Sequential Games

ECON 420: Game Theory

Spring 2018

Centipede game

1. 2 players play for 10 rounds
2. Each round another unit of payoff is added to the pot (starting with one unit)
3. Players alternate turns, choose to either:
   - Stop, and collect the entire pot for themselves
   - Continue, one is added to the pot and next player chooses

Sequential games

- Games where there is a strict order of play
- Games where players take turns moving are sequential
- Real-world games are generally combinations of sequential and simultaneous games

Game trees

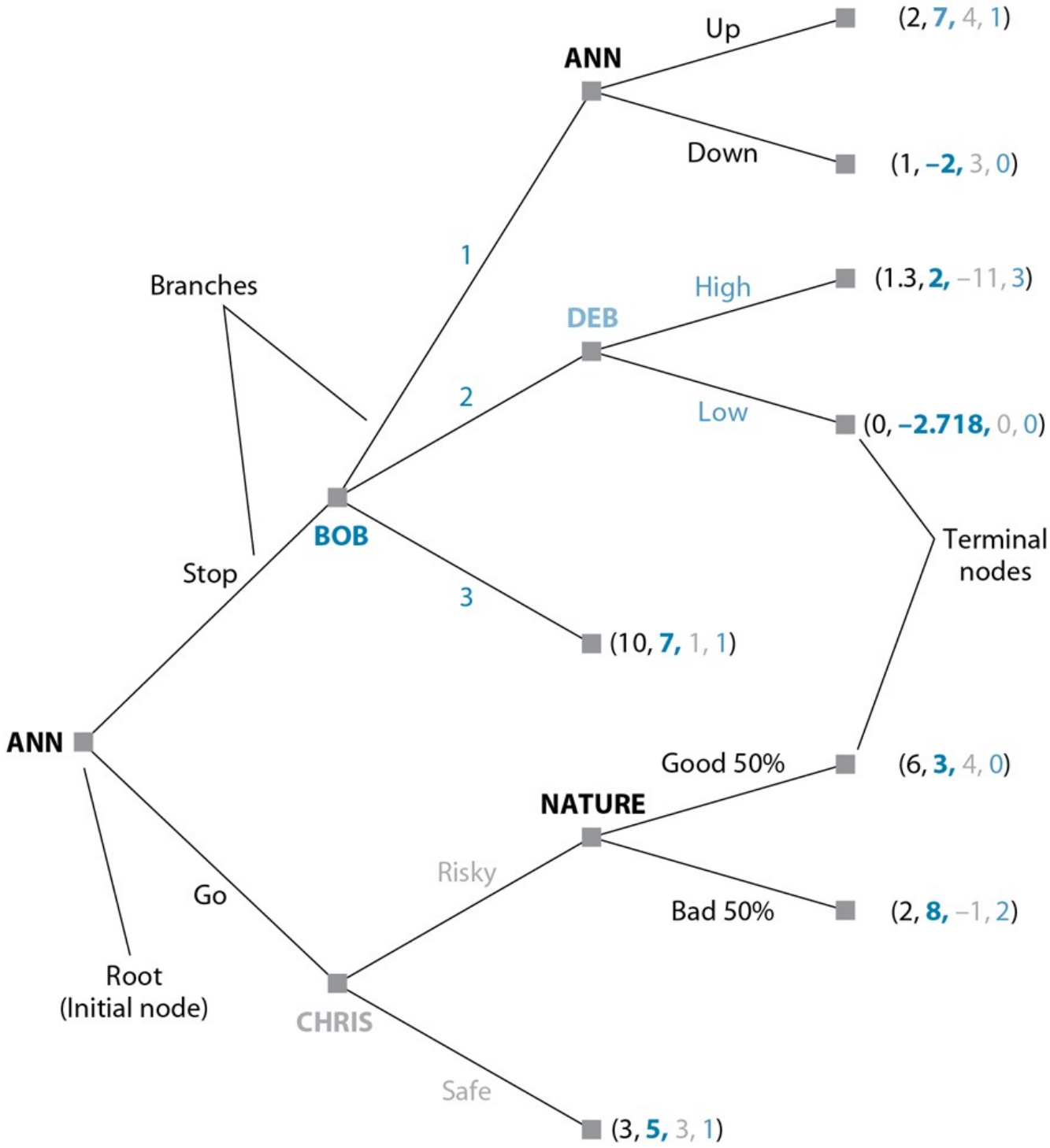
- We will visualize games using **game trees**
- Representing a game as a tree is known as the "extensive form" of a game
- The tree shows all components of a game: players, actions and strategies, payoffs

