Technology

So far: consumers have preferences over consumption goods, we can derive demand given preferences, prices, and income

Now

- Firms supply consumption goods to the market
- How much will a firm supply?
- What price will they charge?

Production process

- Firms take inputs and transform them into output
- The way in which inputs are transformed into outputs is call <u>technology</u>

Factors of Production

- An input to a production process is called a <u>factor</u>

- We will focus on two specific factors,
  1. Capital (K)
  2. Labor (L)

  L: Hours worked per unit of time

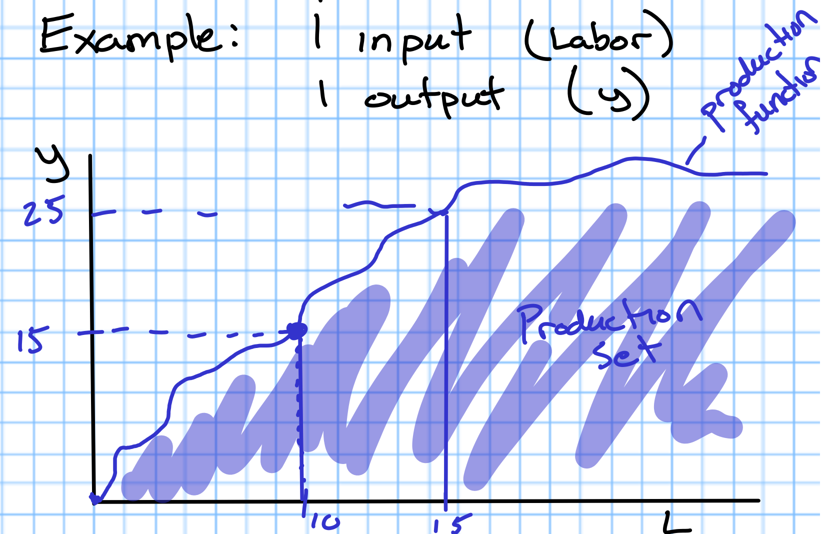
  K: Machine-hours per unit of time

- Capital is any productive factor that is itself an output to some other production process

Production sets

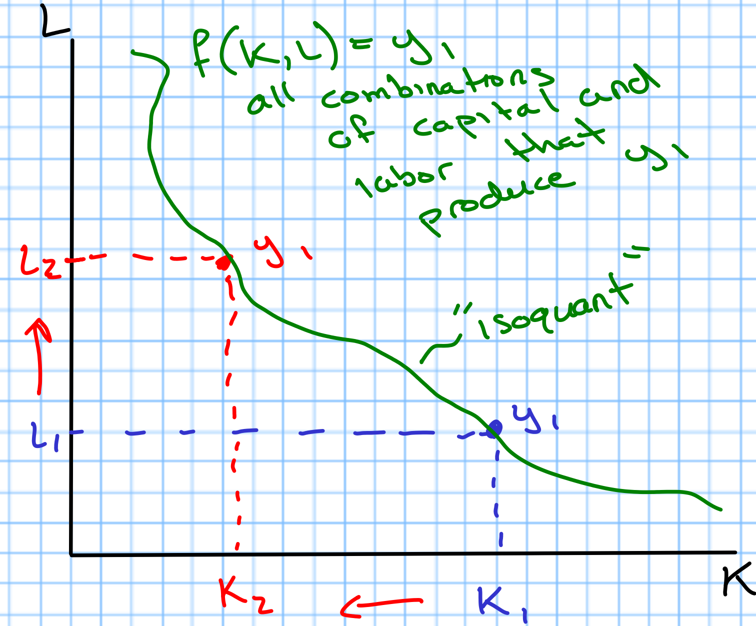
- Given a quantity of inputs (factors), how much output can a firm produce?

