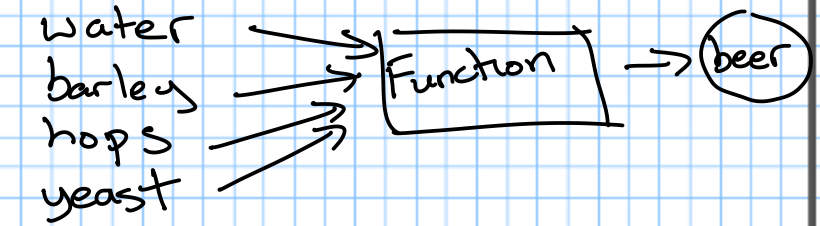
Math "review"

Functions

- A function is a process that transforms inputs into outputs

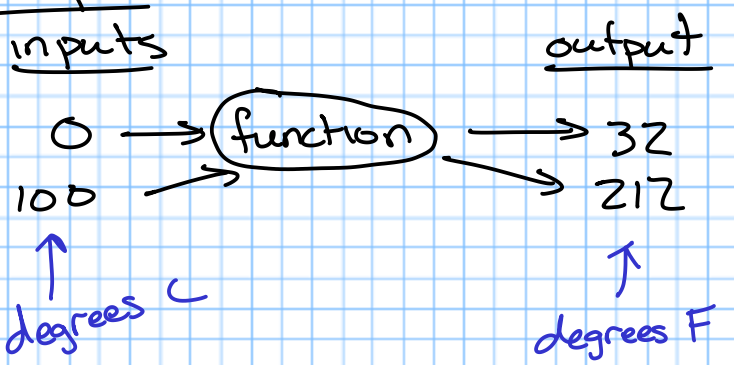
Example



Functions of numbers

- Transfor numbers into other numbers

Example



Notation

$y = f(x)$

input: x

output: y

f is our function

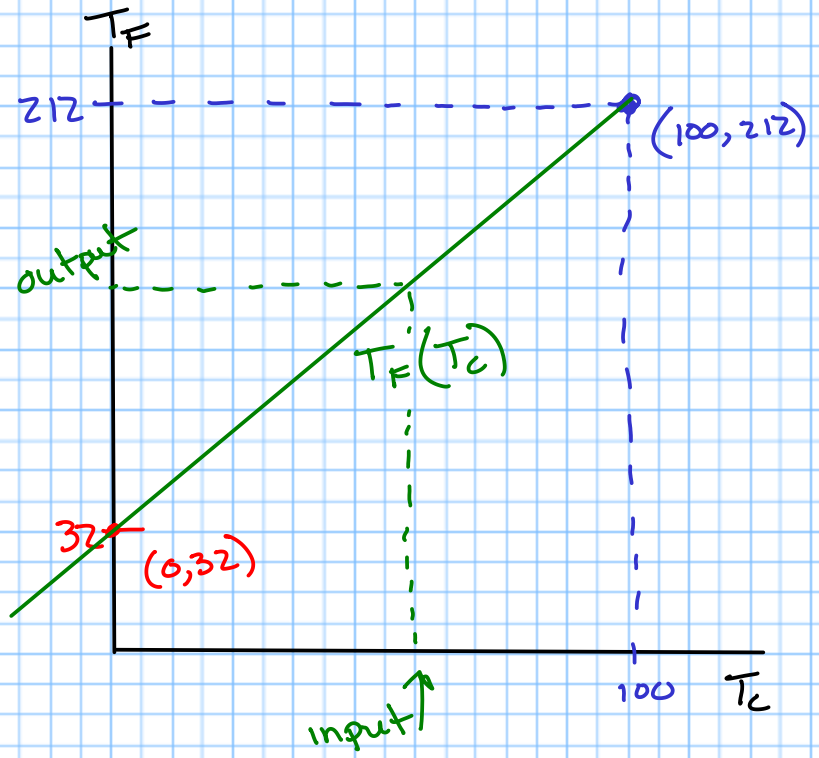
Example

T_F is temp in F

T_C is temp in C

$T_F = T_F(T_C)$

- Here, T_F is the function and the output
- We will rely on context to determine which



