# Nash Equilibrium

ECON 420: Game Theory

Spring 2018

## Pick-a-color game

- ► Two types of teams, A and B
- ➤ Your team will play against the other type (A vs B)
- ► Each team chooses "white" or "blue"
- ► Payoffs for **A** teams:
  - If both teams choose white: 50
  - ▶ If both teams choose blue: 25
  - ▶ If A chooses white and B chooses blue: 75

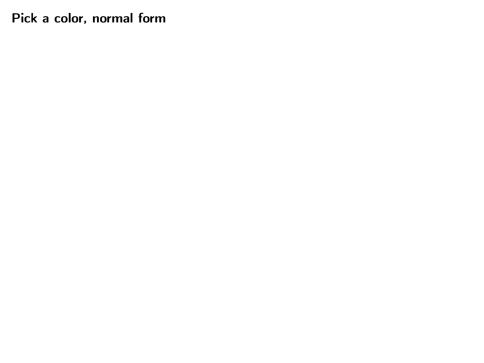
  - ▶ If A chooses blue and B chooses white: 50
- ► Payoffs for **B** teams:
  - If both teams choose white: 50 ▶ If both teams choose blue: 75
  - ▶ If A chooses white and B chooses blue: 25

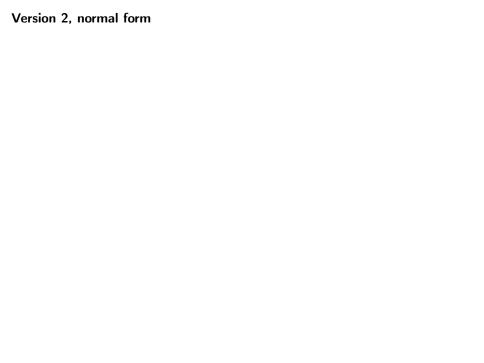
  - ▶ If A chooses blue and B chooses white: 50

### Pick-a-color game (version 2)

- ► Each team chooses "orange" or "black"
- ► Payoffs for **A** teams:
  - ▶ If both teams choose orange: 75
  - ▶ If both teams choose black: 50
  - ▶ If A chooses orange and B chooses black: 25
  - ▶ If A chooses black and B chooses orange: 50
- ► Payoffs for **B** teams:
  - ▶ If both teams choose orange: 25

  - ▶ If both teams choose black: 50 ▶ If A chooses orange and B chooses black: 75 ▶ If A chooses black and B chooses orange: 50





### Prisoners' Dilemma

- ► The story:
  - ► Husband and wife are arrested for a crime and interrogated separately
    - ► Both must choose to confess to the crime or deny that they committed the crime
  - ▶ If both deny, serve 3 years for a crime that police can prove
  - ► A confession is "rewarded" by police if it helps convict the other partner (who denies)
    - ▶ If both confess, both serve a long sentence

## Prisoners' dilemma

		WIFE		
		Confess (Defect)	Deny (Cooperate)	
LILIGO AND	Confess (Defect)	10 yr, 10 yr	1 yr, 25 yr	
HUSBAND	Deny (Cooperate)	25 yr, 1 yr	3 yr, 3 yr	

### Best responses

- 1. Suppose husband believes wife will confess. What is his best response?
- 2. Suppose husband believes wife will deny. What is his best response?

### **Dominance**

- ► If a strategy is *always* a best response, that strategy is a *dominant* strategy
- ▶ If a strategy is never a best response, that strategy is a dominated strategy
   ▶ If both players have a dominant strategy, then these strategies define the
  - Nash equilibrium

    In the prisoners' dilemma, confess is a dominant strategy (deny is dominated)
- In the prisoners' dilemma, confess is a dominant strategy (deny is dominated)What about pick a color?

