Income and Substitution Effects

- Suppose we have a demand curve $x_1^* = x_1(P_1)$
- Suppose price increases from P_1 to P_1' $(P_1' > P_1)$

What happens?

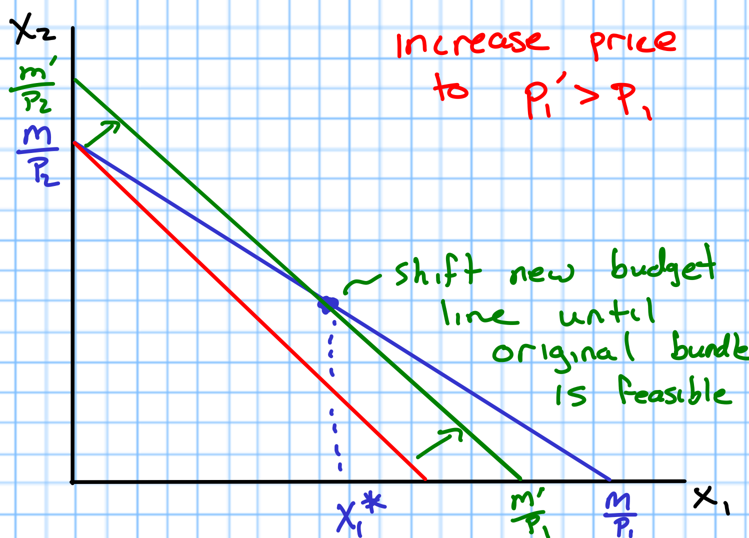
- Overall, demand changes from $x_1^* = x_1(P_1)$ to $x_1^{*\prime} = x_1(P_1')$
- When price increases, I "feel" like I have less income
  - There are bundles I can no longer consume
- Good 1 (x_1) becomes relatively less attractive
  → opportunity cost has increased

Decompose price increases:

1. Decrease in purchasing power (income effect)
2. Increase in opportunity cost (substitution effect)

Substitution effect

- How does a price increase impact consumption assuming that purchasing power remains unchanged?



Original budget line:

$$p_1 x_1 + p_2 x_2 = m \quad \text{(A)}$$

"pivoted" budget line:

$$p_1' x_1 + p_2 x_2 = m' \quad \text{(B)}$$

where m' is the amount of income needed to keep purchasing power unchanged

Subtract B from A:

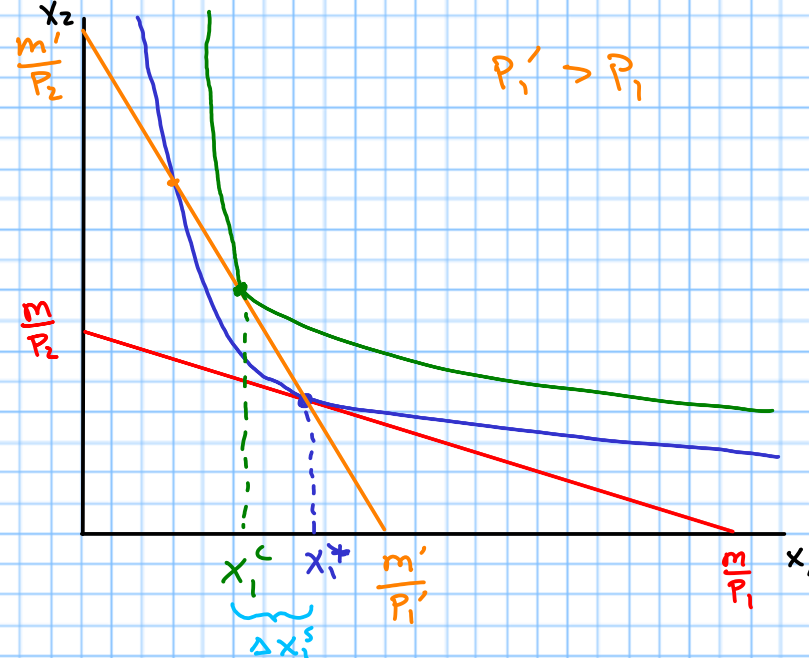
$$\begin{array}{rl} & p_1 x_1 + \cancel{p_2 x_2} = m \\ - & p_1' x_1 + \cancel{p_2 x_2} = m' \\ \hline & p_1 x_1 - p_1' x_1 = m - m' \\ & (p_1 - p_1') x_1 = m - m' \\ & \Delta p_1 x_1 = \Delta m \end{array}$$

- Suppose P_1 increases to P_1'. How much additional income do I need to keep my purchasing power the same?

