Normative Theory

ECON 499: The Economics of Inequality

Winter 2018

Readings (on Canvas):

- Atkinson section 3.4
- Sen (1979) *(recommended)*

Normative theory

- We have seen various ways to measure inequality and different properties of measures
- Indices can disagree with one another!
- Can we project our personal (or societal) values onto different distributions?
- When can we say that one distribution is *better* or *worse* than another?

Positive vs normative

- Much of economics can be broken down into *normative* vs *positive* analysis
- Positive claims can be verified, normative claims reflect values
- Positive claim: The Gini index for the US in 2016 was 45.0
- Normative claim: The Gini index for the US should be lower

Normative theory

- Start with a set of values, then see what the implications of those values are
- Alternatively: Start with result, then see what kind of values we would need to have to make that result hold
- What sort of preferences might *rationalize* certain statements about inequality?

Economic agents

- We assume people derive utility from their consumption of goods and services
- The more goods and services people are able to choose from, the better off they are
- Additional income increases the choice set, more goods and services available to consume
- Income creates *indirect* utility
- $u(x_i)$ is the indirect utility derived from an income of x_i

Utility and social well-being

- Idea: Since utility measures well-being, can we use it to compare income distributions?
- Can the utility of individuals within the income distribution help inform which distributions are preferable?

Diminishing marginal utility of income

- Our enjoyment of additional income gets lower as our income gets higher
- I'd be extremely happy if someone gave me $100 --- Mark Zuckerberg probably wouldn't care very much
- We should weigh lower incomes more heavily than higher incomes

Utilitarianism

- The optimal distribution of income may be achieved by maximizing aggregate utility
- Same as equating marginal utility for each person
- Each additional unit of income will go to the person who will "enjoy" it the most

Problems

- Utility functions are not unique (ECON 311). Which do we choose?
- People can have vastly different marginal utilities of income
- Sen: Consider a person who experiences a physical disability. They will derive less utility from a given level of income, since they have fewer opportunities to spend that money
- Compare that person to a "pleasure-wizard" who derives large satisfaction from small increases in income

Social welfare

- Utility only measures well being from your own income
- People care about the income of other people
- We can capture this with a *social welfare function*
- A SWF is like a utility function for society as a whole
- Rather than think of the utility derived from one income, we think of the total utility that is gained from everyone at all points in the distribution

$$W(x) = W(x_1 + x_2 + ... + x_N)$$

Rawlsian SWF interpretation

- John Rawls (1971): Think about distributions from behind a "veil of ignorance"
- Before you are born, you get to choose which society you live in
- You have an equal chance of being born anywhere within that society
- Societies with high inequality mean that there is a higher risk of being poor
- The SWF is therefore just a measure of *risk-aversion* --- how much of a gamble are you willing to take?

Von-Neumann-Morgenstern Utility

- People have preferences over outcomes that occur probabilistically
- With certain assumptions on preferences (similar to 311 rationality assumptions), we can order preferences over risk

$$U(x, p) = p_1 u(x_1) + p_2 u(x_2) + ... + p_N u(x_N)$$

Risk aversion

- Agents are *risk averse* if they are willing to take an expected loss to avoid uncertainty
- This occurs whenever agents have diminishing marginal utility of income

CRRA Utility

- If an agent's risk aversion is constant for any level of income, then we say that they have *constant relative risk aversion*
- Utility can be represented as

$$u(y) = \frac{y^{1-\epsilon}}{1-\epsilon}$$

- ϵ is the amount of risk aversion
- This is the *same* ϵ from the Atkinson measure!

Atkinson measure, revisited

$$I_A(x, \epsilon) = 1 - \left(\frac{1}{N} \sum_{i=1}^{N} \left(\frac{x_i}{\mu} \right)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}$$

- A person with CRRA utility (given ϵ) will always prefer a distribution with a lower Atkinson index!

Inequality aversion

Another way to interpret ϵ:

- Imagine we are going to transfer $100 from a rich person to a poor person
- We put the money (coins) in a bucket to bring to the poor person
- The bucket has a hole, and the money slowly falls out in transit
- How much money are we prepared to lose before we decide not to go through with the transfer?