Nodes and branches

- *Nodes* are points on the tree where choices are made
  - The first node is called the *root node*
  - The last nodes (without branches) are *terminal nodes*
- *Branches* show the actions available for the player to choose among at any node
- A node (and its branches) represent a "turn" for a player
- Payoffs are listed at the terminal nodes
  - Each player in the game gets a payoff at each node
  - Remember: Higher numbers are always better

External uncertainty

- With external uncertainty, we introduce nature as a "player"
- Nature gets its own node, branches are possible outcomes
- Players calculate expected payoffs across the possible outcomes of nature's "choice"


ANN
Up
(2, 7, 4, 1)
Down
(1, –2, 3, 0)
1
Branches
DEB
High
(1.3, 2, –11, 3)
2
Low
(0, –2.718, 0, 0)
BOB
Terminal nodes
Stop
3
(10, 7, 1, 1)
ANN
Good 50%
(6, 3, 4, 0)
NATURE
Risky
Go
Bad 50%
(2, 8, –1, 2)
Root
(Initial node)
CHRIS
Safe
(3, 5, 3, 1)

Moves vs strategy

- A choice of action at a node is called a *move*
- A strategy is a *complete plan of action*
  - A set of moves that will be performed if a certain situation arises
  - Strategies are collections of statements like "if X then Y" for *any possible* X