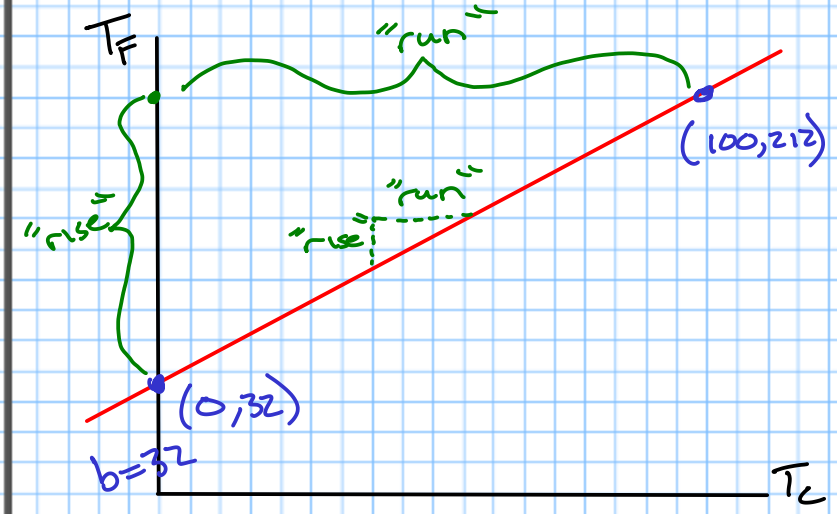
Linear functions

- A function f is <u>linear</u> if we can write it in the form $y = mx + b$, where m and b are numbers (constants)
- What is y when $x = 0$?

$$\begin{aligned} y &= f(x) \\ &= f(0) \\ &= m \cdot 0 + b \\ &= b \end{aligned}$$

(vertical intercept)

- What is m?
  - slope
  - "rate of change"
  - rise over run



rise $= \Delta T_F = 212 - 32$

run $= \Delta T_C = 100 - 0$

$$\text{slope} = m = \frac{\Delta T_F}{\Delta T_C} = \frac{212-32}{100-0}$$

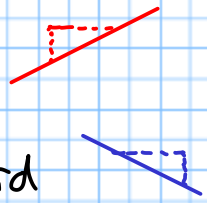
$$m = \frac{180}{100}$$

$$m = \frac{9}{5}$$

$$T_F(T_C) = \frac{9}{5} \cdot T_C + 32$$

Slope

- Graphically, slope represent the "slant" of a line
  - Positive slopes slant upward
  - Negative slopes slant downward



- A slope is the change in output that results from a 1 unit change in input

$y = mx + b$

increase x by 1

$= m(x+1) + b$

$= mx + m + b$

$= \underbrace{mx + b} + \underbrace{m}$

Δ in output

- What is $1°C$ in F?