Leaky bucket interpretation

Distribution A

	Obie	Michael	Llewelyn	Rudy	Kitty
$x:$	2	5	9	20	30

Take 10 from Kitty to give to Obie, but we lose $\delta \times 10$ along the way:

Distribution B

	Obie	Michael	Llewelyn	Rudy	Kitty
$x':$	$\mathbf{2 + (10 - \delta \times 10)}$	5	9	20	$\mathbf{30 - 10}$

Leaky bucket interpretation

- $\delta = 0$: Obie gets all of the transfer
- $\delta = 1$: Obie gets none of the transfer
- What is the highest value of δ that we are willing to have?
- δ is our measure of *inequality aversion*
- $\epsilon = \frac{1}{2\delta}$

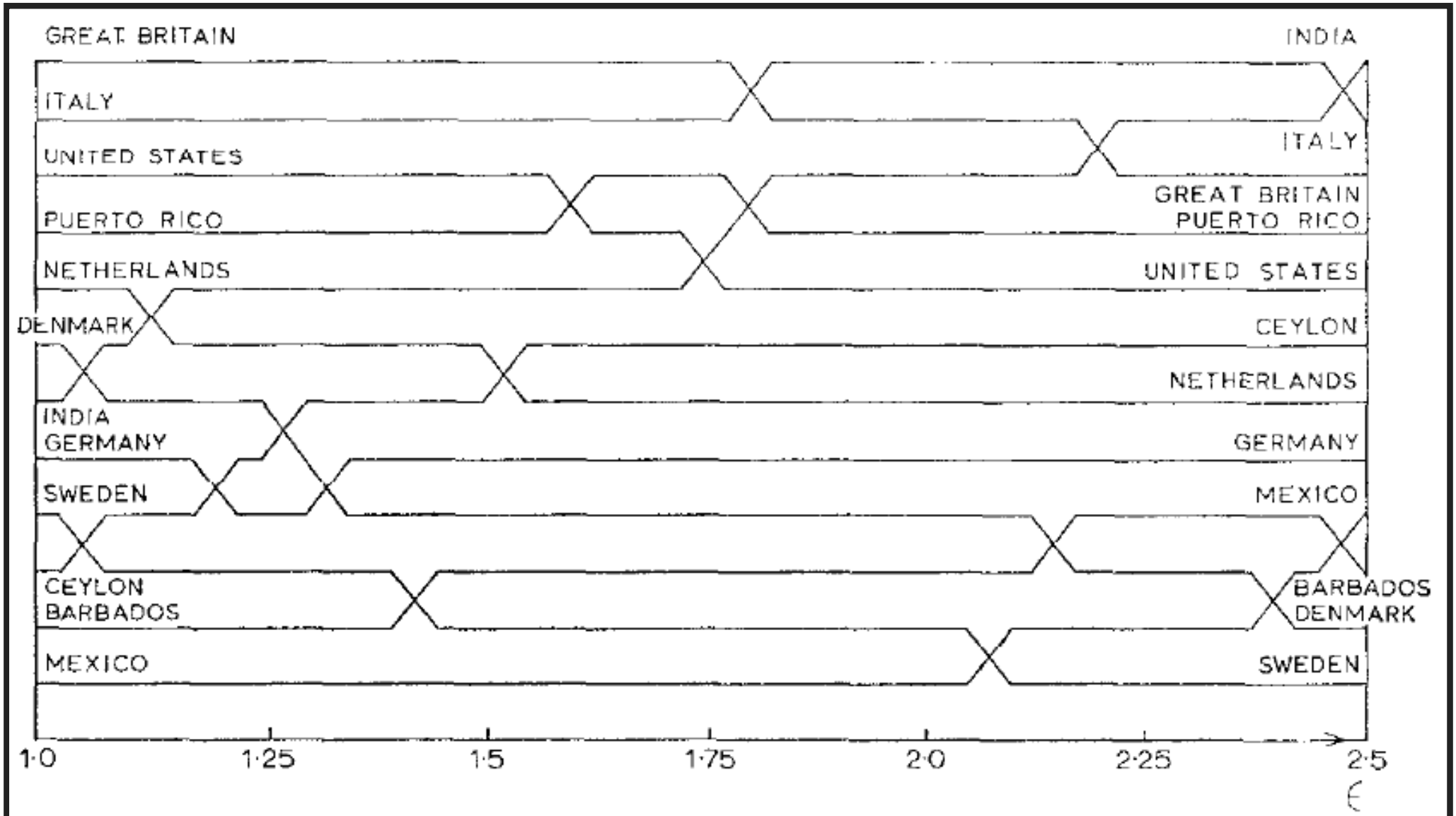


FIG. 7. Ranking of income distributions for different values of ϵ.

