Nash Equilibrium

ECON 420: Game Theory

Spring 2018

Pick-a-color game

- ► Two types of teams, A and B
- ► Your team will play against the other type (A vs B)
- ► Each team chooses "white" or "blue"
- ► Payoffs for **A** teams:
 - ▶ If both teams choose white: 50
 - ▶ If both teams choose blue: 25
 - ▶ If A chooses white and B chooses blue: 75
 - ► If A chooses blue and B chooses white: 50
- ► Payoffs for **B** teams:
 - ► If both teams choose white: 50
 - ▶ If both teams choose blue: 75
 - ▶ If A chooses white and B chooses blue: 25
 - ▶ If A chooses blue and B chooses white: 50

James S

A: white

B: white

A:50 B:50

Came Z

A: white

B: white Payoffs

1:50

Pick-a-color game (version 2)

- ► Each team chooses "orange" or "black"
- ► Payoffs for **A** teams:
 - ▶ If both teams choose orange: 75
 - ▶ If both teams choose black: 50
 - ▶ If A chooses orange and B chooses black: 25
 - ▶ If A chooses black and B chooses orange: 50
- ► Payoffs for **B** teams:
 - ▶ If both teams choose orange: 25
 - ► If both teams choose black: 50
 - ▶ If A chooses orange and B chooses black: 75
 - ▶ If A chooses black and B chooses orange: 50

Same 1

B; Black A: Black

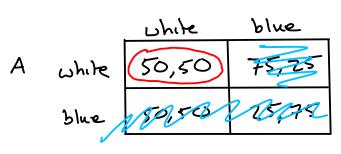
Payoffs

B: 50

A:50

B: Black A: Bbck

B



No team can improve by changing strategy -> NE

	N	E B	
		black	orange
A	black	50,50	50,50
	orange	25, 75	75,25

Prisoners' Dilemma

- ► The story:
 - ► Husband and wife are arrested for a crime and interrogated separately
 - Both must choose to confess to the crime or deny that they committed the crime
 - ► If both deny, serve 3 years for a crime that police can prove
 - ► A confession is "rewarded" by police if it helps convict the other partner (who denies)
 - ► If both confess, both serve a long sentence

Prisoners' dilemma



confess

Best responses

- 1. Suppose husband believes wife will confess. What is his best response?
- 2. Suppose husband believes wife will deny. What is his best response?

Sconfess

Dominance

- ▶ If a strategy is *always* a best response, that strategy is a *dominant* strategy
- ▶ If a strategy is never a best response, that strategy is a dominated strategy
 ▶ If both players have a dominant strategy, then these strategies define the
- Nash equilibrium

 In the pricopers' dilemma, confess is a dominant strategy (deny is dominated)
- In the prisoners' dilemma, confess is a dominant strategy (deny is dominated)What about pick a color?

Prisoners' dilemma

- ► Both players have a dominant strategy
- ► Dominance solution is worse for both players than outcome where both cooperate with each other
- ► The outcome that obtains from rational play (and in practice!) is a bad outcome for the players

Fiscal and monetary policy game

		FEDERAL RESERVE		
		Low interest rates	High interest rates	
	Budget balance	m	mu	
CONGRESS	Budget deficit	mm	2,2	

Dominance Solvable

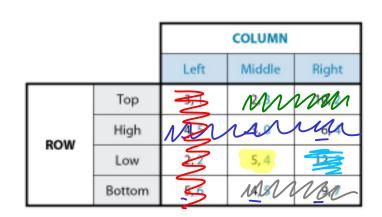
One player has a dominant strategy

- ► Congress has a dominant strategy, Fed does not
- ▶ But Fed knows that Congress has dominant strategy
- ► Fed can choose the best response to Congress's dominant strategy

Successive elimination of dominated strategies

this about each other

- ▶ If a strategy is dominated, then it won't be played at the equilibrium
 - ► Rational players won't play dominated strategies, other rational players know
- ► Removing dominated strategies can simplify the game, makes finding Nash equilibrium easier
- ► A game is *dominance solvable* if successive elimination of dominated strategies ends in a unique outcome (the Nash equilibrium)

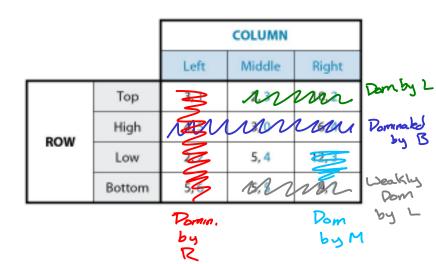


- ► Allows for "ties" in payoffs

- other strategy
- ▶ A strategy is weakly dominant if it never yields a worse outcome than any

- Weak dominance

► Can eliminate weekly dominated strategies to find equilibrium as well



Elimination of weakly dominated strategies

ilmination of weakly dominated strategic
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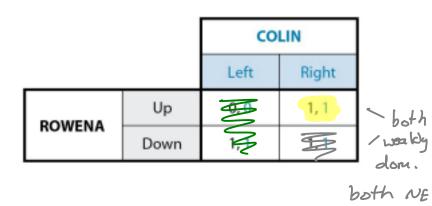
► We can sometimes find a Nash equilibrium by eliminating weakly dominated

strategies

▶ However, we can also eliminate other Nash equilibrium with this strategy!

		COLIN	
		Left	Right
DOWENIA	Up	MM	rown
ROWENA	Down	M	1,1

both weakly, dominated softh NE



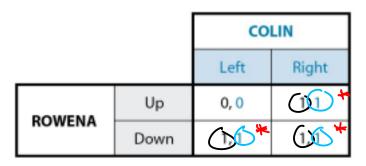
Best response analysis

equilibrium

- ► The Nash equilibrium is a mutual best response
- ▶ We can find the best response for each player for any given opponent strategy
- We can use this to find a Nash equilibrium:
 Find the best responses for each player for all possible opponent strategies
 - ► If one outcome is a best response for both players, then it must be a Nash

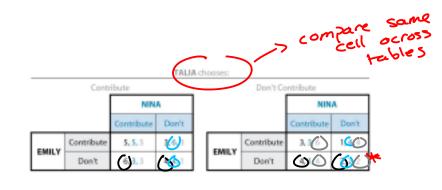
		COLUMN		
		Left	Middle	Right
	Тор	3, 1	2(3)	10, 2
now.	High	4.5	3, 0	6, 4
ROW	Low	2, 2	3 <u>4</u> *	12,3
	Bottom	5,6	4, 5	9(7)





Three player games

- ► We need three dimensions to describe the payoff space with three players
- ► Alternatively, write multiple game matrices for two players, given the choices of a third player
- ► Use best response analysis as before: find best response given strategies of both of the other players



Pure coordination

coordination

- ► Many games have multiple Nash equilibria
 - ▶ If the payoffs are identical across equilibria, then it is a game of *pure*

Pure coordination example

▶ You are to meet someone in Corvallis. You have not been instructed where to meet, you have no prior understanding with the person on where to meet, and you cannot communicate with each other. You are simply told that you will have to guess where to meet, that the other person is being told the same thing, and that you will just have to try to make your guesses coincide. Where do you go?

Games with no pure strategy equilibrium

		NAVRATILOVA	
		DL	CC
EVEDT	DL	50, 50	80, 20
EVERT	CC	90, 10	20, 80

- Games with no pure strategy equilibrium
 - ► What should players do?

▶ More importantly: What *shouldn't* players do?