

Sequential Games

ECON 420: Game Theory

Spring 2018

Centipede game

1. 2 players play for 10 rounds
2. Each round another unit of payoff is added to the pot (starting with one unit)
3. Players alternate turns, choose to either:
 - ▶ Stop, and collect the entire pot for themselves
 - ▶ Continue, one is added to the pot and next player chooses

Sequential games

- ▶ Games where there is a strict order of play
- ▶ Games where players take turns moving are sequential
- ▶ Real-world games are generally combinations of sequential and simultaneous games

Game trees

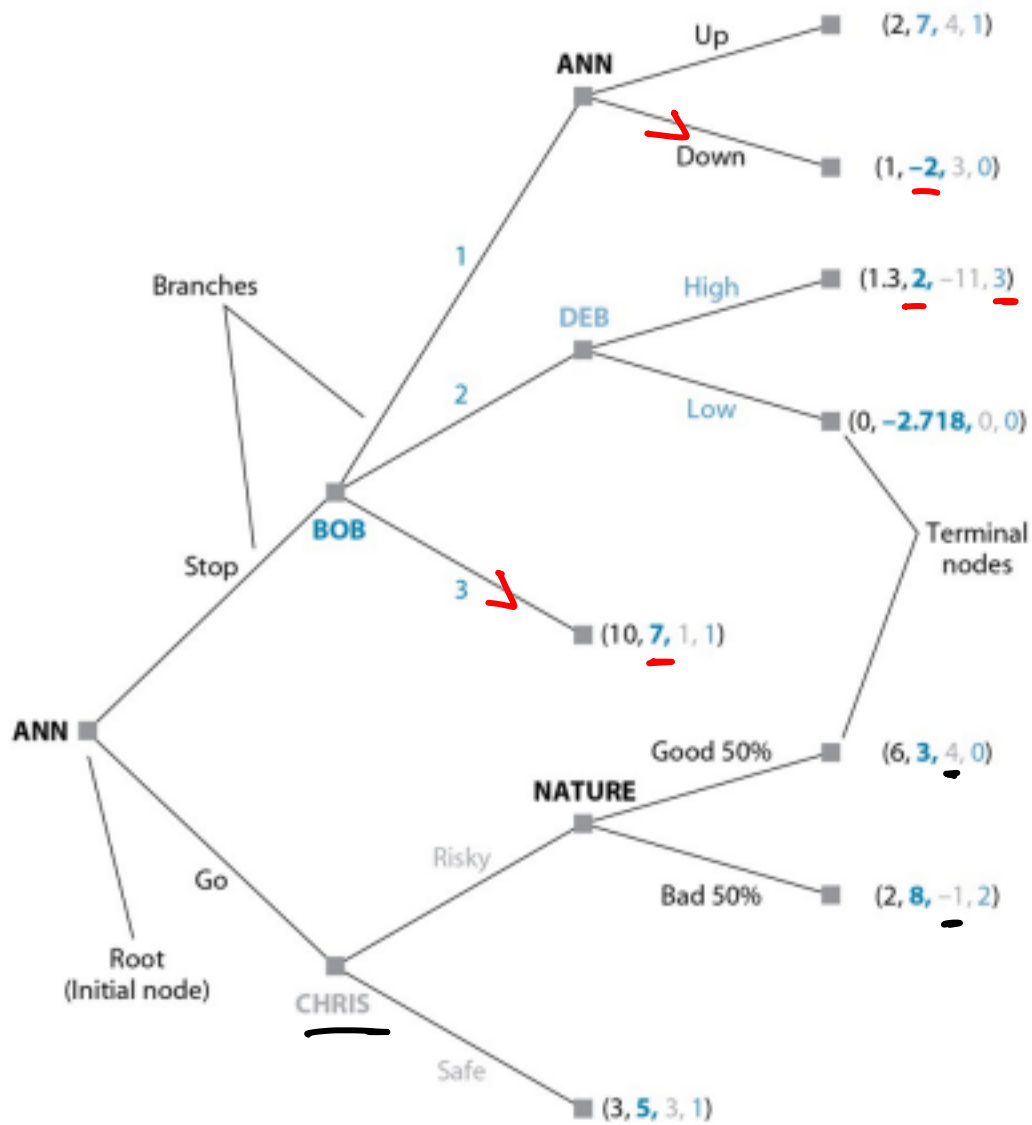
- ▶ We will visualize games using **game trees**
- ▶ Representing a game as a tree is known as the "extensive form" of a game
- ▶ The tree shows all components of a game: players, actions and strategies, payoffs

