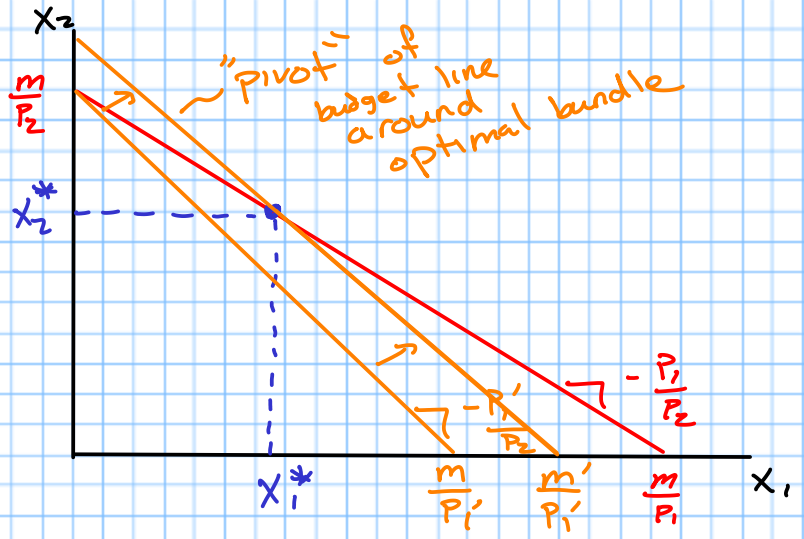
Income and Substitution Effects

- Suppose the price of a good increases from P_1 to P_1'
- Two things happen:
  1. The opportunity cost of the good changes. Other goods become relatively less expensive
  2. As the price increases, there are fewer bundles that we can consume. We feel more poor

Substitution Effect

- How does a price increase affect consumption holding purchasing power constant?



Equation for original budget line

$$P_1 x_1 + P_2 x_2 = m$$

Equation for "pivoted" B.L.

$$P_1' x_1 + P_2 x_2 = m'$$

How much additional income is needed to keep purchasing power constant?

$$\begin{array}{r} P_1' x_1 + P_2 x_2 = m' \\ -\left(P_1 x_1 + P_2 x_2 = m\right) \\ \hline P_1' x_1 - P_1 x_1 = m' - m \\ (P_1' - P_1) x_1 = m' - m \\ \Delta P_1 x_1 = \Delta m \end{array}$$

ΔP_1: change in price; Δm: change in income

Example

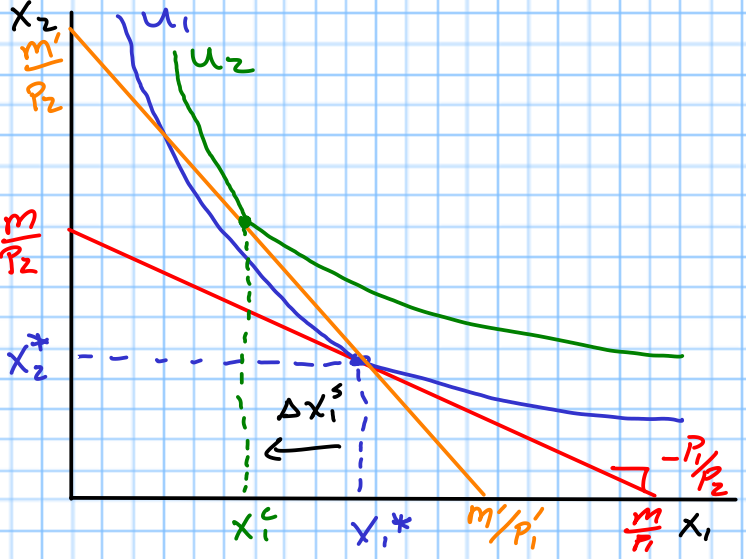
$P_1 = 5$, $m = 20$, $x_1^* = 2$

Price increases to $P_1' = 10$

How much additional income will the consumer need to keep purchasing power constant?

$$\Delta m = \Delta P_1 \cdot x_1$$
$$= (10 - 5)\,2$$
$$= 5 \cdot 2$$
$$= 10$$
$$m' = 20 + 10 = 30$$

- Suppose we increase price to p_1' and simultaneously give the consumer Δm more income

