

Uncertainty

ECON 420: Game Theory

Spring 2018

Announcements

- ▶ Homework 3 on Canvas
 - ▶ Due *Monday, May 21*
- ▶ Reading: Chapter 8

Uncertainty

- ▶ So far: Strategic uncertainty
 - ▶ Some players unaware of the actions of other players
 - ▶ Example: Simultaneous-move games
- ▶ Today: External uncertainty
 - ▶ "Nature" changes aspects of the game
 - ▶ Players cannot control external uncertainty, must take it into account when making decisions

Expected Utility Theory

- ▶ Events that happen according to some probability distribution are called *gambles*
- ▶ Agents are able to rank gambles by comparing the *expected utility* that they would receive from the potential outcomes of the gamble
- ▶ The utility that we will use is *von Neumann-Morgenstern (VNM)* utility

Risk preference

- ▶ When there is uncertainty we can calculate the *expected value* of a gamble
- ▶ But people do not just consider expected value when making decisions
- ▶ Some people might be willing to pay to avoid risk (risk aversion)

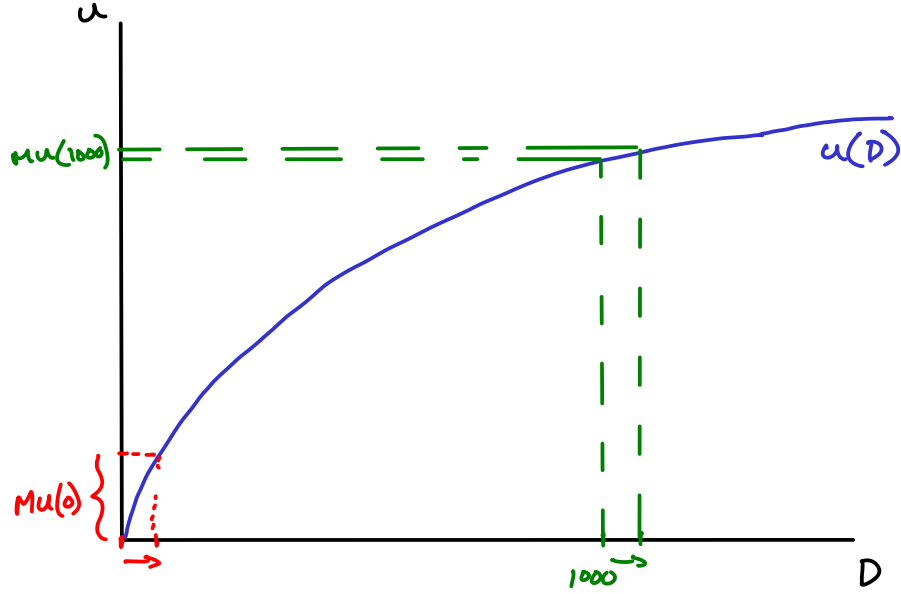
Example

- ▶ Suppose I flip a coin. If heads, you get \$100. If tails, you get \$0.
 - ▶ What is the expected value?
 - ▶ How much would you pay to play this game?
- ▶ Suppose instead the payoffs are \$1 million for heads, \$0 for tails.

VNM Utility and Risk Preference

- ▶ Outcomes are denoted D (dollars)
- ▶ Agents in the model have preferences over outcomes represented by utility $u = u(D)$
- ▶ The risk preference of the agent depends on the concavity of the utility function u
- ▶ Agents with *diminishing marginal utility* are risk averse
 - ▶ Concave utility function

