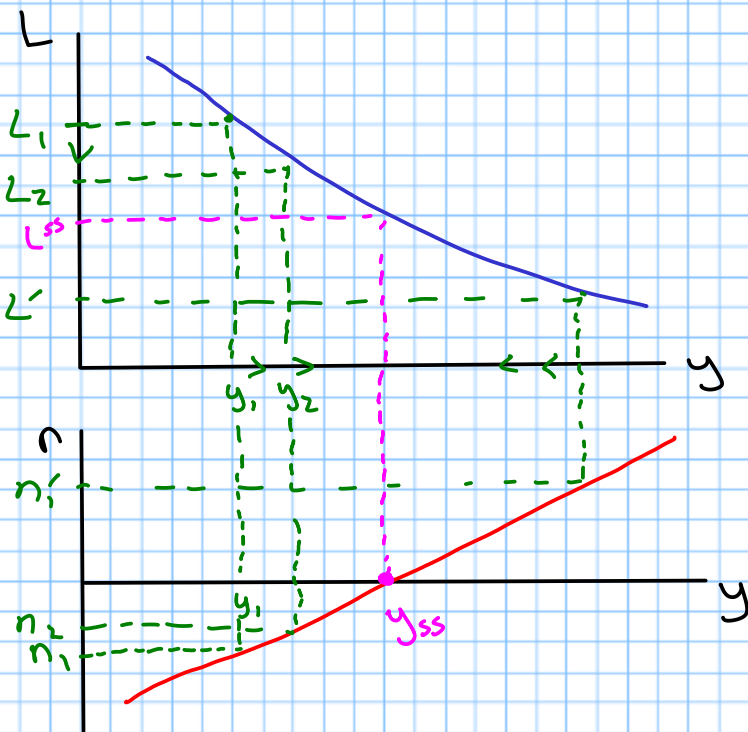
Malthusian Model



L: Number of workers (total population)

y: per capita income

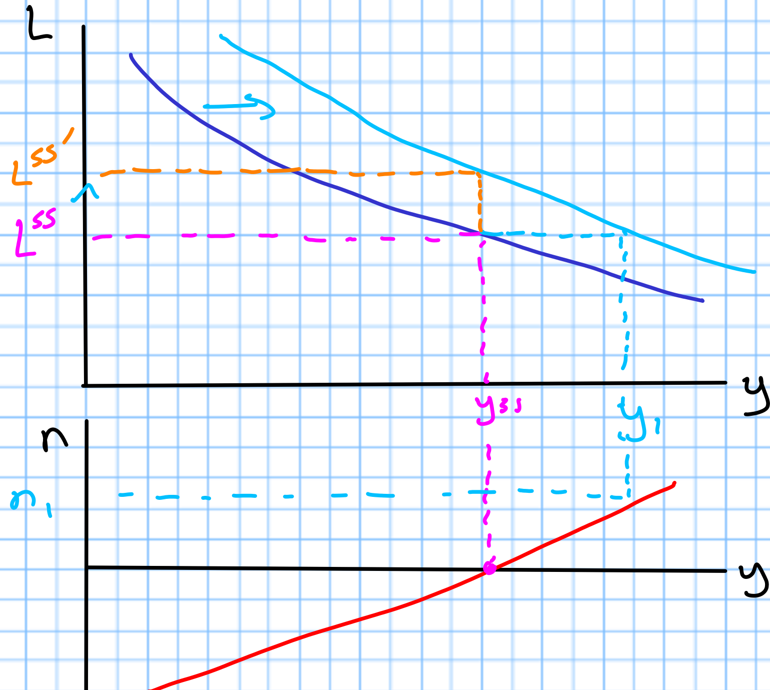
Blue line: relationship between # of people and average income

