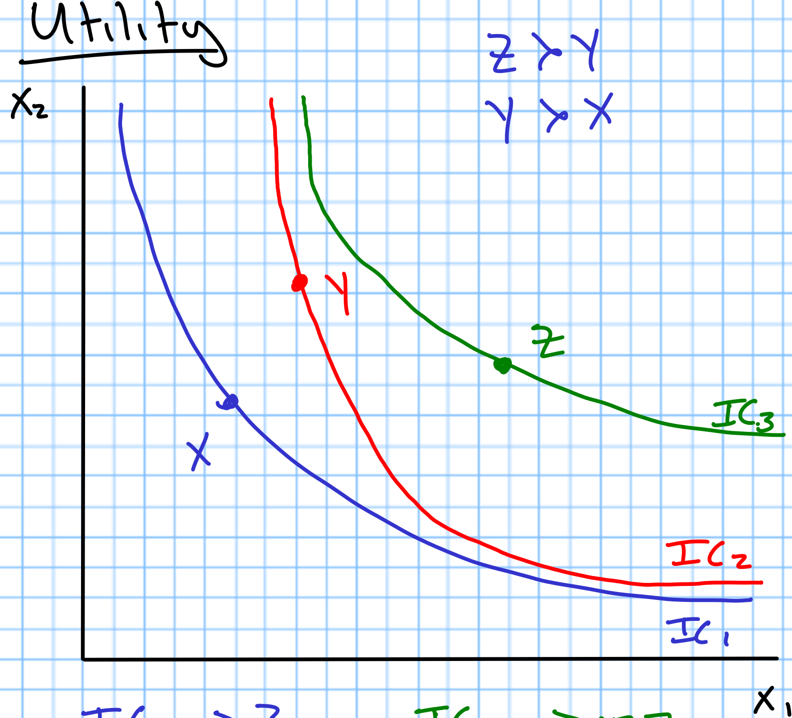
Utility



$IC_1 \rightarrow 3$

$IC_2 \rightarrow 100$

$IC_3 \rightarrow 127$

- Let's imagine we start ranking bundles by assigning numbers to them
- If we would prefer Y to X, we will assign a higher number to Y
- If we are indifferent, we assign both bundles the same number
- Indifference curves will have unique numbers associated with them

- Now we can rank alternative bundles by comparing numbers
  → choose the bundle with a higher number
- Let's create a function that assigns numbers to bundles
  - input: (x_1, x_2) (bundle)
  - output: number u

call this function a utility function

$$u = u(x_1, x_2)$$

Properties of the utility function:

- $X \succ Y$, then $u(x_1, x_2) > u(y_1, y_2)$
- If $Y \succ X$, then $u(y_1, y_2) > u(x_1, x_2)$
- If $X \sim Y$, then $u(x_1, x_2) = u(y_1, y_2)$

Example

$u(x_1, x_2) = 3$

$u(y_1, y_2) = 100$

$u(z_1, z_2) = 127$

Example

Suppose v is a utility fuction

$v(x_1, x_2) = 13.7$

$v(y_1, y_2) = 13.8$

$v(z_1, z_2) = 324{,}874$

- v and u represent the <u>same</u> preferences
- utility functions are <u>not</u> unique (for given preferences, there are infinitely many functions)

- Utility is a "ordinal" number
  → the magnitude doesn't matter

<u>Example:</u> 5K race

Michael: 33 minutes
Rudy: 28 minutes
Llewelyn: 45 minutes

↑ Cardinal numbers

Rank the runners:

ordinal numbers →

M: 2nd (Silver)
R: 1st (Gold)
L: 3rd (Bronze)

Existence

- If preferences are rational, then there is always a function u that represents those preferences
- The utility function is never unique

suppose $u(x_1, x_2) = x_1 + x_2$

Now consider

$$v(x_1, x_2) = u(x_1, x_2) + 5$$
$$= x_1 + x_2 + 5$$

Suppose $(x_1, x_2) = (3, 5)$

$(y_1, y_2) = (0, 7)$

$u(x_1, x_2) = 3+5 = 8$

$u(y_1, y_2) = 7$

$X \succ Y$

$v(x_1, x_2) = 3+5+5 = 13$

$v(y_1, y_2) = 12$

$X \succ Y$

Cobb-Douglas utility

- $u(x_1, x_2) = x_1^a x_2^b$ where a and b are constants

Example:

$u(x_1, x_2) = x_1^2 x_2^3$

suppose $u(x_1, x_2) = 4$

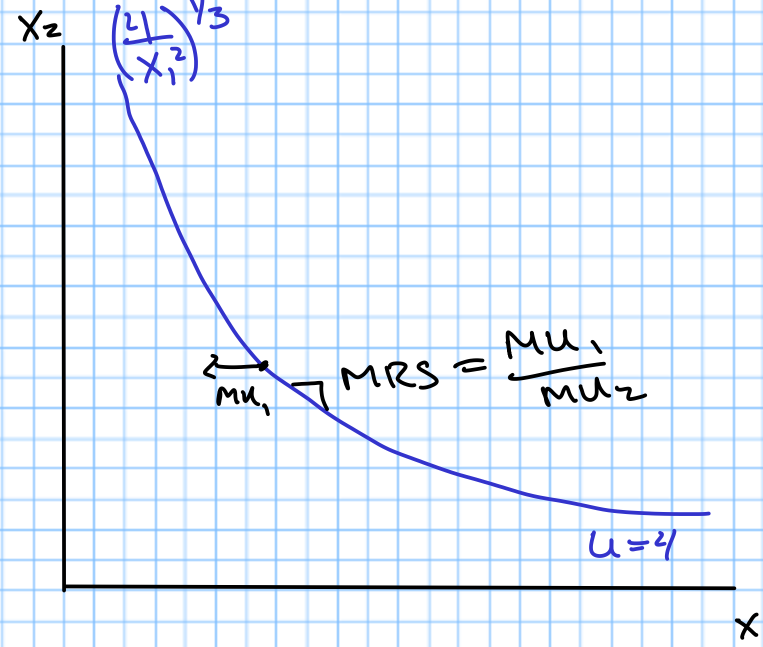
$x_1^2 x_2^3 = 4$

Solve for x_2

$x_2^3 = \frac{4}{x_1^2}$

$x_2 = \left(\frac{4}{x_1^2}\right)^{1/3}$

Plot this function



Result: Cobb-Douglas utility results in well-behaved ICs

Slope of IC

$$\frac{dx_2}{dx_1} \ldots \qquad x_2 = \frac{4^{1/3}}{x_1^{2/3}} = 4^{1/3} x_1^{-2/3}$$

Let's not do this...

Result:

$$MRS = \frac{\dfrac{\partial u(x_1,x_2)}{\partial x_1}}{\dfrac{\partial u(x_1,x_2)}{\partial x_2}} \qquad \begin{array}{l} = MU_1 \\ = MU_2 \end{array}$$

$$\frac{\partial u(x_1,x_2)}{\partial x_1} = 2x_1x_2^3$$

$$\frac{\partial u(x_1,x_2)}{\partial x_2} = 3x_1^2x_2^2$$

$$MRS = \frac{2x_1x_2^3}{3x_1^2x_2^2}$$

$$= \frac{2x_2}{3x_1}$$

Marginal utility

- Slope of the utility function

- $\dfrac{\partial u(x_1, x_2)}{\partial x_1} \rightarrow$ marginal utility of good $\underline{1}$

- Increase x_1 a small amount. How much extra utility do you get?

Example

$$u(x_1, x_2) = x_1^1 x_2^2$$

$$MRS = \frac{MU_1}{MU_2}$$

$$MU_1 = x_2^2 \qquad MU_2 = 2x_1x_2$$

$$MRS = \frac{x_2^2}{2x_1x_2} = \frac{x_2}{2x_1}$$

Example

$$u(x_1, x_2) = x_1^a x_2^b$$

$$MU_1 = a x_1^{a-1} x_2^b$$

$$MU_2 = b x_1^a x_2^{b-1}$$

$$MRS: \frac{a x_1^{a-1} x_2^b}{b x_1^a x_2^{b-1}}$$

$$= \frac{a x_1^{a-1} x_1^{-a}}{b x_2^{b-1} x_2^{-b}}$$

$$= \frac{a x_1^{-1}}{b x_2^{-1}} = \frac{a x_2}{b x_1}$$

Example

$$u(x_1, x_2) = x_1^{-7.3} x_2^{3.14}$$

MRS:

$a = -7.3$

$b = 3.14$

$$MRS = \frac{a x_2}{b x_1} = \frac{-7.3 x_2}{3.14 x_1}$$

Review exponent rules

$$x^{-a} = \frac{1}{x^a}$$

$$x^a = \frac{1}{x^{-a}}$$

$$x^a \cdot x^b = x^{a+b}$$

$$x^2 \cdot x^3 = (x \cdot x) \cdot (x \cdot x \cdot x)$$
$$= x^5$$

$$(x^a)^b = x^{ab}$$

$$(x^2)^3 = (xx)^3$$
$$= (xx)(xx)(xx) = x^6$$