- Health causes income

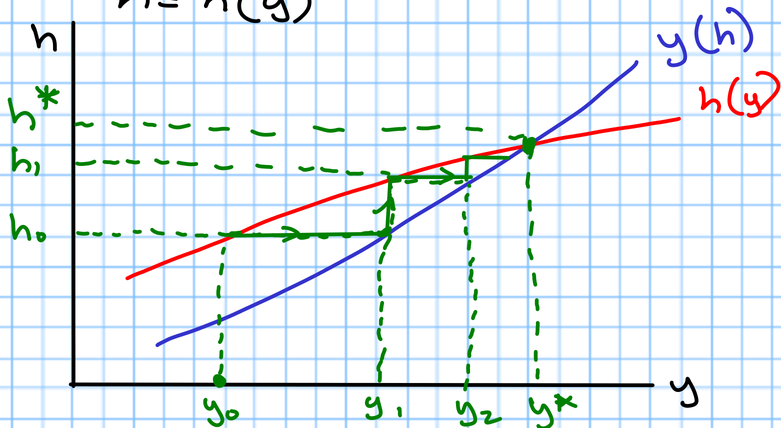
$$y = y(h)$$

$\uparrow$ income $\quad \uparrow$ heath

income is a function of health

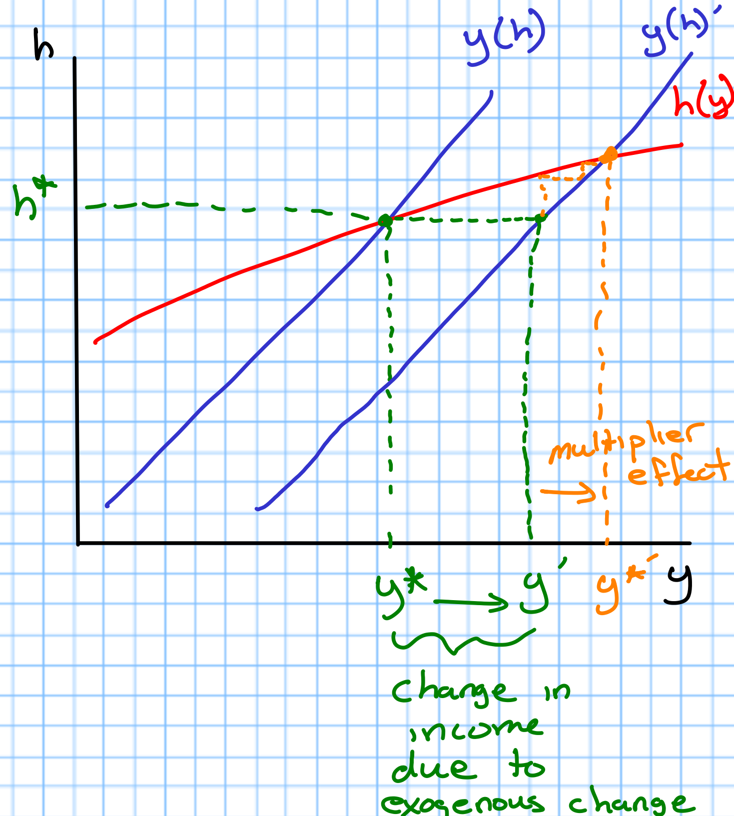
- Income causes health:

$$h = h(y)$$



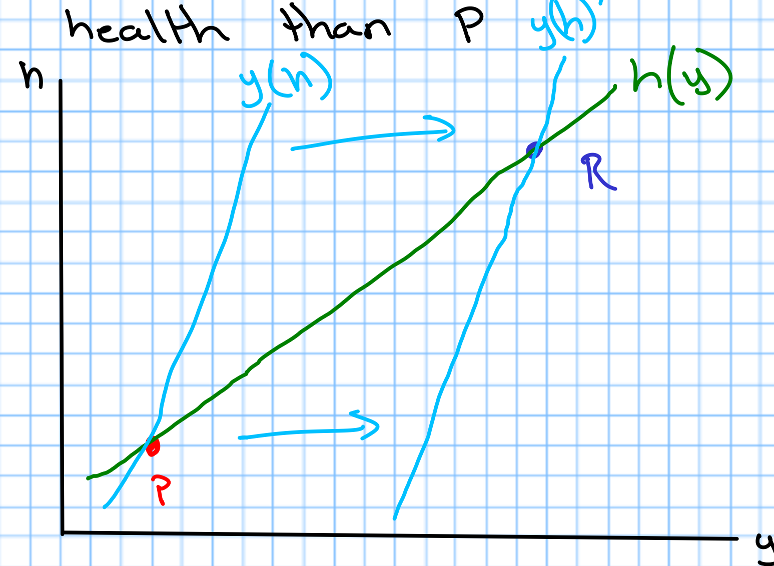
y_0 is initial level of income

- Suppose there is an exogenous change to income
  - → something that causes income to change, but doesn't affect health
  - → productivity increase (technology), business cycle, government expenditures, etc