Data from 1970! Much different today.

TABLE I

Conventional and Equally Distributed Equivalent Measures of Inequality

Country	Year	1[a] Gini Coefficient	2[a] Standard Deviation of Logarithms	3[a] Coefficient of Variation	4[b] Equally Distributed Equivalent Measure $\epsilon = 1.0$	5[b] $\epsilon = 1.5$	6[b] $\epsilon = 2.0$
India	1950	0.410 (8–)	0.305 (3)	0.901 (11)	0.297 (7)	0.359 (5)	0.399 (3)
Ceylon	1952–1953	0.427 (10)	0.341 (6)	0.876 (10)	0.311 (10)	0.395 (6)	0.457 (6)
Mexico	1957	0.498 (12)	0.395 (12)	1.058 (12)	0.401 (12)	0.492 (12)	0.550 (12)
Barbados	1951–1952	0.436 (11)	0.383 (10)	0.842 (9)	0.315 (11)	0.433 (10)	0.524 (10)
Puerto Rico	1953	0.394 (4)	0.317 (4)	0.783 (8)	0.256 (4)	0.341 (4)	0.408 (4)
Italy	1948	0.378 (3)	0.301 (1)	0.748 (3)	0.241 (2)	0.319 (2)	0.379 (1)
Great Britain	1951–1952	0.356 (1)	0.304 (2)	0.673 (1)	0.224 (1)	0.311 (1)	0.384 (2)
West Germany	1950	0.410 (8=)	0.369 (8)	0.773 (6)	0.299 (8)	0.411 (8)	0.498 (8)
Netherlands	1950	0.406 (6=)	0.355 (7)	0.781 (7)	0.290 (5)	0.395 (7)	0.478 (7)
Denmark	1952	0.401 (5)	0.381 (9)	0.751 (4)	0.292 (6)	0.418 (9)	0.521 (9)
Sweden	1948	0.406 (6=)	0.393 (11)	0.752 (5)	0.303 (9)	0.435 (11)	0.540 (11)
United States	1950	0.372 (2)	0.325 (5)	0.705 (2)	0.242 (3)	0.339 (3)	0.421 (5)

