Measuring Inequality

ECON 499: Economics of Inequality

Winter 2018

What is inequality?

- We can use an inequality index to rank distributions
- There might be disagreements on what inequality means
- Different indices have different properties that we might agree on

Notation

- Suppose there are N people in the economy (for the US, $N \approx 242,000,000$)
- Each person is (arbitrarily) assigned an index number i between 1 and N
- The income of person i is x_i
- Our income distribution is then a *vector* $x = (x_1, x_2, x_3, ..., x_N)$

Example

Obie	Michael	Llewelyn	Rudy	Kitty
2	5	9	20	30

- Income distribution:
  $x = (2, 5, 9, 20, 30)$

Inequality indices

- An inequality index is a function that maps from a vector (distribution) to a real number
  $I = I(x)$
- In other words, it's just a number!
- Allows us to rank distributions
- Higher numbers mean more inequality
- $I = 0$ means $x_1 = x_2 = x_3 = ... = x_N$ (perfect equality)

Index properties

- It is common to disagree on which distributions have more inequality
- A possible solution is to consider a set of normative *values* that we think are important
- We can (hopefully) construct indices that reflect these values
- In general, there is no "best" set of values --- reasonable people can disagree!

Pigou-Dalton Principle of Transfers

- Suppose we have a distribution x
- We take some income from a relatively wealthy person i and give it to a relatively poor person k, where $i > k$
- This is called a *progressive transfer*
- Call the new distribution x'
- The Pigou-Dalton transfer principle says $I(x) > I(x')$
- Inequality is higher before the transfer

Examples

Progressive transfers from A to B:

A	**B**
$(2, 5, 9, 20, 30)$	$(2, 6, 8, 20, 30)$
$(2, 5, 9, 20, 30)$	$(3, 5, 9, 20, 29)$
$(2, 5, 9, 20, 30)$	$(2, 6, 9, 20, 29)$
$(10, 10, 10, 10, 30)$	$(10, 10, 10, 20, 20)$
$(2, 5, 9, 20, 30)$	$(2, 6, 9, 19, 30)$

- Column B distributions are formed by a progressive transfer of column A distributions

How important is the Pigou-Dalton principle of transfers?

- Amiel and Cowell (2001) perform an experiment where they asked participants to rank these distributions in terms of inequality
- 1,153 participants

Which distribution is more unequal?

	Inequality	Risk
Equalising Transfer Reduces Inequality?*		
Total	59	61
Male	61	67
Female	57	53
Consistency with Transfer Principle?**		
Total	17	23
Male	21	31
Female	10	11

*Proportion of answer A.

**Proportion of answer A in all six questions

Table 4: Percentage Shares of Response A

Index properties

- We disagree on what equality is when we see it
- Describing inequality in terms of underlying properties can allow us to overcome this disagreement
- Reframing the argument: Discuss *principles* rather than *outcomes*

Common index properties

1. Principle of transfers (Pigou-Dalton)
2. Symmetry
3. Population invariance
4. Scale invariance
5. Transfer sensitivity
6. Subgroup consistency

Symmetry

- Permutations of the same distribution have the same inequality
- The inequality measure is anonymous: an individual's index doesn't matter (only income matters)

Population invariance

- The size of the population shouldn't impact our inequality index
- If we make an identical clone of everyone and add them to our distribution, our inequality index will remain unchanged

Scale invariance

- The inequality index is not sensitive to scaling of income
- If we change the currency that everyone is paid (dollars to euros, for instance), then inequality remains unchanged
- With scale invariance, we don't have to worry about inflation or purchasing power of currency

Transfer sensitivity

- Consider a transfer between two poor people (the relatively richer of the two gives to the relatively poorer)
- Now consider the *same* transfer between two rich people
- For the rich people, the transfer accounts for less of a percentage of their income
- Transfer sensitivity says an index should reflect this difference
- Inequality at the bottom of the distribution is weighted more heavily

Subgroup consistency

- Partition the distribution into disjoint subgroups
- If the inequality in one subgroup goes up and remains the same in all others, then the inequality of the entire population must go up

