

Profit maximization

- We assume that firms are always maximizing profits
- Our firms use labor L and capital K to produce output y
- Sell each unit of y for price p
- Labor costs w per unit (wages)

- K costs r per unit
 r : "rental rate" of capital

- r is the cost of renting machines and tools
- opportunity cost of using existing (already owned) capital

Revenue

$$R = py$$

Costs

$$C = wL + rK$$

Profits (economic profits)

$$\pi = R - C$$

- Suppose a firm employs K , and L , they are able to produce y ,
 $y = f(K, L)$
- Now suppose they decide to increase K by a small amount (1 unit).
 - Additional cost: r
 - Additional revenue: $pMPK$
 - If $pMPK > r$, then profits are increasing
 - If $pMPK < r$, then profits are decreasing

- If the profit maximizing then $pMPK = r$
- Same for labor
- Result: Profit-maximizing firms choose K and L such that

$$\begin{cases} pMPK = r \\ pMPL = w \end{cases}$$

Short run vs Long run

- Long run is the period of time in which there are no fixed inputs
- Short run: At least 1 fixed input

- In our model, we assume that K is fixed in the short run

Short run Π -max

- K is fixed at \bar{K}
- Fixed costs $r\bar{K}$
- The only thing the firm can control is L
- How much L should the firm choose?
- They set $pMPL = w$

Example

$$f(K, L) = K^2 L^{1/2}$$

$$w = 15, \quad r = 2, \quad p = 7$$

$$\bar{K} = 3$$

$$MPL = \frac{1}{2} K^2 L^{-1/2}$$

$$7 \cdot \frac{1}{2} K^2 L^{-1/2} = 15$$

$$7 \cdot \frac{1}{2} 3^2 L^{-1/2} = 15$$

$$63 L^{-1/2} = 30$$

$$(L^{-1/2})^{-2} = \left(\frac{10}{21}\right)^{-2}$$

$$L = \left(\frac{21}{10}\right)^2$$

$$L^* = 4.41$$

$$\begin{aligned} \text{Output: } y &= K^2 L^{1/2} \\ &= 3^2 4.41^{1/2} \end{aligned}$$

$$y = 9 \cdot \frac{21}{10}$$

$$y = \frac{189}{10}$$

$$R = p \cdot y$$
$$= 7 \cdot \frac{189}{10}$$

$$= 132.3$$

$$C = vL + rK$$

$$= 15 \cdot 4.41 + 2 \cdot 3$$

$$\approx 60 + 6$$

$$= 66$$

$$\pi \approx 132.3 - 66 = 66.3$$