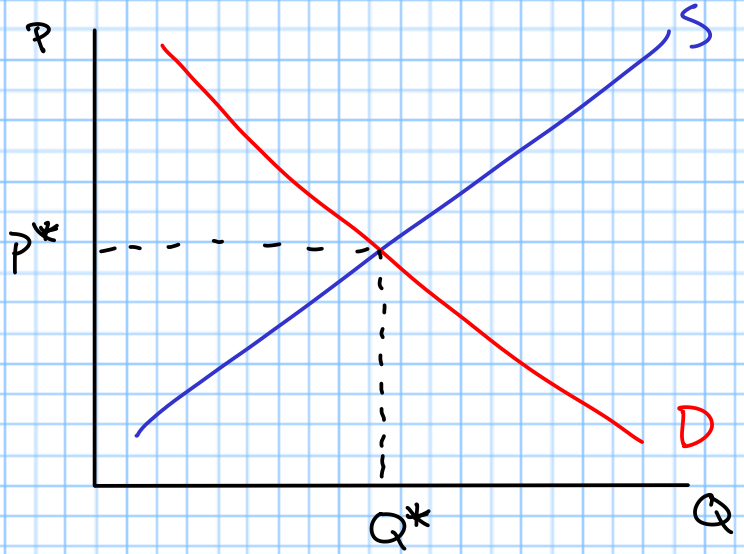
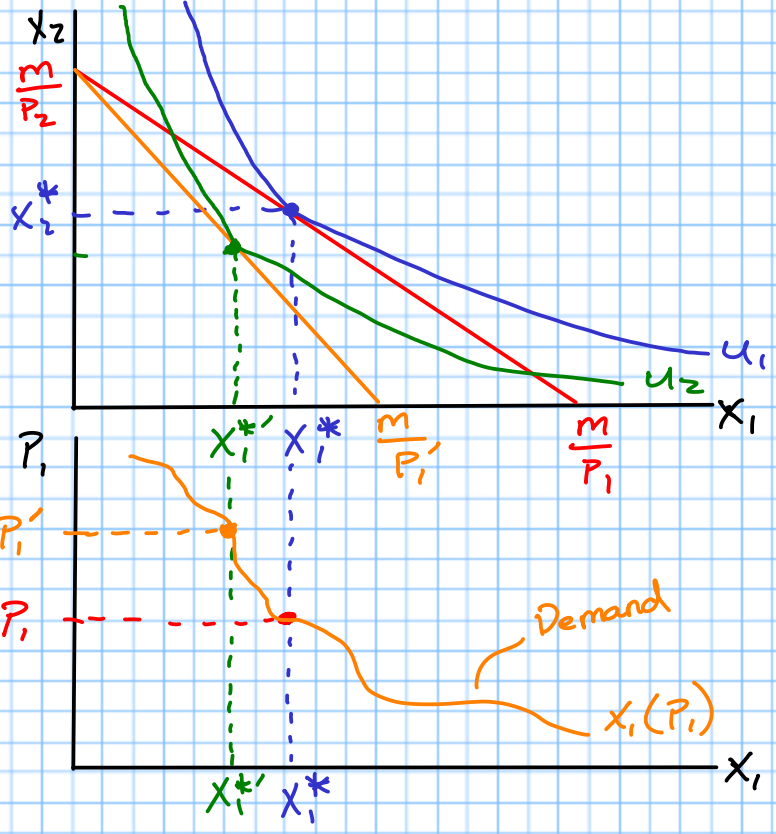
Demand

ECON 201:



- Consider a consumer with rational preferences and utility $u(x_1, x_2)$. There are prices P_1 and P_2, and the consumer has an income m



$x_1(p_1)$ is a function (demand function)

