Mixed Strategies (Chpt 7)

Everest (E) \ Navratilova (N)	DL	CC
DL	50, 50	80, 20
CC	90, 10	20, 80

No pure-strategy NE.

- Suppose E plays DL
  - N's best response: DL
  - E's payoff: 50
- Suppose E plays CC
  - N's best response: CC
  - E's payoff: 20

- Suppose E mixes her strategies
  - choose CC with some probability p
  - chooses DL with probab. 1-p

$p = 0.75$

- What is N's best response?
  - We can calculate N's expected payoff to E's strategy
  - N's expected payoff if N plays DL:

$50(0.25) + 10(0.75) = 20$

- N's payoff if she plays CC:

$20(0.25) + 80(0.75) = 65$

- N's Expected payoff is higher if she plays CC
- N's best response to E playing $p = 0.75$ is CC
- Everet's expected payoff:

$(0.25)80 + (0.75)20 = 35$

- Suppose instead E chooses CC with prob. $p = 0.25$
- N's best response:
  - Payoff to DL:

    $50(0.75) + 10(0.25) = 40$
  - Payoff to CC:

    $20(0.75) + 80(0.25) = 35$
  - BR: DL
- E's expected payoff:

$50(0.75) + 90(0.25) = 60$

- Mixed strategy of CC with probability $p = 0.25$ gives E a higher payoff than playing either pure strat.

Exploiting the opponent's strategy

- Zero sum (fixed-sum), E doing better means N must be doing worse
- N can exploit E's pure strategy and do better
- N also exploits E's mixed strategy of CC with $p = 0.25$
- Question: Can E choose a strategy that can't be exploited?
- In other words, is there a strategy that E can play that makes N indifferent among strategies?
- E plays CC with prob. p
  - What is N's payoff to DL?
    $10p + 50(1-p)$
  - What is N's payoff to CC?
    $80p + 20(1-p)$

$10p + 50(1-p) = 80p + 20(1-p)$

$10p + 50 - 50p = 80p + 20 - 20p$

$10p - 50p - 80p + 20p = 20 - 50$

$-100p = -30$

$100p = 30$

$p = 3/10$

$p = 0.3$

- N's payoff to DL:
  $50(0.7) + 10(0.3) = 38$
- N's payoff to CC:
  $80(0.3) + 20(0.70) = 38$
- E's payoff (if N plays DL) $(q=0)$
  $(0.7)50 + (0.3)90 = 62$
- E's payoff (if N plays CC) $(q=1)$
  $(0.7)80 + (0.3)20 = 62$

Best response

- Both players can choose mixed strategies
  E plays CC with prob p
  N play CC with prob q
- E's payoff if N plays q

$(50(1-p)+90p)(1-q) + (80(1-p)+20p)q$

$(50-50p+90p)(1-q) + (80-80p+20p)q$

$(50+40p)(1-q) + (80-60p)q$

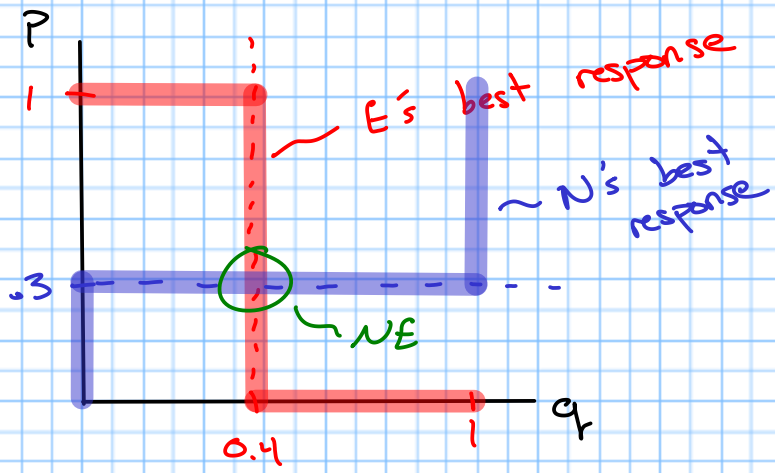
$50+40p-(50q+40pq)+80q-60pq$

$50+40p-100pq+30q$

$50+30q+(40-100q)p$

- What p maximizes this payoff?
  - $p=0$ maximizes when $q=1$
  - $p=1$ maximizes when $q=0$
  - $p=0$ maximizes when $q=0.5$
  - calculate q such that $40-100q=0$
    $\rightarrow q=0.4$

