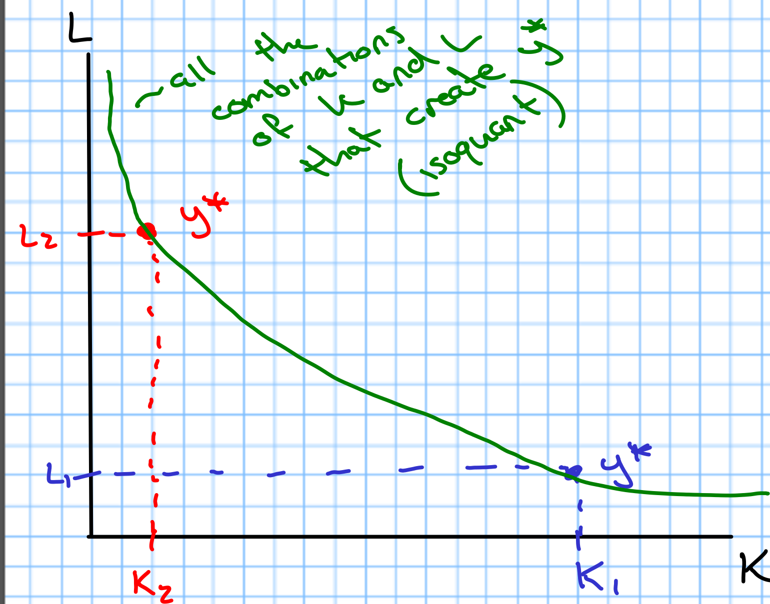
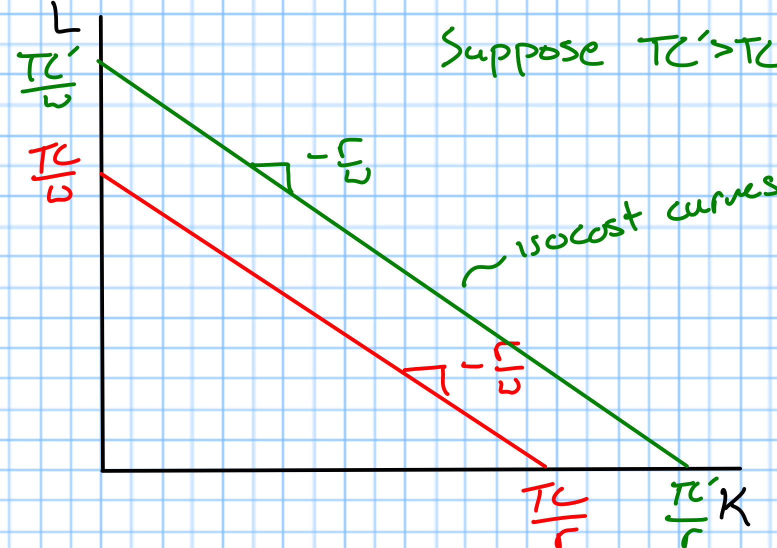
Cost Minimization

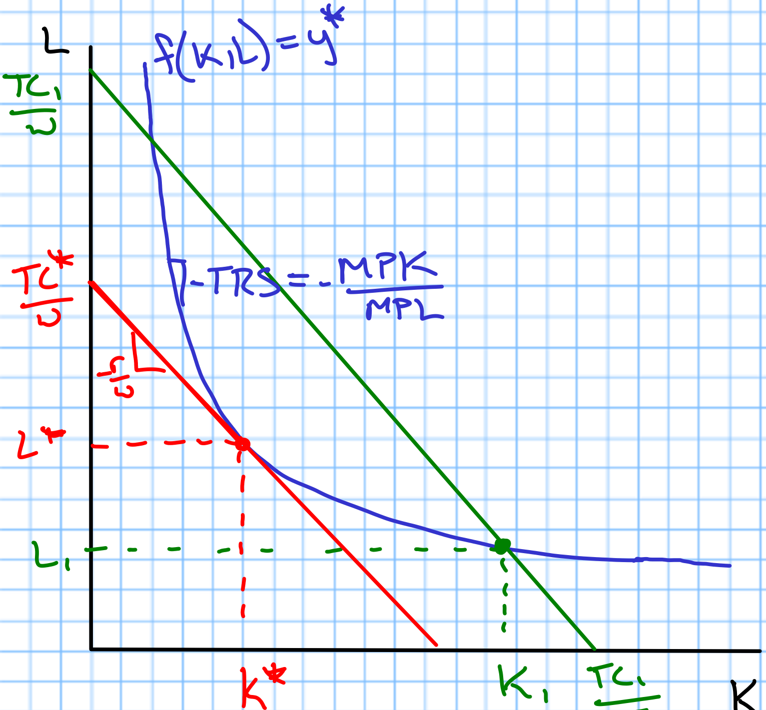
- Suppose that firm decides to produce some level of output $y = y^*$
- production function $y = f(K, L)$
- Think about all of the combinations of K and L that the firm can employ in order to produce y^* units of output



