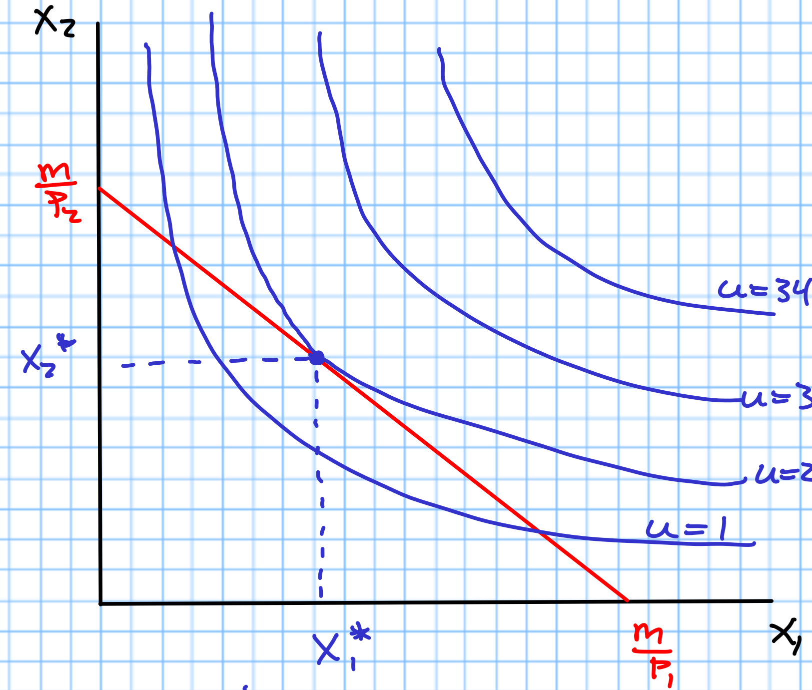
Consumer's Problem

- Consumer has preferences (rational, well-behaved)
- Consumer observes prices, has an income (budget)
- Try to choose the most preferred bundle available to them



(x_1^*, x_2^*) is the <u>optimal choice</u>

- Consumer is trying to get to the "highest" indifference curve subject to their budget constraint
- Consumer's problem:

$$\begin{cases} \max\limits_{x_1, x_2} u(x_1, x_2) \\ \text{subject to } \; P_1 x_1 + P_2 x_2 \leq m \end{cases}$$

In math, this is called a "constrained optimization" problem

2 things must be true at the optimum:

① $P_1 x_1^* + P_2 x_2^* = m$

② $MRS = \frac{P_1}{P_2}$ $\quad \left(\frac{MU_1}{MU_2} = \frac{P_1}{P_2} \right)$

<u>Example</u>

$$u(x_1, x_2) = x_1^2 x_2^3$$

$$P_1 = 7, \quad P_2 = 3, \quad m = 100$$

① $7x_1^* + 3x_2^* = 100$

② $\dfrac{2x_1^* x_2^{*3}}{3x_1^{*2} x_2^{*2}} = \dfrac{7}{3}$

<u>Step 1</u>: simplify $MRS = P_1/P_2$

$$\frac{2x_2}{\cancel{3}x_1} = \frac{7}{\cancel{3}} \rightarrow \frac{2x_2}{x_1} = 7$$

Step 2: Solve for x_1 (or x_2)

$$2x_2 = 7x_1$$

$$x_1 = \frac{2x_2}{7}$$

Step 3: Plug into BL (1)