Social choice theory

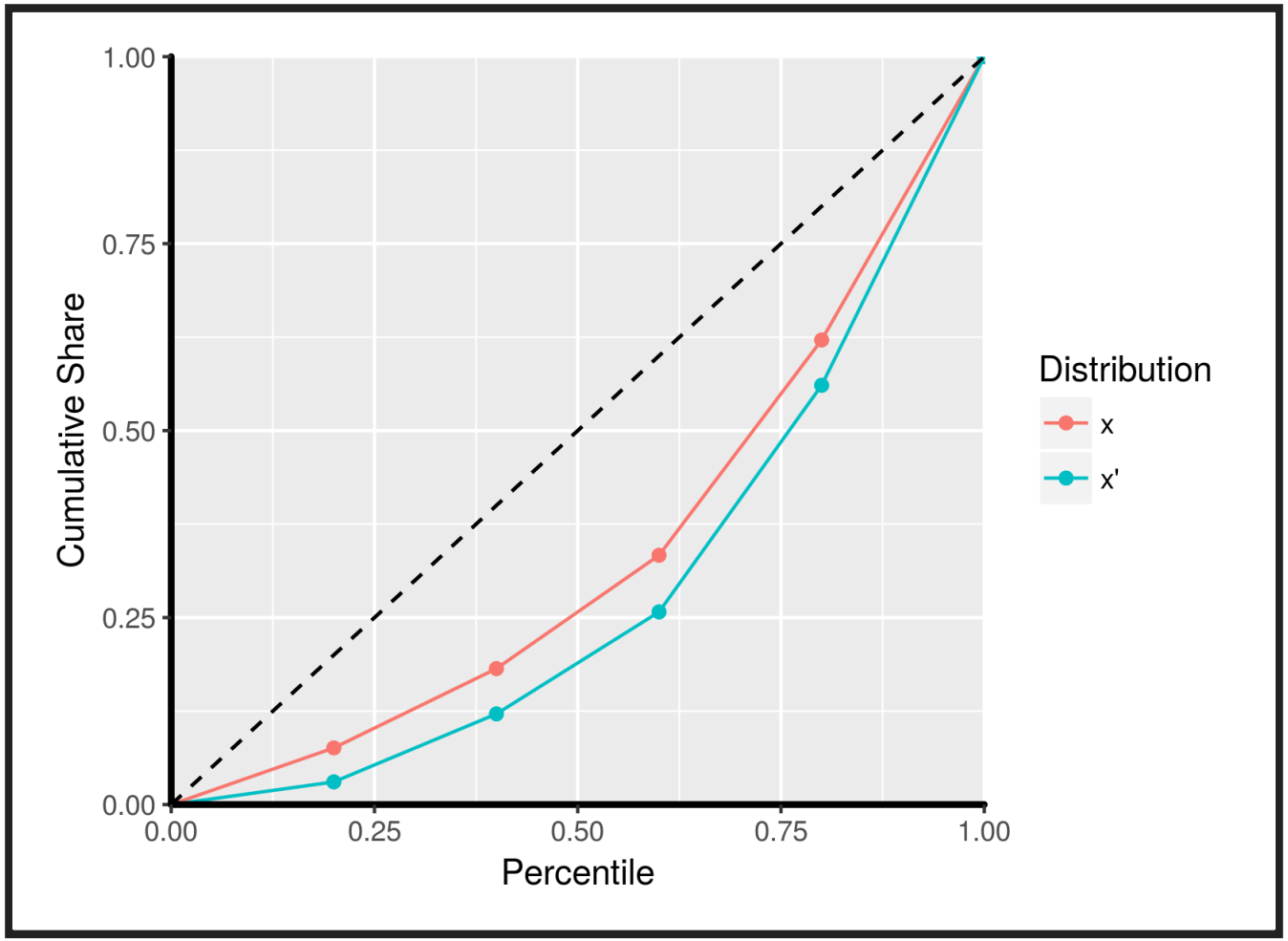
- Level of risk aversion matters for how we rank distributions
- How can we agree on which aversion parameter to use as a society?
- Do we even agree that we have CRRA?
- Arrow's Impossibility Theorem: Under certain (general) conditions, it's impossible to find a "best" social welfare function!
- Can we say anything about more general utility specifications?

Lorenz domination

- A distribution x is said to *Lorenz dominate* distribution x' if the Lorenz curve for x is above x'

Example:

$$x = (5, 7, 10, 19, 25)$$
$$x' = (2, 5, 9, 20, 30)$$



x Lorenz dominates x'

Atkinson theorem

Suppose x and x' are two distributions with the same mean. Any risk-averse person (diminishing marginal utility of income) will *always* prefer a society with distribution x if x Lorenz dominates x'.

- A SWF that weighs the incomes of poorer people more heavily than richer people will be greater under x

Example

Distribution A

	Obie	Michael	Llewelyn	Rudy	Kitty
x :	2	5	9	20	30
Cumulative income:	2	7	16	36	66
Cumulative share:	0.03	0.11	0.24	0.54	1

Distribution B

	Obie	Michael	Llewelyn	Rudy	Kitty
x' :	20	50	90	200	300
Cumulative income:	20	70	160	360	660
Cumulative share:	0.03	0.11	0.24	0.54	1

Generalized Lorenz curves

- The Atkinson theorem doesn't hold if the mean is not the same
- What happens if average income doubles, but inequality is slightly worse?
- We can instead use *generalized Lorenz curves*, which are formed by multiplying the cumulative share by the mean

Generalized Lorenz example

Distribution A (mean = 13.2)

