Uncertainty

ECON 420: Game Theory

Spring 2018

Announcements

- Homework 3 on Canvas
  - Due *Monday, May 21*
- Reading: Chapter 8

Uncertainty

- So far: Strategic uncertainty
  - Some players unaware of the actions of other players
  - Example: Simultaneous-move games
- Today: External uncertainty
  - "Nature" changes aspects of the game
  - Players cannot control external uncertainty, must take it into account when making decisions

Expected Utility Theory

- Events that happen according to some probability distribution are called *gambles*
- Agents are able to rank gambles by comparing the *expected utility* that they would receive from the potential outcomes of the gamble
- The utility that we will use is *von Neumann-Morgenstern (VNM)* utility

Risk preference

- When there is uncertainty we can calculate the *expected value* of a gamble
- But people do not just consider expected value when making decisions
- Some people might be willing to pay to avoid risk (risk aversion)

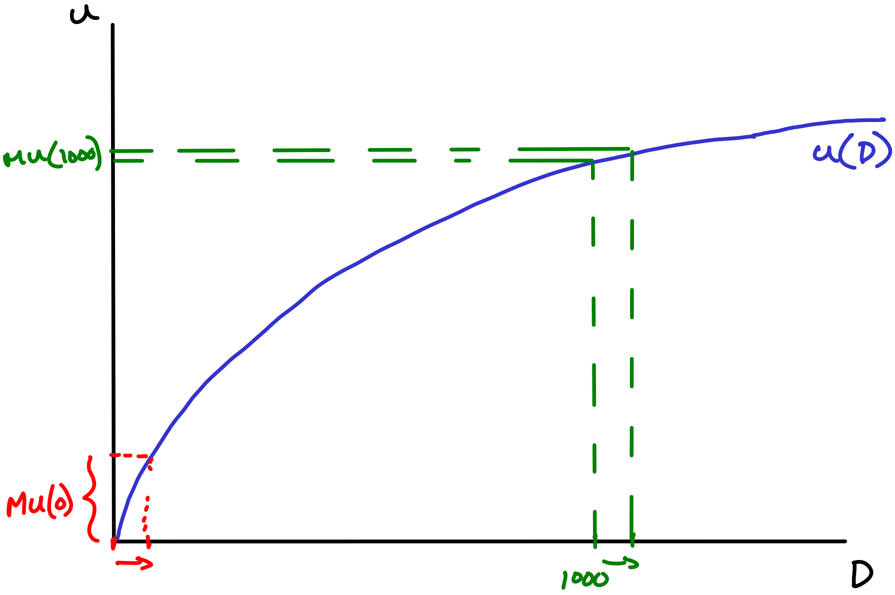
Example

- Suppose I flip a coin. If heads, you get \$100. If tails, you get \$0.
  - What is the expected value?
  - How much would you pay to play this game?
- Suppose instead the payoffs are \$1 million for heads, \$0 for tails.

VNM Utility and Risk Preference

- Outcomes are denoted D (dollars)
- Agents in the model have preferences over outcomes represented by utility $u = u(D)$
- The risk preference of the agent depends on the concavity of the utility function u
- Agents with *diminishing marginal utility* are risk averse
  - Concave utility function

