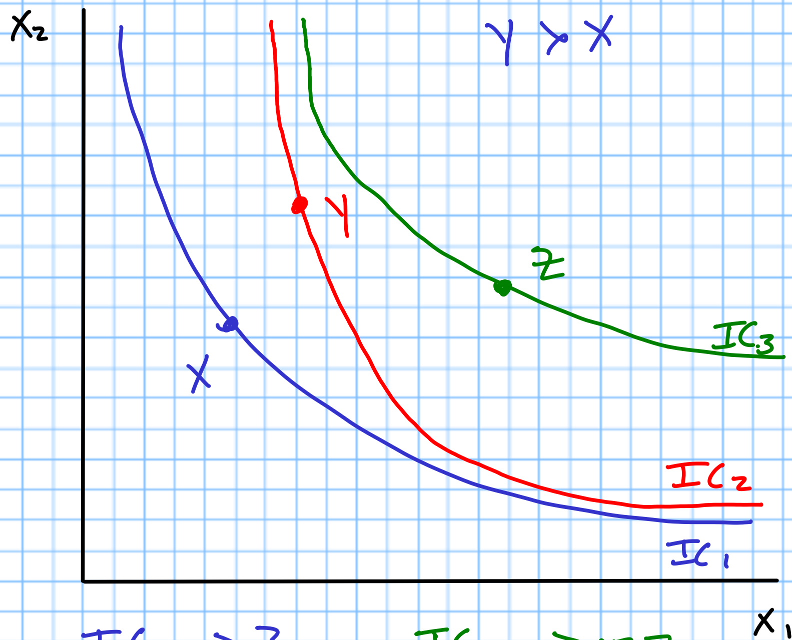
Utility

$Z \succ Y$
$Y \succ X$



$IC_1 \to 3$
$IC_2 \to 100$
$IC_3 \to 127$

- Let's imagine we start ranking bundles by assigning numbers to them
- If we would prefer Y to X, we will assign a higher number to Y
- If we are indifferent, we assign both bundles the same number
- Indifference curves will have unique numbers associated with them

- Now we can rank alternative bundles by comparing numbers
  $\rightarrow$ choose the bundle with a higher number
- Let's create a <u>function</u> that assigns numbers to bundles
  - input: (x_1, x_2) (bundle)
  - output: number u

call this function a <u>utility</u> function

$$u = u(x_1, x_2)$$

Properties of the utility function:

- $X \succ Y$, then $u(x_1, x_2) > u(y_1, y_2)$
- If $Y \succ X$, then $u(y_1, y_2) > u(x_1, x_2)$
- If $X \sim Y$, then $u(x_1, x_2) = u(y_1, y_2)$

<u>Example</u>

$u(x_1, x_2) = 3$

$u(y_1, y_2) = 100$

$u(z_1, z_2) = 127$

Example

Suppose v is a utility fuction

$v(x_1, x_2) = 13.7$

$v(y_1, y_2) = 13.8$

$v(z_1, z_2) = 324{,}874$

- v and u represent the <u>same</u> preferences
- utility functions are <u>not</u> unique (for given preferences, there are infinitely many functions)

- Utility is a "ordinal" number
  → the magnitude doesn't matter

Example: 5K race

Michael : 33 minutes
Rudy : 28 minutes
Llewelyn : 45 minutes

Cardinal numbers

Rank the runners:

ordinal numbers

M : 2^{nd} (Silver)
R : 1^{st} (Gold)
L : 3^{rd} (Bronze)