Example

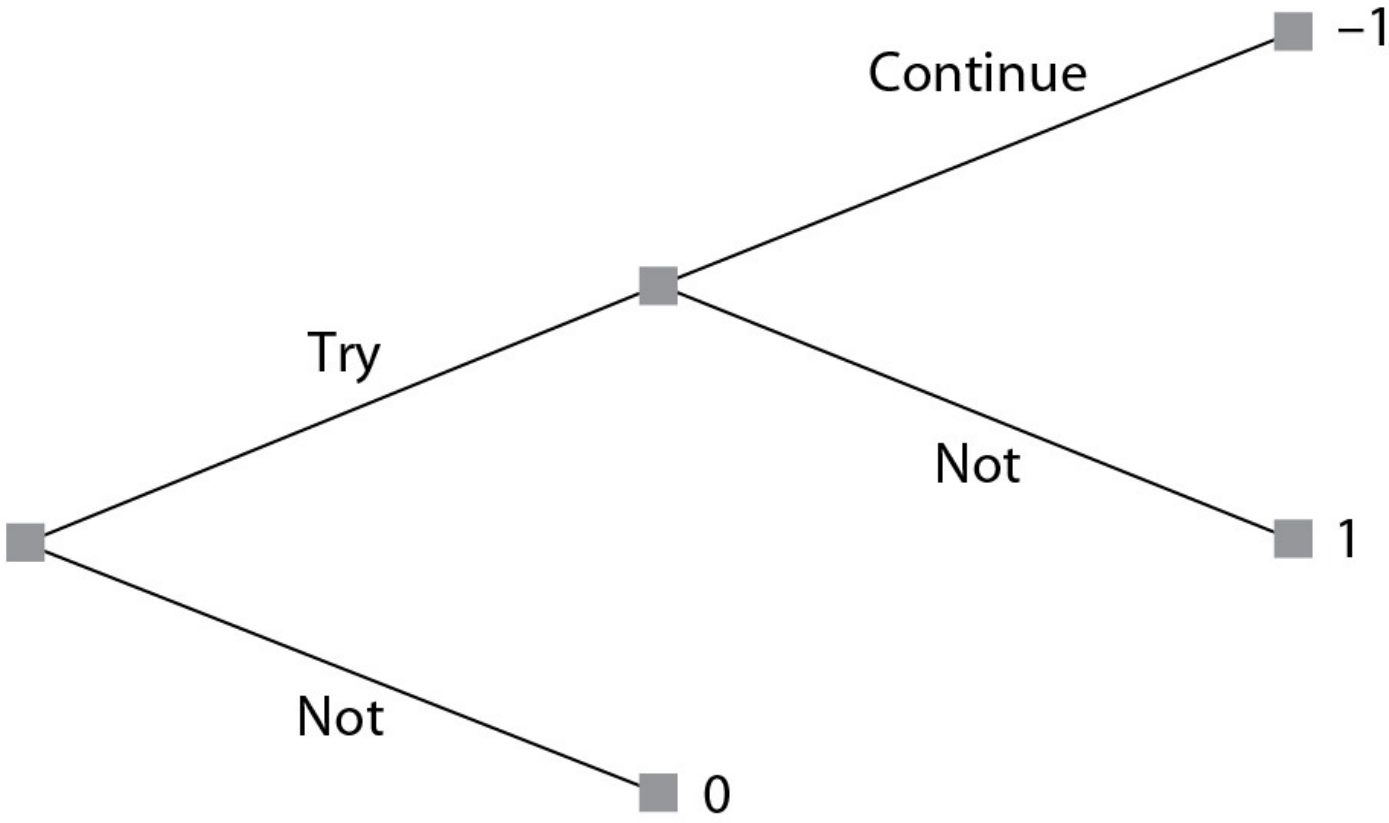
- How many strategies does Ann have?
- What are they?

Strategies

- Strategies must include actions at *each node where a player can move*
- This includes the nodes that won't be reached if a player chooses a particular set of actions
- This is because hypothetical moves might help determine which moves should be chosen at earlier nodes
- Choices early in a game are affected by *expectations* about what will happen later in the game

