Math "review"

Functions

- A function is a process that transforms inputs into outputs

inputs → Function → output

Example

inputs

water → Function → beer
barley → Function
hops → Function
yeast → Function

Functions of numbers

- Transfor numbers into other numbers

Example

inputs		output
0 →	function	→ 32
100 →	function	→ 212
degrees C		degrees F

Notation

$y = f(x)$

input: x

output: y

f is our function

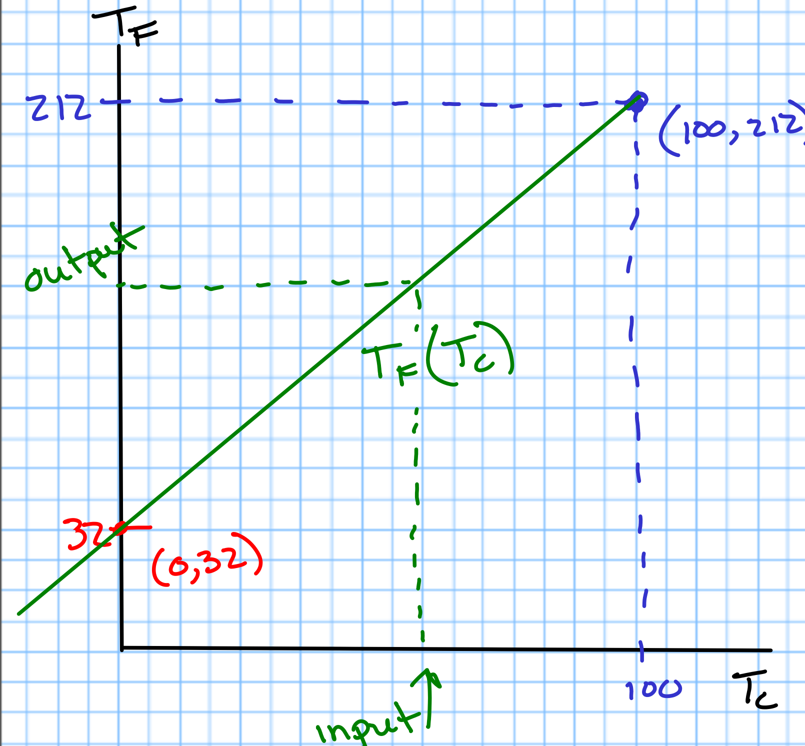
Example

T_F is temp in F

T_c is temp in C

$$T_F = T_F(T_c)$$

- Here, T_F is the function and the output
- We will rely on context to determine which



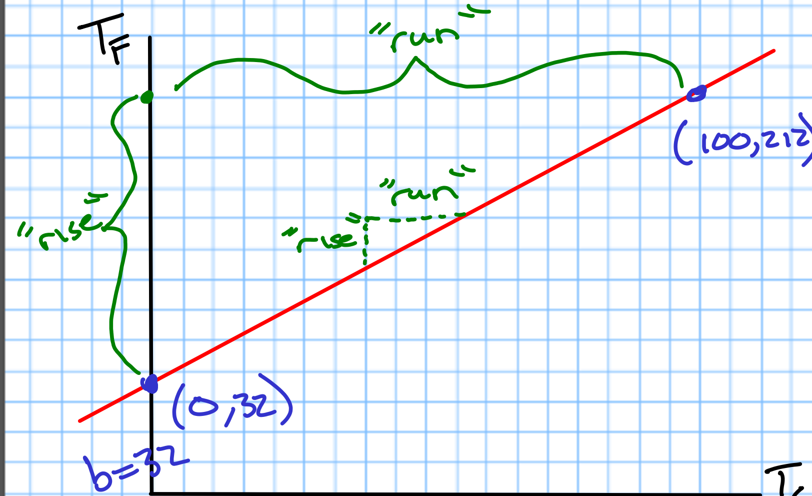
Linear functions

- A function f is <u>linear</u> if we can write it in the form $y = mx + b$, where m and b are numbers (constants)
- What is y when $x = 0$?

$$\begin{aligned} y &= f(x) \\ &= f(0) \\ &= m \cdot 0 + b \\ &= b \end{aligned}$$

