Profit maximization

- We assume that firms are always maximizing profits
- Our firms use labor L and capital K to produce output y
- Sell each unit of y for price p
- Labor costs w per unit (wages)
- K costs r per unit

r: "rental rate" of capital

- r is the cost of renting machines and tools
- opportunity cost of using existing (already owned) capital

Revenue

$$R = py$$

Costs

$$C = wL + rK$$

Profits (economic profits)

$$\pi = R - C$$

- Suppose a firm employs K_1 and L_1 they are able to produce y_1

  $y_1 = f(K_1, L_1)$
- Now suppose they decide to increase K by a small amount (1 unit).
  - Additional cost: r
  - Additional revenue: $pMPK$
  - If $pMPK > r$, then profits are increasing
  - If $pMPK < r$, then profits are decreasing
- If the profit maximizing then $pMPK = r$
- Same for labor
- Result: Profit-maximizing firms choose K and L such that

$$\begin{cases} pMPK = r \\ pMPL = w \end{cases}$$

Short run vs Long run

- Long run is the period of time in which there are no fixed inputs
- Short run: At least 1 fixed input

- In our model, we assume that K is fixed in the short run

Short run π-max

- K is fixed at $\bar{K}$
- Fixed costs $r\bar{K}$
- The only thing the firm can control is L
- How much L should the firm choose?
- They set $pMPL = w$

Example

$f(K,L) = K^2 L^{1/2}$

$w = 15, \quad r = 2, \quad p = \$7$

$\bar{K} = 3$

$MPL = \frac{1}{2} K^2 L^{-1/2}$

$$7 \cdot \frac{1}{2} K^2 L^{-1/2} = 15$$

$$7 \cdot \frac{1}{2} 3^2 L^{-1/2} = 15$$

$$63 L^{-1/2} = 30$$

$$\left(L^{-1/2}\right)^{-2} = \left(\frac{10}{21}\right)^{-2}$$

$$L = \left(\frac{21}{10}\right)^2$$

$$L^* = 4.41$$

Output: $y = K^2 L^{1/2}$

$$= 3^2 \, 4.41^{1/2}$$

$$y = 9 \cdot \frac{21}{10}$$

$$y = \frac{189}{10}$$

$$R = p \cdot y$$
$$= 7 \cdot \frac{189}{10}$$
$$= 132.3$$

$$C = wL + rK$$
$$= 15 \cdot 4.41 + 2 \cdot 3$$
$$\cong 60 + 6$$
$$= 66$$

$$\pi \cong 132.3 - 66 = 66.3$$

Short-run π max (Graphically)

$$\pi = \underbrace{p \cdot y}_{\text{revenue}} - \underbrace{(wL + rK)}_{\text{cost}}$$

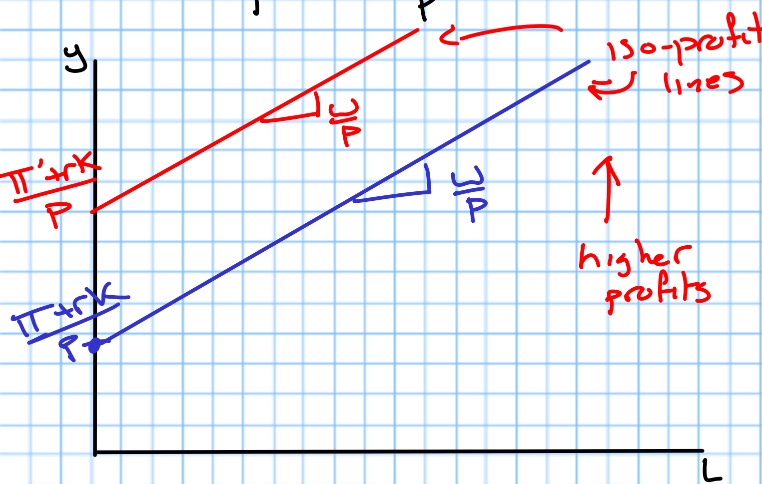
- Graph this function where L on horizontal axis and y on the vertical axis
- Think of this as a function
  - input: L
  - output: y
- solve for y