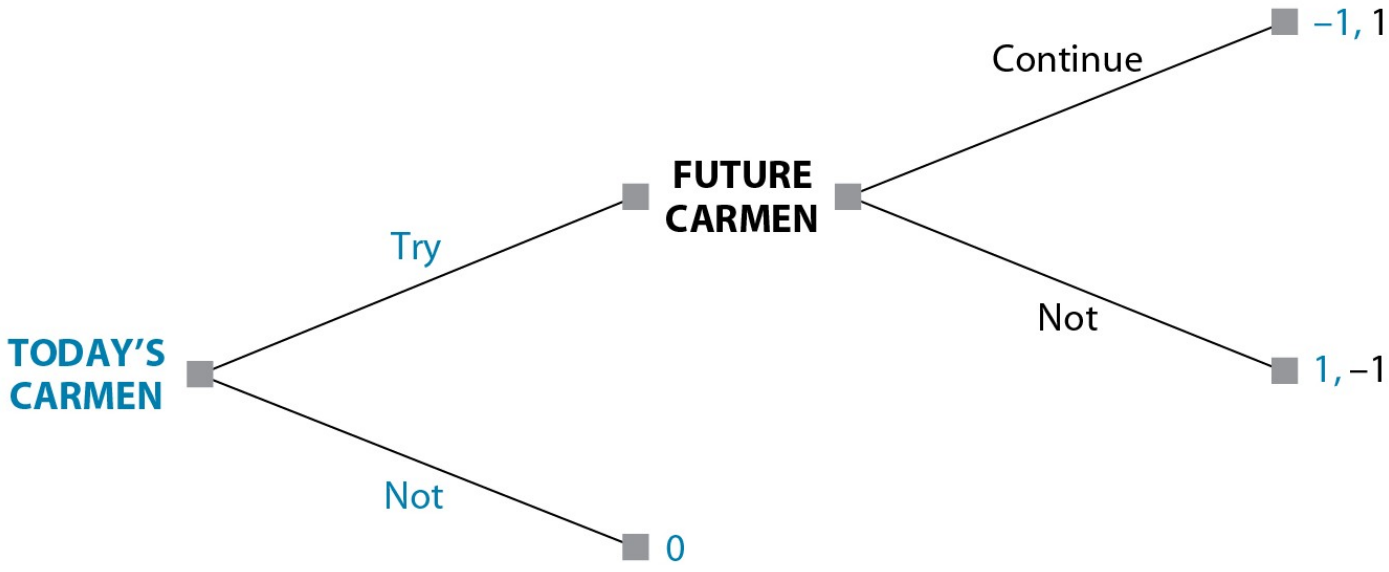
Finding equilibria in game trees

- Consider one person's decision tree (is this a game?)
- The player (Carmen) is considering whether or not to start smoking
- Carmen first decides whether to start, then decides whether to continue
- What should Carmen do?


−1
Continue
Try
Not
1
Not
0

A decision tree as a game

- Previous decision tree ignore that Carmen may become addicted if she starts smoking
  - Once addicted, quitting becomes worse (payoffs are lower)
- Carmen knows she may become addicted and that her payoffs might change if she starts smoking
- We can think of this as a game where the players are Carmen today and Carmen in the future (after the initial decision is made)
- Today's Carmen and future Carmen have different payoffs

