Repeated Games

ECON 420: Game Theory

Spring 2018

Announcements

- Homework 4 due next Wednesday (will be posted later today)
- Final exam: Friday, June 15 at 7:30am (!)

Prisoners' Dilemma

		WIFE	
		Confess (Defect)	Deny (Cooperate)
HUSBAND	Confess (Defect)	10 yr, 10 yr *	1 yr, 25 yr
	Deny (Cooperate)	25 yr, 1 yr	3 yr, 3 yr

Game 1

- You will play the prisoners' dilemma against a random opponent
- Write your name at the top of a sheet of paper
- Choose a strategy to play (Confess or Deny)
- Your opponent will be randomly selected from among your classmates
- The person(s) with the highest payoffs will receive 5 extra-credit points on the homework

Restaurant Pricing Game

		YVONNE'S BISTRO	
		20 (Defect)	26 (Cooperate)
XAVIER'S TAPAS	20 (Defect)	288, 288	360, 216
	26 (Cooperate)	216, 360	324, 324

Game 2

- Pair up with one of your classmates
- Play the restaurant pricing game for 5 rounds
  - Keep track of your payoffs for each round

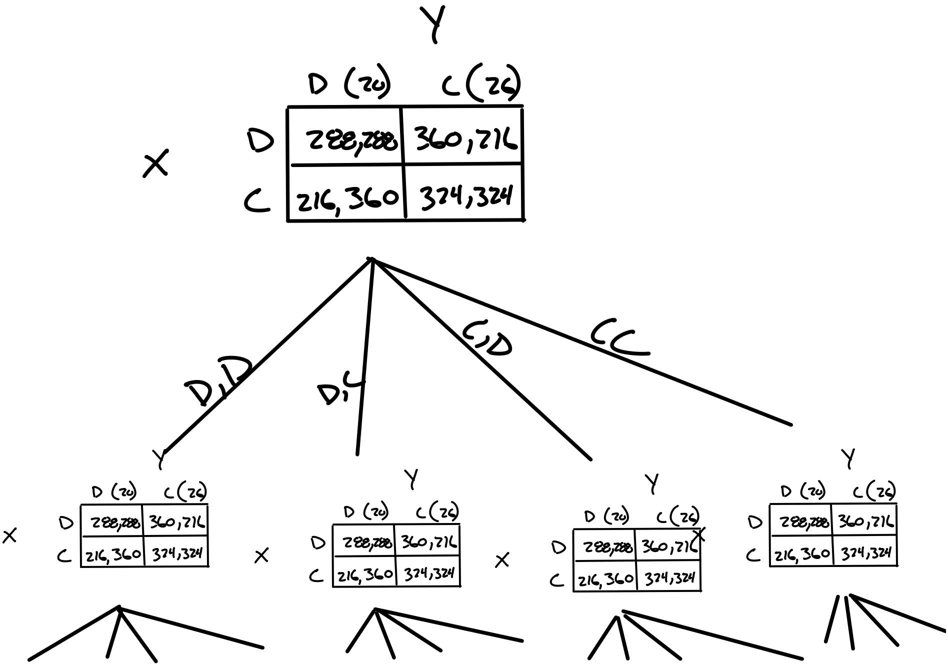
Game 3

- Play the restaurant pricing game again
  - Keep track of your payoffs each round
- Continue playing until I say stop

Repetition and cooperation

- Which version of the game are we most likely to observe cooperation? Why?
- Which version of the game are we *least* likely to observe cooperation?

The game tree



Strategies in repeated games

- Strategies can be extremely complicated in repeated games
  - Strategies can contain infinitely many moves if the game is repeated forever!
- Often useful to simplify the strategy to a "rule"
- *Contingent strategies*: Choose action based on action of opponent in previous round

Rollback equilibrium

- Suppose the game is played a finite number of times
  - What is the rollback equilibrium? Defect always.
- Suppose the game is played an infinite number of times
  - What is the rollback equilibrium? No rollback.