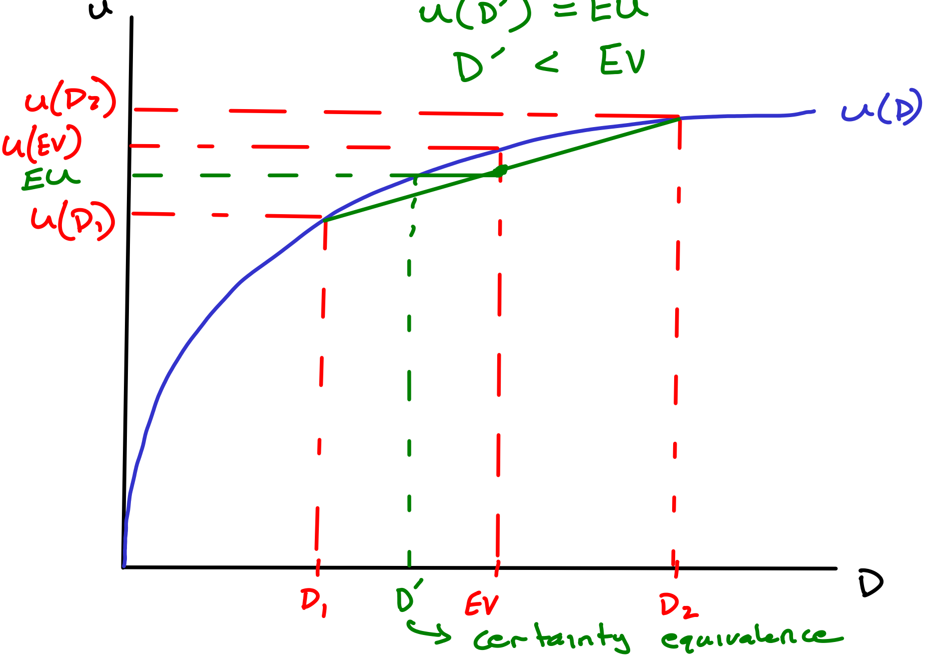
Risk aversion



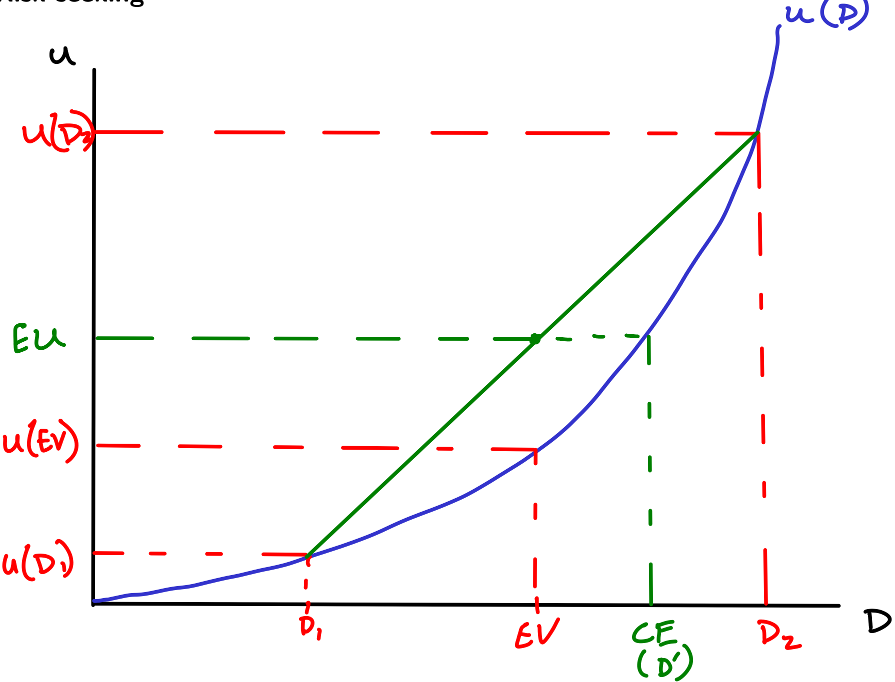
$u(EV) > Eu$

$u(D') = Eu$

$D' < EV$



Risk seeking



$$EV = \frac{1}{2} \cdot 160{,}000 + \frac{1}{2} \cdot 40{,}000$$

$$= 80{,}000 + 20{,}000$$

$$= 100{,}000$$

Example

- A farmer's crop yield depends on weather
- Farmer gets good weather with 50%
- Yield with good weather is \$160,000, yield in bad weather is \$40,000
- Farmer has VNM utility $u(D) = \sqrt{D}$