Nodes and branches

- ▶ *Nodes* are points on the tree where choices are made
 - ▶ The first node is called the *root node*
 - ▶ The last nodes (without branches) are *terminal nodes*
- ▶ *Branches* show the actions available for the player to choose among at any node
- ▶ A node (and its branches) represent a "turn" for a player
- ▶ Payoffs are listed at the terminal nodes
 - ▶ Each player in the game gets a payoff at each node
 - ▶ Remember: Higher numbers are always better

External uncertainty

- ▶ With external uncertainty, we introduce nature as a "player"
- ▶ Nature gets its own node, branches are possible outcomes
- ▶ Players calculate expected payoffs across the possible outcomes of nature's "choice"



$$\frac{1}{2} \cdot 4 + \frac{1}{2}(-1)$$

$$2 - \frac{1}{2}$$

$$1 \frac{1}{2}$$

Moves vs strategy

- ▶ A choice of action at a node is called a *move*
- ▶ A strategy is a *complete plan of action*
 - ▶ A set of moves that will be performed if a certain situation arises
 - ▶ Strategies are collections of statements like "if X then Y " for *any possible X*

- ① choose stop, then up (stop, up)
- ② choose stop, then down (stop, down)
- ③ (go, up)
- ④ (go, down)

Example

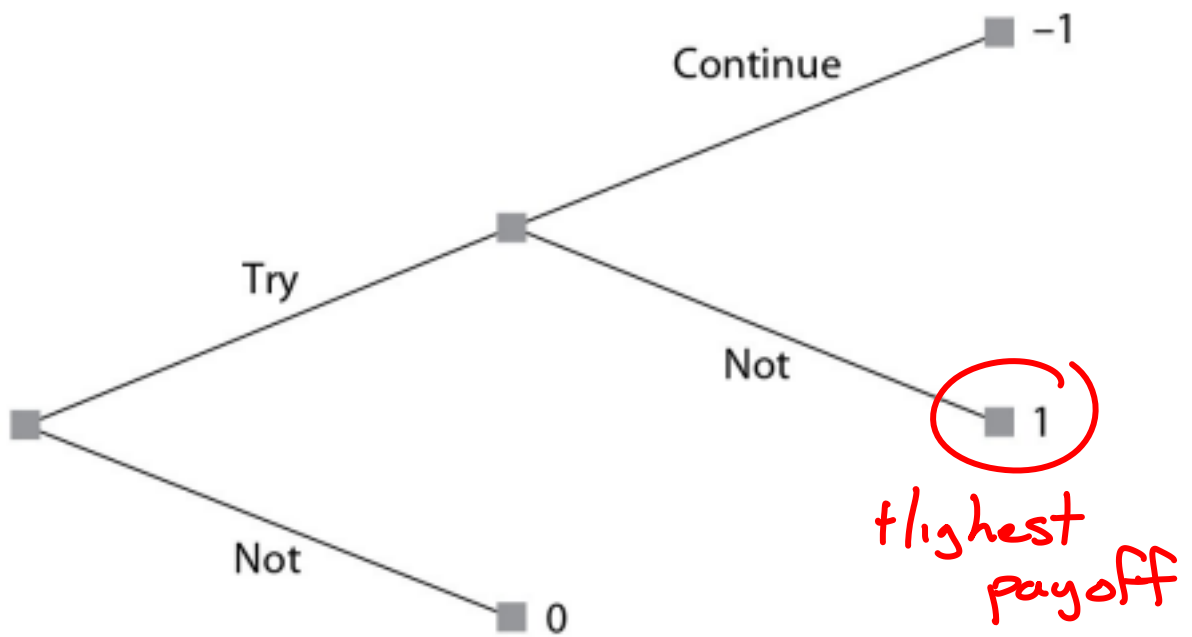
- How many strategies does Ann have? 4
- What are they?