Example: 1 input (Labor)
1 output (y)



Production Function

- The most output that can be produced from a given level of input
- Shape of the production function is determined by technology
- Inputs: K, L
- Output: y
  $y = f(K, L)$



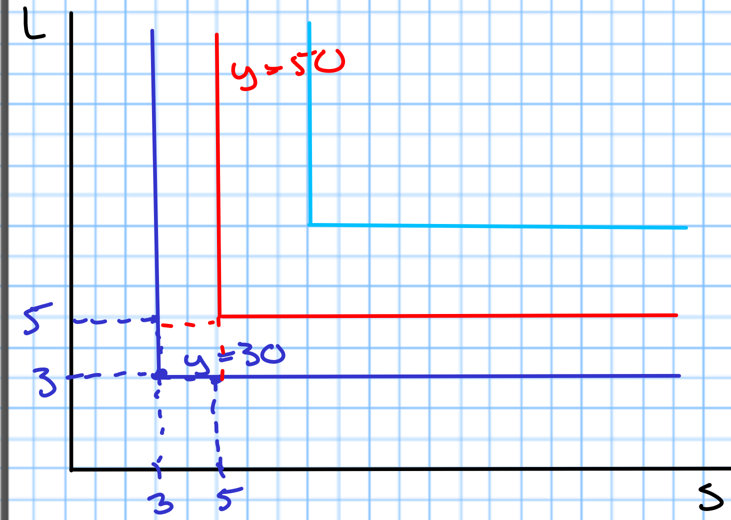
$y_1 = f(K_1, L_1)$

$y_1 = f(K_2, L_2)$

Examples

Fixed-proportion technology

- Output: Digging holes
- Inputs: workers
  shovels
- Workers without shovels can't do anything
- Shovels without workers can't do anything

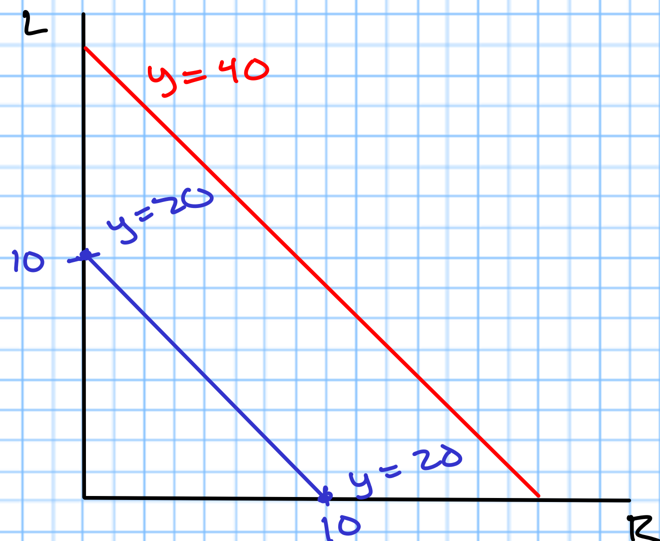


$f(3,3) = 30$

Example

Perfect substitutes

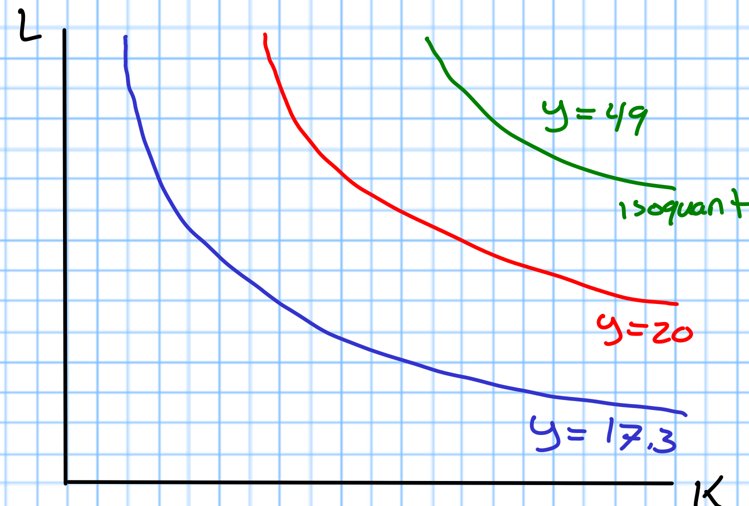
- Left-handed lawyers (L)
- Right-handed lawyers (R)



