Profit maximization

- Firms use labor (L) and capital (K) to produce output (y)
- Firms sell output y for price p
- Each unit of labor (L) costs ω (wages)
- Each unit of capital (K) costs r (rental rate)
  - cost of renting capital
  - opportunity cost of K
- Revenue

  Total revenue = TR = py

- Cost

  Total cost = TC = $\underbrace{\omega L}_{\text{total amount paid to labor}} + \underbrace{rK}_{\text{total amount paid to capital}}$

- Profit

  Profit = Total revenue − Total cost

  $\pi = TR - TC$

  $\pi = py - (rK + \omega L)$

We will assume firms are maximizing profits

- Firm's problem

  $\max \; p \cdot y - (\omega L + rK)$

Firms are choosing K and L
then $y = f(K, L)$
- output is determined by choice of K, L

- Suppose firm is using K_0 capital and L_0 labor and produces $y_0 = f(K_0, L_0)$

Question: when should the firm hire another unit of labor? (keep one more unit working)

- If we keep someone working 1 more hour, we have to pay them ω
  - ω is marginal cost of labor

- If we keep someone working one more hour, we get additional revenue

Marginal revenue
- How much more output do we get from that hour?

MPL

$MR = p \cdot MPL$

- Hire when $MR > MC$

$pMPL > \omega$

π is increasing

Question: When should I fire some worker? (or have them work one hour less?)

• $pMPL < w$

The firm is profit maximizing when: $pMPL = w$

$MR = MC$

What about capital?

$$pMPK = r$$

(for the same reasons)

$$\begin{cases} pMPL = w \\ pMPK = r \end{cases}$$ Firm's π-max conditions

Short-run vs Long-run

- Long-run: Period of time in which there are no fixed inputs
- Short-run: At least one fixed input
- In this class: Assume K is fixed in the short run. $(K = \bar{K})$

Short-run profit max:

$$\max_{L} \; py - (wL + rK)$$

Short-run profit max condition:

$pMPL = w$

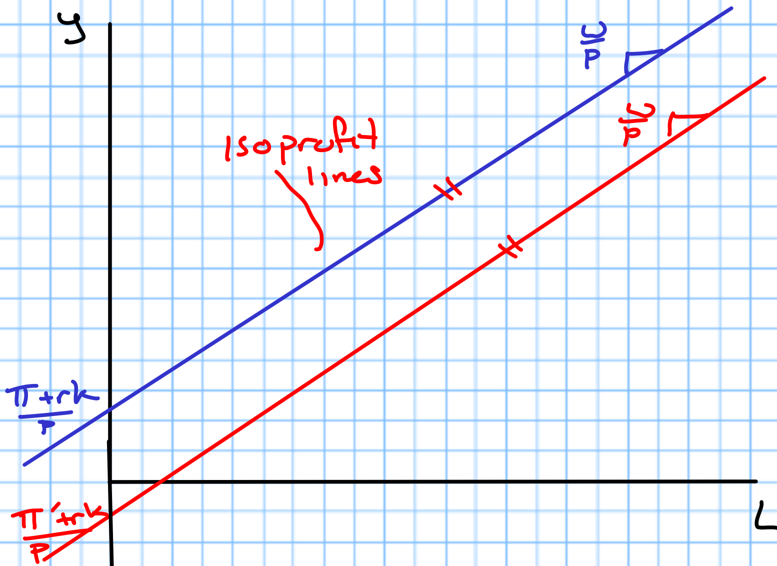
Let's plot our profit function with y on the vertical axis and L on the horizontal axis