(vertical intercept)

- What is m?
  - slope
  - "rate of change"
  - rise over run



$$\text{rise} = \Delta T_F = 212 - 32$$

$$\text{run} = \Delta T_C = 100 - 0$$

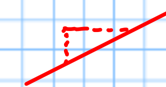
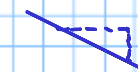
$$\text{slope} = m = \frac{\Delta T_F}{\Delta T_C} = \frac{212-32}{100-0}$$

$$m = \frac{180}{100}$$

$$m = \frac{9}{5}$$

$$T_F(T_C) = \frac{9}{5} \cdot T_C + 32$$

<u>Slope</u>

- Graphically, slope represent the "slant" of a line
  - Positive slopes slant upward 
  - Negative slopes slant downward 
- A slope is the change in output that results from a 1 unit change in input

$$y = mx + b$$

increase x by 1

$$= m(x+1) + b$$

$$= mx + m + b$$

$$= \underbrace{mx + b} + \underbrace{m}_{\Delta \text{ in output}}$$

- What is 1°C in F?

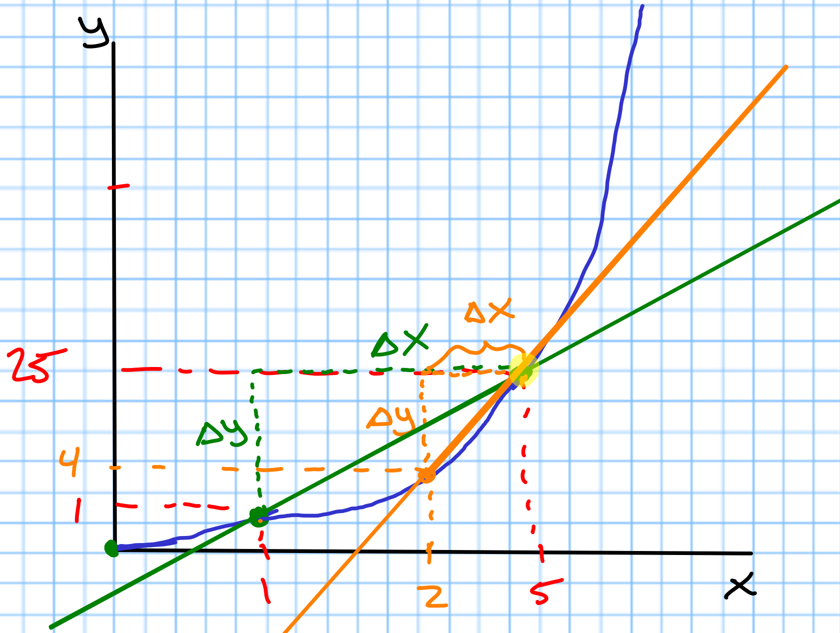
$$32 + 9/5 = 33\,4/5$$

Nonlinear functions

Example

$$y = f(x) = x^2$$

input	output
1	1
5	25
2	4



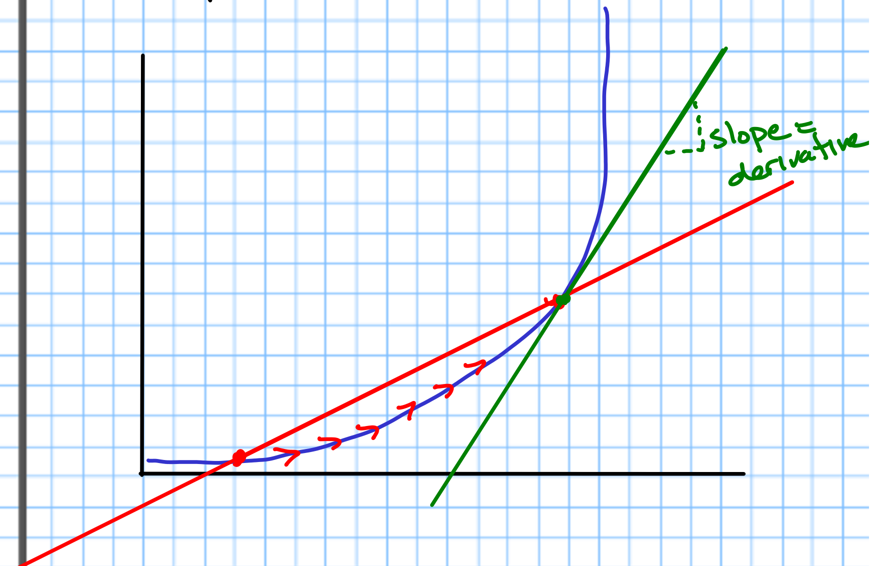
$\Delta y = 24$ $\quad$ slope $= \frac{24}{4} = 6$