Strategies

- ▶ Strategies must include actions at *each node where a player can move*
- ▶ This includes the nodes that won't be reached if a player chooses a particular set of actions
- ▶ This is because hypothetical moves might help determine which moves should be chosen at earlier nodes
- ▶ Choices early in a game are affected by *expectations* about what will happen later in the game

Finding equilibria in game trees

- ▶ Consider one person's decision tree (is this a game?)
- ▶ The player (Carmen) is considering whether or not to start smoking
- ▶ Carmen first decides whether to start, then decides whether to continue
- ▶ What should Carmen do?



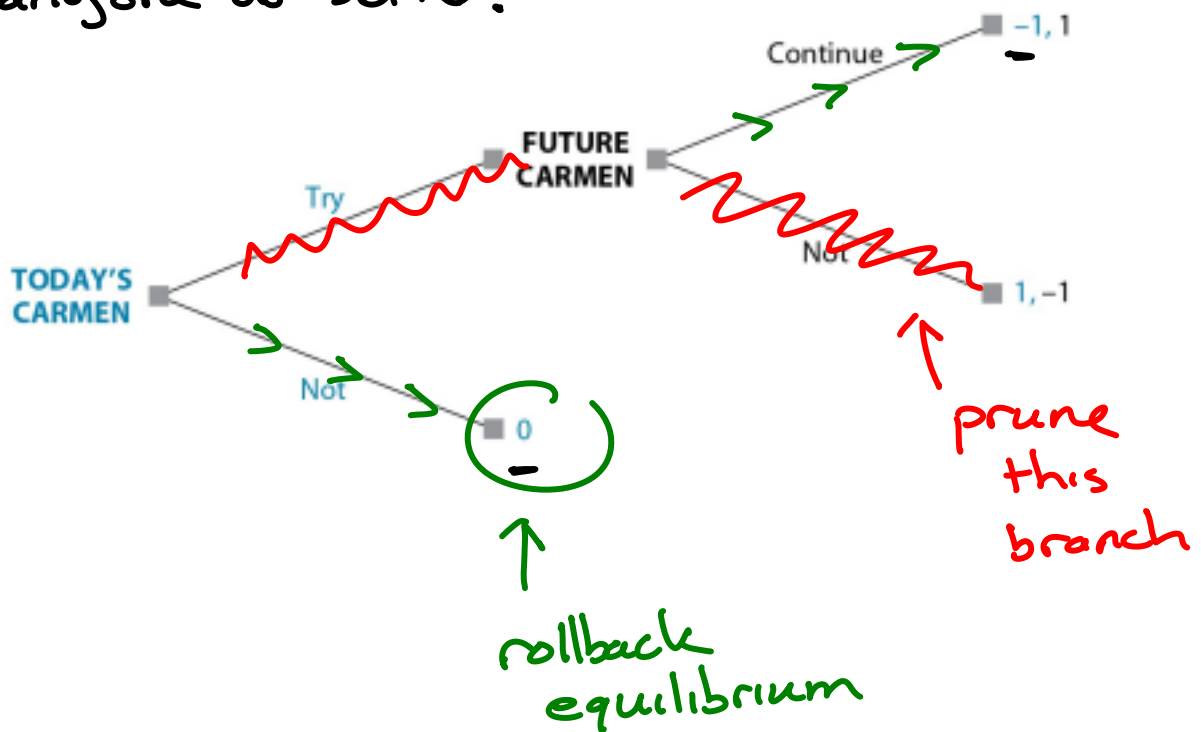
A decision tree as a game

- ▶ Previous decision tree ignore that Carmen may become addicted if she starts smoking
 - ▶ Once addicted, quitting becomes worse (payoffs are lower)
- ▶ Carmen knows she may become addicted and that her payoffs might change if she starts smoking
