Introduction

Images can be degraded by a number of different mechanisms, including noise, blurring and distortion, which are errors in the pixel values. Noise is present in any imaging device by optical imperfections and instrumentation noise (for example, thermal noise in CCD devices) results in more noise. Sampling causes noise due to aliasing of high-frequency signal components, and digitization produces quantization errors. Further noise can be introduced by communication errors and compression.

Blurring is present in any imaging system which uses electromagnetic radiation (for example, visible light and x-rays). Diffraction limits the resolution of an imaging device to features on the order of the illuminating wavelength. Scattering of light between the target object and imaging system (for example, by the atmosphere) introduces additional blurring. Lenses and mirrors cause blurring because they have limited spatial extent and optical imperfections. