Nash Equilibrium

ECON 420: Game Theory

Spring 2018

Pick-a-color game

- Two types of teams, A and B
- Your team will play against the other type (A vs B)
- Each team chooses "white" or "blue"
- Payoffs for **A** teams:
  - If both teams choose white: 50
  - If both teams choose blue: 25
  - If A chooses white and B chooses blue: 75
  - If A chooses blue and B chooses white: 50
- Payoffs for **B** teams:
  - If both teams choose white: 50
  - If both teams choose blue: 75
  - If A chooses white and B chooses blue: 25
  - If A chooses blue and B chooses white: 50

Pick-a-color game (version 2)

- Each team chooses "orange" or "black"
- Payoffs for **A** teams:
  - If both teams choose orange: 75
  - If both teams choose black: 50
  - If A chooses orange and B chooses black: 25
  - If A chooses black and B chooses orange: 50
- Payoffs for **B** teams:
  - If both teams choose orange: 25
  - If both teams choose black: 50
  - If A chooses orange and B chooses black: 75
  - If A chooses black and B chooses orange: 50

Pick a color, normal form

Version 2, normal form

Prisoners' Dilemma

- The story:
  - Husband and wife are arrested for a crime and interrogated separately
  - Both must choose to confess to the crime or deny that they committed the crime
  - If both deny, serve 3 years for a crime that police can prove
  - A confession is "rewarded" by police if it helps convict the other partner (who denies)
  - If both confess, both serve a long sentence

Prisoners' dilemma

		WIFE	
		Confess (Defect)	Deny (Cooperate)
HUSBAND	Confess (Defect)	10 yr, 10 yr	1 yr, 25 yr
	Deny (Cooperate)	25 yr, 1 yr	3 yr, 3 yr

Best responses

1. Suppose husband believes wife will confess. What is his best response?
2. Suppose husband believes wife will deny. What is his best response?

Dominance

- If a strategy is *always* a best response, that strategy is a *dominant* strategy
- If a strategy is *never* a best response, that strategy is a *dominated* strategy
- If both players have a dominant strategy, then these strategies define the Nash equilibrium
- In the prisoners' dilemma, confess is a dominant strategy (deny is dominated)
- What about pick a color?

Prisoners' dilemma

- Both players have a dominant strategy
- Dominance solution is worse for both players than outcome where both cooperate with each other
- The outcome that obtains from rational play (and in practice!) is a bad outcome for the players

Fiscal and monetary policy game

		FEDERAL RESERVE	
		Low interest rates	High interest rates
CONGRESS	Budget balance	3, 4	1, 3
	Budget deficit	4, 1	2, 2

One player has a dominant strategy

- Congress has a dominant strategy, Fed does not
- But Fed knows that Congress has dominant strategy
- Fed can choose the best response to Congress's dominant strategy

Successive elimination of dominated strategies

- If a strategy is dominated, then it won't be played at the equilibrium
  - Rational players won't play dominated strategies, other rational players know this about each other
- Removing dominated strategies can simplify the game, makes finding Nash equilibrium easier
- A game is *dominance solvable* if successive elimination of dominated strategies ends in a unique outcome (the Nash equilibrium)

		COLUMN		
		Left	Middle	Right
ROW	Top	3, 1	2, 3	10, 2
	High	4, 5	3, 0	6, 4
	Low	2, 2	5, 4	12, 3
	Bottom	5, 6	4, 5	9, 7

Weak dominance

- A strategy is *weakly dominant* if it never yields a worse outcome than any other strategy
  - Allows for "ties" in payoffs
- Can eliminate weekly dominated strategies to find equilibrium as well

		COLUMN		
		Left	Middle	Right
ROW	Top	3, 1	2, 3	10, 2
	High	4, 5	3, 0	6, 4
	Low	2, 2	5, 4	12, 3
	Bottom	5, 6	5, 5	9, 7

Elimination of weakly dominated strategies

- We can sometimes find a Nash equilibrium by eliminating weakly dominated strategies
- However, we can also eliminate other Nash equilibrium with this strategy!

		COLIN	
		Left	Right
ROWENA	Up	0, 0	1, 1
	Down	1, 1	1, 1

Best response analysis

- The Nash equilibrium is a mutual best response
- We can find the best response for each player for any given opponent strategy
- We can use this to find a Nash equilibrium:
  - Find the best responses for each player for *all possible* opponent strategies
  - If one outcome is a best response for both players, then it must be a Nash equilibrium

		COLUMN		
		Left	Middle	Right
ROW	Top	3, 1	2, 3	10, 2
	High	4, 5	3, 0	6, 4
	Low	2, 2	5, 4	12, 3
	Bottom	5, 6	4, 5	9, 7

		COLIN	
		Left	Right
ROWENA	Up	0, 0	1, 1
	Down	1, 1	1, 1

Three player games

- We need three dimensions to describe the payoff space with three players
- Alternatively, write multiple game matrices for two players, given the choices of a third player
- Use best response analysis as before: find best response given strategies of *both* of the other players

TALIA chooses:

Contribute

		NINA	
		Contribute	Don't
EMILY	Contribute	5, 5, 5	3, 6, 3
	Don't	6, 3, 3	4, 4, 1

Don't Contribute

		NINA	
		Contribute	Don't
EMILY	Contribute	3, 3, 6	1, 4, 4
	Don't	4, 1, 4	2, 2, 2

Pure coordination

- Many games have multiple Nash equilibria
- If the payoffs are identical across equilibria, then it is a game of *pure coordination*

Pure coordination example

- You are to meet someone in Corvallis. You have not been instructed where to meet, you have no prior understanding with the person on where to meet, and you cannot communicate with each other. You are simply told that you will have to guess where to meet, that the other person is being told the same thing, and that you will just have to try to make your guesses coincide. Where do you go?

Games with no pure strategy equilibrium

		NAVRATILOVA	
		DL	CC
EVERT	DL	50, 50	80, 20
	CC	90, 10	20, 80

Games with no pure strategy equilibrium

- What should players do?
- More importantly: What *shouldn't* players do?