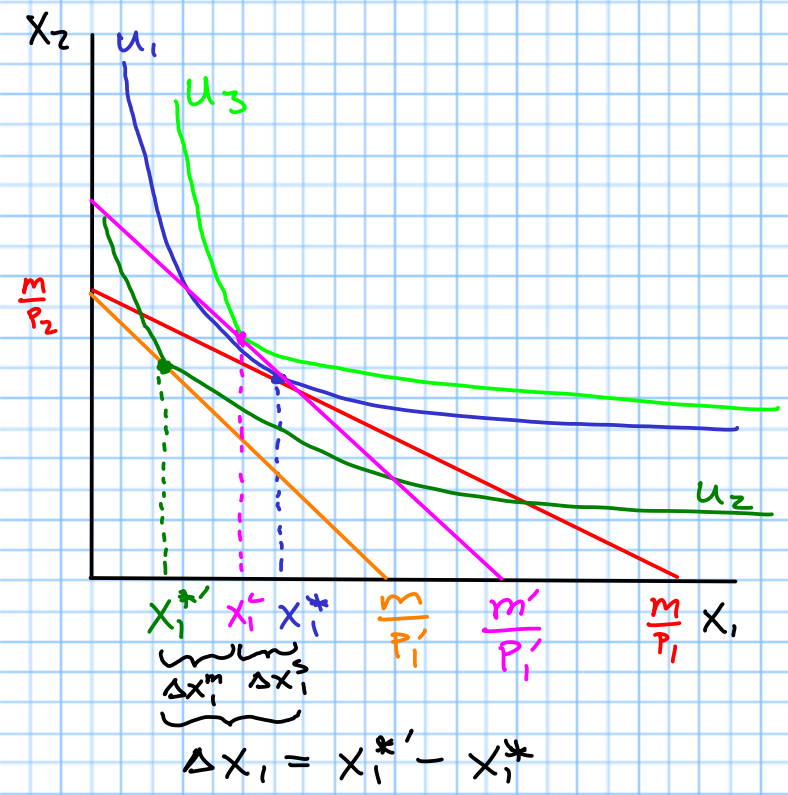


x_1^c is the compensated demand for good 1

Note: $x_1^c < x_1^*$ because x_1 is now relatively more expensive than x_2

- $x_1^c - x_1^*$ is the substitution effect

$$\Delta x_1^s = x_1^c - x_1^*$$



$$\Delta X_1 = X_1^{*\prime} - X_1^* \quad + X_1^c - X_1^c$$

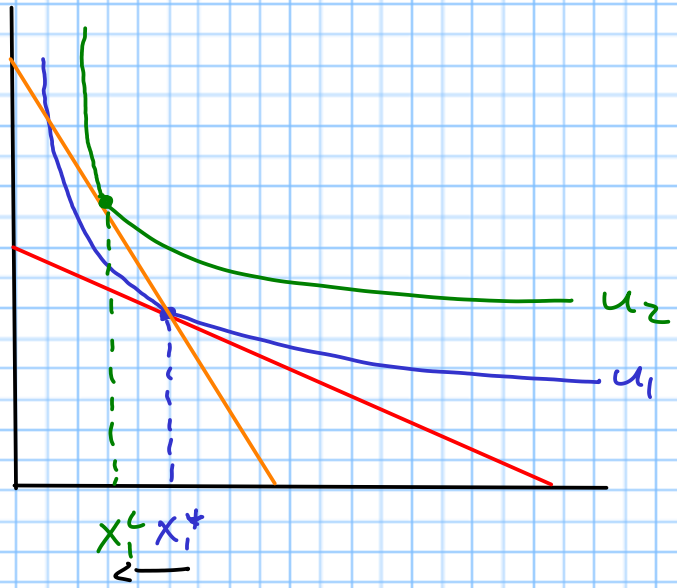
$$\Delta X_1 = (X_1^c - X_1^*) + (X_1^{*\prime} - X_1^c)$$

