Mixed Strategies (Chpt 7)

Everet (E) \ Navratilova (N)	DL	CC
DL	50, 50	80, 20
CC	90, 10	20, 80

No pure-strategy NE.

- Suppose E plays DL
  - N's best response: DL
  - E's payoff: 50
- Suppose E plays CC
  - N's best response: CC
  - E's payoff: 20

- Suppose E mixes her strategies
  - choose CC with some probability p
  - chooses DL with probab. 1 - p

p = 0.75

- What is N's best response?
  - We can calculate N's expected payoff to E's strategy
  - N's expected payoff if N plays DL:

$50(0.25) + 10(0.75) = 20$

- N's payoff if she plays CC:
  $20(0.25) + 80(0.75) = 65$
- N's Expected payoff is higher if she plays CC
- N's best response to E playing $p = 0.75$ is CC
- Everet's expected payoff:
  $(0.25)80 + (0.75)20 = 35$

- Suppose instead E chooses CC with prob. $p = 0.25$
- N's best response:
  - Payoff to DL:
    $50(0.75) + 10(0.25) = 40$
  - Payoff to CC:
    $20(0.75) + 80(0.25) = 35$
  - BR: DL
- E's expected payoff:
  $50(0.75) + 90(0.25) = 60$
  - Mixed strategy of CC with probability $p = 0.25$ gives E a higher payoff than playing either pure strat.

Exploiting the opponent's strategy

- Zero sum (fixed-sum), E doing better means N must be doing worse
- N can exploit E's pure strategy and do better
- N also exploits E's mixed strategy of CC with $p = 0.25$
- Question: Can E choose a strategy that can't be exploited?
- In other words, is there a strategy that E can play that makes N indifferent among strategies?
- E plays CC with prob. p
  - What is N's payoff to DL?
    $10p + 50(1-p)$
  - What is N's payoff to CC?
    $80p + 20(1-p)$

$$10p + 50(1-p) = 80p + 20(1-p)$$
$$10p + 50 - 50p = 80p + 20 - 20p$$
$$10p - 50p - 80p + 20p = 20 - 50$$

$-100p = -30$

$100p = 30$

$p = 3/10$

$p = 0.3$

- N's payoff to DL:

$50(0.7) + 10(0.3) = 38$

- N's payoff to CC:

$80(0.3) + 20(0.70) = 38$

- E's payoff (if N plays DL) $(q=0)$

$(0.7)50 + (0.3)90 = 62$

- E's payoff (if N plays CC) $(q=1)$

$(0.7)80 + (0.3)20 = 62$

Best response

- Both players can choose mixed strategies

  E plays CC with prob p

  N play CC with prob q

- E's payoff if N plays q

$(50(1-p)+90p)(1-q) + (80(1-p)+20p)q$

$(50-50p+90p)(1-q) + (80-80p+20p)q$

$(50+40p)(1-q) + (80-60p)q$

$50+40p-(50q+40pq)+80q-60pq$

$50+40p-100pq+30q$

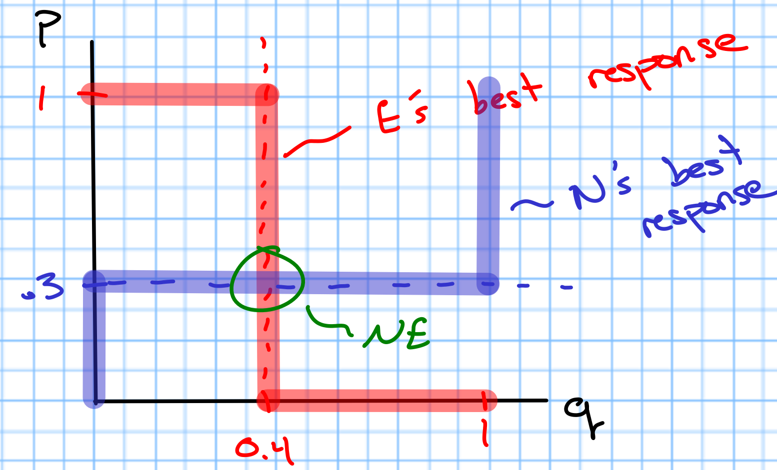
$50+30q+(40-100q)p$

- What p maximizes this payoff?
  - $p=0$ maximizes when $q=1$
  - $p=1$ maximizes when $q=0$
  - $p=0$ maximizes when $q=0.5$
  - calculate q such that

    $40-100q=0$

    $\longrightarrow q=0.4$

Best response: $p^* = \begin{cases} 0 & \text{if } q>.4 \\ \text{anything} & \text{if } q=.4 \\ 1 & \text{if } q<.4 \end{cases}$



- N's payoff:

$[50(1-q)+20(q)](1-p)+[10(1-q)+80q]p$

$(50-50q+20q)(1-p)+(10-10q+80q)p$

$50-30q-50p+30pq+10p+70pq$

$50 - 30q + 100pq - 40p$

$50 - 40p + (100p - 30)q$

N's best response:

$$q = \begin{cases} 0 & \text{if } p < .3 \\ \text{anything} & \text{if } p = .3 \\ 1 & \text{if } p > .3 \end{cases}$$

Nash equilibrium:

E: $p = 0.3$

N: $q = 0.4$