- Revenue: py^*
  Costs: $rK + wL$
  π: $py^* - (rK + wL)$
- Given that the firm <u>must</u> produce y^*, how do the maximize profits?
  → minimize cost
- Result: A profit <u>maximizing</u> firm must always be <u>minimizing</u> their costs
- Total cost:
  $wL + rK = TC$
- Solve for L:

$$wL + rK = TC$$
$$wL = TC - rK$$
$$L = \frac{TC}{w} - \frac{r}{w}K$$





- If the firm is cost minimizing, then:

(1) $TRS = \frac{r}{w} \Rightarrow \frac{MPK}{MPL} = \frac{r}{w}$

(2) $f(K^*,L^*) = y^*$

Example

$$f(K,L) = K^3L^2$$

$$r = 4, \quad w = 2, \quad y^* = 10$$

$$MPK = \frac{\partial f(K,L)}{\partial K} = 3K^2L^2$$

$$MPL = 2K^3L$$

$$TRS = \frac{3\cancel{K^2}L^{\cancel{2}}}{2K^{\cancel{3}}\cancel{L}} = \frac{3L}{2K}$$

① : $\frac{3L}{2K} = \frac{4}{2}$

$6L = 8K$

$L = \frac{4}{3}K$

② $K^3 L^2 = 10$

$K^3\left(\frac{4}{3}K\right)^2 = 10$

$K^3 K^2 \cdot \left(\frac{4}{3}\right)^2 = 10$

$K^5 \cdot \frac{16}{9} = 10$

$K^5 = \frac{90}{16}$

$K = \left(\frac{90}{16}\right)^{1/5} = 1.41$

$L = \frac{4}{3}(1.41)$

Example

$f(K,L) = K^{1/3} L^{1/2}$

$w = 4 \quad r = 2, \quad y = 10$

$TRS = \frac{\frac{1}{3}L}{\frac{1}{2}K} = \frac{2L}{3K}$

① $\frac{2L}{3K} = \frac{2}{4}$

$L = \frac{3}{4}K$

② $K^{1/3}\left(\frac{3}{4}K\right)^{1/2} = 10$

$K^{1/3} K^{1/2} \left(3/4\right)^{1/2} = 10$

$K^{5/6} \left(3/4\right)^{1/2} = 10$

$K^{5/6} = 10 \cdot \left(\frac{3}{4}\right)^2$

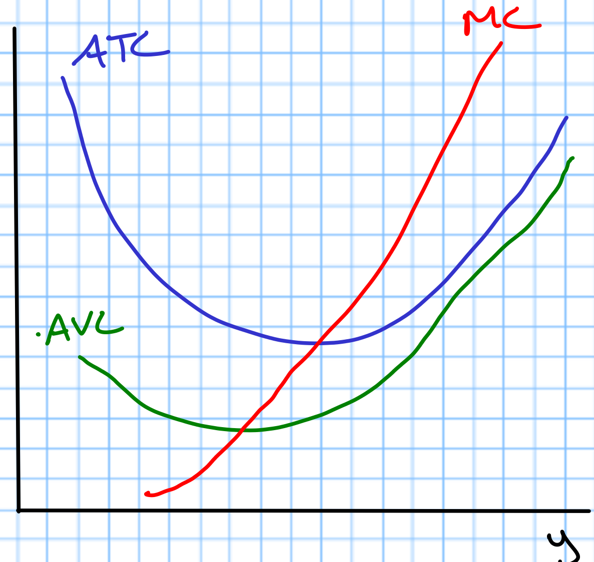
$K^{5/6} = \frac{90}{16}$

$K = \left(\frac{90}{16}\right)^{6/5}$

Plug K into $L = \frac{3}{4}K$

$\rightarrow$ we're done

Recall ECON 201:



Goal: find a function that tells us Total cost given some quantity y

Example $f(K,L) = KL$

$w = 9$, $r = 4$, $y = ?$

$TRS = \frac{L}{K}$

(1) $\frac{L}{K} = \frac{4}{9}$

$L = \frac{4}{9}K$

(2) $KL = y$

$K\left(\frac{4}{9}K\right) = y$

$K^2 \cdot \frac{4}{9} = y$

$$K^2 = \frac{9}{4} y$$

$$K = \frac{3}{2}\sqrt{y}$$

$$K(y) = \frac{3}{2}\sqrt{y}$$

function:

input: quantity produced

output: cost-minimizing level of K

$$L(y) = \frac{4}{9} \cdot \frac{3}{2}\sqrt{y}$$

$$= \frac{2}{3}\sqrt{y}$$

Total cost function:

$$c(y) = wL(y) + rK(y)$$

$$= 9 \cdot \frac{2}{3}\sqrt{y} + 4 \cdot \frac{3}{2}\sqrt{y}$$

Fixed costs: $4 \cdot \bar{K}$

variable costs: $9 \cdot \frac{2}{3}\sqrt{y}$

$$c(y) = \underbrace{c_v(y)}_{\text{variable cost}} + \underbrace{F}_{\text{Fixed cost}}$$

<u>Example</u>

$$c(y) = y^2 + 1$$

$$c_v(y) = y^2$$

$$F = 1$$

Average Total cost:

$$\frac{c(y)}{y} = y + \frac{1}{y}$$

Average variable cost:

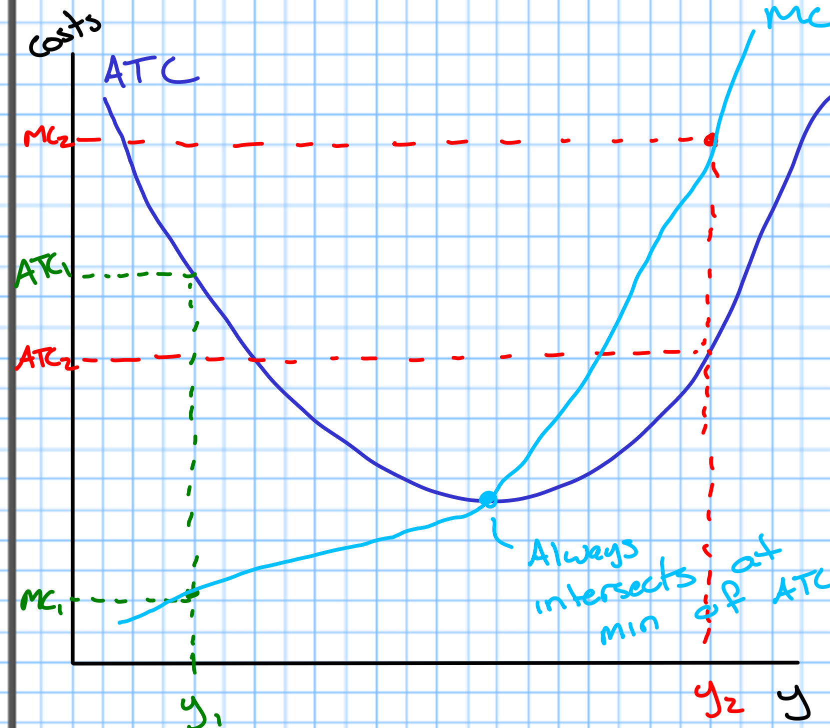
$$\frac{C_v(y)}{y} = y$$

Average Fixed cost:

$$\frac{F}{y} = \frac{1}{y}$$

Marginal cost:

$$\frac{dc(y)}{dy} = 2y$$



$$ATC = \frac{c(y)}{y}$$

$$\frac{dATC}{dy} = 0 \quad \text{when } MC = ATC$$

$$ATC = c(y)\,y^{-1}$$

$$\frac{dATC}{dy} = \underbrace{\frac{dc(y)}{dy}}_{MC} y^{-1} + (-1y^{-2})c(y)$$

$$MC\,y^{-1} - y^{-2}c(y) = 0$$

$$MC\,y^{-1} = y^{-2}c(y)$$

$$\frac{MC}{y} = \frac{c(y)}{y^2}$$

$$\frac{MC}{y} = \frac{c(y)}{y^2}$$

$$MC = \frac{c(y)}{y} \qquad MC = ATC$$