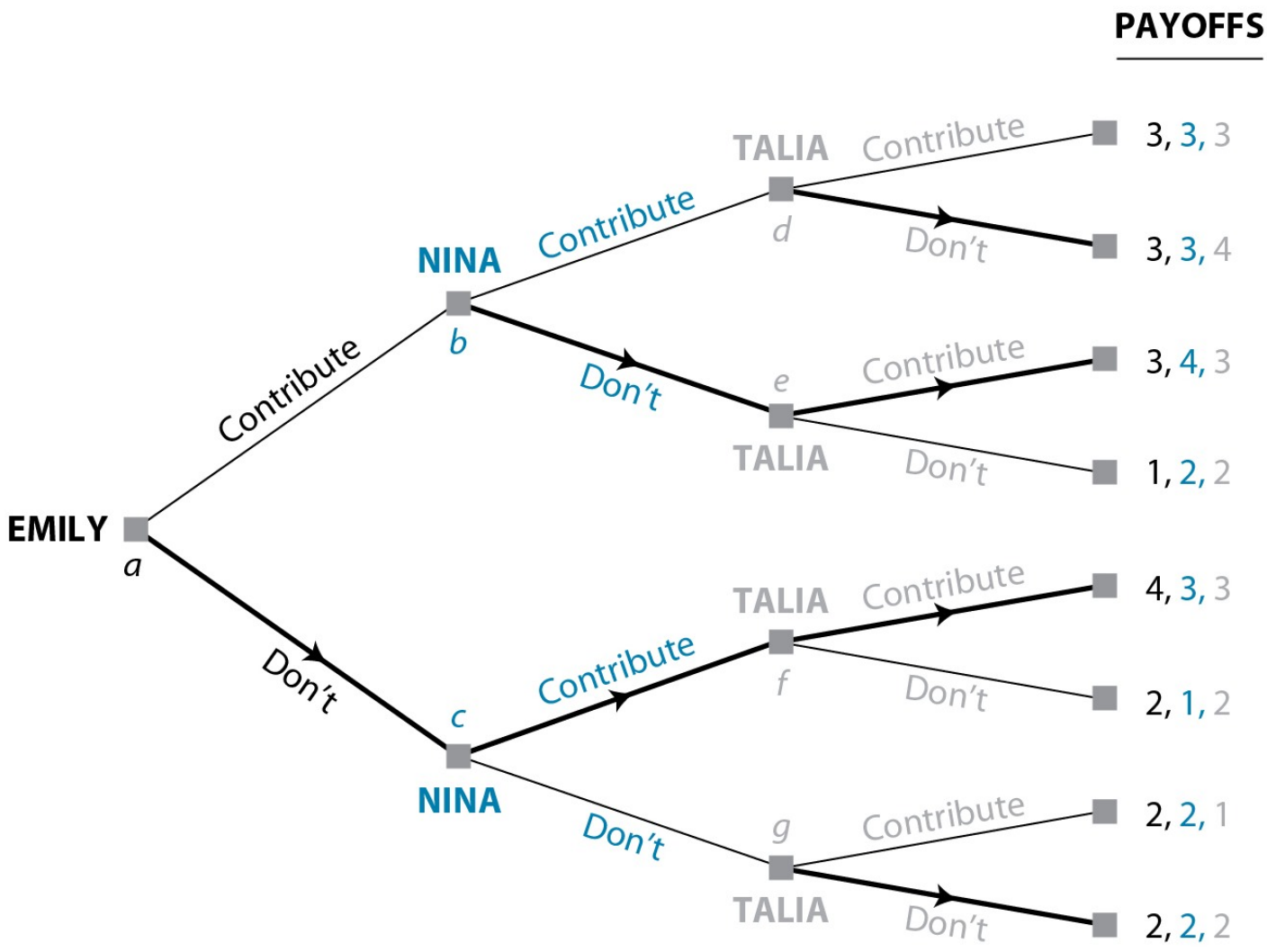

TODAY'S CARMEN
Try
Not
0
FUTURE CARMEN
Continue
Not
–1, 1
1, –1

Pruning

- Starting at the end, we can "prune" the branches that we know will not be chosen
- When there is one action remaining at the final nodes, this means that the "final" decision moves back to the previous node (rollback)
- Starting at the end and moving backward by pruning allows today's Carmen to choose the best option for herself
- When all players use rollback analysis, the result of the game is called a *rollback equilibrium*

Smoking game

- What are the rollback equilibrium strategies?
- Can either player do better by changing their strategies?


PAYOFFS
EMILY
a
Contribute
Don't
NINA
b
Contribute
Don't
NINA
c
Contribute
Don't
TALIA
d
Contribute
Don't
TALIA
e
Contribute
Don't
TALIA
f
Contribute
Don't
TALIA
g
Contribute
Don't
3, 3, 3
3, 3, 4
3, 4, 3
1, 2, 2
4, 3, 3
2, 1, 2
2, 2, 1
2, 2, 2

Three-player game

- How many strategies does each player have?
- What is the rollback equilibrium?
- What are the rollback equilibrium strategies?