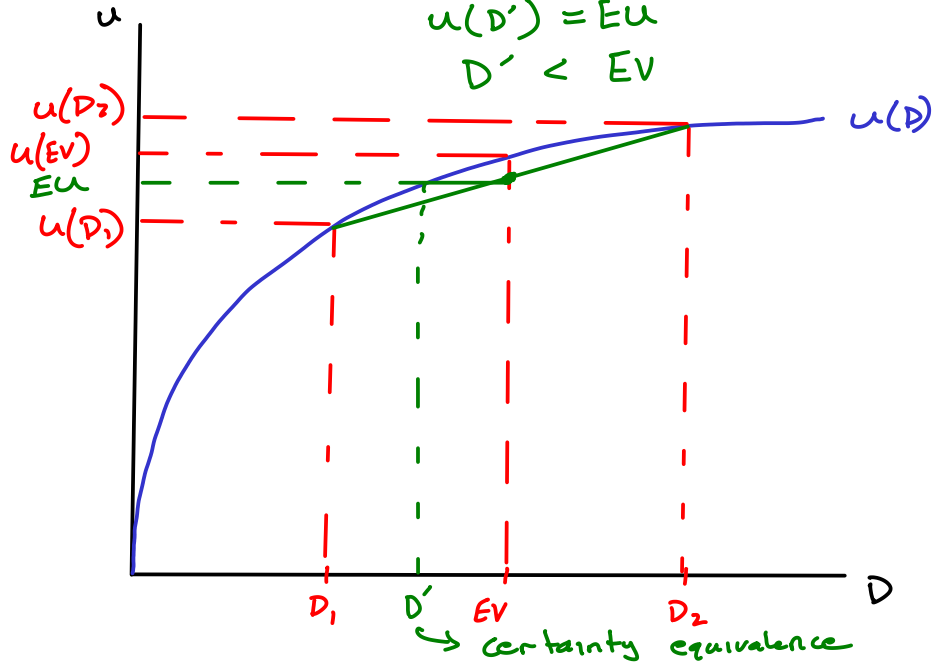
Risk aversion



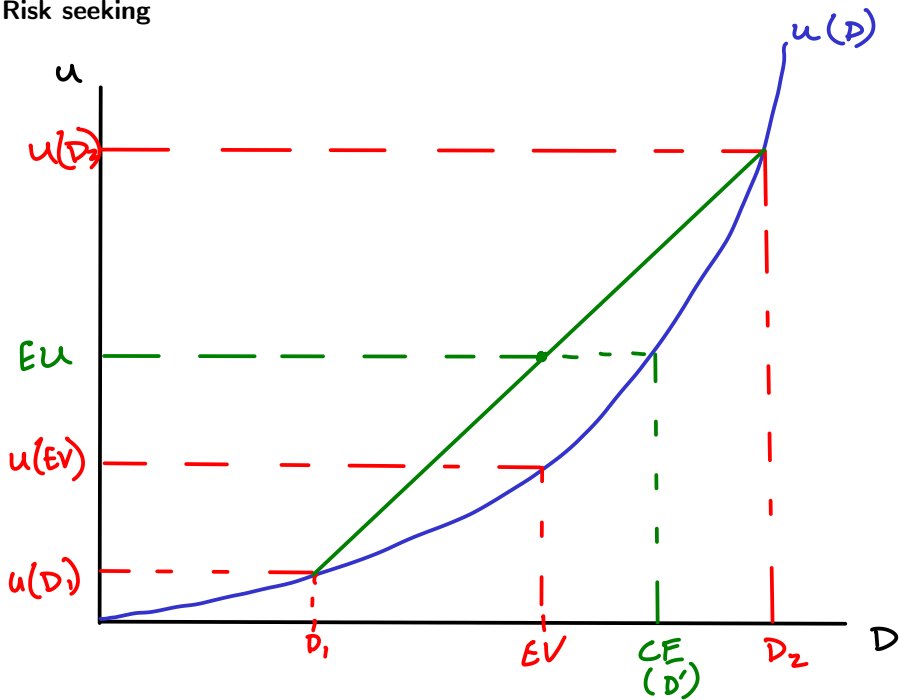
$$u(EV) > Eu$$

$$u(D') = Eu$$

$$D' < EV$$



Risk seeking



$$\begin{aligned}
 EV &= \frac{1}{2} \cdot 160,000 + \frac{1}{2} \cdot 40,000 \\
 &= 80,000 + 20,000 \\
 &= 100,000
 \end{aligned}$$

Example

- ▶ A farmer's crop yield depends on weather
- ▶ Farmer gets good weather with 50%
- ▶ Yield with good weather is \$160,000, yield in bad weather is \$40,000
- ▶ Farmer has VNM utility $u(D) = \sqrt{D}$

↳ risk averse

$$\begin{aligned}
 EU &= \frac{1}{2} u(160,000) + \frac{1}{2} u(40,000) \\
 &= \frac{1}{2} \sqrt{160,000} + \frac{1}{2} \sqrt{40,000} \\
 &= \frac{1}{2} 400 + \frac{1}{2} \cdot 200 = 300
 \end{aligned}$$