Inequality indices

There are many, many different inequality indices. We will focus on the 4 most common:

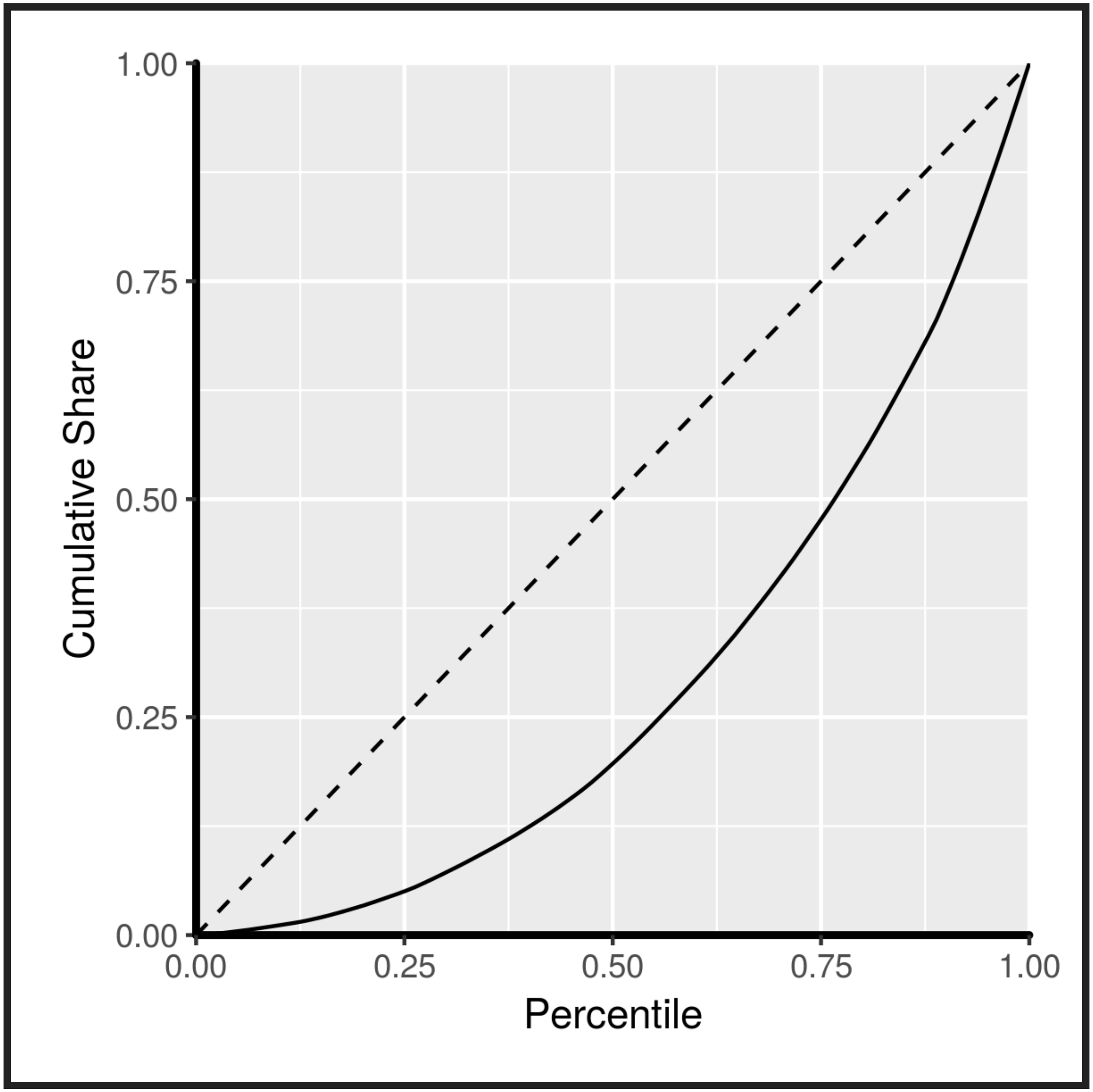
1. Gini coefficient
2. Quantile ratio
3. Atkinson measure
4. Theil index