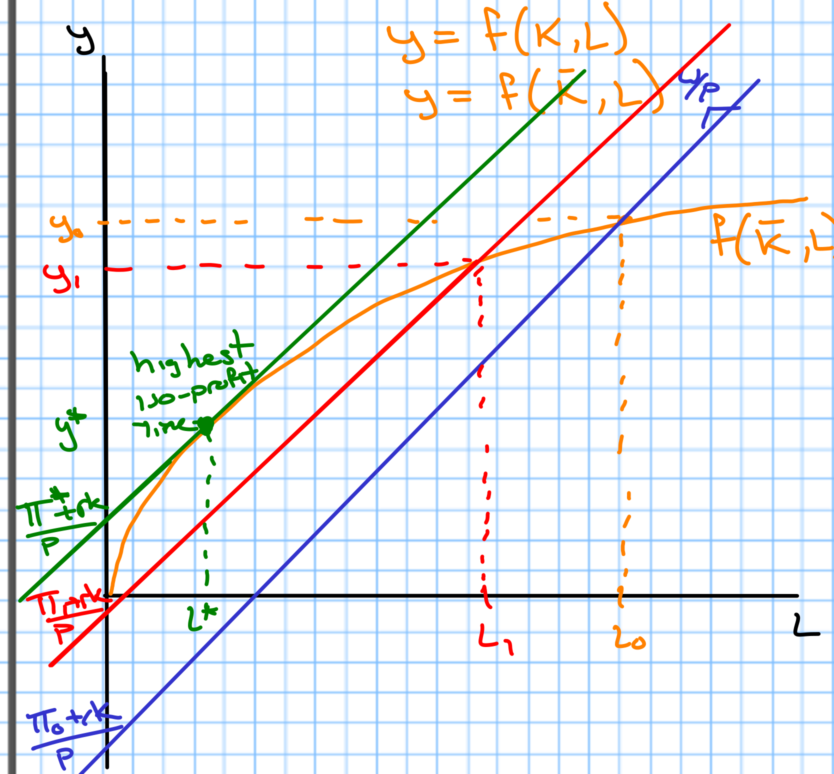
$$\pi = py - wL - rK$$

solve for y

$$py = \pi + wL + rK$$

$$y = \frac{\pi + rK}{p} + \frac{w}{p} L$$





What is the slope of the production function?

$$\frac{\partial f(\bar{K},L)}{\partial L} \equiv MPL$$

$$MPL = \frac{w}{p} \longrightarrow pMPL = w$$

<u>Example</u>

$$f(K,L) = K^{1/3} L^{1/2}$$

$p=5$, $w=3$, $\bar{K}=8$, $r=1$

What is the firm's short-run profit maximizing L, y, π?

$$pMPL = w$$

$$MPL = \frac{\partial f(K,L)}{\partial L}$$

$$= \frac{1}{2}\bar{K}^{1/3} L^{-1/2}$$

$$p \cdot \left(\frac{1}{2}\bar{K}^{1/3} L^{-1/2}\right) = w$$

$$L^{1/2}\left(5 \cdot \frac{1}{2} 8^{1/3} L^{-1/2}\right) = (3) L^{1/2}$$

$$5 \cdot \cancel{\frac{1}{2}} \cancel{2} = 3L^{1/2}$$

$$\frac{5}{3} = L^{1/2}$$

$$L^* = \frac{25}{9} = 2\,7/9$$

$$y^* = f(\bar{K}, L^*)$$

$$= 8^{1/3}\left(\frac{25}{9}\right)^{1/2}$$

$$= 2 \cdot \frac{5}{3} = \frac{10}{3} = 3\,1/3$$

$$\pi = p \cdot y^* - (wL^* + r\bar{K})$$

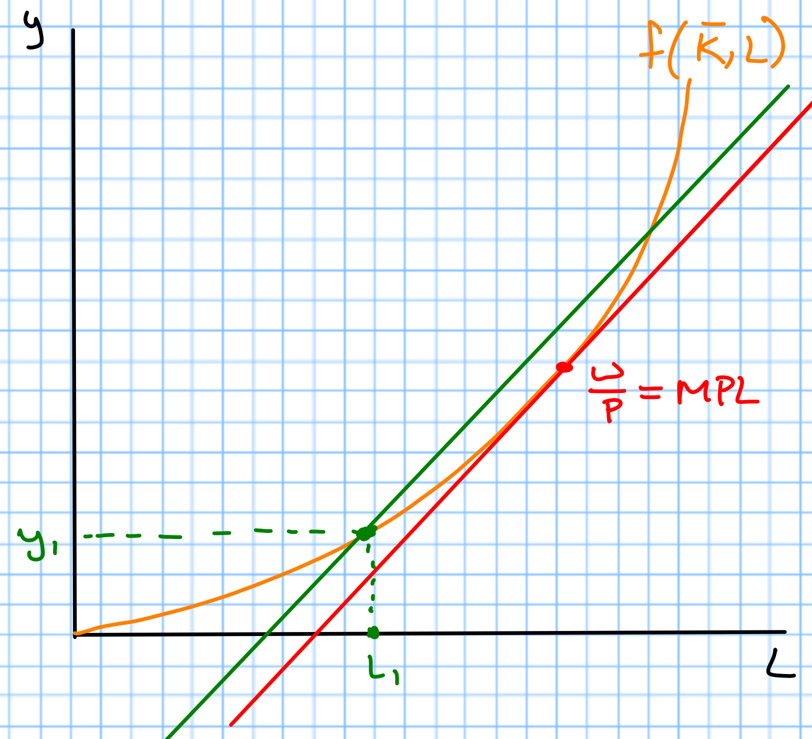
$$= 5 \cdot \frac{10}{3} - \left(3 \frac{25}{9} + 1 \cdot 8\right)$$

$$= \frac{50}{3} - \frac{25}{3} - 8$$

$$= \frac{50 - 25 - 24}{3}$$

$$\pi^* = \frac{1}{3}$$

RTS & π-max



Result

$pMPL = w$ is the profit-maximizing condition only when $f(K,L)$ has diminishing RTS

Suppose we have increasing RTS

Double inputs $\rightarrow$ more than double output

$TR = p \cdot y = p \cdot f(K,L)$

$\rightarrow$ revenue more than doubles

$TC = wL + rK$

$\rightarrow$ cost ... doubles

$w(2L) + r(2K) = 2(wL+rK)$

$\rightarrow$ $TR - TC$ increases

$\rightarrow$ π increases

Increasing RTS means profits always increase as K and L are increased

$\rightarrow$ there is no (finite) solution to the firm's problem!