Existence

- If preferences are rational, then there is always a function u that represents those preferences
- The utility function is never unique

suppose $u(x_1, x_2) = x_1 + x_2$

Now consider

$$v(x_1, x_2) = u(x_1, x_2) + 5$$
$$= x_1 + x_2 + 5$$

Suppose $(x_1, x_2) = (3, 5)$

$(y_1, y_2) = (0, 7)$

$u(x_1, x_2) = 3 + 5 = 8$

$u(y_1, y_2) = 7$

$X \succ Y$

$v(x_1, x_2) = 3 + 5 + 5 = 13$

$v(y_1, y_2) = 12$

$X \succ Y$

Cobb-Douglas utility

- $u(x_1, x_2) = x_1^a x_2^b$ where a and b are constants

Example:

$$u(x_1, x_2) = x_1^2 x_2^3$$

Suppose $u(x_1, x_2) = 4$

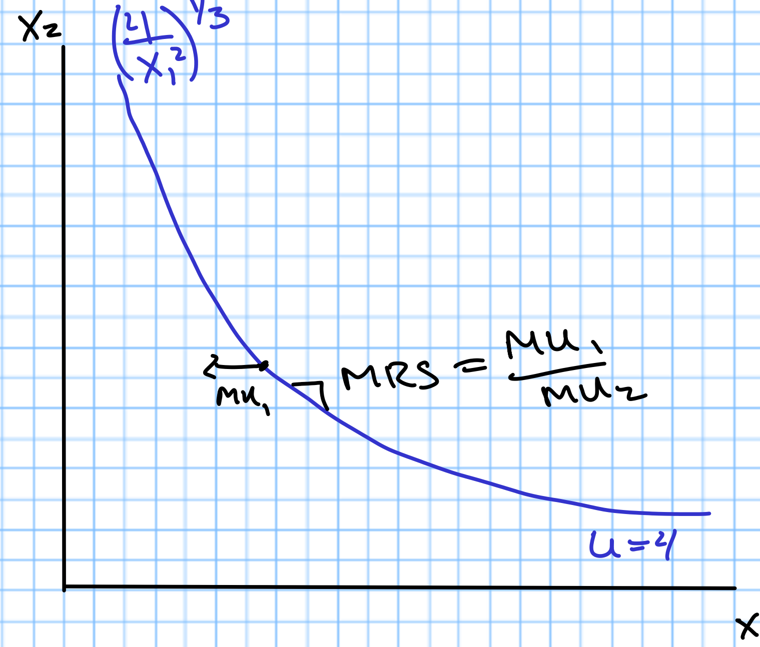
$$x_1^2 x_2^3 = 4$$

Solve for x_2

$$x_2^3 = \frac{4}{x_1^2}$$

$$x_2 = \left(\frac{4}{x_1^2}\right)^{1/3}$$

Plot this function



Result: Cobb-Douglas utility results in well-behaved ICs

Slope of IC

$$\frac{dx_2}{dx_1} \;\ldots \qquad x_2 = \frac{4^{1/3}}{x_1^{2/3}} = 4^{1/3} x_1^{-2/3}$$

Let's not do this...

Result:

$$MRS = \frac{\dfrac{\partial u(x_1,x_2)}{\partial x_1}}{\dfrac{\partial u(x_1,x_2)}{\partial x_2}} \qquad \begin{matrix} = MU_1 \\ \\ = MU_2 \end{matrix}$$

$$\frac{\partial u(x_1,x_2)}{\partial x_1} = \underline{2x_1x_2^3}$$

$$\frac{\partial u(x_1,x_2)}{\partial x_2} = \underline{3x_1^2x_2^2}$$

$$MRS = \frac{2x_1x_2^3}{3x_1^2x_2^2}$$

$$= \frac{2x_2}{3x_1}$$

Marginal utility

- Slope of the utility function

- $\dfrac{\partial u(x_1, x_2)}{\partial x_1} \to$ marginal utility of good $\underline{1}$

- Increase x_1 a small amount. How much extra utility do you get?