- ▶ We can think of this as a game where the players are Carmen today and Carmen in the future (after the initial decision is made)
- ▶ Today's Carmen and future Carmen have different payoffs

Future carmen's strategy: C

Today's carmen's strategy: N

Can anyone do better?

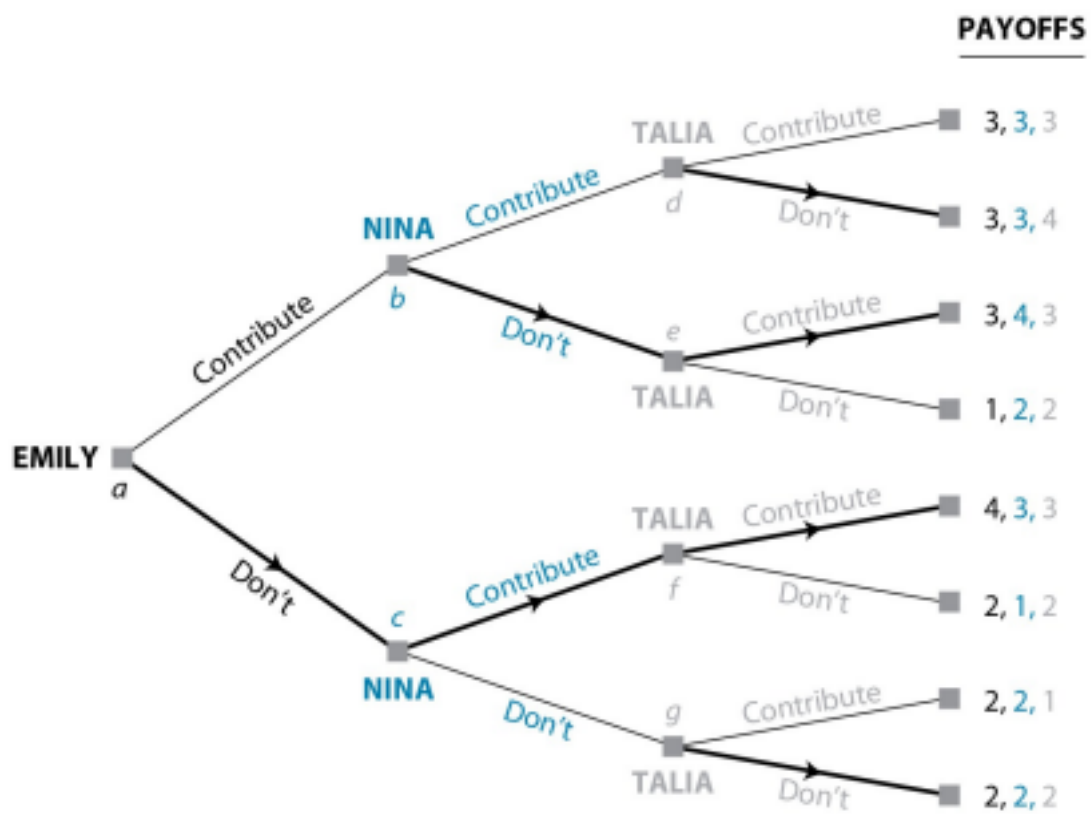


Pruning

- ▶ Starting at the end, we can "prune" the branches that we know will not be chosen
- ▶ When there is one action remaining at the final nodes, this means that the "final" decision moves back to the previous node (rollback)
- ▶ Starting at the end and moving backward by pruning allows today's Carmen to choose the best option for herself
- ▶ When all players use rollback analysis, the result of the game is called a *rollback equilibrium*

Smoking game

- ▶ What are the rollback equilibrium strategies?
- ▶ Can either player do better by changing their strategies?



