Algebra Review

Functions

- A function is a process that transforms inputs into outputs

inputs → function → output

Example:

inputs

water
barley
yeast
hops

→ function → beer

Functions of numbers

- transforms one number into another number
- inputs and outputs are numbers

Example

A function that transforms temperature in C to temp. in F

inputs			output
0	→	function	→ 32
100	→	function	→ 212

Notation

$y = f(x)$

x: input, y: output, f: function

f transforms x into y

Example

$u = u(x)$

→ same letter for output and function

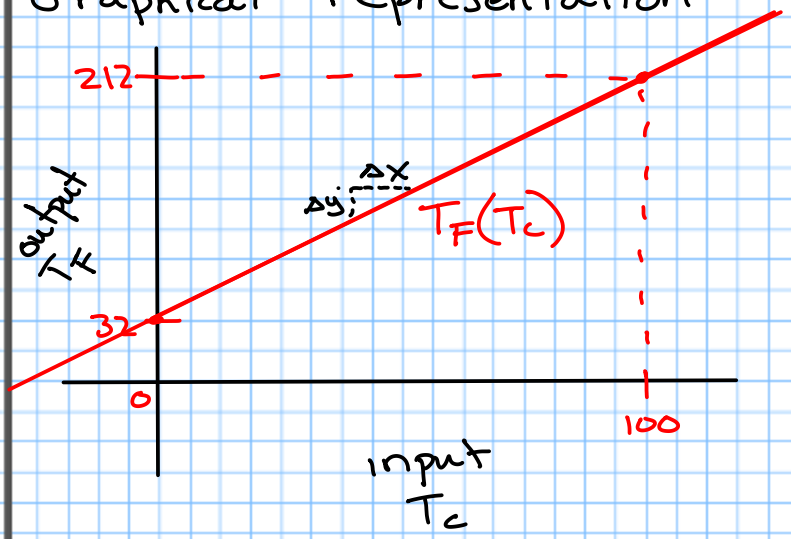
Example

T_F: temp. in F

T_C: temp in C

$T_F = T_F(T_C)$

Graphical representation



Linear functions

A function is linear if it can be written as

$y = mx + b$

b: vertical intercept

m: slope

$m = \frac{\Delta y}{\Delta x}$

Example $(T_C, T_F) = (0, 32)$

$(T_C', T_F') = (100, 212)$

$$m = \frac{\Delta T_F}{\Delta T_C} = \frac{T_F' - T_F}{T_C' - T_C}$$

$$= \frac{212 - 32}{100 - 0}$$

$$= \frac{180}{100}$$

$$= \frac{9}{5}$$

$$\boxed{T_F = \frac{9}{5} T_C + 32}$$

Slope

- Graphically, it is the "slant" of a line
  - positive slopes slant upward
  - negative slopes slant downward
- change in output that results from a change in input

Example

$$T_F = \frac{9}{5} T_C + 32$$

$$= T_F(T_C)$$

$T_C = 0$

$T_F(T_C) = T_F(0) = \frac{9}{5} \cdot 0 + 32$

$= 32$

Increase T_C by 1:

$T_C' = 1$

$T_F(1) = \frac{9}{5} 1 + 32 = 32\frac{9}{5}$

Change in output:

$T_F(1) - T_F(0) = 32\frac{9}{5} - 32$

$= 9/5$

$=$ slope

Algebra Rules

x is a variable,

a and b are numbers (constants)

$$x^a = \frac{1}{x^{-a}}$$

$$x^{-a} = \frac{1}{x^a}$$

$$x^a x^b = x^{a+b}$$

$$\frac{x^a}{x^b} = x^a x^{-b} = x^{a-b}$$

$$(x^a)^b = x^{ab}$$

Example

$x^2 \cdot x^3 = xxxxx = x^5$

$(x^2)^3 = x^2 x^2 x^2 = xxxxxx$

$= x^6$

$x = x^1$

$x^0 = 1$

$x^{-1} = \frac{1}{x}$

$x^2 x^{-2} = x^0 = 1$

$2^{-1/2} = \frac{1}{2^{1/2}}$

$(x^{1/2})^2 = x^{1/2 \cdot 2} = x$