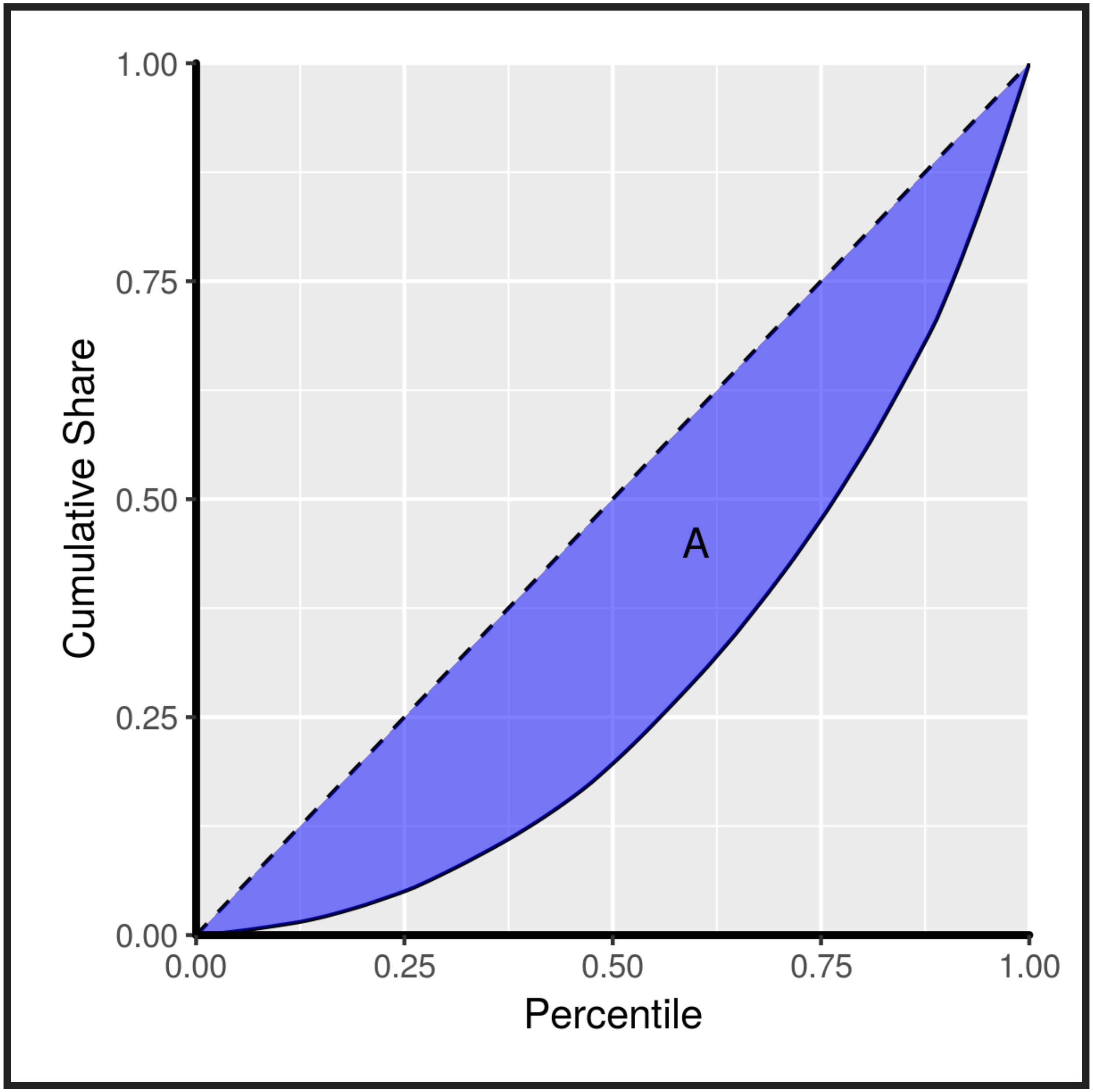
Gini coefficient

Given a distribution $x = (x_1, x_2, x_3, ..., x_N)$, the Gini index is:

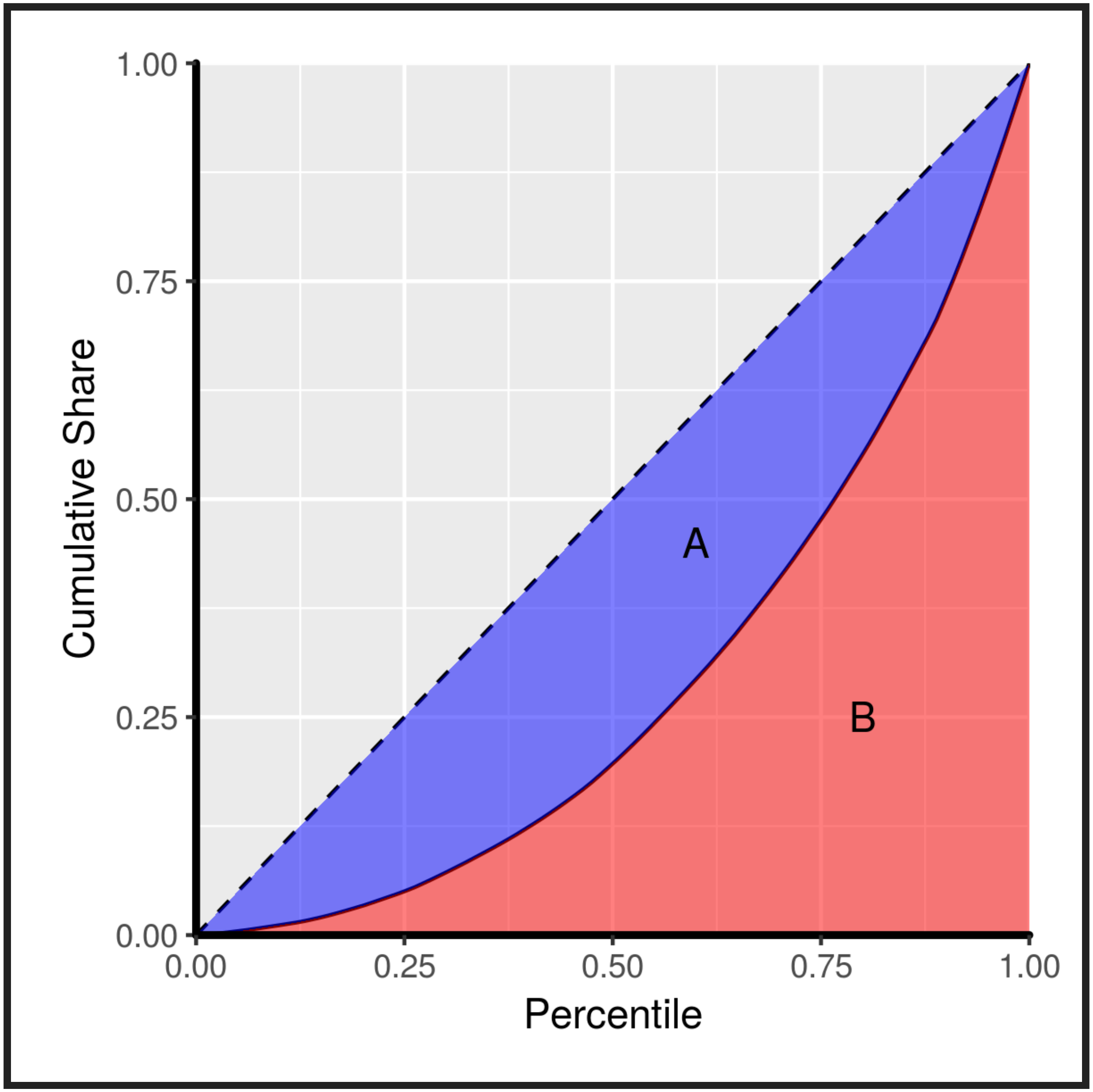
$$I_G(x) = \frac{1}{2N^2\mu} \sum_{i=1}^{N} \sum_{j=1}^{N} |x_i - x_j|$$