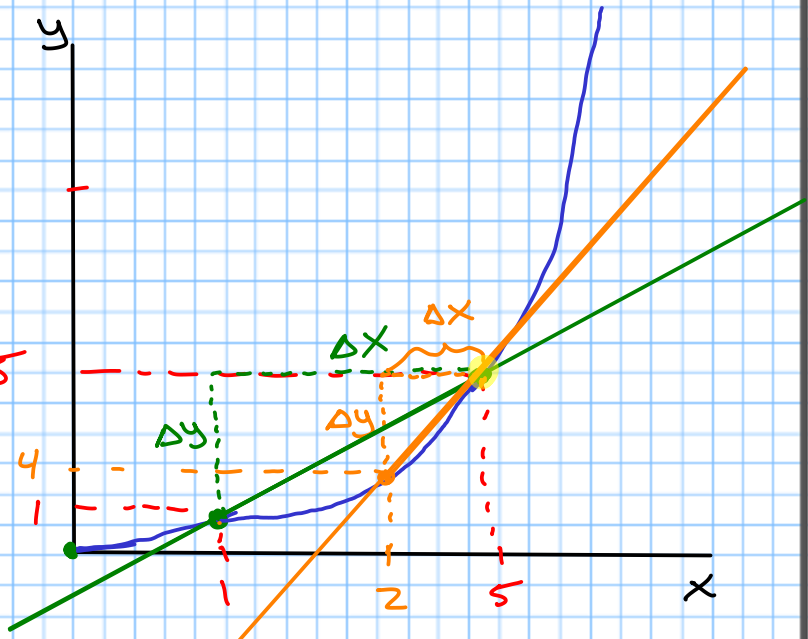
$32 + 9/5 = 33\,4/5$

Nonlinear functions

Example

$y = f(x) = x^2$

input	output
1	1
5	25
2	4



$\Delta y = 24$

$\Delta x = 4$

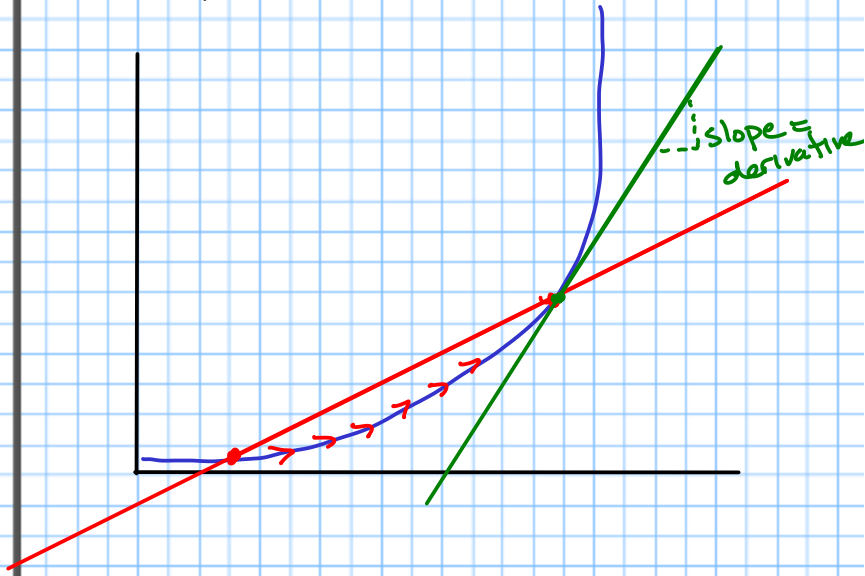
slope $= \frac{24}{4} = 6$

$\Delta y = 21$

$\Delta x = 3$

slope $= \frac{21}{3} = 7$

Which slope is the "correct" slope?



The slope of the line that touches our function in one spot (tangent line) is called a derivative

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(5-h) - f(5)}{h}$$

ignore this

Notation:

the derivative of a function $y = f(x)$ is $\frac{dy}{dx}$

Rules

Power rule

If $f(x) = ax^b$

then $\frac{df(x)}{dx} = a \cdot b x^{b-1}$

Examples

- $y = x^2$

$$\frac{dy}{dx} = 2x^{2-1}$$
$$= 2x^1$$
$$= 2x$$

- $y = 3x^{-3}$

$$\frac{dy}{dx} = (-3)\cdot 3x^{-3-1}$$
$$= -9x^{-4}$$

- $y = x^{1/3}$

$$\frac{dy}{dx} = \frac{1}{3}x^{1/3-1}$$
$$= \frac{1}{3}x^{\frac{1}{3}-\frac{3}{3}}$$
$$= \frac{1}{3}x^{-\frac{2}{3}}$$

- $y = x^2$

what is the slope of the tangent line that passes through $(5, 25)$?

$$\frac{dy}{dx} = 2x \rightarrow 2\cdot 5 = 10$$