$\hookrightarrow$ risk averse

$$EU = \frac{1}{2} u(160{,}000) + \frac{1}{2} u(40{,}000)$$

$$= \frac{1}{2} \sqrt{160{,}000} + \frac{1}{2} \sqrt{40{,}000}$$

$$= \frac{1}{2} 400 + \frac{1}{2} \cdot 200 = 300$$

$$u(EV) = \sqrt{100{,}000}$$

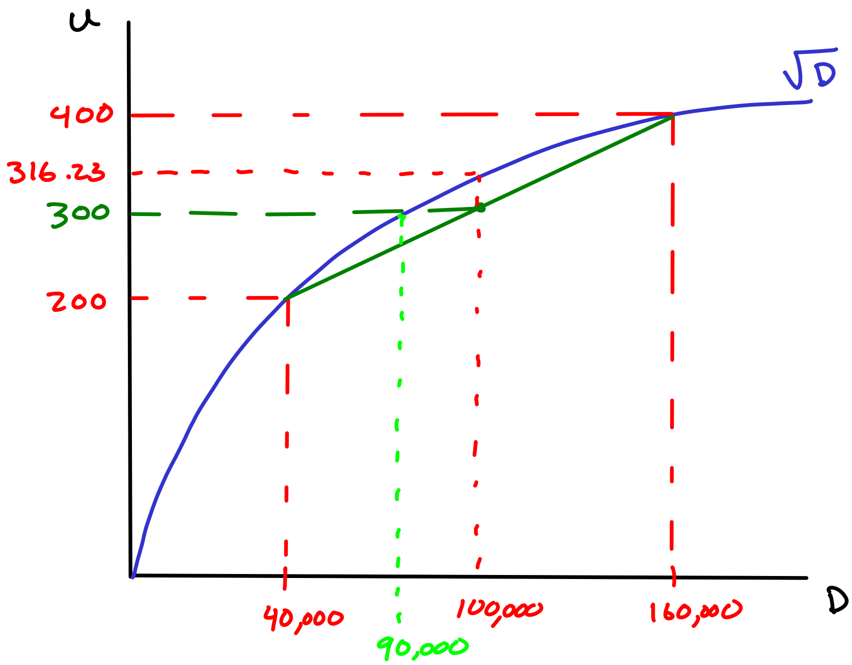
$$= 316.23$$

CE: amount that the farmer needs with certainty (prob 1) to be indifferent between that and taking on the risk

$$EU = u(CE)$$

$$300 = \sqrt{CE}$$

$$CE = 90{,}000$$


u
√D
400
316.23
300
200
D
40,000
90,000
100,000
160,000

Risk sharing

- Risk averse agents willing to pay to remove risk
- Agents can therefore benefit from trading *state-contingent claims* with one another
  - You agree to pay someone else if you have a good outcome, someone else pays you if you have a bad outcome

Example

- Suppose there is another farmer that has the same weather probability and outcomes (weather probability is independent of first farmer)
- Farmers agree to a contract: If one farmer gets good luck and the other gets bad luck, lucky farmer pays $60,000 to the unlucky farmer
- Are the farmers better off?

Possible states:

Farmer 1	Farmer 2	Probability	Farmer 1's payoff
Good	Good	1/4	160,000
Bad	Bad	1/4	40000
Good	Bad	1/4	100,000
Bad	Good	1/4	100,000

Farmer 1's utility	Farmer 1's payoff
400	160,000
200	40000
316.23	100,000
316.23	100,000

← from last page

Farmer 1's EU:

$$EU = \frac{1}{4} \cdot 400 + \frac{1}{4} \cdot 200 + \frac{1}{2} \cdot 316.23$$

$$= 100 + 50 + 158.12$$

$$= 308.12 > 300$$

↑ utility without risk sharing

Example

- Now suppose the other farmer faces no uncertainty and will earn \$100,000 with probability 1
- The farmer with risk is willing to accept their certainty equivalence instead of the gamble
- Is the riskless farmer willing to buy the risk in exchange for the certainty equivalence?