Example

- Cobb-Douglas production functions

$$y = f(K, L) = K^a L^b$$



Marginal Product

- Extra output that is produced after a small increase in one of the factors

$y = f(K, L)$

- MPK = Marginal productivity of capital

$$= \frac{\partial f(K,L)}{\partial K}$$

- $MPL = \dfrac{\partial f(K,L)}{\partial L}$

Example

$$y = K^2 L^3$$

$$MPK = 2KL^3$$

$$MPL = 3K^2L^2$$

Example:

$$y = K^{1/3} L^{2/3}$$

$$MPK = \frac{1}{3} K^{-2/3} L^{2/3}$$

$$= \frac{L^{2/3}}{3K^{2/3}}$$

$$MPL = \frac{2}{3} K^{1/3} L^{-1/3}$$

Technical Rate of Substitution (TRS)

- Slope of the isoquant

- The rate at which the firm can substitute capital for labor such that production remains unchanged
- $TRS = \frac{MPK}{MPL}$
- Example:

$$y = K^{1/3} L^{1/5}$$
$$MPK = \frac{1}{3} K^{-2/3} L^{1/5}$$
$$MPL = \frac{1}{5} K^{1/3} L^{-4/5}$$

$$TRS = \frac{MPK}{MPL} = \frac{\frac{1}{\cancel{3}} K^{-2/3} L^{1/5}}{\frac{1}{\cancel{5}} K^{1/3} L^{-4/5}} \cdot \frac{(5)(\cancel{3})}{(\cancel{5})(3)}$$
$$= \frac{5 L^{1/5} L^{4/5}}{3 K^{1/3} K^{2/3}}$$
$$= \frac{5L}{3K}$$

<u>Returns to scale</u> (RTS)

- Suppose there is a production function $y = f(K, L)$
- What happens if we double our inputs?
- If output <u>more</u> than doubles, then we have <u>increasing</u> RTS

- If our output less than doubles, decreasing RTS
- If our output exactly doubles, constant RTS

Example (Cobb-Douglas)

$$y = K^2 L^4$$

Double inputs

$$(2K)^2 (2L)^4 = 2^2 K^2 2^4 L^4$$

how I remember: $(xy)^3 = (xy)(xy)(xy)$

$= xxxyyy$

$= x^3 y^3$

$$= 2^2 2^4 K^2 L^4$$

$$= 2^6 K^2 L^4$$

$x^3 x^4 = (xxx)(xxxx) = x^7$

$$= 2^6 y > 2y$$

$\rightarrow$ output has <u>more</u> than doubles

Example

$$y = K^{1/3} L^{1/5}$$

$$(2K)^{1/3} (2L)^{1/5} = 2^{1/3} K^{1/3} 2^{1/5} L^{1/5}$$

$$= 2^{1/3} 2^{1/5} K^{1/3} L^{1/5}$$

$$= 2^{5/15} 2^{3/15} y$$

$$= 2^{8/15} y < 2y$$

$\rightarrow$ decreasing RTS

Example

$$y = K^{1/3} L^{2/3}$$

$$(2K)^{1/3} (2L)^{2/3} = 2^{1/3} 2^{2/3} K^{1/3} L^{2/3}$$

$= 2y$

$\rightarrow$ Constant RTS

In general:

$y = K^a L^b$

If $a+b > 1$, increasing RTS

If $a+b < 1$, decreasing RTS

If $a+b = 1$, CRTS

Note: In macro we sometimes write $K^\alpha L^{1-\alpha}$

$\alpha + (1-\alpha) = 1$

$\rightarrow$ CRTS