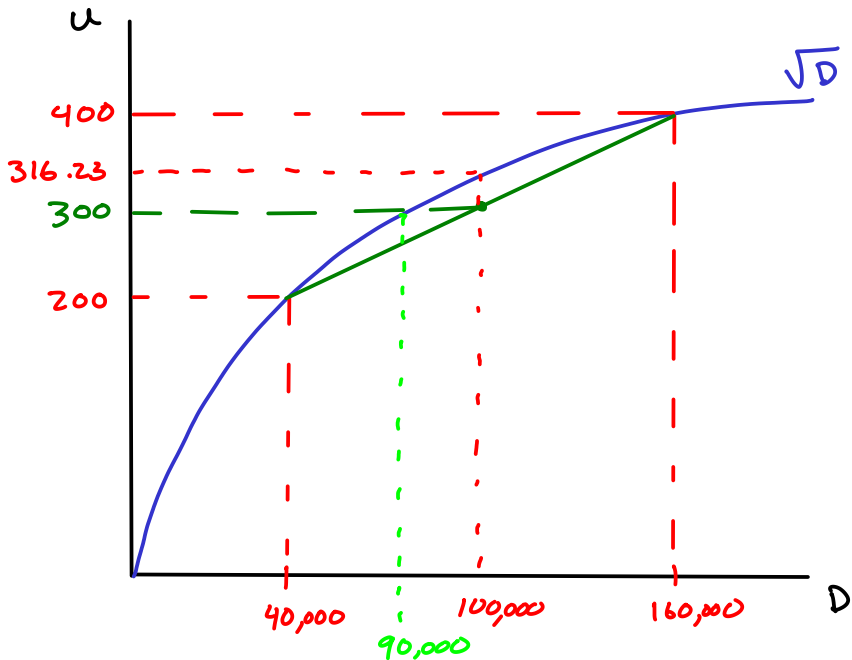
$$u(EV) = \sqrt{100,000}$$
$$= 316.23$$

CE: amount that the farmer needs with certainty (prob 1) to be indifferent between that and taking on the risk

$$EU = u(CE)$$

$$300 = \sqrt{CE}$$

$$CE = 90,000$$



Risk sharing

- ▶ Risk averse agents willing to pay to remove risk
- ▶ Agents can therefore benefit from trading *state-contingent claims* with one another
 - ▶ You agree to pay someone else if you have a good outcome, someone else pays you if you have a bad outcome

Example

- ▶ Suppose there is another farmer that has the same weather probability and outcomes (weather probability is independent of first farmer)
- ▶ Farmers agree to a contract: If one farmer gets good luck and the other gets bad luck, lucky farmer pays \$60,000 to the unlucky farmer
- ▶ Are the farmers better off?

Possible states:

| <u>Farmer 1</u> | <u>Farmer 2</u> | <u>Probability</u> | <u>Farmer 1's payoff</u> |
|-----------------|-----------------|--------------------|------------------------------|
| Good | Good | $1/4$ | 160,000 |
| Bad | Bad | $1/4$ | 40,000 |
| Good | Bad | $1/4$ | 100,000 |
| Bad | Good | $1/4$ | 100,000 |

Farmer 1's
utility

400

200

316.23

316.23

Farmer 1's
payoff

160,000

40000

100,000

100,000

← from last
page

Farmer 1's EU:

$$EU = \frac{1}{4} \cdot 400 + \frac{1}{4} \cdot 200 + \frac{1}{2} \cdot 316.23$$

$$= 100 + 50 + 158.12$$

$$= 308.12 > 300$$

↑ utility without
risk sharing

Example

- ▶ Now suppose the other farmer faces no uncertainty and will earn \$100,000 with probability 1
- ▶ The farmer with risk is willing to accept their certainty equivalence instead of the gamble
- ▶ Is the riskless farmer willing to buy the risk in exchange for the certainty equivalence?

Riskless Farmer's expected utility:

$$EV = \frac{1}{2}(40,000 - 90,000) + \frac{1}{2}(160,000 - 90,000) + 100,000$$

$$EV = \frac{1}{2}(40,000) + \frac{1}{2}(160,000) + 100,000 - 90,000$$

$$EV = \frac{1}{2}(40,000) + \frac{1}{2}(160,000) + 10,000 = 110,000$$

$$EU = \frac{1}{2} \sqrt{40,000} + \frac{1}{2} \sqrt{160,000} + \sqrt{10,000}$$

$$= \frac{1}{2} \cdot 200 + \frac{1}{2} \cdot 400 + 100$$

$$EU = 400$$

If they don't make the deal:

$$EU = \sqrt{100,000}$$

$$= 316.23$$

Example

- ▶ Now suppose the farmer without risk is *risk neutral*
- ▶ What is the maximum that this farmer is willing to pay for the gamble?

100,000 \rightarrow EV of the gamble

Insurance and risk

- ▶ Suppose there are thousands of farmers with identical risk/outcomes
- ▶ A single entity (insurance company) can buy the risk of all of the farmers and make them better off
- ▶ Law of large numbers says that the insurance company will earn the expected value of the gamble

Manipulating Risk

- ▶ Sometimes agents have control over risk and can use it to their advantage
- ▶ By increasing risk, the probability of "tail events" increases
- ▶ This is why underdogs in sports often choose risky actions

Example

- ▶ A basketball team scores 60 points per game on average
- ▶ They are playing a better opponent and must score at least 80 points to win
- ▶ How can this team maximize their chances of winning?

Cheap Talk

- ▶ In coordination games, players may be able to costlessly communicate before the game begins
- ▶ This might allow players to better coordinate on preferred outcomes

| | | SALLY | |
|-------|-------------|-----------|-------------|
| | | Starbucks | Local Latte |
| HARRY | Starbucks | 1, 1 | 0, 0 |
| | Local Latte | 0, 0 | 2, 2 |