It looks scary, but it's just an average of the difference in income between each pair of people in the economy


1.00
0.75
0.50
0.25
0.00
Cumulative Share
0.00
0.25
0.50
0.75
1.00
Percentile



1.00
0.75
0.50
0.25
0.00
Cumulative Share
A
0.00
0.25
0.50
0.75
1.00
Percentile


1.00
0.75
0.50
0.25
0.00
Cumulative Share
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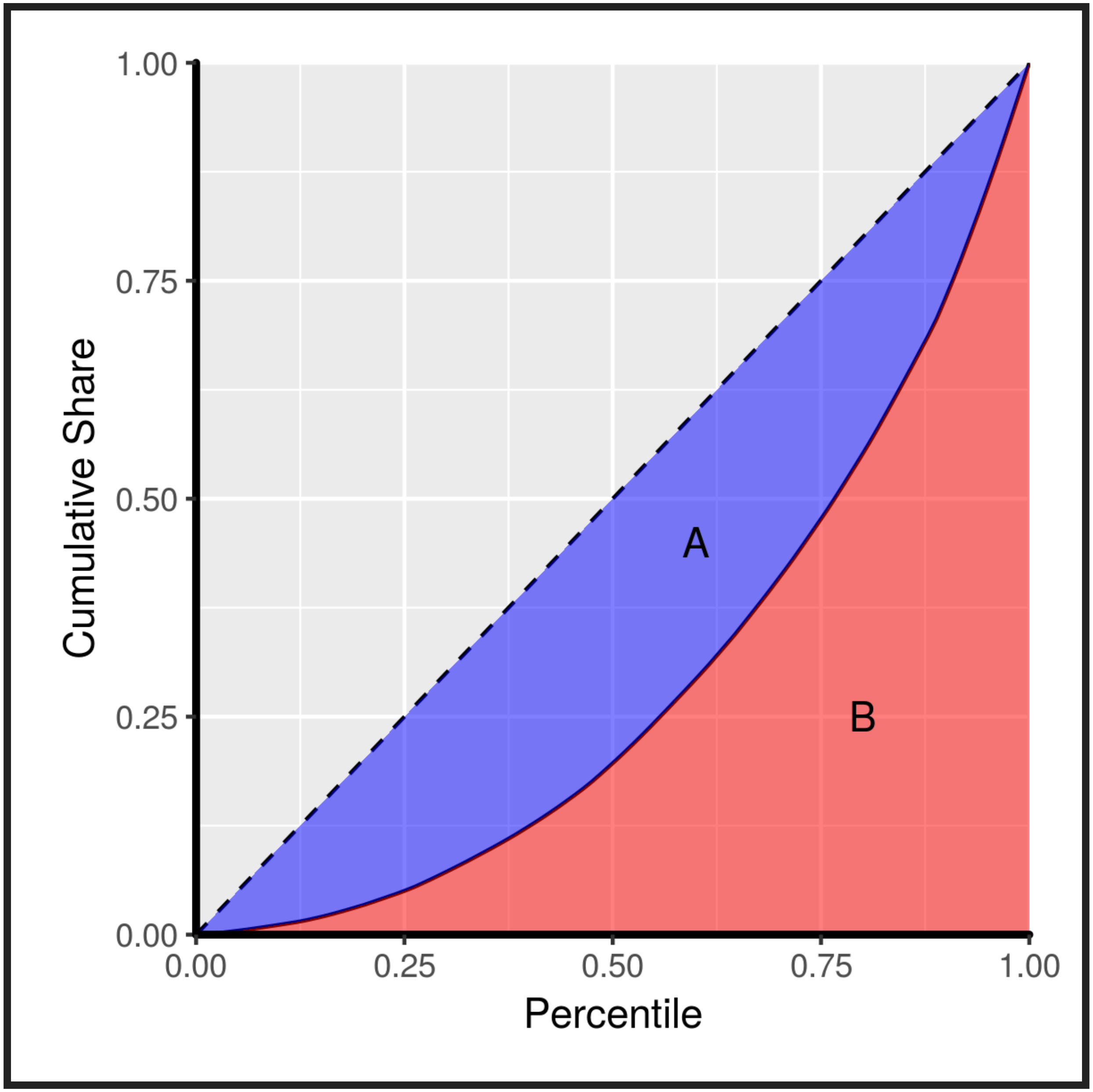
0.25
0.50
0.75
1.00
Percentile
A
B



$$\text{Gini} = A/(A+B)$$

Gini properties

The Gini index has the following properties:

- Principle of transfers
- Symmetry
- Population invariance
- Scale invariance

The Gini does **not** satisfy:

- Transfer sensitivity
- Subgroup consistency

Quantile ratio

- A *quantile* is ranked income
- Example: A person with income at the 90th percentile of the income distribution has more income than 90% of the population, less income than 10% of the population. This income of this person is the 90% quantile.
- A quantile ratio is a simple fraction of the incomes of people at various points in the distribution
- Common ratios: 90/10, 90/50, 50/10, 80/20 (Kuznets ratio)
- Intuition for the 90/10 ratio: how much more income does a person richer than 90% of the population have than someone who is poorer than 90% of the population?
- Sometimes called "income gap"

Example

	Obie	Michael	Llewelyn	Rudy	Kitty
x :	2	5	9	20	30
Percentile:	20	40	60	80	100

$$I_{80/20}(x) = 20/2 = 10$$

Quantile ratio properties

The quantile ratio has the following properties:

- Symmetry
- Population invariance
- Scale invariance

The quantile ratio does **not** satisfy:

- Principle of transfers
- Transfer sensitivity
- Subgroup consistency

Principle of transfers example:

	Obie	**Michael**	**Llewelyn**	**Rudy**	**Kitty**
x :	2	5	9	20	30
x' :	2	4	9	20	31
Percentile:	20	40	60	80	100

$$I_{80/20}(x) = 20/2 = 10$$

$$I_{80/20}(x') = 20/2 = 10$$

Atkinson measure

$$I_A(x,\epsilon) = 1 - \left(\frac{1}{N} \sum_{i=1}^{N} \left(\frac{x_i}{\mu} \right)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}$$

- A "weighted average" of each income's deviation from the mean
- Index goes up as incomes become more "spread out"
- Depends on an additional parameter ϵ, which captures the amount of inequality aversion
- $\epsilon = 0 \Rightarrow I_A(x,\epsilon) = 0$, for all distributions. This means we don't care about inequality
- $\epsilon = \infty \Rightarrow I_A(x,\epsilon) = 1$, for all distributions. Inequality is infinitely bad!

Atkinson properties

- The Atkinson measure satisfies all 6 of our properties!

Theil Index

$$I_T(x) = \frac{1}{N}\sum_{i=1}^{N}\frac{x_i}{\mu}\ln\left(\frac{x_i}{\mu}\right)$$

- In a class of measures called "generalized entropy measures"
- *Entropy* is a concept from physics that measures the amount of disorder in a system
- Here, disorder is a deviation from perfect equality

Theil properties

- The Theil measure also satisfies all 6 of our properties!

Pros and cons

Gini:

- Pros: Commonly used, easy to interpret in terms of Lorenz curves
- Cons: Does not satisfy transfer sensitivity or subgroup consistency

Quantile ratio:

- Pros: Very easy to calculate, very easy to interpret
- Cons: Does not capture much information about the distribution, does not satisfy Pigou-Dalton

Pros and cons, continued

Atkinson:

- Pros: Satisfies all properties, has important normative interpretation (we'll see this later)
- Cons: Difficult to interpret, depends on normative parameter

Theil:

- Pros: Satisfies all properties
- Cons: Difficult to interpret, can be greater than 1, normative properties not clear

Additional reading (optional, on Canvas)

- Foster_et al, chapter 2