

Suppose we are in time  $t$   
and we have some amount  
of capital ( $K_t$ ) and income  
(output,  $Y_t$ )

How much capital will we have  
tomorrow?

- Some of today's capital will  
depreciate

$$K_t \text{ today} \rightarrow K_t - \delta K_t = (1-\delta)K_t$$

- We will create new capital  
 $Y_t$  today  $\rightarrow sY_t$  capital  
tomorrow

Putting it all together:

$$K_{t+1} = sY_t + (1-\delta)K_t$$

"Capital law of motion"

Production function

$$Y_t = f(K_t), \quad f''(K_t) < 0$$



For simplicity, assume  
 $F(K_t) = K_t^\alpha$   $0 < \alpha < 1$

CLM:  $K_{t+1} = sY_t + (1-\delta)K_t$   
 $K_{t+1} = sK_t^\alpha + (1-\delta)K_t$   
 $K_{t+1} = sK_t^\alpha + K_t - \delta K_t$   
 $K_{t+1} - K_t = sK_t^\alpha - \delta K_t$   
 $\Delta K_t = sK_t^\alpha - \delta K_t$

$\Delta K_t > 0 \iff sK_t^\alpha > \delta K_t$

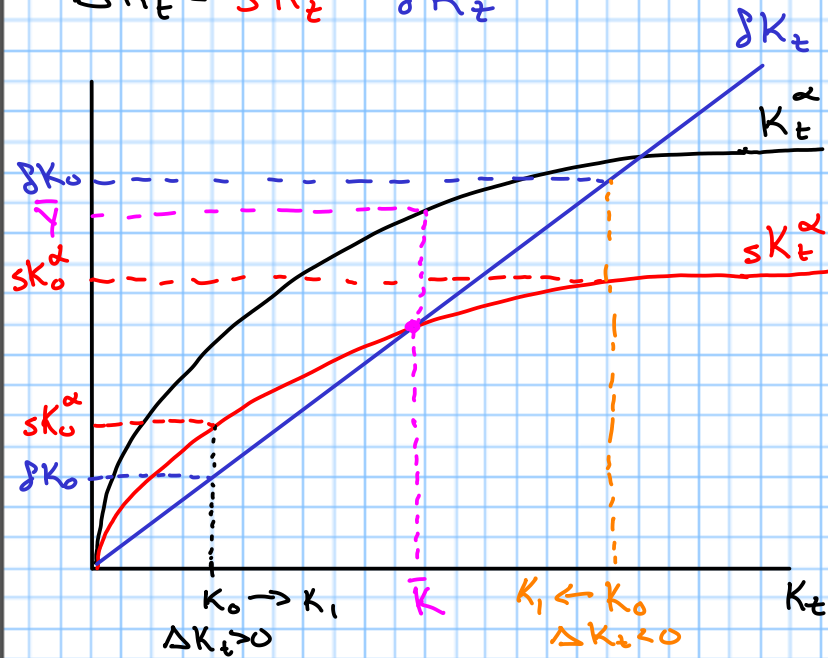
$\Delta K_t < 0 \iff sK_t^\alpha < \delta K_t$

$\Delta K_t = 0 \iff sK_t^\alpha = \delta K_t$

When  $\Delta K_t = 0$ , we say that  
 we are at "steady state"

$K_{t+1} = K_t = \bar{K}$  is the  
 steady-state level of capital

$\Delta K_t = sK_t^\alpha - \delta K_t$



$$\Delta K_t = sK_t^\alpha - \delta K_t$$

$$\Delta K_t = 0 \Rightarrow 0 = s\bar{K}^\alpha - \delta \bar{K}$$

$$\frac{\delta \bar{K}}{\bar{K}^\alpha} = \frac{s \bar{K}^\alpha}{\bar{K}^\alpha}$$