$\longrightarrow \Delta m = \Delta P_1 \cdot x$

Example

$$x_1^* = x_1(P_1)$$
$$P_1 = 5$$
$$x_1^* = x_1(5) = 2$$
$$P_1' = 7$$
$$\Delta m = \Delta P_1 \cdot x_1^*$$
$$= (7-5)\cdot 2$$
$$= 2 \cdot 2$$
$$= 4$$



x_1^c is the compensated demand

Think of demand as a function of price and income:

$x_1^* = x_1(p_1, m)$

$x_1^c = x_1(p_1', m')$

Define: $\Delta x_1^s = x_1^c - x_1^*$

$\Delta x_1^s = x_1(p_1', m') - x_1(p_1, m)$

Δx_1^s is the substitution effect of a price increase

→ it is a compensated change in demand

Example

$m = 100$

$p_1 = 5$

$x_1(p_1, m) = 10 + \frac{m}{5p_1}$

Calculate substitution effect of a price increase to $p_1' = 7$

$\Delta x_1^s = x_1^c - x_1^*$

$= x_1(p_1', m') - \underbrace{x_1(p_1, m)}_{14}$

$x_1^* = x_1(p_1, m)$

$= 10 + \frac{100}{5 \cdot 5}$

$= 10 + 4$

$= 14$

$$x_1(p_1', m') = 10 + \frac{m'}{5p_1'}$$

$$m' = m + \Delta m$$

$$m' = 100 + \Delta p_1 \cdot x_1$$

$$m' = 100 + (7-5) \cdot 14$$

$$m' = 100 + 2 \cdot 14$$

$$m' = 128$$

$$x_1^c = x_1(p_1', m')$$

$$= x_1(7, 128)$$

$$= 10 + \frac{128}{5 \cdot 7}$$

$$= 10 + \frac{128}{35}$$

$$= 13\tfrac{23}{35}$$

$$\cong 13.7$$

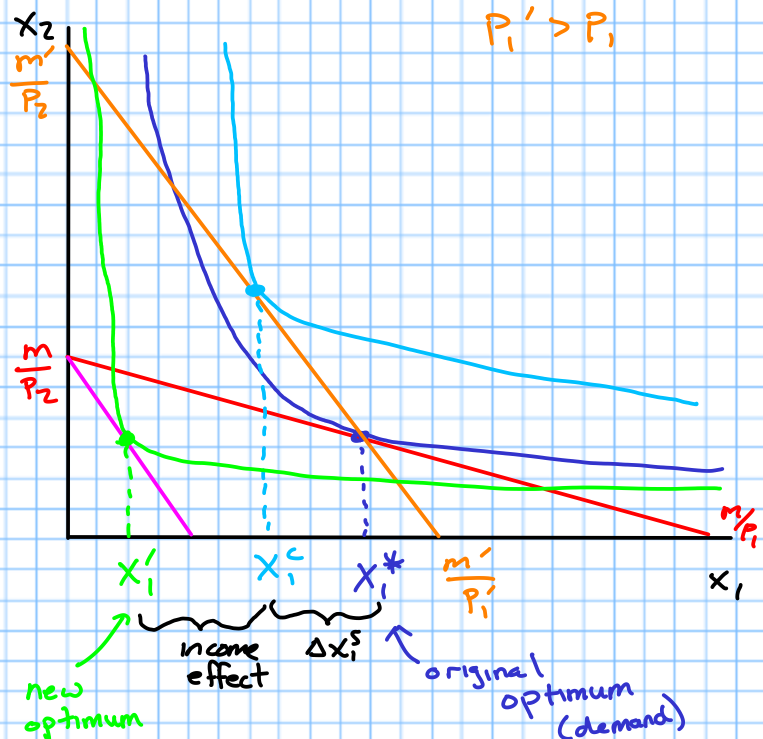
$$\Delta x_1^s = x_1^c - x_1^*$$

$$= 13.7 - 14$$

$$\cong -0.3$$

Income effect

- Given the substitution effect, how much does the change in purchasing power affect demand?



Suppose we have $x_1(p_1, m)$ and p_1 increases to p_1'. What is the change in demand?

$$\Delta x_1 = x_1(p_1', m) - x(p_1, m)$$

change in demand — new demand — original demand

$$x_1' - x_1^*$$

$$\Delta x_1 = x_1' - x_1^* + x_1^c - x_1^c$$

$$\Delta x_1 = (x_1^c - x_1^*) + (x_1' - x_1^c)$$

$$\Delta x_1 = \Delta x_1^s + \Delta x_1^m$$

total change in demand = substitution effect + income effect

$\Delta x_1 = \Delta x_1^s + \Delta x^m$

$\rightarrow$ "Slutsky identity"

Example, continued

$x_1(p_1, m) = 10 + \frac{m}{5p_1}$

$p_1 = 5, \quad m = 100, \quad \underline{x_1^*} = 14$

$p_1' = 7, \quad m' = 128, \quad x_1^c = 13.7$

$$x_1' = x_1(7, 100)$$
$$= 10 + \frac{100}{5 \cdot 7}$$
$$= 10 + \frac{100}{35}$$
$$= 10 + \frac{20}{7}$$
$$\cong 12.85$$

$\underline{\Delta x_1} = \Delta x_1^s + \Delta x_1^m$

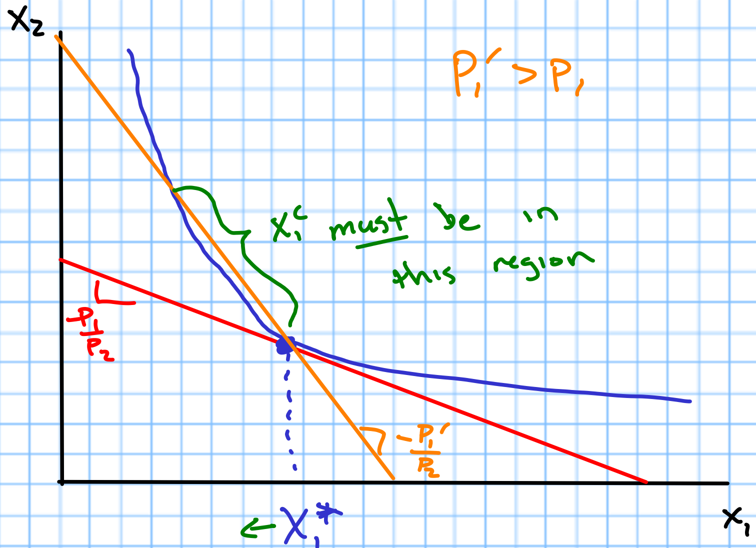
$-1.15 = -0.3 + \Delta x_1^m$

$\Delta x_1^m = -0.85$

$$\Delta x_1^m = x_1' - x_1^c$$
$$= 12.85 - 13.7$$
$$= -0.85$$

(hopefully!)

Sign of the substitution effect



Can x_1^c be greater than x_1^*

– No. Substitution effect always negative

Sign of the income effect

x_1', x_1'', x_1''' are potential bundles

- If they choose x_1', then $\Delta x_1^m < 0$
- If they choose x_1'' or x_1''', $\Delta x_1^m > 0$

Result:

- If $\Delta x_1^m < 0$, then x_1 is a normal good
- If $\Delta x_1^m > 0$, then x_1 is an inferior good