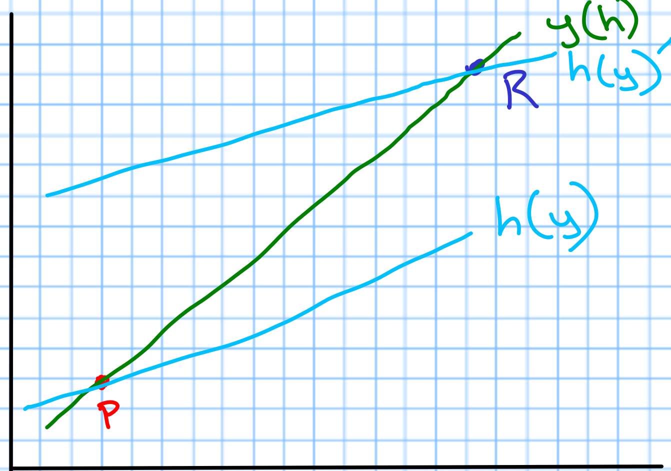
Let's suppose we observe two countries, R and P

We observe that R has higher income and better health than P



If P wants to increase y and h, what should they do?

→ Target policies that promote income growth



→ P should target health policies

→ It's difficult (impossible?) to determine which of these two situations is describing the difference between R and P

Cobb-Douglas production with human capital

Before: $Y_t = K_t^{\alpha} L_t^{1-\alpha}$

Now: $Y_t = K_t^{\alpha} (hL_t)^{1-\alpha}$

Assume $L_t = \bar{L}$ $\underbrace{\forall t}_{\text{at every point in time}}$

Express $F(K_t, hL_t)$ in per-worker terms

$$\frac{Y_t}{\bar{L}} = \frac{K_t^{\alpha}(h\bar{L})^{1-\alpha}}{\bar{L}}$$

$$y_t = K_t^{\alpha} h^{1-\alpha} (\bar{L})^{1-\alpha} (\bar{L})^{-1}$$

$$y_t = K_t^{\alpha} h^{1-\alpha} (\bar{L})^{-\alpha}$$

$$y_t = \frac{K_t^{\alpha} h^{1-\alpha}}{(\bar{L})^{\alpha}}$$

$$\underline{y_t = k_t^{\alpha} h^{1-\alpha}} \checkmark$$

Solow model

$$K_{t+1} = K_t + I_t - D_t$$

$$\frac{K_{t+1}}{\bar{L}} = \frac{K_t}{\bar{L}} + \frac{\gamma Y_t}{\bar{L}} - \frac{\delta K_t}{\bar{L}}$$

$$k_{t+1} = k_t + \gamma y_t - \delta k_t$$

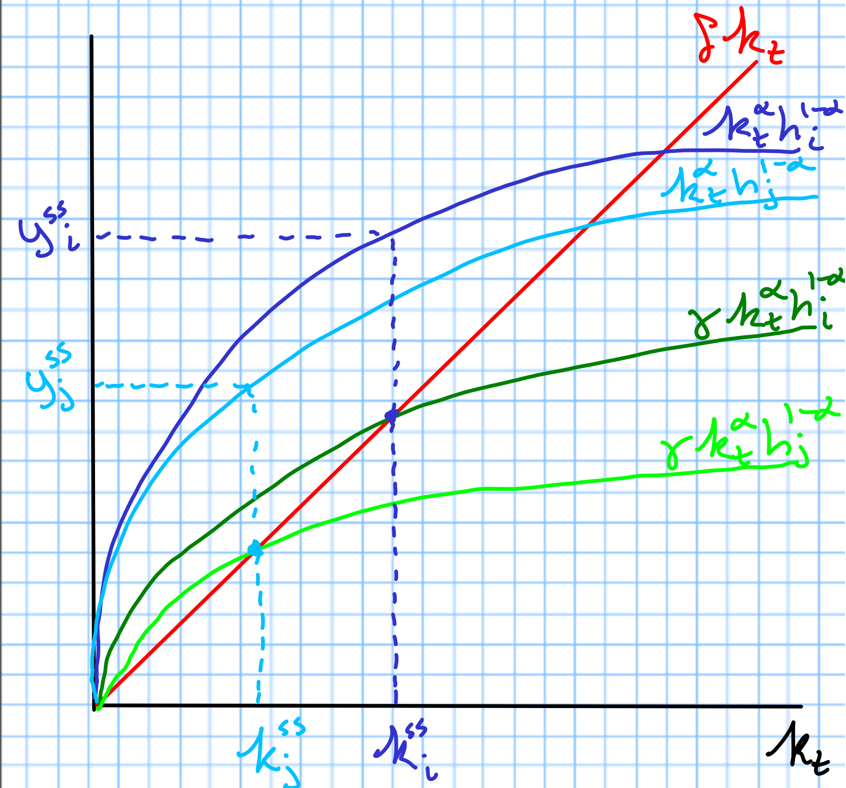
$$k_{t+1} = k_t + \gamma k_t^{\alpha} h^{1-\alpha} - \delta k_t$$