$$7\left(\frac{2x_2}{7}\right) + 3x_2 = 100$$

Step 4: Solve for x_2

$$2x_2 + 3x_2 = 100$$

$$5x_2 = 100$$

$$x_2^* = 20$$

Step 5: Plug into (2), solve for x_1

$$x_1^* = \frac{2x_2}{7}$$

$$x_1^* = \frac{2(20)}{7}$$

$$x_1^* = \underline{40/7} \quad (5\tfrac{5}{7})$$

<u>Example 2</u>

$$u(x_1, x_2) = x_1^5 x_2^6$$

$$P_1 = 13 \quad P_2 = 10, \quad m = 130$$

(1) $P_1x_1 + P_2x_2 = m$

$$13x_1 + 10x_2 = 130$$

(2) $\dfrac{5x_2}{6x_1} = \dfrac{13}{10}$

$$50x_2 = 78x_1$$

$$x_1 = \frac{50\,x_2}{78}$$

$$\cancel{13}\left(\frac{50\,x_2}{\cancel{78}_{6}}\right) + 10x_2 = 130$$

$$\frac{50}{6}x_2 + 10x_2 = 130$$

$$\frac{50}{6}x_2 + \frac{60}{6}x_2 = 130$$

$$\frac{11\cancel{0}}{6}x_2 = 13\cancel{0}$$

$$11x_2 = 78$$

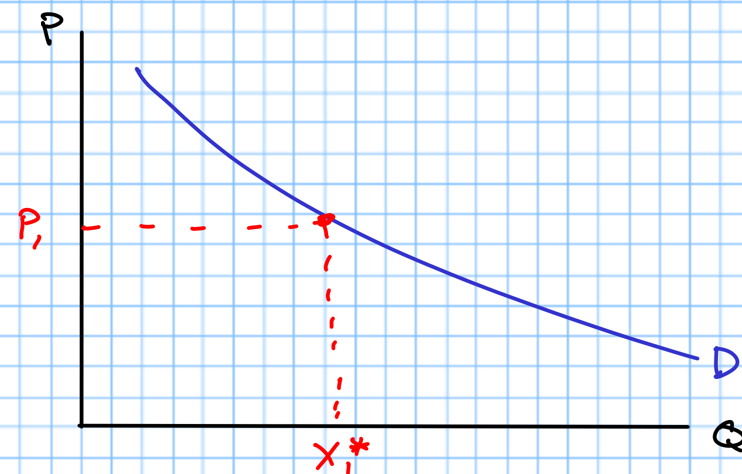
$$x_2^* = \frac{78}{11} \qquad \left(7\tfrac{1}{11}\right)$$

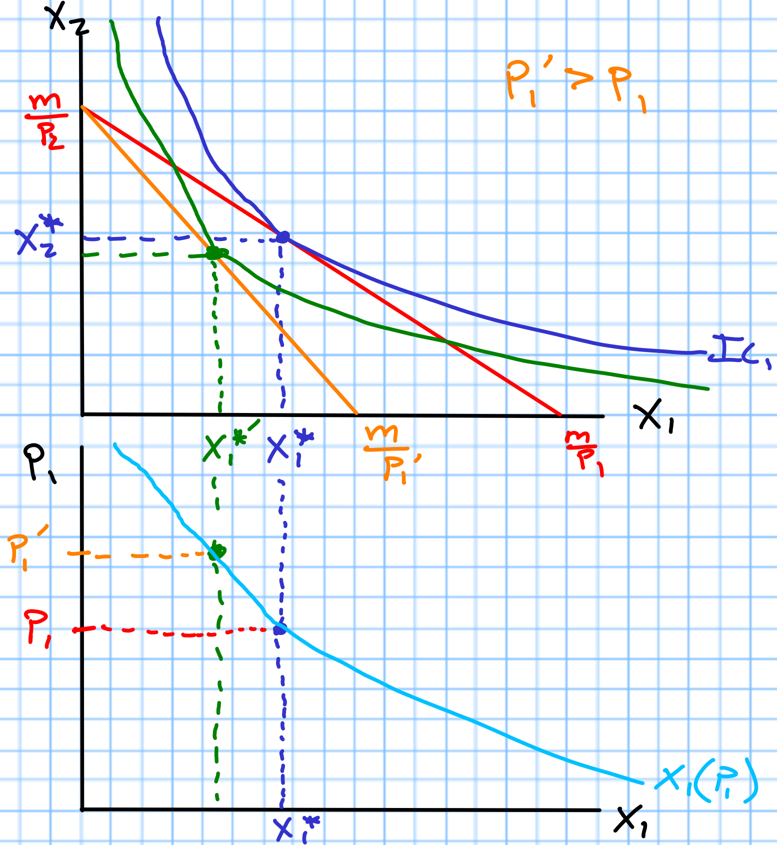
$$x_1 = \frac{50}{\cancel{78}}\,\frac{\cancel{78}}{11}$$

$$= 50/11 \qquad \left(4\tfrac{6}{11}\right)$$

Demand

The solution to the consumer's problem is the quantity demanded of good 1 and good 2





