Thinking about Games

ECON 420: Game Theory

Spring 2018

Reading

- This week: Chapters 1 and 2
- Next week: Chapter 3

Nim

- Today we'll play an alternative version of nim
- One row of 20 lines
- Each turn, the player must choose to remove 1, 2, or 3 lines
- The last person to remove a line wins

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Decisions vs games

- *Decisions* are choices that can be made "without concern for reaction or response from others"
- *Strategic games* (or just *games*) are choices that occur among "mutually aware players"
  - Players in strategic games take into account the cross-effects of their actions and the actions of other players

Classifying games

Games can be:

- Sequential or simultaneous
- Zero sum or non-zero sum
- Single state or repeated
  - Infinite or finite repetition
- Perfect or imperfect information
- Fixed or manipulable rules

Sequential vs simultaneous games

- Sequential games
  - Players take turns (one after another)
  - Players look ahead at what might happen in the future to make choices
- Simultaneous games
  - Players make choices at the same time
  - Must predict what other players will do contemporaneously

Nim

Sequential or simultaneous?

- *sequential*

Rock, paper, scissors

Sequential or simultaneous?

- *simultaneous*

Chess

Sequential or simultaneous?

- *sequential*

A single play in American football

Sequential or simultaneous?

- *simultaneous*

A soccer penalty kick

Sequential or simultaneous?

- *simultaneous*

Registering for classes

Sequential or simultaneous?

- ?

Constant-sum vs non-constant-sum games

- Constant sum
  - The sum total payoffs are fixed
  - Playing the game only determines the allocation of payoffs, not the total amount
- Zero sum
  - A special case of constant sum where total payoffs are zero
  - Often used to refer to constant-sum games
- Non-constant sum
  - Total payoffs depend on choices of players

Nim

Constant or non-constant sum?

- *constant (zero)*

Rock, paper, scissors

Constant or non-constant sum?

- *constant (zero)*

Splitting the last piece of cake with someone

Constant or non-constant sum?

- *constant (not zero)*

Chicken (stay straight or swerve)

Constant or non-constant sum?

- *non-constant*

International trade

Constant or non-constant sum?

- *non-constant*

Example

All-pay auction

- You will bid to receive a $5 bill
- You have to pay me your highest bid *regardless if you win or lose the auction*
- Everyone who bids has to pay, but only one person will win the $5

Constant-sum games

- Constant-sum games can be either:
  - Negative sum (war, household chores)
  - Zero sum (sports, games with a "winner" and "loser")
  - Positive sum (eating cake)
- Non-constant-sum games can be any of the above, too
  - Sometimes positive *and* negative sums are possible in the same game (all-pay auction)

Single-stage and repeated games

- Single-stage games are played (against some particular opponents) and never again
- Repeated games are played over and over
  - Can be finitely or infinitely repeated
  - Choices in one round (stage) might affect later rounds (and vice versa)

Golden Balls (split or steal)

Single-stage or repeated?

- *single-stage*

"Battle of wits" (poison cups)

Single-stage or repeated?

- *single-stage*

A baseball plate appearance (pitcher vs batter)

Single-stage or repeated?

- *repeated*

OPEC oil production

Single-stage or repeated?

- *repeated*

Perfect and imperfect information

- Perfect information
  - Players know exactly what choice are available to each player and what the payoffs will be (given choices)
- Imperfect information
  - Uncertainty over choices or payoffs (or both)
  - External uncertainty: "Nature" (the state of the world) changes choices or payoffs
  - Strategic uncertainty: Imperfect information about what other players are doing or have done in the past

Nim

Perfect or imperfect information?

- *perfect information*

Chess

Perfect or imperfect information?

- *perfect information*

Vacation planning

Perfect of imperfect information?

- *imperfect information (external uncertainty)*

Applying for jobs

Perfect of imperfect information?

- *imperfect information (strategic uncertainty)*

Poker

Perfect of imperfect information?

- *imperfect information (strategic uncertainty)*

Fixed and manipulable rules

- Fixed rules can't be altered by players
- The choices available to each player are constant and known

Nim

Fixed or manipulable rules?

- *fixed*

Political campaigns

Fixed or manipulable rules?

- *manipulable*

Advertising

Fixed or manipulable rules?

- *manipulable*

Defining a game

- A strategic game must contain three elements:
  1. Players
  2. Strategies
  3. Payoffs
- We make various assumptions about these elements

Players

- The participants in the game who make choices
- Humans, firms, "nature", etc
- We assume players are *rational*
  - They can calculate outcomes from different strategies and will choose the optimum
- We assume players have *common knowledge of the rules*
  - Rules are fixed *at some level*
    - Example: Releasing tax returns when running for president
    - Example: Battle of wits
  - Whether or not to follow rules is *itself* part of a larger game

Example

Who are the players in a game of poker?

Strategies

- The set of choices available to the player
- A complete strategy is a "map" (set of instructions) on how to play a game given any possible set of choices from the other players
- Strategies are collections of choices
- A strategy is complete if you could give your instructions to someone else (or a machine) to do

Payoffs

- The outcomes of the game
- Can be profits, utility, money, wins and losses, etc
- In this class we will assume that higher payoffs are more desirable
- We will often need to calculate *expected payoffs* if there is some randomness or uncertainty (imperfect information)
  - For any possible outcome i with payoff π_i, expected payoffs are $\sum_i p_i \pi_i$, where p_i is the probability of i occurring

Examples

1. Suppose I flip a coin
   - Heads: You get $100
   - Tails: You lose $50
2. Suppose I flip 2 coins:
   - Heads/heads: You get $100
   - Heads/tails: You get $20
   - Tails/tails: You lose $40

Equilibrium

- The outcome where nobody can do better by *unilaterally* changing their strategy
- At equilibrium, nobody can say "I wish I had done that differently"
- Equilibrium does *not* mean that the outcome is optimal
- We don't expect to always obtain an equilibrium
- There is *at least* one equilibrium in every game

One half the average game

What is an equilibrium of this game?

- Everyone picks 0

Extra-credit game (choosing points)

What is an equilibrium of this game?

- Everyone chooses 1 point
- Any other equilibria?

Guess the average

- Everyone write down a number between 0 and 100
- The winning number is the average number guessed
- Trade papers after writing down your number

Extra credit game, part 2

- Get a blank sheet of paper and write your full name at the top
- You will choose to get either 2 points or 10 points extra credit on your first homework assignment
  - If 2 or fewer people choose 10 points, everyone will get their chosen number of points
  - If more than 2 people choose 10 points, *nobody* gets any points
- You can communicate with each other if you'd like (be respectful!)
- I will not reveal the choices (but you can reveal your own...)
- Bring your paper to the front when you've decided