Uncertainty

ECON 420: Game Theory

Spring 2018

Announcements

- Homework 3 on Canvas
  - Due *Monday, May 21*
- Reading: Chapter 8

Uncertainty

- So far: Strategic uncertainty
  - Some players unaware of the actions of other players
  - Example: Simultaneous-move games
- Today: External uncertainty
  - "Nature" changes aspects of the game
  - Players cannot control external uncertainty, must take it into account when making decisions

Expected Utility Theory

- Events that happen according to some probability distribution are called *gambles*
- Agents are able to rank gambles by comparing the *expected utility* that they would receive from the potential outcomes of the gamble
- The utility that we will use is *von Neumann-Morgenstern (VNM)* utility

Risk preference

- When there is uncertainty we can calculate the *expected value* of a gamble
- But people do not just consider expected value when making decisions
- Some people might be willing to pay to avoid risk (risk aversion)

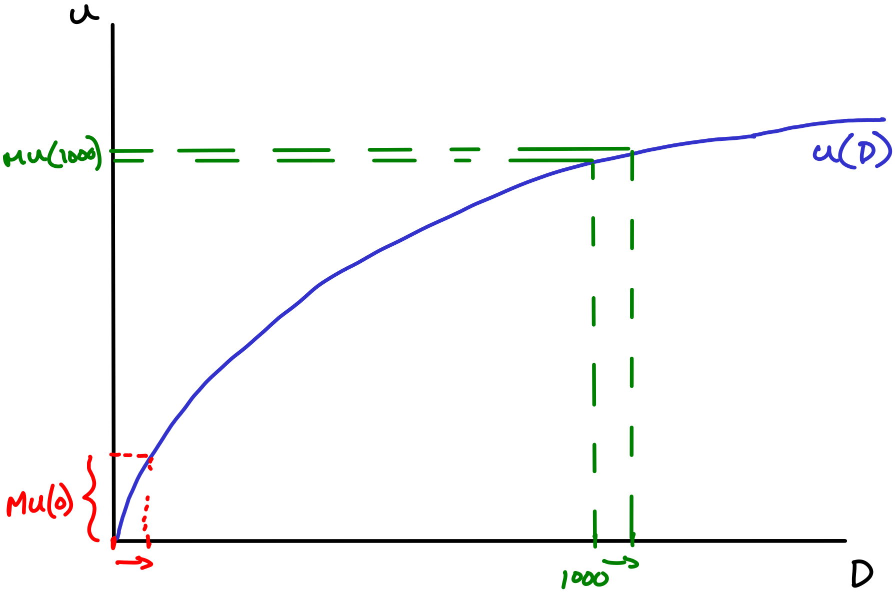
Example

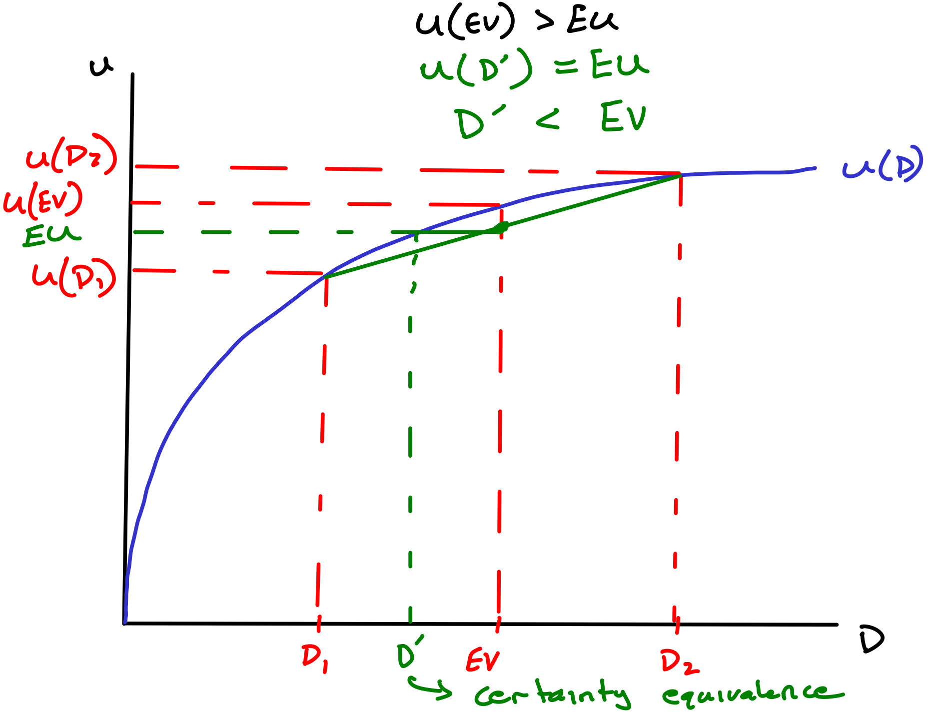
- Suppose I flip a coin. If heads, you get \$100. If tails, you get \$0.
  - What is the expected value?
  - How much would you pay to play this game?
- Suppose instead the payoffs are \$1 million for heads, \$0 for tails.