<u>Example</u>

$$u(x_1, x_2) = x_1^1 x_2^2$$

$$MRS = \frac{MU_1}{MU_2}$$

$$MU_1 = x_2^2 \qquad MU_2 = 2x_1x_2$$

$$MRS = \frac{x_2^2}{2x_1x_2} = \frac{x_2}{2x_1}$$

<u>Example</u>

$$u(x_1, x_2) = x_1^a x_2^b$$

$$MU_1 = a x_1^{a-1} x_2^b$$

$$MU_2 = b x_1^a x_2^{b-1}$$

$$MRS: \frac{a x_1^{a-1} x_2^b}{b x_1^a x_2^{b-1}}$$

$$= \frac{a x_1^{a-1} x_1^{-a}}{b x_2^{b-1} x_2^{-b}}$$

$$= \frac{a x_1^{-1}}{b x_2^{-1}} = \frac{a x_2}{b x_1}$$

Example

$u(x_1, x_2) = x_1^{-7.3} x_2^{3.14}$

MRS:

$a = -7.3$

$b = 3.14$

$MRS = \frac{a x_2}{b x_1} = \frac{-7.3 x_2}{3.14 x_1}$

Review exponent rules

$x^{-a} = \frac{1}{x^a}$

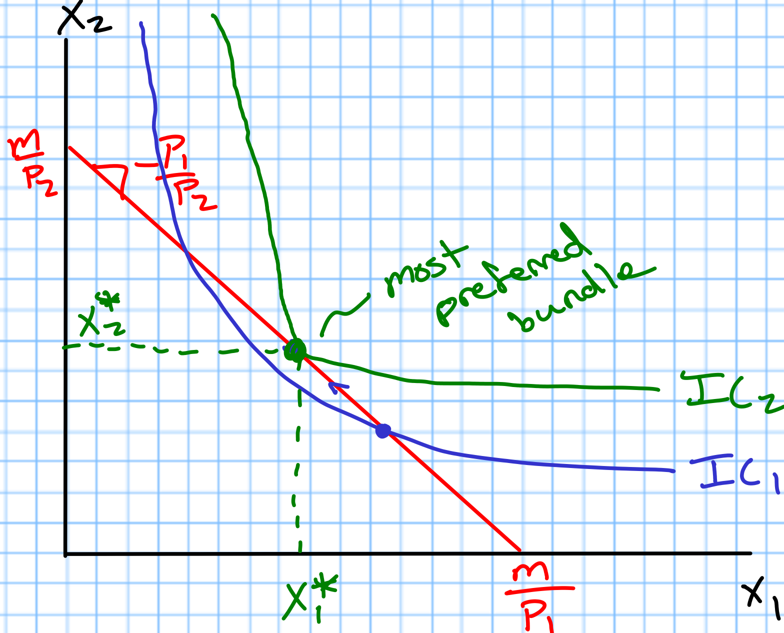
$x^a = \frac{1}{x^{-a}}$

$x^a \cdot x^b = x^{a+b}$

$x^2 \cdot x^3 = (x \cdot x) \cdot (x \cdot x \cdot x)$
$= x^5$

$(x^a)^b = x^{ab}$

$(x^2)^3 = (xx)^3$
$= (xx)(xx)(xx) = x^6$



2 things must be true:

① : $P_1 X_1^* + P_2 X_2^* = m$

Total expenditure = Total income

② MRS = P_1/P_2

willingness to trade good 1 for good 2

=

The opportunity cost of good 1 in terms of good 2

Quick review:

$P_1 X_1 + P_2 X_2 = \underline{m}$

We want to write this as a function $X_2 = X_2(X_1)$

Solve for X_2:

$$P_2 X_2 = m - P_1 X_1$$

$$X_2 = \frac{m}{P_2} - \underbrace{\frac{P_1}{P_2}}_{\text{slope}} X_1$$

intercept

Suppose a consumer has preferences represented by the utility function

$$U(x_1, x_2) = x_1^3 x_2^6$$

The price of good 1 is $P_1 = 4$ and $P_2 = 2$

And consumer has income $m = 15$

How much x_1 and x_2 will they consume?