Best response: $p^* = \begin{cases} 0 & \text{if } q>.4 \\ \text{anything} & \text{if } q=.4 \\ 1 & \text{if } q<.4 \end{cases}$



- N's payoff:

$$[50(1-q)+20(q)](1-p)+[10(1-q)+80q]p$$
$$(50-50q+20q)(1-p)+(10-10q+80q)p$$
$$50-30q-50p+30pq+10p+70pq$$

$50 - 30q + 100pq - 40p$

$50 - 40p + (100p - 30)q$

N's best response:

$$q = \begin{cases} 0 & \text{if } p < .3 \\ \text{anything} & \text{if } p = .3 \\ 1 & \text{if } p > .3 \end{cases}$$

Nash equilibrium:

E: $p = 0.3$

N: $q = 0.4$

NE as beliefs

- At the NE, each player forms a belief about the other player's strategies, and chooses their best response to that belief, and the beliefs are accurate

Result

- If there is a mixed strategy equilibrium, all players will indifferent among their various strategies

To find E's NE mixed strategy, choose p (prob of cc) such that N has the same expected payoff

regardless of their choice

- N's expected payoff when playing DL:

$$10p + 50(1-p)$$

- N's expected payoff when playing CC:

$$80p + 20(1-p)$$

$$10p + 50(1-p) = 80p + 20(1-p)$$

$$10p + 50 - 50p = 80p + 20 - 20p$$

$$-100p = -30$$

$$p = \frac{30}{100} = 0.3$$

N's NE strategy:
Make E indifferent

$$\underbrace{80q + 50(1-q)}_{\text{payoff when playing DL}} = \underbrace{20q + 90(1-q)}_{\text{payoff when playing CC}}$$

$$80q + 50 - 80q = 20q + 90 - 90q$$

$$100q = 40$$

$$q = \frac{40}{100} = 0.4$$

Prisoners' Dilemma

		Wife: Confess	Wife: Deny
Husband	C	-10, -10*	-1, -25
	D	-25, -1	-3, -3

Mixed-strategy

H chooses C with prob p

U chooses C with prob q

(Pure strategy NE: p=1, q=1)

$$-10p + (-1)(1-p) = -25p + (-3)(1-p)$$

$$-10p - 1 + p = -25p - 3 + 3p$$

$$13p = -2$$

$$p = -2/13$$

but p must be between 0 and 1, Therefore no mixed strategy NE exists

Coordination game

		Sally: Starbucks	Sally: Local
H	S	1, 1 *	0, 0
	L	0, 0	2, 2 *

2 pure-strategy NE

p: prob H chooses Starbucks

q: prob Sally chooses Starbucks

H: $1p + 0(1-p) = 0p + 2(1-p)$

$$p = 2 - 2p$$

$$3p = 2$$

$$p = 2/3$$

Sally: $1q + 0(1-q) = 0q + 2(1-q)$

$q = 2/3$

NE: $p = 2/3$, $q = 2/3$

$p = 1$, $q = 1$

$p = 0$, $q = 0$

Payoffs when $p = q = 0$

H: 2

Sally: 2

Payoffs when $p = q = 1$

H: 1

Sally: 1

Payoffs when $p = q = 2/3$

H: $\left(\frac{2}{3}\cdot 1 + \frac{1}{3}0\right)\frac{2}{3} + \left(0\cdot\frac{2}{3} + 2\cdot\frac{1}{3}\right)\frac{1}{3}$

$\frac{2}{3}\cdot\frac{2}{3} + \frac{2}{3}\cdot\frac{1}{3} = \frac{4}{9} + \frac{2}{9}$

$= \frac{6}{9} = \frac{2}{3}$

Sally: $\frac{2}{3}$

Chicken

J \ D	Swerve	Straight
Swerve	0, 0	−1, 1
Straight	1, −1	−2, −2

p: prob J plays Swerve

q: Prob D plays Swerve

J: $0p + (-1)(1-p) = 1p + (-2)(1-p)$

$-1 + p = p - 2 + 2p$

$1 = 2p$

$p = 1/2$

D: $q = 1/2$

Payoffs: $P=1, q=0$

J: -1

D: 1

$P=0, q=1$

J: 1

D: -1

$P=q=1/2$

J: $(0 \cdot \frac{1}{2} + (-1)\frac{1}{2})\frac{1}{2} + (1 \cdot \frac{1}{2} + (-2)\frac{1}{2})\frac{1}{2}$

$-1/4 + -1/4 = -1/2$

D: $-1/2$