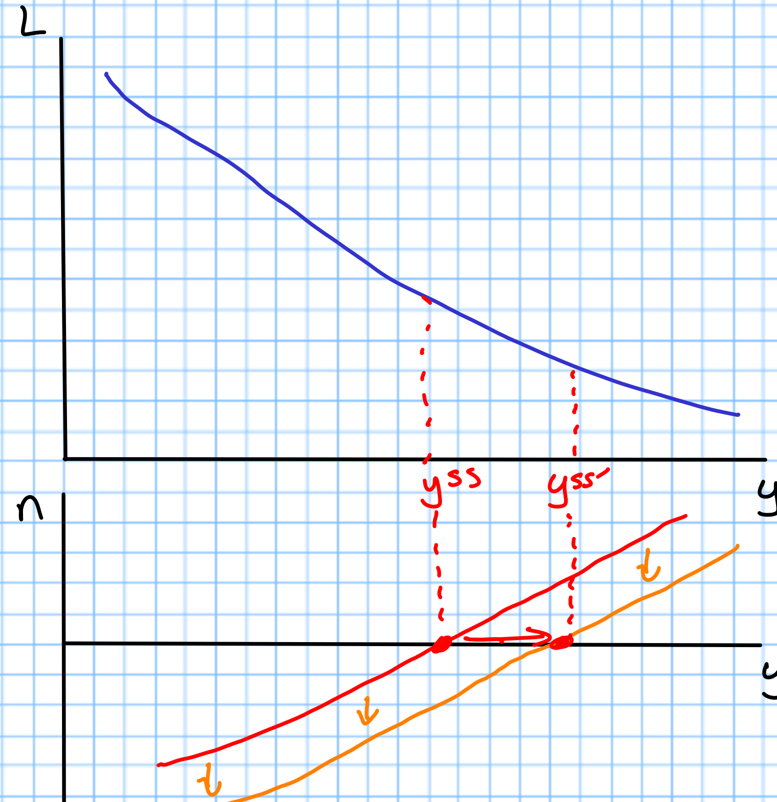
n: growth rate of population

$$n = \frac{L_{t+1} - L_t}{L_t}$$

Red line: relationship between fertility and average income

Productivity growth in Malthusian model





Malthus: The only way to increase long-run income per person is reduce fertility for a given level of income.

→ "Moral restraint"

Population growth in the Solow model

- Capital law of motion

$$K_{t+1} = K_t + I_t - D_t$$

$$K_t = K_{t-1} + I_{t-1} - D_{t-1}$$

- constant savings rate γ
- constant depreciation δ

$$I_t = \gamma Y_t = \gamma K_t^{\alpha} L_t^{1-\alpha}$$

$$D_t = \delta K_t$$

$$K_{t+1} = K_t + \gamma K_t^{\alpha} L_t^{1-\alpha} - \delta K_t$$

Divide both sides by L_t to get in per-worker terms

$$\frac{K_{t+1}}{L_t} = \frac{K_t}{L_t} + \frac{\gamma K_t^{\alpha} L_t^{1-\alpha}}{L_t} - \frac{\delta K_t}{L_t}$$

k_t

$\gamma K_t^{\alpha} L_t^{1-\alpha} L_t^{-1}$

$\gamma K_t^{\alpha} L_t^{-\alpha}$

$\dfrac{\gamma K_t^{\alpha}}{L_t^{\alpha}}$

$\gamma \left(\dfrac{K_t}{L_t}\right)^{\alpha} = \gamma k_t^{\alpha}$

δk_t

$\frac{K_{t+1}}{L_t}$ ← different from chapter 3

Assume population grows at rate n

$$n = \frac{L_{t+1} - L_t}{L_t}$$

Solve for L_t:

$$nL_t = L_{t+1} - L_t$$
$$nL_t + L_t = L_{t+1}$$
$$(1+n)L_t = L_{t+1}$$
$$L_t = \frac{L_{t+1}}{1+n}$$