## Prisoners' dilemma

► Both players have a dominant strategy

outcome for the players

- Dominance solution is worse for both players than outcome where both cooperate with each other
- ► The outcome that obtains from rational play (and in practice!) is a bad

# Fiscal and monetary policy game

		FEDERAL RESERVE			
		Low interest rates High interest ra			
CONCRE	Budget balance	3, 4	1, 3		
CONGRESS	Budget deficit	4, 1	2, 2		

### One player has a dominant strategy

- ► Congress has a dominant strategy, Fed does not
- ▶ But Fed knows that Congress has dominant strategy
- ► Fed can choose the best response to Congress's dominant strategy

## Successive elimination of dominated strategies

this about each other

- ▶ If a strategy is dominated, then it won't be played at the equilibrium
  - ► Rational players won't play dominated strategies, other rational players know
- ▶ Removing dominated strategies can simplify the game, makes finding Nash equilibrium easier
- ► A game is *dominance solvable* if successive elimination of dominated strategies ends in a unique outcome (the Nash equilibrium)

		COLUMN			
		Left	Middle	Right	
	Тор	3, 1	2, 3	10, 2	
ROW	High	4, 5	3, 0	6, 4	
ROW	Low	2, 2	5, 4	12, 3	
	Bottom	5, 6	4, 5	9, 7	

- Weak dominance
- ▶ A strategy is weakly dominant if it never yields a worse outcome than any
- ► Allows for "ties" in payoffs

► Can eliminate weekly dominated strategies to find equilibrium as well

other strategy

		COLUMN			
		Left	Middle	Right	
	Тор	3, 1	2, 3	10, 2	
ROW	High	4, 5	3, 0	6, 4	
ROW	Low	2, 2	5, 4	12, 3	
	Bottom	5, 6	5, 5	9, 7	

limination of weakly dominated strates	316	38
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▶ However, we can also eliminate other Nash equilibrium with this strategy!

strategies

▶ We can sometimes find a Nash equilibrium by eliminating weakly dominated

		CO	LIN
		Left	Right
ROWENA	Up	0, 0	1, 1
KOWENA	Down	1, 1	1, 1

### Best response analysis

equilibrium

- ► The Nash equilibrium is a mutual best response
- ▶ We can find the best response for each player for any given opponent strategy
- We can use this to find a Nash equilibrium:
   Find the best responses for each player for all possible opponent strategies
  - If one outcome is a best response for both players, then it must be a Nash

		COLUMN			
		Left	Middle	Right	
	Тор	3, 1	2, 3	10, 2	
ROW	High	4, 5	3, 0	6, 4	
ROW	Low	2, 2	5, 4	12, 3	
	Bottom	5, 6	4, 5	9, 7	

		CO	LIN
		Left	Right
ROWENA	Up	0, 0	1, 1
KOWENA	Down	1, 1	1, 1

### Three player games

- ► We need three dimensions to describe the payoff space with three players
- ► Alternatively, write multiple game matrices for two players, given the choices of a third player
- ► Use best response analysis as before: find best response given strategies of both of the other players

### TALIA chooses:

Contribute				Don't Co	ntribute		
NINA				NIN	Α		
	Contribute	Don't				Contribute	Don't
Contribute	5, 5, 5	<b>3, 6,</b> 3		FMILV	Contribute	3, 3, 6	1, 4, 4
Don't	6, 3, 3	<b>4, 4,</b> 1		EMILY	Don't	4, 1, 4	2,2,2
	Contribute	Contribute 5, 5, 5	NINA Contribute Don't Contribute 5, 5, 5 3, 6, 3	NINA  Contribute Don't  Contribute 5, 5, 5 3, 6, 3	NINA Contribute Don't  Contribute 5, 5, 5 3, 6, 3  EMILY	NINA Contribute Don't  Contribute 5, 5, 5 3, 6, 3  EMILY  Contribute	NINA Contribute Don't  Contribute 5, 5, 5 3, 6, 3  Contribute 3, 3, 6  EMILY  Contribute 3, 3, 6

# Pure coordination

coordination

- ► Many games have multiple Nash equilibria
- ► If the payoffs are identical across equilibria, then it is a game of *pure*

### Pure coordination example

▶ You are to meet someone in Corvallis. You have not been instructed where to meet, you have no prior understanding with the person on where to meet, and you cannot communicate with each other. You are simply told that you will have to guess where to meet, that the other person is being told the same thing, and that you will just have to try to make your guesses coincide. Where do you go?

# Games with no pure strategy equilibrium

		NAVRATILOVA		
		DL	CC	
FVEDT	DL	50, 50	80, 20	
EVERT	CC	90, 10	20, 80	

- Games with no pure strategy equilibrium

▶ More importantly: What *shouldn't* players do?

► What should players do?