Continuous Strategies and Rationalizability

ECON 420: Game Theory

Spring 2018

Announcements

- Reading: Chapter 5 and 6
- Homework due next Monday
- Midterm exam next Wednesday

Continuous strategies

- So far: Games with *discrete* strategies
  - Choosing from a finite set of actions
- Many games have many (or infinite) available actions
- Can we generalize the notion of *best response* to these settings?

Price-setting game

- Suppose there are two competing restaurants (they make only one dish)
- Both firms must choose their prices p_1 and p_2
- The number of dishes each restaurant sells is $Q_i = 44 - 2p_i + p_j$ Demand
  - After a price change, half of your usual customers will leave to go to the other restaurant
- The dishes cost $8 to make for each restaurant
- Which price should each restaurant choose?

Both firms are maximizing profits

$\rightarrow$ $MR = MC$

$R(P_1)$: revenue for firm 1 as a function of price

$$R(P_1) = P_1 Q_1$$
$$= P_1(44 - 2P_1 + P_2)$$
$$= 44P_1 - 2P_1^2 + P_1 P_2$$

$$C(P_1) = 8 \cdot Q_1$$
$$= 8(44 - 2P_1 + P_2)$$
$$= 8 \cdot 44 - 16P_1 + 8P_2$$

$$MR = 44 - 4P_1 + P_2$$
$$MC = -16$$
$$44 - 4P_1 + P_2 = -16$$
$$4P_1 = 60 + P_2$$
$$P_1 = 15 + \frac{1}{4} P_2$$

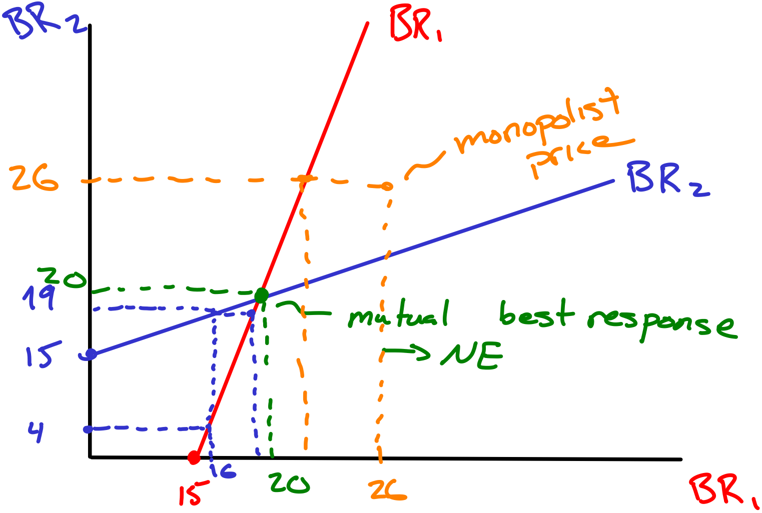
profit-maximizing price given P_2

Best response

- Profit depends on the pricing choice of the other firm
- Restaurants try to profit maximize given the price that they think the other will choose
- This pricing strategy is the *best response* of the restaurant

$BR_1 = 15 + \frac{1}{4} P_2$

$BR_2 = 15 + \frac{1}{4} P_1$ (Firms are the same)



BR2
BR1
monopolist price
26
20
19
15
4
mutual best response
→ NE
BR2
15
16
20
26
BR1

$$BR_1 = 15 + \frac{1}{4}P_2$$

$$BR_2 = 15 + \frac{1}{4}P_1$$

At NE:

$$\left.\begin{aligned} BR_1 &= 15 + \frac{1}{4}BR_2 \\ BR_2 &= 15 + \frac{1}{4}BR_1 \end{aligned}\right\} \text{both true simultaneously}$$

$$4(P_1) = \left[15 + \frac{1}{4}\left(15 + \frac{1}{4}P_1\right)\right]4$$

$$4(4P_1) = \left(60 + 15 + \frac{1}{4}P_1\right)4$$

$$16P_1 = 240 + 60 + P_1$$

$$15P_1 = 300$$

$$P_1 = 20, \quad P_2 = 20 \quad \text{(firms are the same)}$$

$P_1 = P_2 = 20$

$$\pi_1 = P_1 Q_1 - 8Q_1$$
$$= 20(44 - 2(20) + 20) - 8(44 - 2(20) + 20)$$
$$= 20(24) - 8(24)$$
$$= (20 - 8)24$$
$$= 12 \cdot 24$$
$$= \underline{288}$$

Suppose $P_1 = 19$, $P_2 = 20$

$$Q_1 = 44 - 2(19) + 20$$
$$= 26$$

$$\pi_1 = (P_1 - 8)Q_1$$
$$= (19 - 8)26$$
$$= 11 \cdot 26$$
$$= 271$$

Can the restaurants do better?