Three-player game

- ▶ How many strategies does each player have?
- ▶ What is the rollback equilibrium?
- ▶ What are the rollback equilibrium strategies?

Example: Ultimatum game

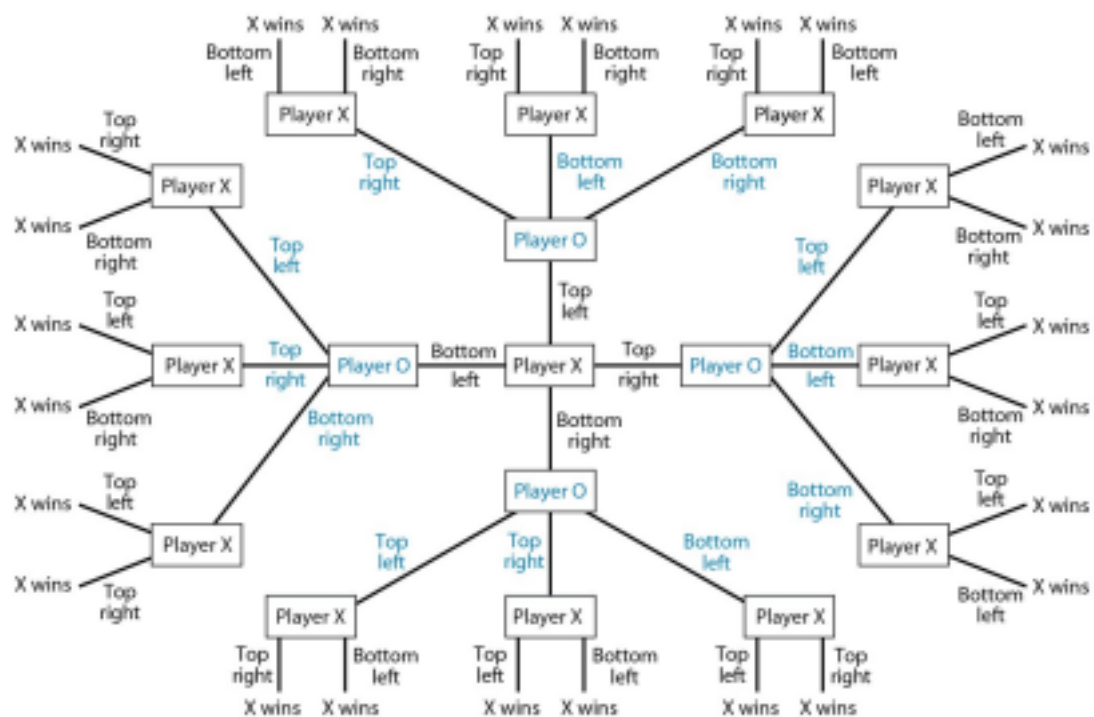
- ▶ Player 1:
 - ▶ Choose how to split 10 units so that both players get at least one unit
- ▶ Player 2:
 - ▶ Choose to either:
 1. Accept the split (you get what player 1 chooses for you)
 2. Reject the split (neither player gets anything)