VNM Utility and Risk Preference

- Outcomes are denoted D (dollars)
- Agents in the model have preferences over outcomes represented by utility $u = u(D)$
- The risk preference of the agent depends on the concavity of the utility function u
- Agents with *diminishing marginal utility* are risk averse
  - Concave utility function

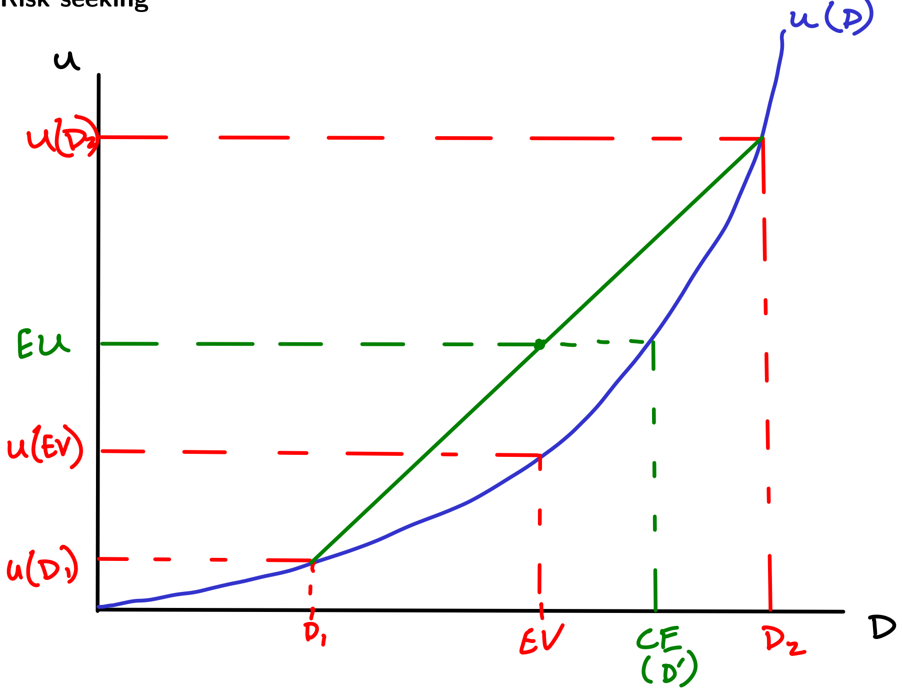
Risk aversion





u(EV) > Eu
u(D') = Eu
D' < EV
u
u(D2)
u(EV)
Eu
u(D1)
u(D)
D
D1
D'
EV
D2
certainty equivalence

Risk seeking



Example

- A farmer's crop yield depends on weather
- Farmer gets good weather with 50%
- Yield with good weather is \$160,000, yield in bad weather is \$40,000
- Farmer has VNM utility $u(D) = \sqrt{D}$

Risk sharing

- Risk averse agents willing to pay to remove risk
- Agents can therefore benefit from trading *state-contingent claims* with one another
  - You agree to pay someone else if you have a good outcome, someone else pays you if you have a bad outcome

Example

- Suppose there is another farmer that has the same weather probability and outcomes (weather probability is independent of first farmer)
- Farmers agree to a contract: If one farmer gets good luck and the other gets bad luck, lucky farmer pays $60,000 to the unlucky farmer
- Are the farmers better off?

Example

- Now suppose the other farmer faces no uncertainty and will earn \$100,000 with probability 1
- The farmer with risk is willing to accept their certainty equivalence instead of the gamble
- Is the riskless farmer willing to buy the risk in exchange for the certainty equivalence?

Example

- Now suppose the farmer without risk is *risk neutral*
- What is the maximum that this farmer is willing to pay for the gamble?

Insurance and risk

- Suppose there are thousands of farmers with identical risk/outcomes
- A single entity (insurance company) can buy the risk of all of the farmers and make them better off
- Law of large numbers says that the insurance company will earn the expected value of the gamble

Manipulating Risk

- Sometimes agents have control over risk and can use it to their advantage
- By increasing risk, the probability of "tail events" increases
- This is why underdogs in sports often choose risky actions

Example

- A basketball team scores 60 points per game on average
- They are playing a better opponent and must score at least 80 points to win
- How can this team maximize their chances of winning?

Cheap Talk

- In coordination games, players may be able to costlessly communicate before the game begins
- This might allow players to better coordinate on preferred outcomes

		SALLY	
		Starbucks	Local Latte
HARRY	Starbucks	1, 1	0, 0
	Local Latte	0, 0	2, 2