

## Backpropagate ( $y, A, \eta$ )

- Given weight matrices  $W^2, \dots, W^L$  and bias vectors  $\vec{b}^2, \dots, \vec{b}^L$  at each layer,

• Declare vars:

- $\vec{d}_{\vec{a}} C$ , the derivative of cost w.r.t current layer's activations
- $\vec{d}_{\vec{z}} C$ , same w.r.t current layer's  $\vec{z}$  values
- $\Delta \vec{b}$ , change in  $\vec{b}$  at current layer
- $\Delta W$ , same for  $W$
- $\vec{a}$ , current layer's activations
- $\vec{a}'$ , previous layer's activations
- $\vec{y}$ , vector corresponding to classification  $y$

Note:  $\sigma'(x) = \sigma(x)(1 - \sigma(x))$

We use this substitution in the algorithms.

$$\cdot \vec{y} = \vec{e}_y$$

•  $\vec{a}$  = last element of  $A$

$$\cdot \vec{d}_{\vec{a}} C = \vec{a} - \vec{y}$$

$$\cdot \vec{d}_{\vec{z}} C = \vec{d}_{\vec{a}} C \odot [\vec{a} \odot (\vec{I} - \vec{a})]$$

• For each layer  $l$  from  $L$  down to 2:

$$\left[ \begin{array}{l} \cdot \vec{a} = l^{\text{th}} \text{ elem. of } A \end{array} \right.$$

$$\left[ \begin{array}{l} \cdot \vec{a}' = (l-1)^{\text{th}} \text{ elem. of } A \end{array} \right.$$

$$\left[ \begin{array}{l} \cdot \Delta \vec{b} = -\eta \vec{d}_{\vec{z}} C \end{array} \right.$$

$$\left[ \begin{array}{l} \cdot \Delta W = -\eta \vec{d}_{\vec{z}} C \otimes \vec{a}' \end{array} \right.$$

$$\left[ \begin{array}{l} \cdot \vec{d}_{\vec{a}} C = (W^l)^t \vec{d}_{\vec{z}} C \end{array} \right.$$

$$\left[ \begin{array}{l} \cdot \vec{d}_{\vec{z}} C = \vec{d}_{\vec{a}} C \odot [\vec{a}' \odot (\vec{I} - \vec{a}')] \end{array} \right.$$

$$\left[ \begin{array}{l} \cdot \vec{b}^l += \Delta \vec{b} \\ \cdot W^l += \Delta W \end{array} \right.$$

## Train ( $\vec{x}_1, \dots, \vec{x}_n, y_1, \dots, y_n, \eta, T$ )

• Declare vars:

•  $A$ , the list of activations returned by Feedforward

• Repeat  $T$  times:

• For each  $\vec{x}, y$ :

•  $A = \text{Feedforward}(\vec{x})$

•  $\text{Backpropagate}(y, A, \eta)$