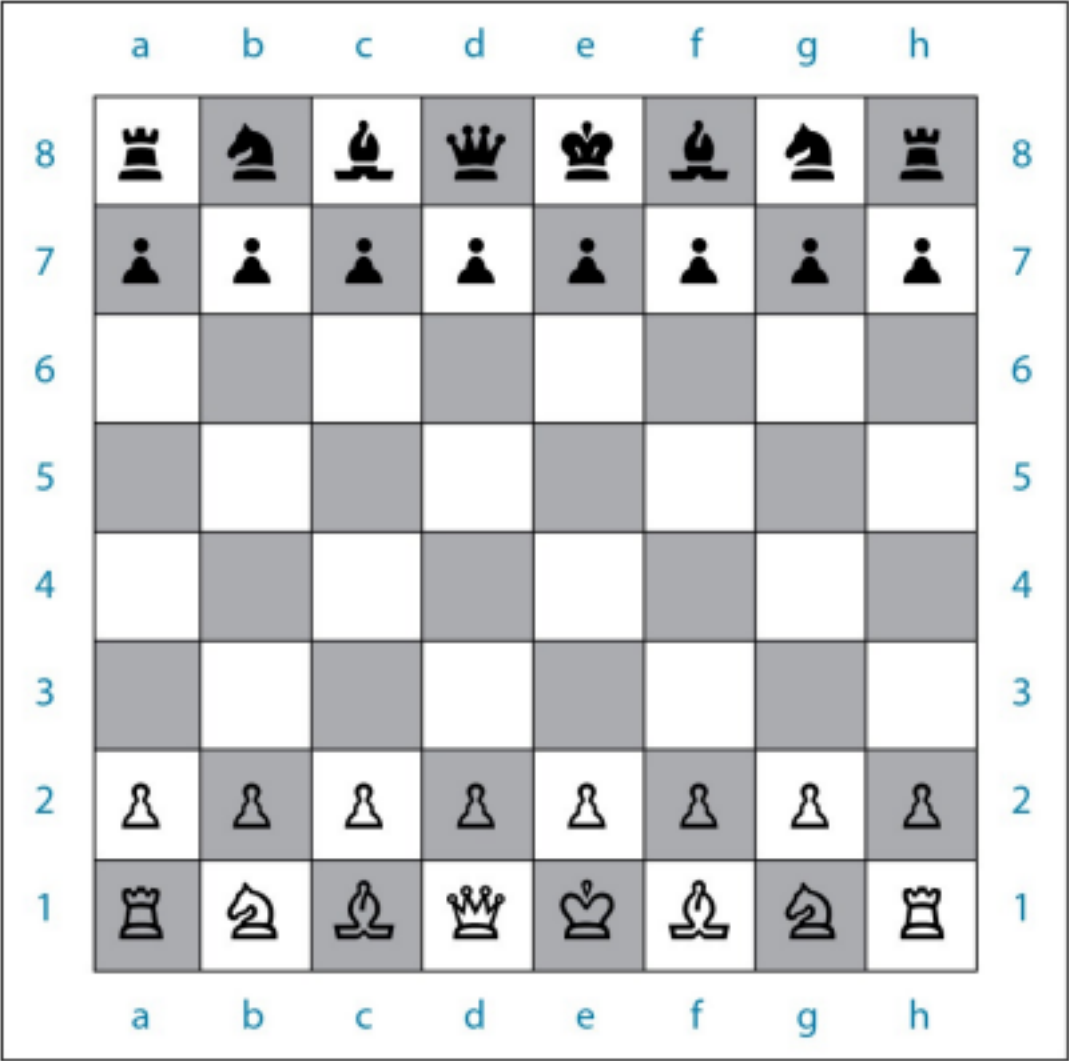
Example: Centipede game

- ▶ What does the game tree look like?
- ▶ What are the strategies for each player?
- ▶ What is the rollback equilibrium outcome?
- ▶ What are the rollback equilibrium strategies?
- ▶ Is this the outcome we observe in practice?

Limitations of rollback analysis

- ▶ Simple games can become difficult to express in extensive form
 - ▶ How many moves does the first player have in tic-tac-toe?
 - ▶ How many moves does the second player have?
- ▶ Some sequential games are *impossible* to express in extensive form!





Chess

- ▶ 400 possible positions (nodes) after each player moves once
- ▶ 9 million after the third move
- ▶ 288 billion after the forth move
- ▶ 40 move game: More possible positions than fundamental particles in the universe