$$k_{t+1} - k_t = \gamma k_t^{\alpha} h^{1-\alpha} - \delta k_t$$

$$\Delta k_t = \underbrace{\gamma k_t^{\alpha} h^{1-\alpha}} - \underbrace{\delta k_t}$$

Suppose there are two countries i and j identical in every way except

$$h_i > h_j$$



$\Delta k_t = \gamma k_t^{\alpha} h^{1-\alpha} - \delta k_t$

Solve for steady-state analytically:

$\Delta k_t = 0$

$0 = \gamma k_t^{\alpha} h^{1-\alpha} - \delta k_t$

$k_t^{-\alpha}(\delta k_t) = \gamma k_t^{\alpha} h^{1-\alpha} k_t^{-\alpha}$

$\delta k_t^{1-\alpha} = \gamma h^{1-\alpha}$

$k_t^{1-\alpha} = \frac{\gamma}{\delta} h^{1-\alpha}$

$(k_t^{1-\alpha})^{1/1-\alpha} = \left(\frac{\gamma}{\delta} h^{1-\alpha}\right)^{1/1-\alpha}$

$k^{ss} = \left(\frac{\gamma}{\delta}\right)^{1/1-\alpha} h$

$y^{ss} = (k^{ss})^{\alpha} h^{1-\alpha}$

$y^{ss} = \left[\left(\frac{\gamma}{\delta}\right)^{1/1-\alpha} h\right]^{\alpha} h^{1-\alpha}$

$y^{ss} = \left(\frac{\gamma}{\delta}\right)^{\alpha/1-\alpha} \underbrace{h^{\alpha} h^{1-\alpha}}$

$y^{ss} = \left(\frac{\gamma}{\delta}\right)^{\alpha/1-\alpha} h$

Suppose there are two countries that are identical in every way except for human capital. How much more income will the country with higher human capital have?

$$y_i^{ss} = \left(\frac{\gamma}{\delta}\right)^{\alpha/1-\alpha} h_i$$

$$y_j^{ss} = \left(\frac{\gamma}{\delta}\right)^{\alpha/1-\alpha} h_j$$

$$\frac{y_i^{ss}}{y_j^{ss}} = \frac{h_i}{h_j}$$

Returns to education:

$$\begin{cases} 1^{st}\ 4 \text{ years: } 13.4\% \text{ per year} \\ \text{years } 5-8\text{: } 10.1\% \text{ per year} \\ >8 \text{ years: } 6.8\% \text{ per year} \end{cases}$$

There's some "raw" human capital $h_0 \rightarrow$ labor input of a worker with no education

Example: 2 countries i and j, Average years i: 10 years

Average years j: 4 years

$$h_i = h_0 \cdot (1.134)^4 (1.101)^4 (1.068)^2$$

$$h_j = h_0 \cdot (1.134)^4$$

$$\frac{y_i}{y_j} = (1.101)^4 (1.068)^2$$

$$= 1.68$$

$$h_i = \underset{\text{initial}}{h_0} \cdot \underset{\text{return } 1^{st} \text{ year}}{(1+.134)} \underset{2^{nd}}{(1+.134)} \underset{3^{rd}}{(1+.134)} \underset{4^{th}}{(1+.134)} \underset{5^{th}}{(1+.101)}$$