Simultaneous Games

ECON 420: Game Theory

Spring 2018

Announcements

- Homework due on Wednesday
- Reading: Chapter 4

Simultaneous games

- Players move at the same time
- No knowledge of what other players choose when own choices are made
- Simultaneous games have *imperfect information*

Example

- Your firm is competing against another firm (randomly matched)
- Your team is tasked with setting prices for you firm's product
  - You can choose a price of \$5 or \$10 per unit
  - Both firms are choosing price simultaneously
- Profits:
  - If both firms choose p=\$10, profits are \$8
  - If both firms choose p=\$5, profits are \$6
  - If one firm chooses \$5 and the other chooses \$10:
    - ⋆ Firm that chooses \$5 gets \$10 profit
    - ⋆ Firm that chooses \$10 gets \$5 profit

Strategies

- Recall that strategies are *complete plans of action*
- In simultaneous games, only one choice can be made
  - Little difference between action and strategy
- But players may have a strategy that choose probabilistic actions
  - Example: Rock, paper, scissors
- For now, consider only *pure strategies* (non-probabilistic)

Game table

- Simultaneous move games can be represented with a *game table*
- Contains information on players, strategies, and payoffs
- Games represented as game tables are said to be in *normal form* (or *strategic form*)

Players: "Row", "Column"

Players →

↓

		COLUMN		
		Left	Middle	Right
ROW	Top	3, 1	2, 3	10, 2
	High	4, 5	3, 0	6, 4
	Low	2, 2	5, 4	12, 3
	Bottom	5, 6	4, 5	9, 7

actions for column player

payoff if Row plays B and column plays R

Actions (pure strategies) available to the row player

1st number in payoffs corresponds to row player

Example: American football play

- Offensive team chooses from among types of plays
- Defensive team simultaneously chooses type of play to counter offense
- Payoffs are zero-sum: whatever offense gains (in yards) is equal to what the defense loses

		DEFENSE		
		Run	Pass	Blitz
OFFENSE	Run	2, –2	5, –5	13, –13
	Short Pass	6, –6	5.6, –5.6	10.5, –10.5
	Medium Pass	6, –6	4.5, –4.5	1, –1
	Long Pass	10, –10	3, –3	–2, 2

Example: Pricing game

Players: Firm 1 and Firm 2

Strategies: $P = 5$ or $P = 10$ (same for both players)

		Firm 2: $P = 5$	Firm 2: $P = 10$
Firm 1	$P = 5$	6, 6	10, 5
	$P = 10$	5, 10	8, 8

Best response

- We can analyze simultaneous move games by describing player's *best response* actions
- The best response is the action that maximizes a player's payoffs *given* the action of the opposing player(s)
- *If* the other player does X, *then* I should do Y

Firm 1's best response to Firm 2 choosing $p=10$ is $p=5$

Suppose Row plays top:

		COLUMN		
		Left	Middle	Right
ROW	Top	3, 1	2, 3	10, 2
	High	4, 5	3, 0	6, 4
	Low	2, 2	5, 4	12, 3
	Bottom	5, 6	4, 5	9, 7

Column's best response is middle

"mutual best response"

Nash equilibrium

- Nash equilibrium occurs when both players are *simultaneously* choosing their best-response actions
- Neither player can achieve higher payoffs by changing their actions *given* the action of the other player
- Not necessarily the best outcome for both players

At the N.E., neither player has (personal) incentive to change their strategy

Nash equilibrium: (Low, Middle)

Row's best response to Middle : Low

		COLUMN		
		Left	Middle	Right
ROW	Top	3, 1	2, 3	10, 2
	High	4, 5	3, 0	6, 4
	Low	2, 2	5, 4	12, 3
	Bottom	5, 6	4, 5	9, 7

Column's best response to Low: Middle

Still a Nash equilibrium?

Is (low, middle) still a NE? Yes

		COLUMN		
		Left	Middle	Right
ROW	Top	3, 1	2, 3	10, 2
	High	4, 5	3, 0	6, 4
	Low	2, 2	5, 4	12, 3
	Bottom	5, 6	5, 5	9, 7

Question: Can either player do better given the other player's action?

→ Yes → Not a NE

→ No → NE

Beliefs

- How can a player choose a best "response" if they are playing simultaneously?
  - What are they responding to?
- Players form *beliefs* about what other players will do
- At the Nash equilibrium:
  - All players choose optimal actions given their beliefs about what other players are doing
  - The beliefs are accurate
- If either condition doesn't hold, then not a Nash equilibrium

Will Row play Low if they believe Column isn't playing Middle?

		COLUMN		
		Left	Middle	Right
ROW	Top	3, 1	2, 3	10, 2
	High	4, 5	3, 0	6, 4
	Low	2, 2	5, 4	12, 3
	Bottom	5, 6	4, 5	9, 7

Row believes Column plays Left. → Bottom

Row believes Column Plays Right. → Low

What is the Nash equilibrium?

		DEFENSE		
		Run	Pass	Blitz
OFFENSE	Run	2, −2	5, −5	13, −13
	Short Pass	6, −6	5.6, −5.6	10.5, −10.5
	Medium Pass	6, −6	4.5, −4.5	1, −1
	Long Pass	10, −10	3, −3	−2, 2

only square where neither can do better

Extra credit

- Write your name at the top of a blank sheet of paper
- Answer the following questions. For each answer that is the same for *everyone else*, you will receive 2 points extra credit on the homework
  1. Select "Heads" or "Tails"
  2. Choose one of the numbers 7, 100, 13, 261, 99, 555
  3. You are to meet someone in Corvallis. You have not been instructed where to meet, you have no prior understanding with the person on where to meet, and you cannot communicate with each other. You are simply told that you will have to guess where to meet, that the other person is being told the same thing, and that you will just have to try to make your guesses coincide. Where do you go?
  4. You are told the date but not the hour of the meeting in question 3. The two of you must guess the exact minute of the day for the meeting. At what time will you appear?
  5. Write some positive number.