$$\pi = py - (wL + rK)$$

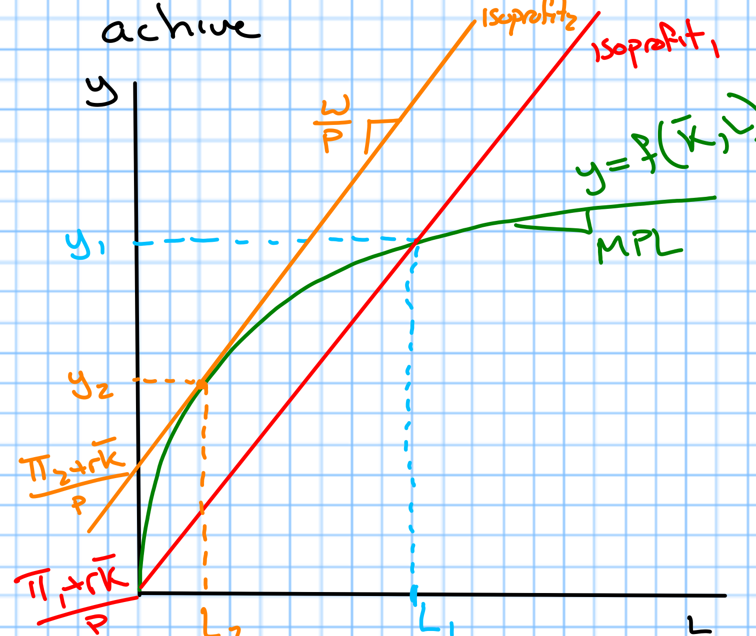
$$\pi + (wL + rK) = py$$

$$y = \frac{\pi + (wL + rK)}{p}$$

$$y = \frac{\pi + rK}{p} + \frac{w}{p} L$$



- Firm is trying to get to the highest isoprofit line that they can achieve

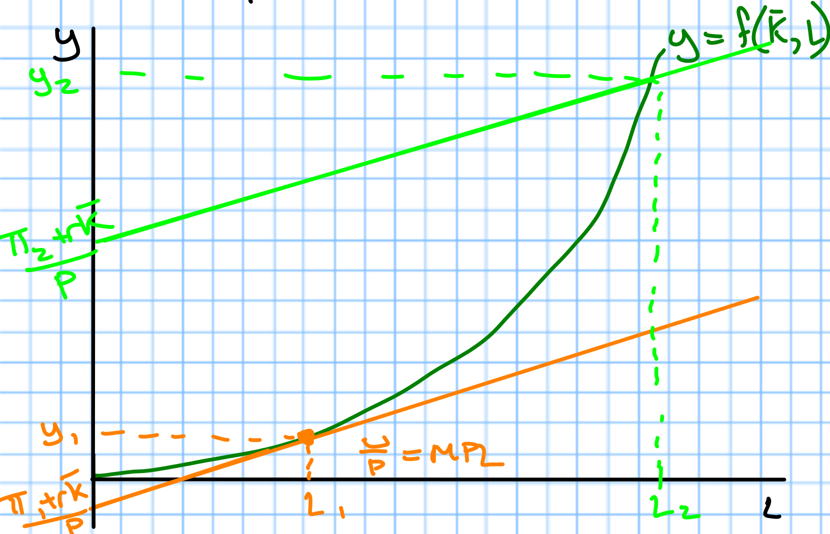


L_2, y_2 maximizes profits

At that point,

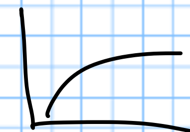
$$\frac{w}{p} = MPL$$

$$\to w = pMPL$$

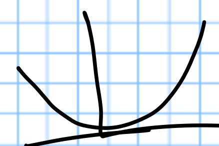


Here, the firm can do better than setting $pMPL = w$

$$y = \bar{K} L^{1/2} \quad (= \sqrt{L})$$



$$y = \bar{K} L^2$$



Short-run returns to scale:

- Firms <u>only</u> control L in the short run

- If the exponent on L is less than 1, then there are short-run decreasing RTS
- If the exponent > 1, then increasing RTS

Increasing RTS:

- Double input (2)
- Output more than doubles
- Revenue more than doubles
- Costs double

With increasing RTS,
Revenue increases faster than costs

- It's always profitable for a firm with increasing RTS to increase inputs (and outputs)
- pMPL = w is profit maximizing <u>only</u> when the firm has decreasing RTS (in the short run)