Example: Ultimatum game

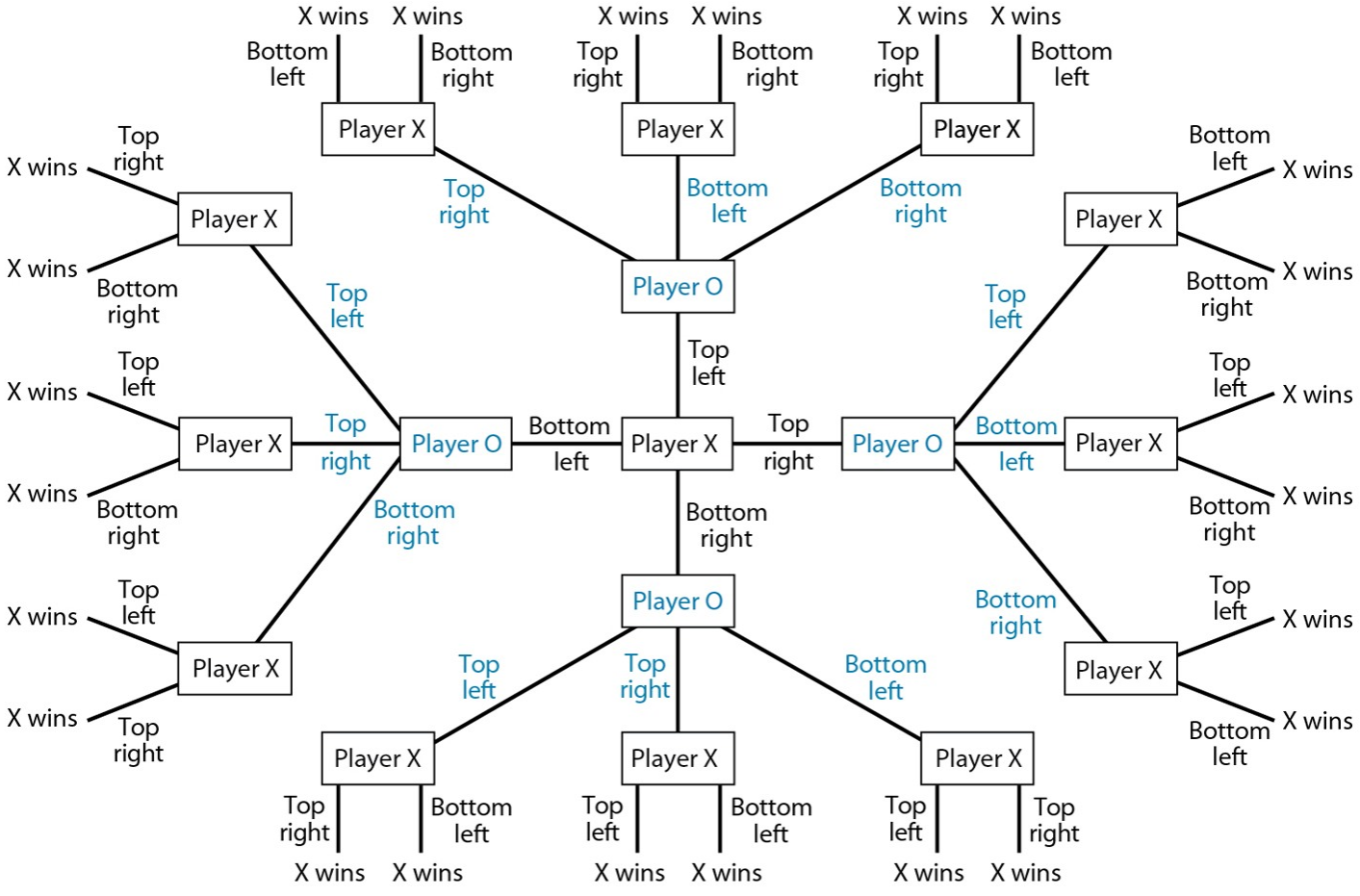
- Player 1:
  - Choose how to split 10 units so that both players get at least one unit
- Player 2:
  - Choose to either:
    1. Accept the split (you get what player 1 chooses for you)
    2. Reject the split (neither player gets anything)