$$\delta \bar{K}^{1-\alpha} = s$$

$$\bar{K}^{1-\alpha} = \frac{s}{\delta}$$

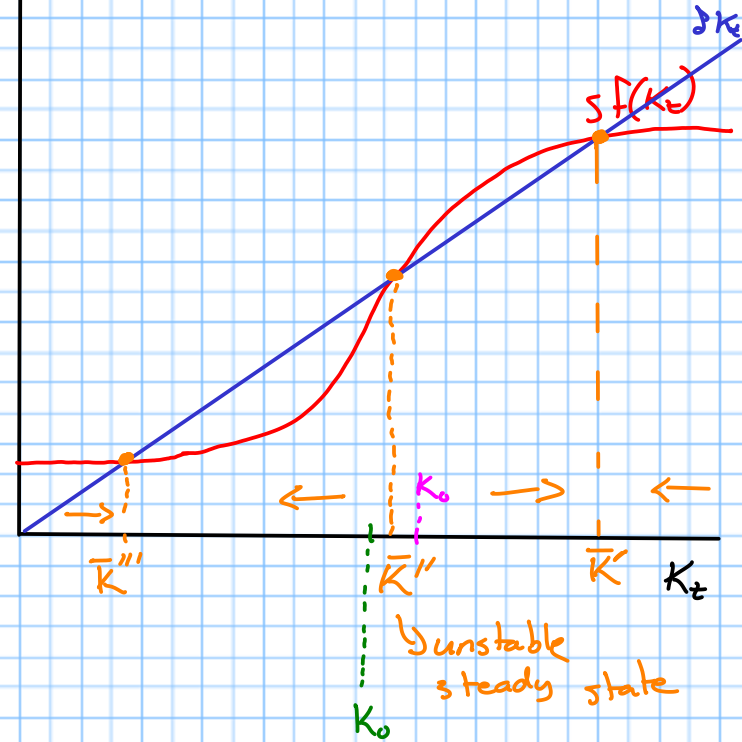
$$\bar{K} = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}$$

$$\bar{Y} = \bar{K}^\alpha$$

$$= \left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

$s f(K_t)$   
 $\delta K_t$

$$Y_t = f(K_t)$$



$$CLM: K_{t+1} = (1-\delta)K_t + I_t$$

$$I_t = s Y_t \rightarrow \text{"Gross investment"}$$

$$s \rightarrow \text{"Gross" savings rate}$$

$$Y_t \rightarrow \text{Gross income}$$

Instead consider "net"

variables

$$\tilde{Y}_t = Y_t - \delta K_t \quad (\text{"income net of depreciation"})$$

$$CLM: K_{t+1} = K_t - \delta K_t + I_t$$

$$K_{t+1} = K_t + (I_t - \delta K_t)$$

$$K_{t+1} = K_t + \tilde{I}_t$$

↑  
net investment

$$K_{t+1} = K_t + \tilde{S} \tilde{Y}_t$$

On the "balanced growth path"  
(steady-state growth)

$\tilde{Y}_t$  grow at rate  $g$

$K_t$  grow at rate  $g$

$\frac{K_t}{\tilde{Y}_t}$  is constant on BGP

$$\frac{K_{t+1}}{\tilde{Y}_t} = \frac{K_t}{\tilde{Y}_t} + \tilde{S}$$

Since  $K_t/\tilde{Y}_t$  is constant,

$$\frac{K_t}{\tilde{Y}_t} = \frac{K_{t+1}}{\tilde{Y}_{t+1}} = \dots$$

Growth rate of  $g$  means

$$\frac{\tilde{y}_{t+1}}{\tilde{y}_t} = (1+g) \quad \left( g = \frac{\tilde{y}_{t+1} - \tilde{y}_t}{\tilde{y}_t} \right)$$

$$\frac{K_{t+1}}{\tilde{y}_{t+1}} - \frac{\tilde{y}_{t+1}}{\tilde{y}_t} = \frac{K_t}{\tilde{y}_t} + \tilde{s}$$

$$\frac{K_{t+1}}{\tilde{y}_{t+1}} (1+g) = \frac{K_t}{\tilde{y}_t} + \tilde{s}$$

$$\frac{K_t}{\tilde{y}_t} (1+g) = \frac{K_t}{\tilde{y}_t} + \tilde{s}$$

$$\frac{K_t}{\tilde{y}_t} g = \tilde{s}$$

$$\frac{K_t}{\tilde{y}_t} = \frac{\tilde{s}}{g}$$

Piketty's "Second Fundamental Law of Capitalism"

Wealth to (net) income ratio  
is equal to the (net)  
saving to growth ratio

Gross terms:

$$\frac{K_t}{Y_t} = \frac{s}{g+s}$$