① $4x_1 + 2x_2 = 15$

② $MRS = \frac{4}{2}$

$$MRS = \frac{MU_1}{MU_2} = \frac{3\cancel{x_1^{2}}\, x_2^{\cancel{6}}}{6x_1^{3}\,\cancel{x_2^{5}}}$$

$$= \frac{3x_2}{6x_1}$$

② $\frac{3x_2}{6x_1} = \frac{4}{2}$

System of equations

We solve by simplifying one equation and substituting into the other equation

Start with ②:

$$\frac{3x_2}{6x_1} = \frac{4}{2}$$

$$\frac{x_2}{2x_1} = 2$$

$$\boxed{x_2 = 4x_1}$$

Substitute into Budget Line

$4x_1 + 2(4x_1) = 15$

$4x_1 + 8x_1 = 15$

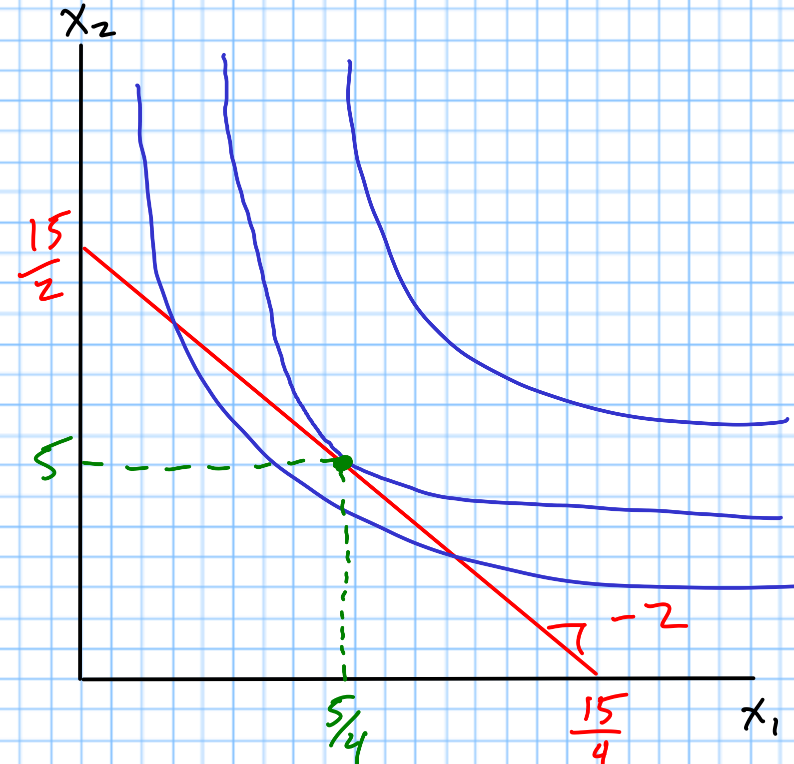
$12x_1 = 15$

$x_1 = 15/12$

$x_1^* = 5/4$

$x_2^* = \cancel{4}(5/\cancel{4})$

$x_2^* = 5$



Example

$$u(x_1, x_2) = x_1^{17} x_2^{3}$$

$$P_1 = 5, \quad P_2 = 11, \quad m = 55$$

① $5x_1 + 11x_2 = 55$

② $\dfrac{17x_2}{3x_1} = \dfrac{5}{11}$

Note:

$$\frac{MU_1}{MU_2} = \frac{P_1}{P_2}$$

index must match!

index must match!

$$MU_1 = 17x_1^{16}x_2^{3}$$

$$MU_2 = 3x_1^{17}x_2^{2}$$

Cross-multiply ②:

$$(11)(17)x_2 = 15x_1$$

$$x_2 = \frac{15}{11 \cdot 17} x_1$$

$$5x_1 + \cancel{11}\left(\frac{15}{\cancel{11} \cdot 17} x_1\right) = 55$$

$$5x_1 + \frac{15}{17} x_1 = 55$$

$$\frac{85}{17} x_1 + \frac{15}{17} x_1 = 55$$

$$\frac{100}{17} x_1 = 55$$

$$100x_1 = 17 \cdot 55$$

$$100x_1 = 935$$

$$x_1 = 9.35$$

$$x_2 = \frac{15}{11 \cdot \cancel{17}} \cdot \frac{\cancel{17} \cdot 55}{100}$$

$$x_2 = \frac{825}{1100}$$

$$= 0.75$$