Intertemporal choice

- So far: well-behaved prefs.
  - Consumers are spending all of their income
  - Model just describes behavior at one point in time
- In the real world, people spend more or less than their income in any given period of time

Example: Suppose I get a paycheck today and another one tomorrow

m_1: income today

m_2: income tomorrow

c_1: composite of all consumption today

c_2: consumption tomorrow

Example: Consumption this year vs consumption next year

- Consumers can invest income this year
- Consumers can borrow against future income
- Assume one interest rate for saving and borrowing (r)

Suppose I borrow \$x today. How much will I have to pay back next year?

$$\underbrace{x}_{\text{amount borrowed}} + \underbrace{rx}_{\text{interest}} = (1+r)x$$

- What is the most our consumer can consume this year?

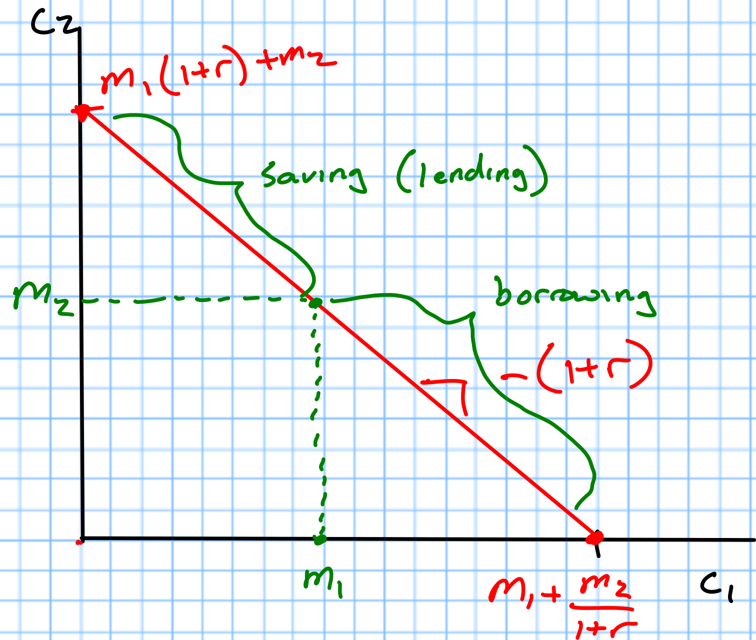
$$c_2 = 0$$

$$c_1 = m_1 + \frac{m_2}{1+r}$$

- What is the most I can consume next year?

$$c_1 = 0$$

$$c_2 = m_1 + m_2 + r m_1$$



Present value

- Consumers earn income each period (this year and next year). Now imagine we pay them this year only but we want to keep their total purchasing power unchanged.

Pay $m_1 + \frac{m_2}{1+r}$ today

Suppose they consume everything tomorrow:

$$c_2 = \left(m_1 + \frac{m_2}{1+r}\right)(1+r)$$

If the consumer makes m_1 today and m_2 tomorrow, we could give them $m_1 + \frac{m_2}{1+r}$ today and the consumer would be just as well off.

- We say that $m_1 + \frac{m_2}{1+r}$ is the <u>present value</u> of the consumer's income today and tomorrow

- Suppose I'm paid my PV today and I want to consume it all today.

$$c_2 = 0$$
$$c_1 = m_1 + \frac{m_2}{1+r}$$

- Suppose I spend none of it today, and all of it tomorrow:

$$c_1 = 0$$
$$c_2 = \left(m_1 + \frac{m_2}{1+r}\right)(1+r)$$
$$c_2 = m_1(1+r) + m_2$$

Same Budget line

More than 2 periods

income m_1, m_2, m_3

We wake up tomorrow and calculate PV of m_2 and m_3

$$PV_2 = m_2 + \frac{m_3}{1+r}$$

$$PV_1 = m_1 + \frac{PV_2}{1+r}$$

$$PV_1 = m_1 + \frac{\left(m_2 + \frac{m_3}{1+r}\right)}{1+r}$$

$$= m_1 + \frac{m_2}{1+r} + \frac{\frac{m_3}{1+r}}{1+r}$$

$$PV_1 = m_1 + \frac{m_2}{1+r} + \frac{m_3}{1+r} \cdot \frac{1}{1+r}$$

$$PV_1 = m_1 + \frac{m_2}{1+r} + \frac{m_3}{(1+r)^2}$$

Suppose there are 4 periods

$$PV_1 = m_1 + \frac{m_2}{1+r} + \frac{m_3}{(1+r)^2} + \frac{m_4}{(1+r)^3}$$

Suppose there are N periods

$$PV_1 = \sum_{i=1}^{N} \frac{m_i}{(1+r)^{i-1}}$$

Intertemporal Choice

- We know what the consumers can consume
- We need consumers to have preferences over consumption today vs consumption tomorrow
- As always, preferences are rational
- Monotonicity: More c_1 and c_2 better
- Convexity: Consumers prefer to spend something this year and next year rather that all one year