Tit-for-tat

- Strategy: Cooperate in first round, then do whatever opponent does in previous round
- Allows for cooperative outcomes, but "punishes" opponent for defecting

Grim-trigger

- Strategy: Cooperate in every round if opponent also cooperates, defect forever if opponent defects once
- Most severe punishment for opponent

Time value

- Suppose the restaurant pricing game is repeated monthly
  - Your opponent is playing a tit-for-tat strategy
- Should you defect in the first round?
  - Cooperate every round after
- Gain in the first month 36
- Lose *more* in the second month 108
- But money is more valuable today than next month!

Present value

- To compare money now with money later, we need to calculate the *present value* of money later
- The PV of future money is the amount we'd be willing to accept today instead
- For a discount rate r, the present value of future income I is

$$PV = \frac{I}{1+r}$$

Example

$r = 6\%$

Loss next month 108

Present value of the loss:

$$\frac{108}{1+0.06} = \frac{108}{1.06} = 101.88$$

Gain this month: $36 < 101.88$

$\longrightarrow$ bad idea to defect first round

Defecting against a grim trigger

- Receive the higher payoff at first, non-cooperative outcome forever after
- Is immediate payoff the long-run loss?
  - What is the immediate gain? 36
  - What is the PV of future losses? 1800

$r = 0.06$

$$\frac{108}{1.06} + \frac{108}{1.06^2} + \frac{108}{1.06^3} + \dots = \frac{108}{0.06}$$

$$S = \frac{108}{1+0.06} + \frac{108}{(1+0.06)^2} + \frac{108}{(1+0.06)^3} + \cdots$$

$$S = \frac{1}{1+0.06}\left(108 + \frac{108}{1+0.06} + \frac{108}{(1+0.06)^2} + \cdots\right)$$

$$S(1+0.06) = 108 + \frac{108}{1+0.06} + \frac{108}{(1+0.06)^2} + \cdots$$

$$S(1+0.06) = 108 + S$$

$$\cancel{S} + 0.06S - \cancel{S} = 108 \longrightarrow S = \frac{108}{0.06}$$

Penalties and rewards

- Perhaps there is a social cost to defecting (snitches get stitches?)
- In this case, the payoff table is poorly specified
- Properly specifying the payoffs may mean that the game is not a prisoners' dilemma at all
- Perhaps threats or promises in a new first round can change the payoffs of a game (chapter 9)

Experiments with repeated games

- Robert Axelrod created a computer "tournament" where teams could submit computer programs to play a repeated prisoners' dilemma
- Teams chose a strategy for the programs, then they play other randomly selected programs
  - Which strategy was best? Tit-for-tat
- After first round, teams could submit *new* strategies knowing what the optimum was
  - Which strategy was the best this time? Tit-for-tat!

Axelrod:

- "Don't be envious. Don't be the first to defect. Reciprocate both cooperation and defection. Don't be too clever."

		Y	
		(defect) 20	(cooperate) 26
X	20	288, 288 *	360, 216
	26	216, 360	324, 324

→ both players better, not NE

2 types of players: (phenotype)

1. Always defect (D)
2. Always Cooperate (C)

- Suppose the entire population is type D
  - random mutation creates a type C
  - type D payoff: 288
  - type C payoff: 216
    - ↳ not as successful as the D types
- Suppose we have entire population of type C
  - random mutatition → D
  - $\pi_C = 324$ $\pi_D = 360$ <u>D takes over</u>

Repeat the game 3 times

Two types:

D: Always defect

T: Tit-for-tat

	D	T
D	864, 864	936, 792
T	792, 936	972, 972

- Suppose only type D, and a single type T

$\pi_D = 864$

$\pi_T = 792$

Type T cannot invade type D

- Suppose instead only type T and one type D

$\pi_D = 936$

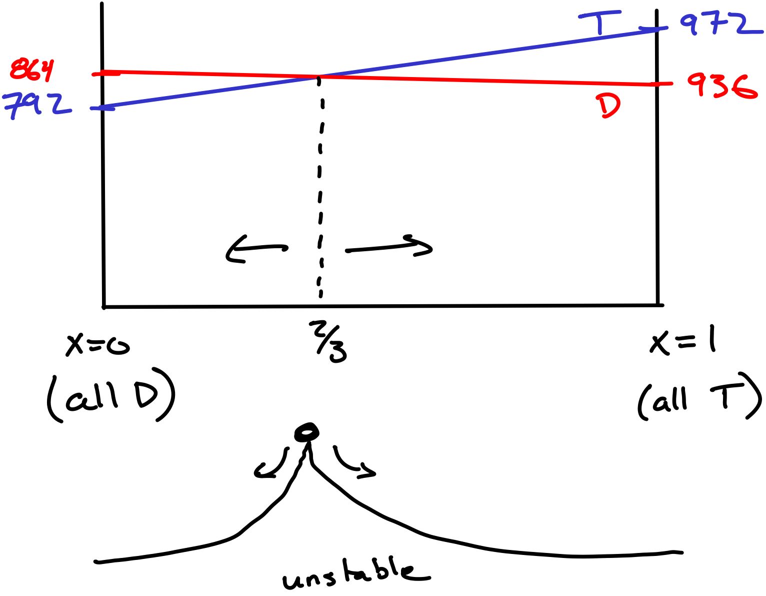
$\pi_T = 972$

D can't invade population of T

- Both all T and all D are evolutionarily(c) stable

- Suppose the proportion of T is x
- when do the type Ts outcompete D?

$$972x + 792(1-x) > 936x + 864(1-x)$$
$$36x > 72(1-x)$$
$$108x > 72$$
$$x > 2/3$$


T
972
864
792
936
D
x=0
(all D)
2/3
x=1
(all T)
unstable

Chicken

2 types:

Wimps and Macho

	W	M
W	0, 0	-1, 1
M	1, -1	-2, -2

- Suppose all W and 1 M

$\pi_W = 0$

$\pi_M = 1$

$\rightarrow$ M can invade W

- Suppose all M, 1 W

  $\pi_M = -2$

  $\pi_W = -1$

  $\to$ W can invade M

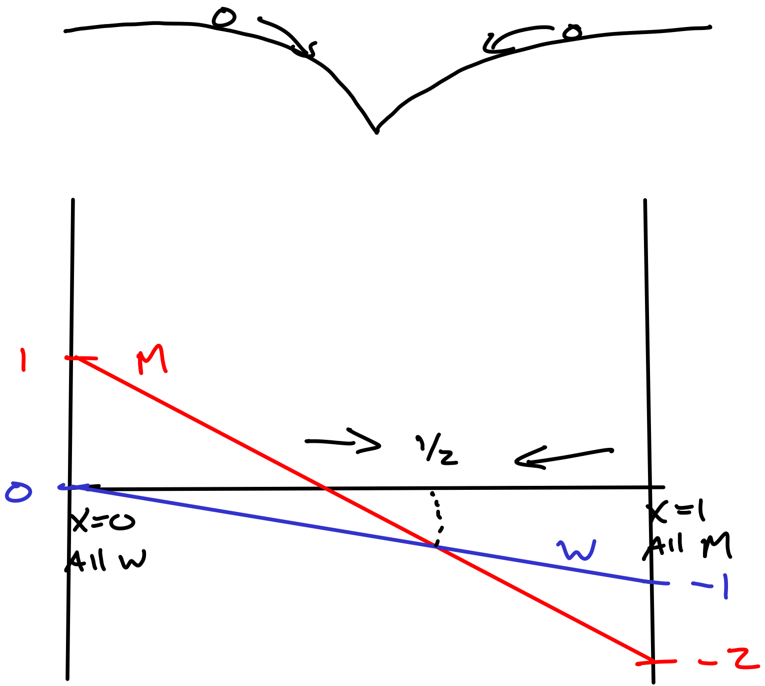
- Neither extreme is stable
- Let x be proportion of M

$$-2x + 1(1-x) = -1x + 0(1-x)$$

$$1 - x = x$$

$$1 = 2x$$

$$x = 1/2$$



0
0
1
M
1/2
0
X=0
All W
W
X=1
All M
-1
-2