input: price p_1

output: optimal quantity given the price

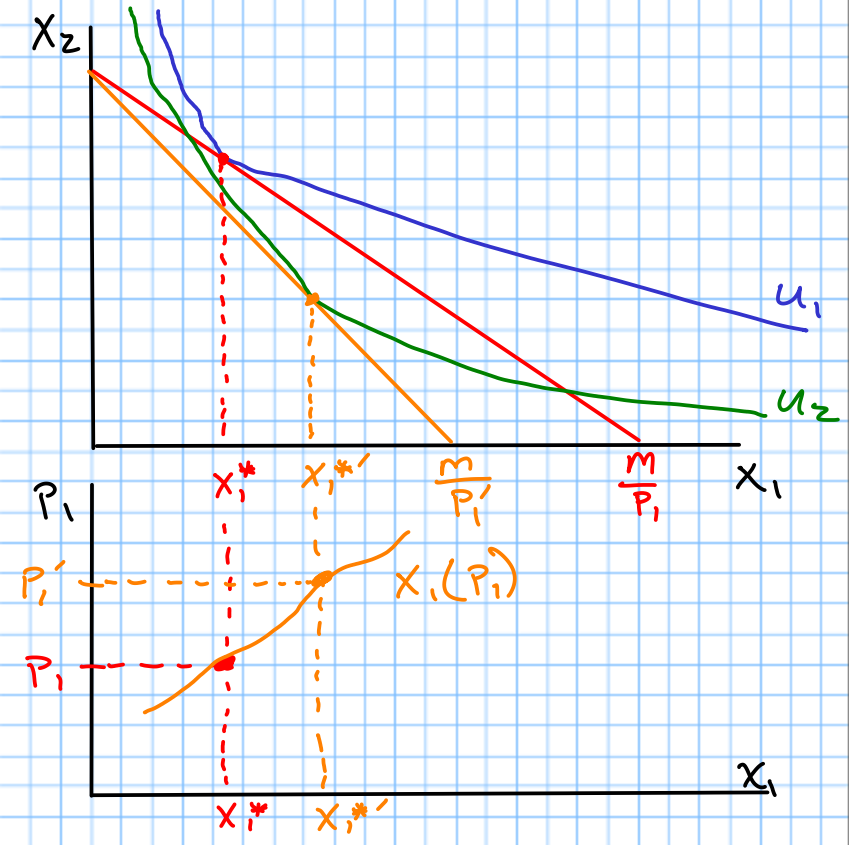
Slope of demand curves

$$\text{slope} = \frac{dx_1}{dp_1}$$

"Law of demand":

$$\frac{dx_1}{dp_1} < 0$$

- Demand curves are downward sloping



- In the rational choice model, the law of demand does not necessarily hold
- Definition: If $\frac{dx_1}{dP_1} > 0$,

  then x_1 is said to be a Giffen good

Deriving demand mathematically

Consumer has utility

$$u(x_1, x_2) = x_1^3 x_2^6$$

$m = 36$ $P_1 = ?$ (leave it as a variable)

$P_2 = 9$

① $P_1x_1 + 9x_2 = 36$

② $\frac{3x_2}{6x_1} = \frac{P_1}{9}$

$\frac{x_2}{2x_1} = \frac{P_1}{9}$

$9x_2 = 2P_1x_1$

$x_2 = \frac{2}{9}P_1x_1$

$P_1x_1 + 9\left(\frac{2}{9}P_1x_1\right) = 36$

$P_1x_1 + 2P_1x_1 = 36$

$3P_1x_1 = 36$

$P_1x_1 = 12 \longrightarrow \underline{x_1 = \frac{12}{P_1}}$

Slope:

$$\frac{dx_1}{dP_1} = -\frac{12}{P_1^2} < 0$$

$x_1 = 12P_1^{-1}$

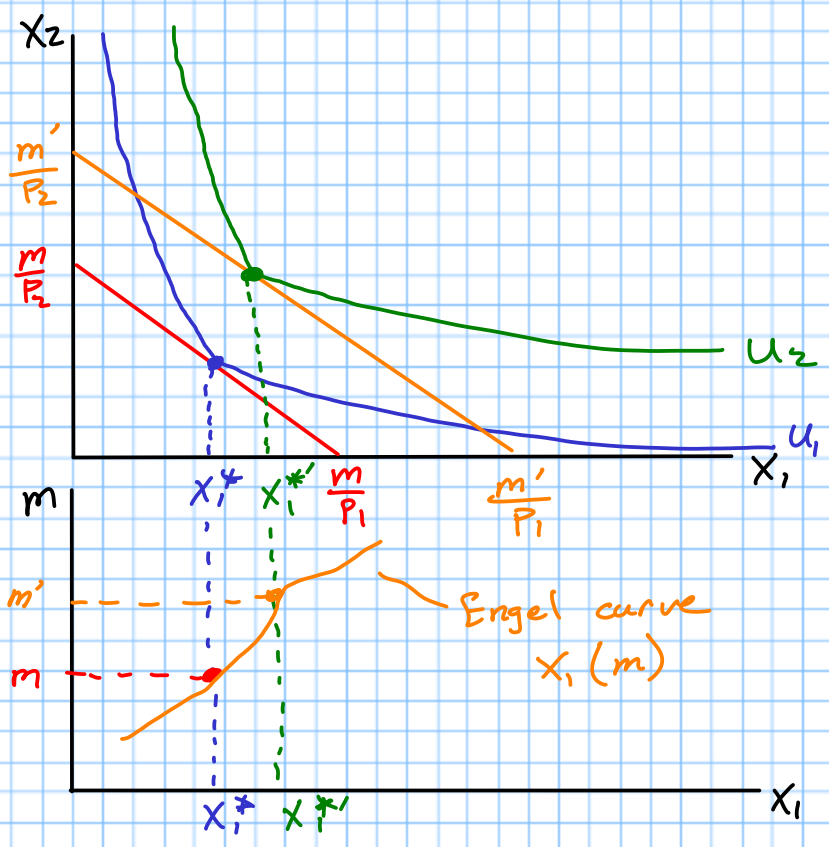
$\frac{dx_1}{dP_1} = (-1)12P_1^{-2}$

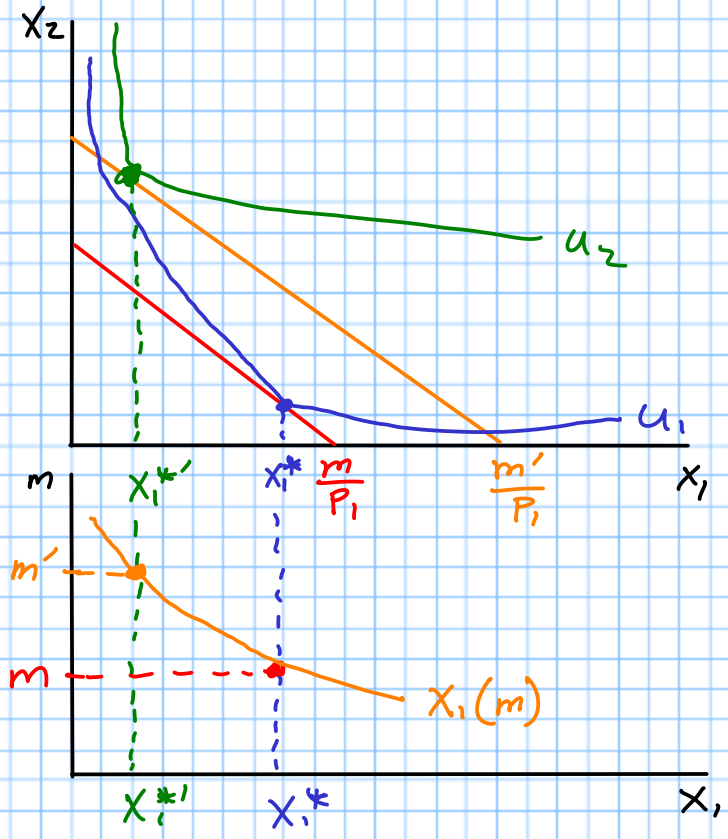
$= -\frac{12}{P_1^2}$

Cobb-Douglas utility functions never represent Giffen goods

Engel curves

- Demand curve: x_1 as a function of p_1
- Engel curve: x_1 as a function of m





Normal goods:

$$\frac{dx_1}{dm} > 0$$

→ Engel curve slopes upward

Inferior goods

$$\frac{dx_1}{dm} < 0$$

→ Engel curve slopes downward

Engel curves mathematically

$u(x_1, x_2) = x_1^5 x_2^{25}$

$p_1 = 10 \quad p_2 = 7 \quad m = ?$

① $10x_1 + 7x_2 = m$

② $\frac{5x_2}{25x_1} = \frac{10}{7}$

$\frac{x_2}{5x_1} = \frac{10}{7}$

$7x_2 = 50x_1$

$x_2 = \frac{50}{7}x_1$

$10x_1 + 7\left(\frac{50}{7}x_1\right) = m$

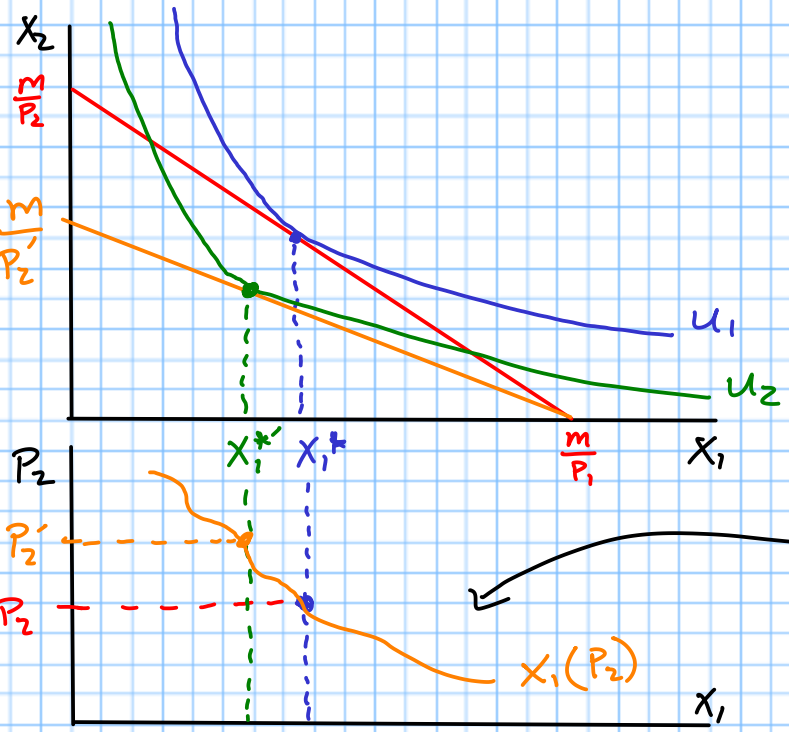
$10x_1 + 50x_1 = m$

$60x_1 = m$

$x_1 = \frac{m}{60}$

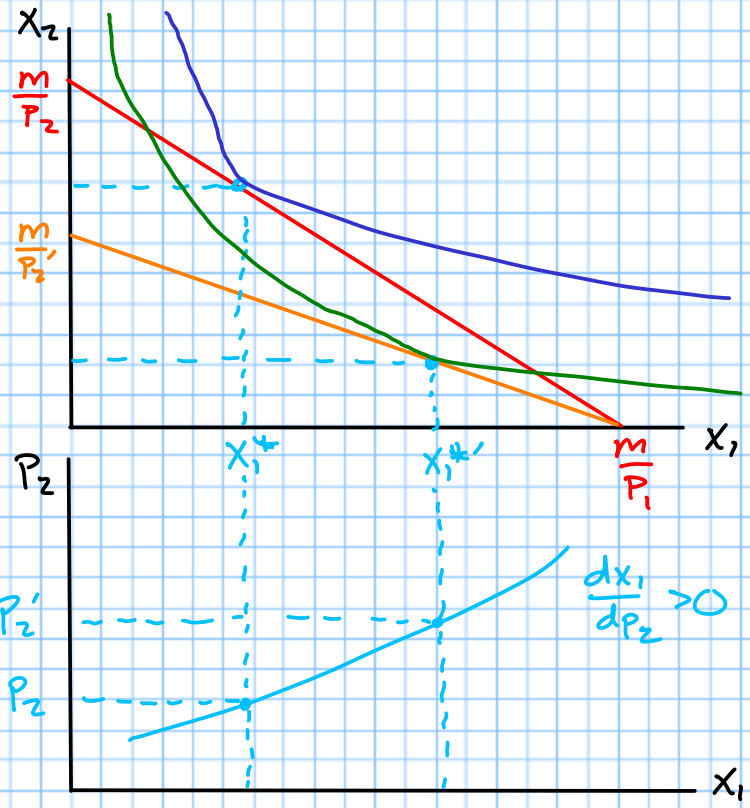
Result: Cobb-Douglas functions always represent normal goods

Complements and Substitutes



- If $\frac{dx_1}{dP_2} < 0$ → complements
- If $\frac{dx_1}{dP_2} > 0$ → substitutes

where $\frac{dx_1}{dP_2}$ is the slope of this line


x_2
$\frac{m}{p_2}$
$\frac{m}{p_2'}$
x_1
x_1^*
$x_1^{*\prime}$
$\frac{m}{p_1}$
p_2
p_2'
p_2
$\frac{dx_1}{dp_2} > 0$
x_1