Riskless Farmer's expected utility:

$$EV = \frac{1}{2}(40{,}000 - 90000) + \frac{1}{2}(160{,}000 - 90000) + 100{,}000$$

$$EV = \frac{1}{2}(40{,}000) + \frac{1}{2}(160{,}000) + 100{,}000 - 90{,}000$$

$$EV = \frac{1}{2}(40{,}000) + \frac{1}{2}(160{,}000) + 10{,}000 = 110{,}000$$

$$EU = \frac{1}{2}\sqrt{40{,}000} + \frac{1}{2}\sqrt{160{,}000} + \sqrt{10{,}000}$$

$$= \frac{1}{2}\cdot 200 + \frac{1}{2}\cdot 400 + 100$$

$$EU = 400$$

If they don't make the deal:

$$EU = \sqrt{100{,}000}$$

$$= 316.23$$

Example

- Now suppose the farmer without risk is *risk neutral*
- What is the maximum that this farmer is willing to pay for the gamble?

100,000 $\rightarrow$ EV of the gamble

Insurance and risk

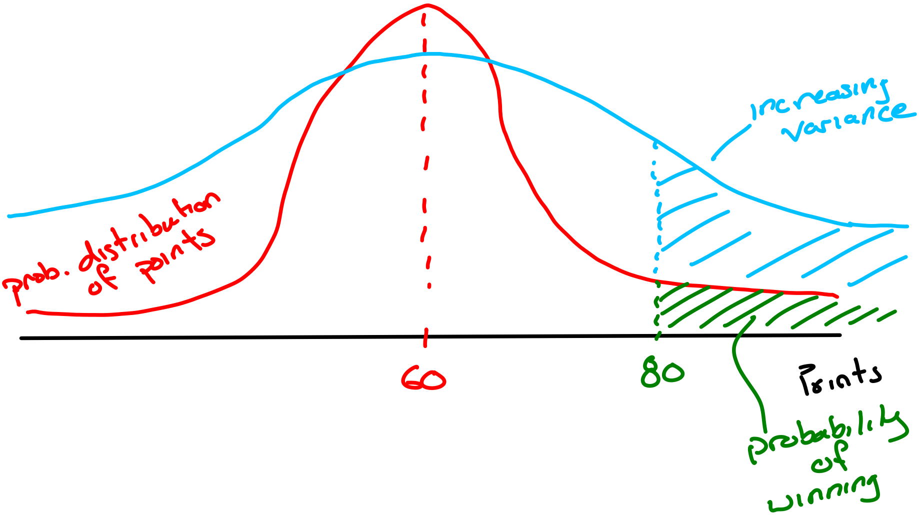
- Suppose there are thousands of farmers with identical risk/outcomes
- A single entity (insurance company) can buy the risk of all of the farmers and make them better off
- Law of large numbers says that the insurance company will earn the expected value of the gamble

Manipulating Risk

- Sometimes agents have control over risk and can use it to their advantage
- By increasing risk, the probability of "tail events" increases
- This is why underdogs in sports often choose risky actions

Example

- A basketball team scores 60 points per game on average
- They are playing a better opponent and must score at least 80 points to win
- How can this team maximize their chances of winning?


increasing
variance
prob. distribution
of points
60
80
Points
probability
of
winning

Cheap Talk

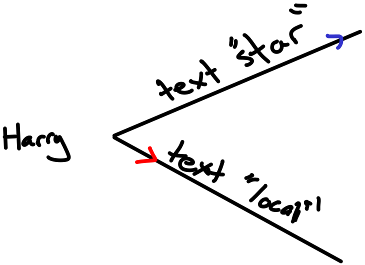
- In coordination games, players may be able to costlessly communicate before the game begins
- This might allow players to better coordinate on preferred outcomes

		SALLY	
		Starbucks	Local Latte
HARRY	Starbucks	1, 1	0, 0
	Local Latte	0, 0	2, 2

Rollback 2:

H: star, local, star

S: local, star



Harry (after text "star"):

	Sally: star	Sally: local
Harry: star	1, 1	0, 0
Harry: local	0, 0	2, 2

Harry (after text "local"):

	Sally: star	Sally: local
Harry: star	1, 1	0, 0
Harry: local	0, 0	2, 2

Rollback 1: H: text local, star if star, local if local

S: star if star, local if local