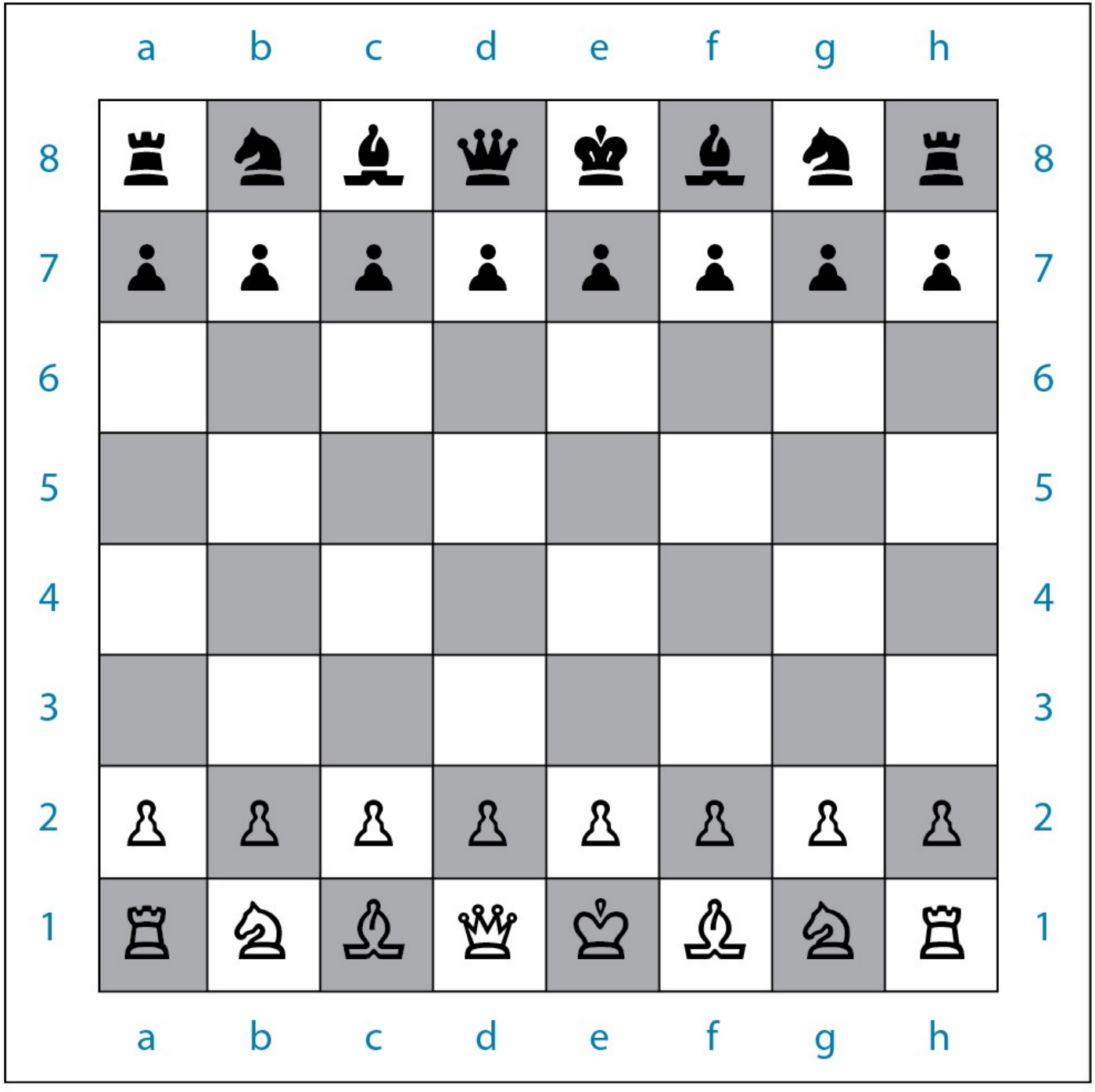
Example: Centipede game

- What does the game tree look like?
- What are the strategies for each player?
- What is the rollback equilibrium outcome?
- What are the rollback equilibrium strategies?
- Is this the outcome we observe in practice?

Limitations of rollback analysis

- Simple games can become difficult to express in extensive form
  - How many moves does the first player have in tic-tac-toe?
  - How many moves does the second player have?
- Some sequential games are *impossible* to express in extensive form!


Player X
Player O
Top left
Top right
Bottom left
Bottom right
X wins


a b c d e f g h
8 7 6 5 4 3 2 1

Chess

- 400 possible positions (nodes) after each player moves once
- 9 million after the third move
- 288 billion after the forth move
- 40 move game: More possible positions than fundamental particles in the universe