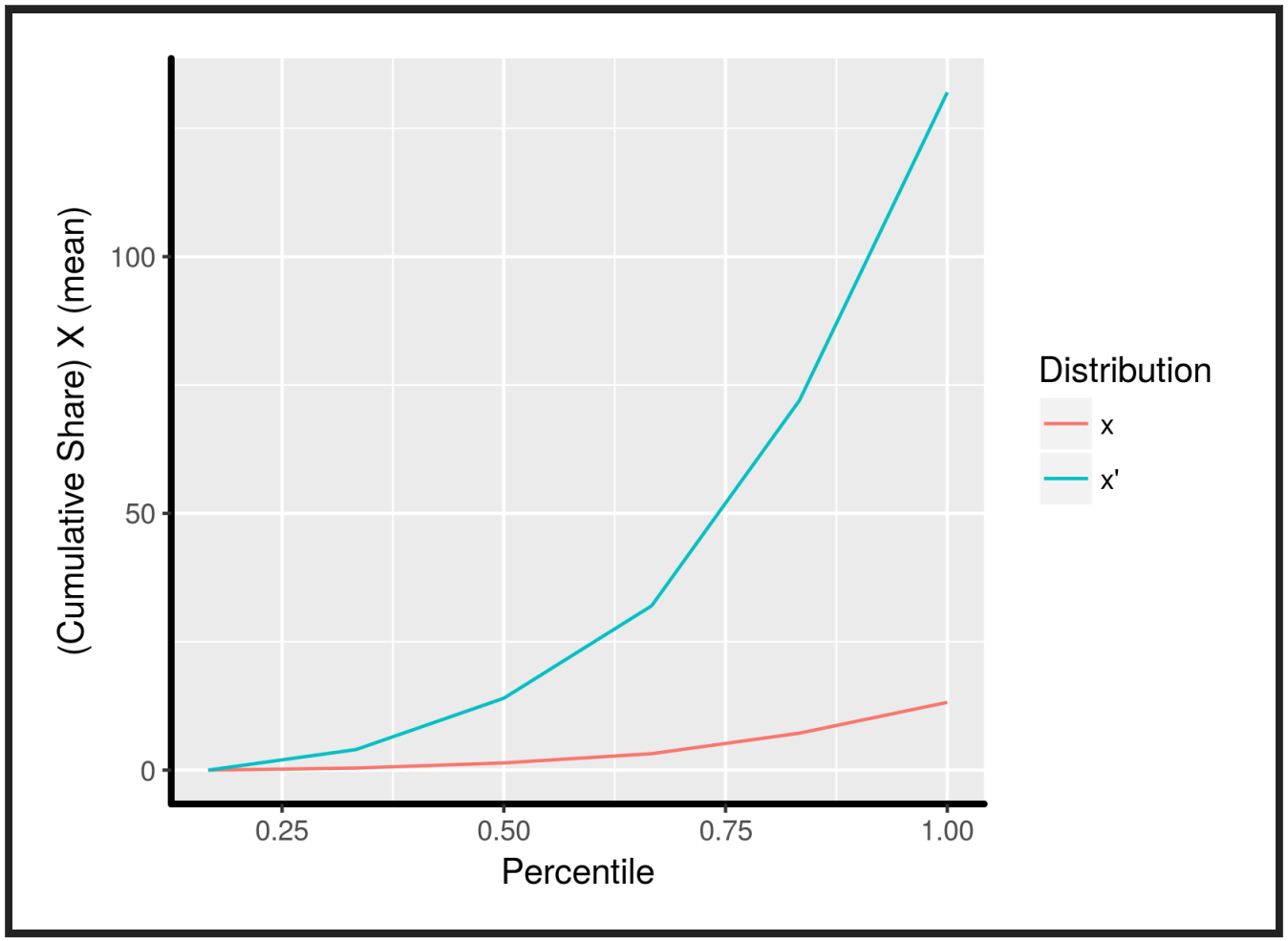
	Obie	Michael	Llewelyn	Rudy	Kitty
x :	2	5	9	20	30
Cumulative income:	2	7	16	36	66
Cumulative share:	0.03	0.11	0.24	0.54	1
Cum. share $\times$ mean:	0.4	1.4	3.2	7.2	13.2

Distribution B (mean = 132)

	Obie	Michael	Llewelyn	Rudy	Kitty
x' :	20	50	90	200	300
Cumulative income:	20	70	160	360	660
Cumulative share:	0.03	0.11	0.24	0.54	1
Cum. share $\times$ mean:	4	14	32	72	132

Generalized Lorenz curves

$$x = (2, 5, 9, 20, 30)$$
$$x' = (20, 50, 90, 200, 300)$$



Shorrocks Theorem

Suppose x and x' are two distributions. Any risk averse person (diminishing marginal utility of income) will always prefer a society with distribution x if the generalized Lorenz curve for x dominates that of x'

Generalized Lorenz curves in the real world

- We can't compare distributions with the Shorrocks theorem if generalized Lorenz curves intersect
- Generalized Lorenz domination is not something we observe with great frequency in the real world
- In practice we must make more strict normative assumptions (i.e. level of risk aversion)