

⑧ (D,C,C,C)

⑭ CCDC

② (D,D,D,D)

⑨ (C,D,D,C)

⑮

③ (C,C,C,C)

⑩ (C,C,C,D)

⋮

④ (C,D,C,D)

⑪ (C,C,D,D)

use  
a  
system

⑤ (D,C,D,C)

⑫ (C,D,D,D)

⑥ (D,D,D,C)

⑬ (D,D,C,D)

⑦ (D,D,C,C)

CCCC

CCCD

CCDC

CCDD

C D C C

C D C D

C D D C

C D D D

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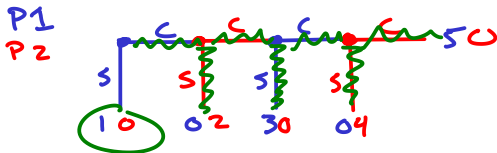
D D C D

D D D C

D D D D

## **Example: Ultimatum game**

- ▶ Player 1:
  - ▶ Choose how to split 10 units so that both players get at least one unit
- ▶ Player 2:
  - ▶ Choose to either:
    1. Accept the split (you get what player 1 chooses for you)
    2. Reject the split (neither player gets anything)



## Example: Centipede game (5 round version)

- ▶ What does the game tree look like?
- ▶ What are the strategies for each player?
- ▶ What is the rollback equilibrium outcome? 1, 0
- ▶ What are the rollback equilibrium strategies?
- ▶ Is this the outcome we observe in practice?

P1: SS, P2: SS

unique roll-back equilibrium

strategies:

P1: C, C

C, S

S, C

S, S

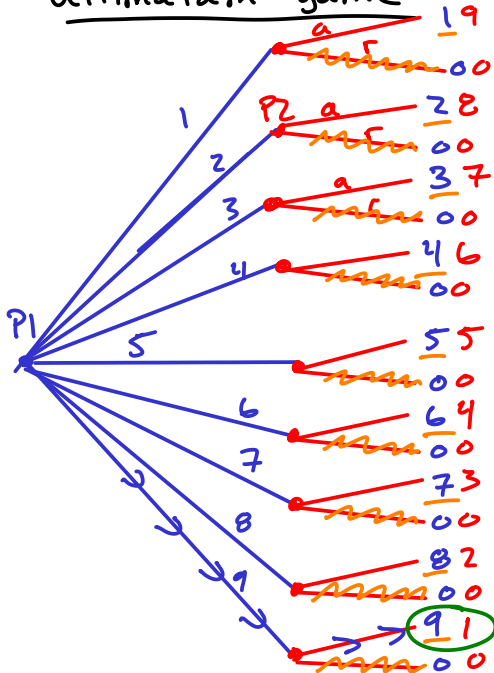
P2: C C

C S

S C

S S

# Ultimatum game



## Equilibrium strategies

P1: 9

P2: (a, a, a, a, ...)

Suppose P2 chooses:  
(a, a, a, a, a, a, a, a, r)

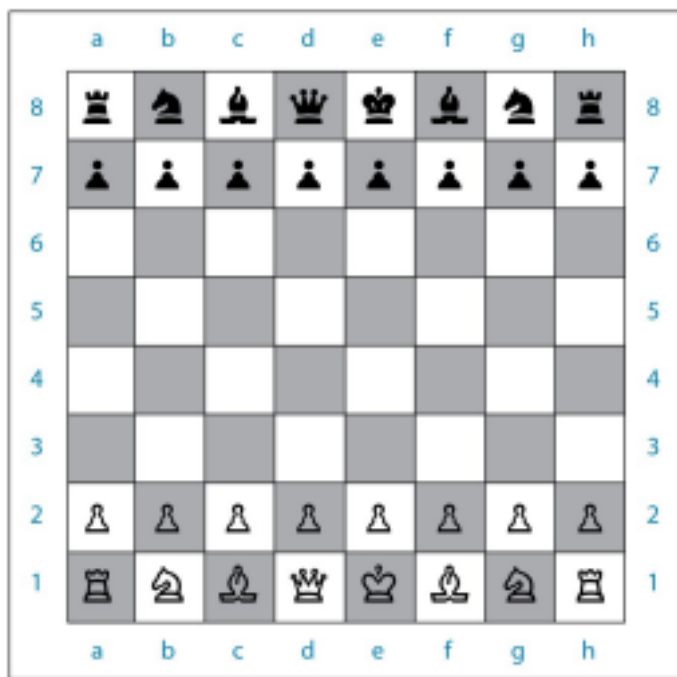
rollback  
equilibrium

## Limitations of rollback analysis

- ▶ Simple games can become difficult to express in extensive form
  - ▶ How many moves does the first player have in tic-tac-toe?
  - ▶ How many moves does the second player have?
- ▶ Some sequential games are *impossible* to express in extensive form!







## Chess

- ▶ 400 possible positions (nodes) after each player moves once
- ▶ 9 million after the third move
- ▶ 288 billion after the forth move
- ▶ 40 move game: More possible positions than fundamental particles in the universe