- Suppose an outside company buys both restaurants
- The firm is now a monopolist, chooses one price for both locations
- What is the optimal price? What are the profits?

$$Q_1 = 44 - 2P + P$$

$$= 44 - P$$

$$R(P) = P(44 - P)$$

$$= 44P - P^{2} \quad -1$$

$$MR = 44 - 2P$$

$$c(P) = 8(44 - P)$$

$$= 8 \cdot 44 - 8P$$

$$MC = -8$$

$$MR = MC$$

$$44 - 2P = -8$$

$$2P = 52$$

$$P = 26$$

$$\pi_1 = PQ_1 - 8Q_1 \qquad Q_1 = 44 - P$$

$$= (P-8)(44-P)$$

$$= (26-8)(44-26)$$

$$= 18 \cdot 18$$

$$\pi_1 = 324$$

$BR_1 = 15 + \frac{1}{4}P_2$

$P_2 = 26$

$BR_1 = 15 + \frac{1}{4}(26)$

$= 15 + 6\frac{1}{2}$

$= 21\frac{1}{2}$

$\pi_1 = Q_1(P_1 - 8)$

$Q_1 = 44 - 2(21.5) + 26$

$= 44 - 43 + 26$

$= 27$

$\pi_1 = 27(21.5 - 8)$

$= 27(13.5)$

$= 364.5$

Collusion

- The pricing game is a form of a prisoners' dilemma (with continuous strategies)
- The firms could cooperate to split the monopolist profits
- But each can do better (individually) by choosing something *other* than the monopolist price
- Cooperation is *never* a best response

Limitations of NE?

Example:

- Player A: Chooses "Up" or "Down"
- Player B: Chooses "Left" or "Right"
- Payoffs (A, B):
  - Up, Left: (2 chocolates, 2 chocolates)
  - Up, Right: (1 chocolates, 1 chocolates)
  - Down, Left: (3 chocolates, 2 chocolates)
  - Down, Right: (50% penalty on midterm, 1 chocolate)

B

		Left	Right
A	Up	2, 2	1, 1
	Down	3, 2	F, ~~1~~ 3

✡ outcome we observe

✱ NE

Why might we not see a NE?

- Often, player A won't choose Down, because it is risky
- Why is it risky?
  - A might think B doesn't like chocolate
  - A might be concerned the B will try to "spite" them
- These options might mean that the game is *misspecified*
  - A has uncertainty about B's payoffs

Example

		COLUMN		
		A	B	C
ROW	A	2, 2	3, 1	0, 2
	B	1, 3	2, 2	3, 2
	C	2, 0	2, 3	2, 2

Rationalization

- Suppose games are properly specified
- Nash equilibrium:
  - The choice of each player is their best response given their beliefs about what the other players are doing
  - The beliefs are accurate
- Does this mean that purely rational players will achieve the NE?

		COLUMN		
		C1	C2	C3
ROW	R1	0, 7	2, 5	7, 0
	R2	5, 2	3, 3	5, 2
	R3	7, 0	2, 5	0, 7

Is it rational for R to play R1?
Yes, if they believe C plays C3
Is it rational for C to play C3?
Yes, if the believe R plays R3.

Rationalizability

- Multiple outcomes can be supported by rational "chains" of thought
  - Not necessarily NE
- But not *every* outcome is supported by rationality
- For instance: It is never rational to play a strategy that is *never a best response*

		COLUMN			
		C1	C2	C3	C4
ROW	R1	0, 7	2, 5	7, 0	0, 1
	R2	5, 2	3, 3	5, 2	0, 1
	R3	7, 0	2, 5	0, 7	0, 1
	R4	0, 0	0, −2	0, 0	10, −1

Rationalizability

- Note: Not all strategies that are never a best response are dominated by some other strategy
- Sometimes rationalizability can lead to a NE (but not always)

Cournot competition

- Suppose there are two fishing boats that choose how many fish to catch each day
- The local fish market buys the fish for a price $P = 60 - Y$
- Boat one has costs of 30 per fish and boat 2 has costs 36 per fish