$$\Delta X_1 = \Delta X_1^s + \Delta X_1^m$$

Total change in demand = substitution effect + income effect

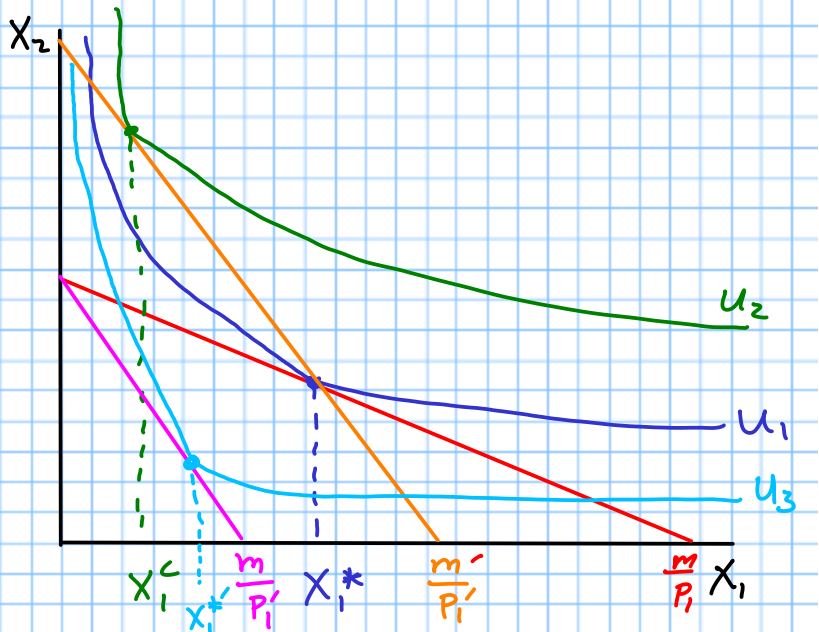
"Slutsky Identity"

Sign of ΔX_1^s



Result: $\Delta X_1^s < 0$ always

Sign of ΔX_1^m



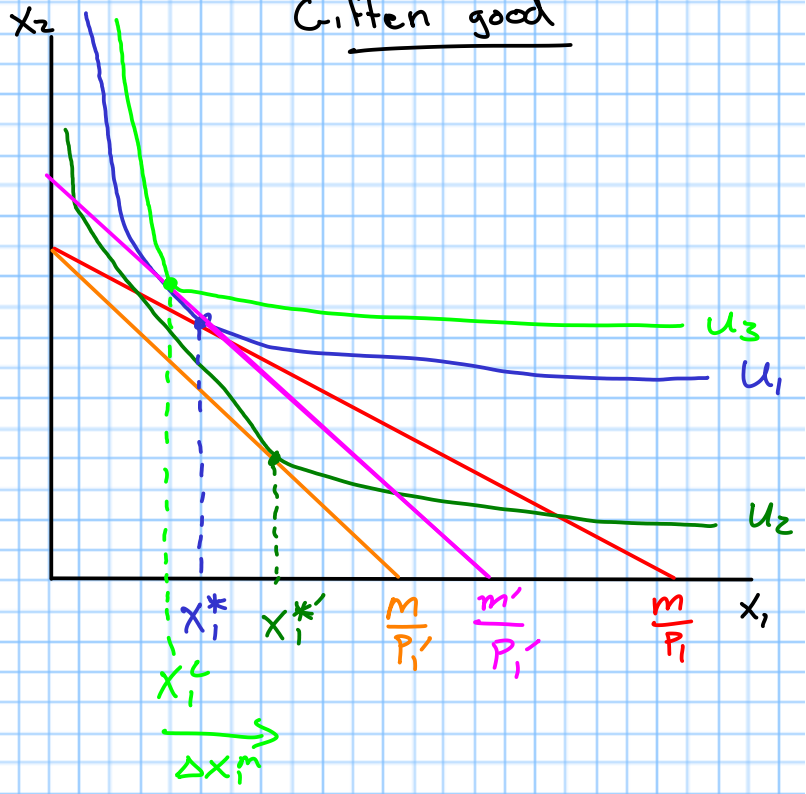
$\Rightarrow \Delta X_1^m$

$\Leftarrow \Delta X_1^s$

Result

- If $\Delta x_1^m > 0$ (income effect is opposite the substitution effect) then x_1 is an inferior good
- If $\Delta x_1^m < 0$ (same sign as Δx_1^s) then x_1 is a normal good

Giffen good



Result

For Giffen goods, $\Delta x_i^m > 0$. Therefore, Giffen goods are necessarily inferior