$\Delta x = 4$

$\Delta y = 21$ $\quad$ slope $= \frac{21}{3} = 7$

$\Delta x = 3$

Which slope is the "correct" slope?



The slope of the line that touches our function in one spot (tangent line) is called a derivative

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(5-h) - f(5)}{h}$$

Ignore this

Notation:

the derivative of a function $y = f(x)$ is $\frac{dy}{dx}$

Rules

Power rule

If $f(x) = ax^b$

then $\frac{d f(x)}{dx} = a \cdot b x^{b-1}$

Examples

- $y = x^2$

$$\frac{dy}{dx} = 2x^{2-1}$$
$$= 2x^1$$
$$= \underline{2x}$$

- $y = 3x^{-3}$

$$\frac{dy}{dx} = (-3)\cdot 3x^{-3-1}$$
$$= -9x^{-4}$$

- $y = x^{1/3}$

$$\frac{dy}{dx} = \frac{1}{3}x^{1/3-1}$$
$$= \frac{1}{3}x^{\frac{1}{3}-\frac{3}{3}}$$
$$= \frac{1}{3}x^{-\frac{2}{3}}$$

- $y = x^2$

what is the slope of the tangent line that passes through $(5,25)$?

$$\frac{dy}{dx} = 2x \rightarrow 2\cdot 5 = 10$$

Derivatives are functions
input: independent variable
(input to "parent" function)
output: slope of a tangent line

Functions of more than one variable

- Univariate functions ("one variable")

$$y = f(x)$$

1 input
1 output

- Multivariate functions

$$z = f(x, y)$$

inputs: x and y
outputs: z

- Slope of multivariate functions
→ we will use "partial derivatives" to think about slope for multivariate functions
- Suppose we have a function $z = f(x, y)$

Define "the partial derivative of f with respect to x" as:

$$\frac{\partial f(x,y)}{\partial x} = \frac{\partial z}{\partial x}$$

Define "... with respect to y"

$$\frac{\partial f(x,y)}{\partial y} = \frac{\partial z}{\partial y}$$

- Partial derivatives tell us how the output variable changes as <u>just one</u> of our input variables change
- The other input variable is held constant

<u>Example</u>

$$z = f(x,y) = x^1 y^2$$

$$\frac{\partial z}{\partial y} = 2x^1 y$$

$$\frac{\partial z}{\partial x} = y^2$$

Define $g(x) = ax$

$$\frac{dg(x)}{dx} = a$$

Suppose $a = y^2$

then $\frac{dg(x)}{dx} = y^2$

Example

$$f(x,y) = 2x^2 + y$$

$$\frac{\partial f}{\partial x} = 4x$$

$$f(x,y) = 2x^2 + yx^0$$

$$\frac{\partial f}{\partial x} = 4x + \underbrace{0 \cdot yx^{-1}}$$

$$= 4x$$

Example

$$u(x_1, x_2) = x_1^2 x_2^3$$

$$\frac{\partial u}{\partial x_1} = 2x_1 x_2^3$$

$$\frac{\partial u}{\partial x_2} = x_1^2 \, 3x_2^2 \quad = 3x_1^2 x_2^2$$

Example

$$f(K,L) = K^{1/3} L^{2/3}$$

$$\frac{\partial f}{\partial K} = \frac{1}{3} K^{-2/3} L^{2/3}$$

$$\frac{\partial f}{\partial L} = \frac{2}{3} K^{1/3} L^{-1/3}$$

what is this?

$\longrightarrow$ MPL

(much more about this in the second half of the course)