$$\frac{K_{t+1}}{L_t} = \frac{K_{t+1}}{\left(\frac{L_{t+1}}{1+n}\right)}$$

$$= \frac{K_{t+1}}{1} \cdot \frac{1+n}{L_{t+1}} = \frac{K_{t+1}(1+n)}{1 \cdot L_{t+1}}$$

$$= \frac{K_{t+1}}{L_{t+1}} \cdot (1+n)$$

$$= k_{t+1}(1+n)$$

$$k_{t+1}(1+n) = k_t + \gamma k_t^{\alpha} - \delta k_t$$

- Try to get Δk_t on LHS

$$k_{t+1} + nk_{t+1} = k_t + \gamma k_t^{\alpha} - \delta k_t$$
$$\underline{k_{t+1} - k_t} + nk_{t+1} = \gamma k_t^{\alpha} - \delta k_t$$

$$\Delta k_t + n k_{t+1} - n k_t = \gamma k_t^\alpha - \delta k_t - n k_t$$

$$\Delta k_t + n \Delta k_t = \gamma k_t^\alpha - (\delta k_t + n k_t)$$

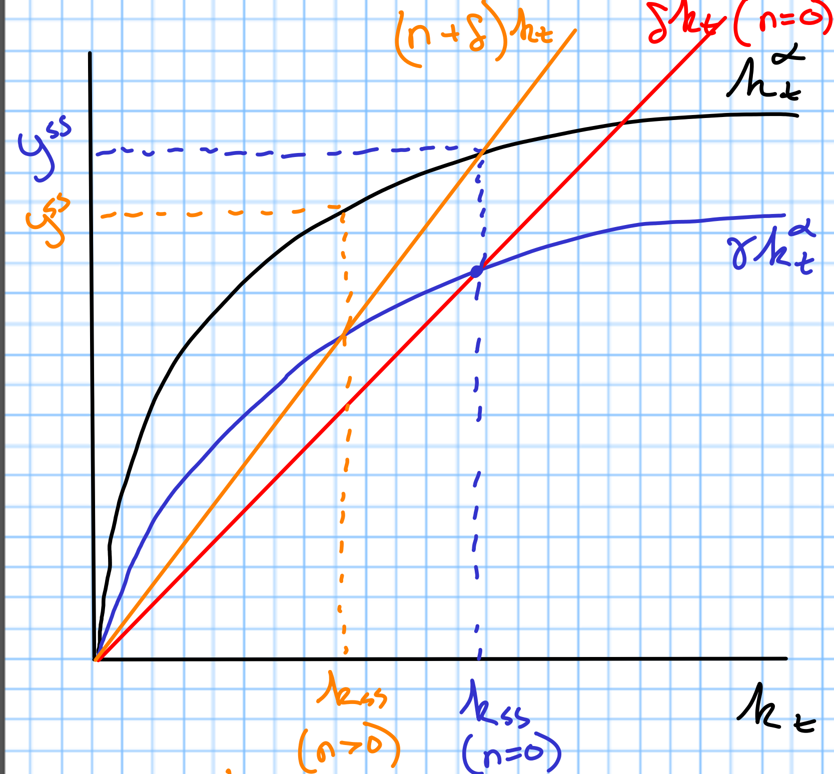
$$(1+n)\Delta k_t = \gamma k_t^\alpha - (\delta + n) k_t$$

Recall:

Without population growth,

$$\boxed{\Delta k_t = \gamma k_t^\alpha - \delta k_t}$$

- Suppose $\gamma k_t^\alpha > (\delta + n) k_t$
  then $\Delta k_t > 0$
- Suppose $\gamma k_t^\alpha < (\delta + n) k_t$
  then $\Delta k_t < 0$
- $\Delta k_t = 0$ (steady-state)
  $\longrightarrow \gamma k_t^\alpha = (\delta + n) k_t$



Capital dilution

At the steady-state:
y_{ss} is not changing

$\rightarrow \frac{Y_t}{L_t}$ is constant

but $L_t \uparrow$ at rate n

$\rightarrow Y_t$ must also be increasing at rate n