$x_1(p_1)$ is a demand <u>function</u>

inputs: price

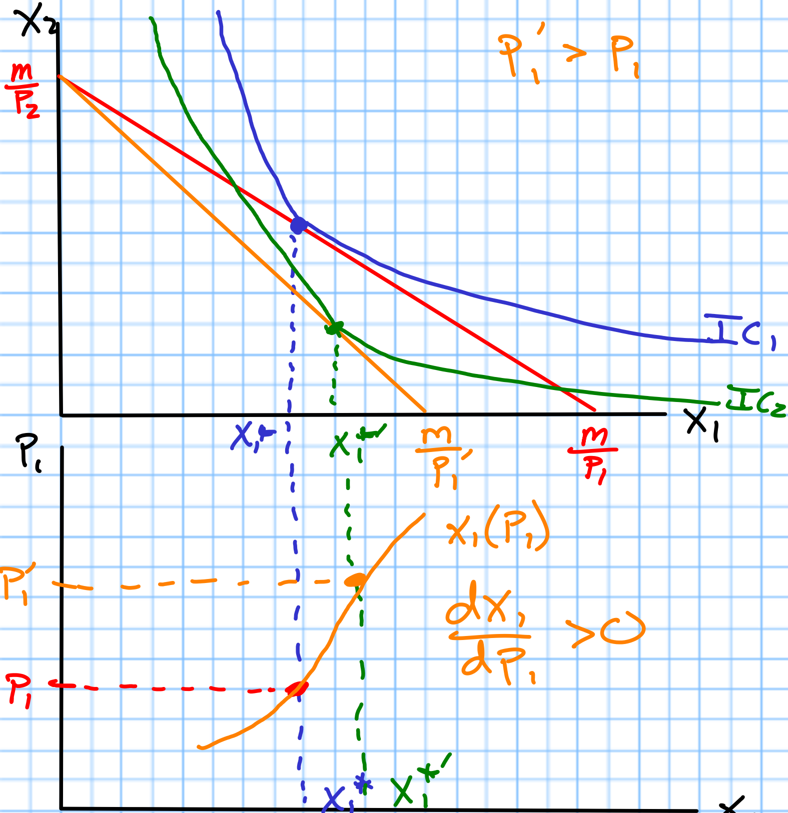
output: optimal bundle

Slope of the demand curve

slope : $\frac{dx_1}{dp_1}$

"Law of demand"

- as price goes up, quantity demanded goes down
- $\frac{dx_1}{dp_1} < 0$



In the rational choice model, the "law of demand" does not necessarily hold

Define: If $dx_1/dp_1 > 0$, then we say that x_1 is a Giffen good

Finding demand numerically

$$u(x_1, x_2) = x_1^2 x_2^3$$

$$p_1 = p_1 \quad p_2 = 3 \quad m = 100$$

Solve: $\max\limits_{x_1, x_2} \; x_1^2 x_2^3$

subject to $p_1 x_1 + 3x_2 \leq 100$

(1) $P_1 x_1 + 3x_2 = 100$

(2) $\dfrac{2x_2}{3x_1} = \dfrac{P_1}{3}$

$6x_2 = 3P_1x_1$

$x_2 = \frac{1}{2}P_1x_1$

$P_1x_1 + 3\left(\frac{1}{2}P_1x_1\right) = 100$

$\frac{2}{2}P_1x_1 + \frac{3}{2}P_1x_1 = 100$

$\frac{5}{2}P_1x_1 = 100$

$P_1x_1 = 40$

$x_1 = \dfrac{40}{P_1}$

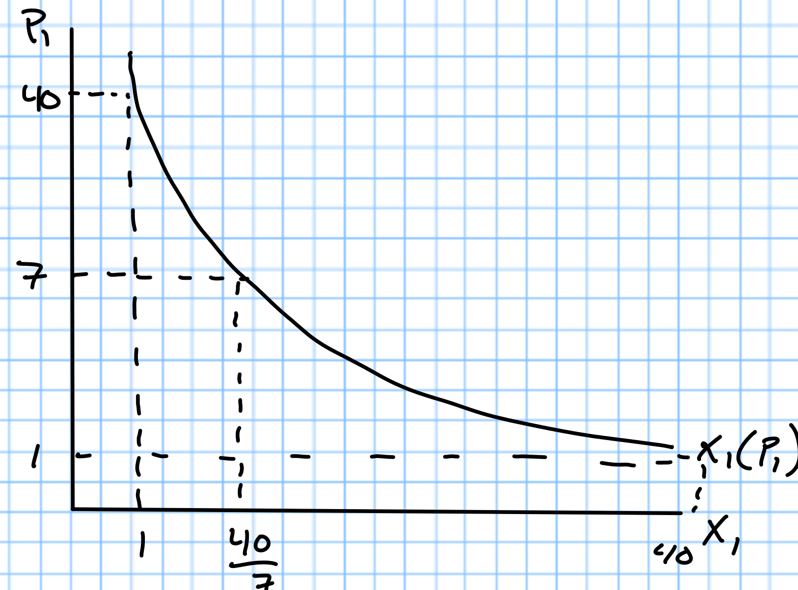
write this as:

$x_1(P_1) = \dfrac{40}{P_1}$

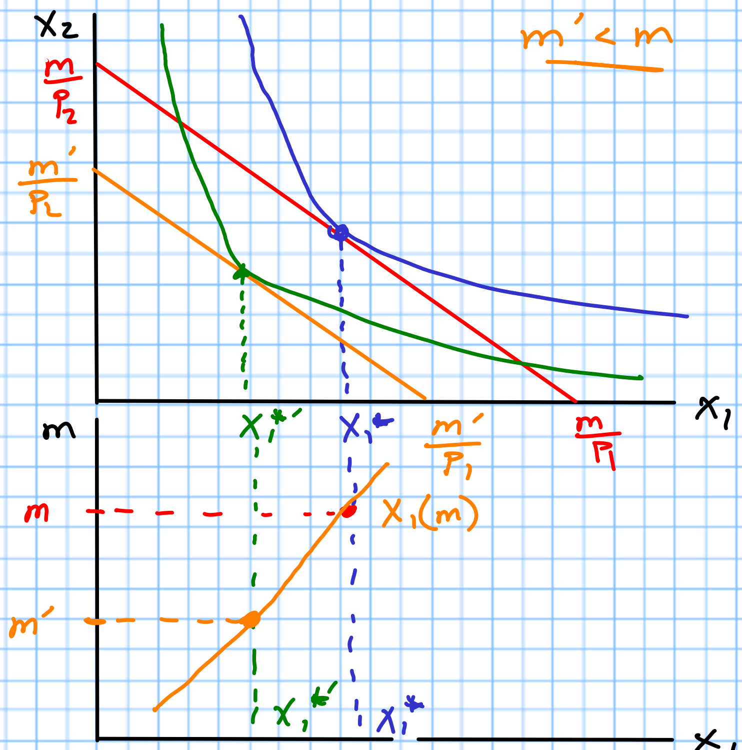
Suppose $P_1 = 7$

$x_1(7) = 40/7$

$= 5\,5/7$



Income changes



$x_1(m)$ is called an **Engel curve**

input: income

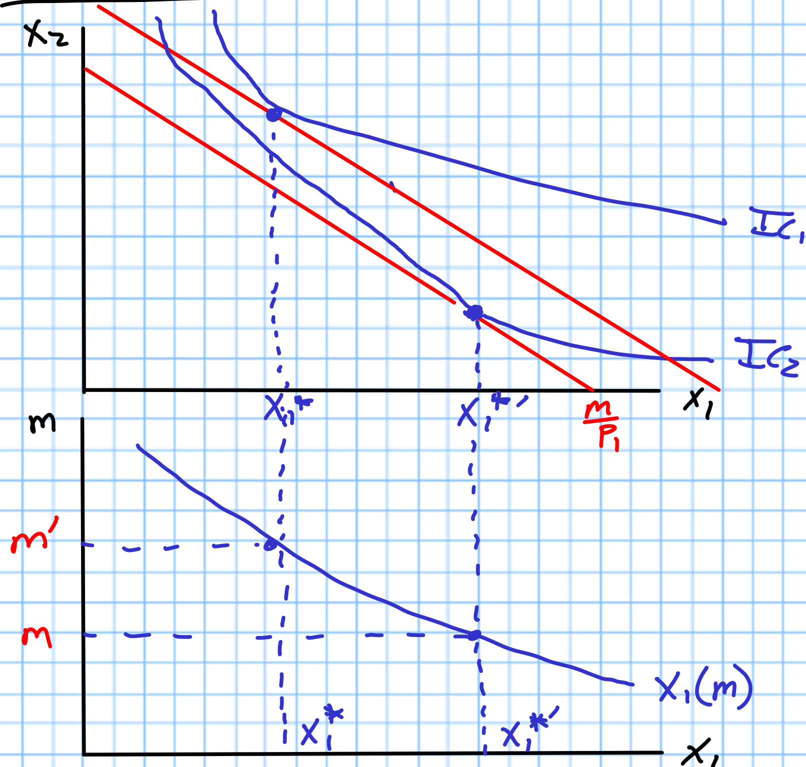
output: consumption of x_1

Slope of the Engel curve

$$\frac{dx_1}{dm} > 0 : \text{Normal good}$$

$$\frac{dx_1}{dm} < 0 : \text{Inferior good}$$

Inferior goods



Deriving Engel curves numerically

$$u(x_1, x_2) = x_1^2 x_2^3$$

$$P_1 = 7 \quad P_2 = 3 \quad m = m$$

(1) $7x_1 + 3x_2 = m$

(2) $\frac{2x_2}{3x_1} = \frac{7}{3}$

$$6x_2 = 21x_1$$

$$x_2 = \frac{21}{6}x_1$$

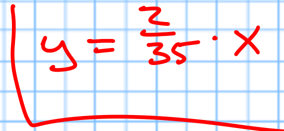
$$7x_1 + 3\left(\frac{21}{\cancel{6}_2}x_1\right) = m$$

$$\frac{14}{2}x_1 + \frac{21}{2}x_1 = m$$

$$\frac{35}{2}x_1 = m$$

$$x_1 = \frac{2m}{35} \longrightarrow x_1(m) = \frac{2m}{35}$$

$$\frac{dx_1}{dm} = \frac{2}{35} > 0$$

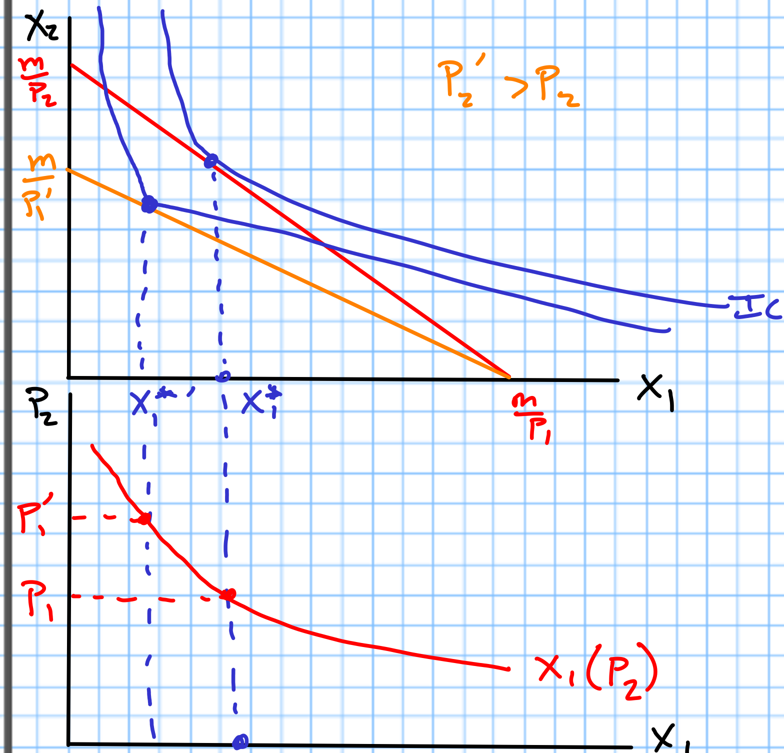


$\longrightarrow$ Normal good

Caution

The opposite of a Giffen good is not a normal good

Complements and substitutes



$x_1(p_2)$ is a function

$\frac{dx_1}{dp_2} < 0$: complements

$\frac{dx_1}{dp_2} > 0$: substitutes

Generalized demand

Solve: $\max\limits_{x_1, x_2} \; x_1^a x_2^b$

subject to: $p_1 x_1 + p_2 x_2 \leq m$

① $p_1 x_1 + p_2 x_2 = m$

② $\frac{a x_2}{b x_1} = \frac{p_1}{p_2}$

$a x_2 p_2 = b x_1 p_1$

$$x_2 = \frac{b p_1 x_1}{a p_2}$$

$$p_1 x_1 + \cancel{p_2}\left(\frac{b p_1 x_1}{a \cancel{p_2}}\right) = m$$

$$\frac{a}{a} p_1 x_1 + \frac{b}{a} p_1 x_1 = m$$

$$\frac{(a+b) p_1 x_1}{a} = m$$

$$(a+b) p_1 x_1 = am$$

$$x_1 = \frac{am}{(a+b)p_1}$$

$$\underbrace{x_1(p_1, p_2, m)}_{\text{general demand function}} = \frac{am}{(a+b)p_1} = \frac{am\, p_1^{-1}}{a+b}$$

$$\frac{\partial x_1}{\partial P_1} = -\frac{a m P_1^{-2}}{a+b}$$

$$= -\frac{am}{(a+b)P_1^2} \overset{?}{\gtrless} 0$$

$< 0 \rightarrow$ "regular" good (not Giffen)

To find if x_1 is normal or inferior, simply calculate $\frac{\partial x_1}{\partial m}$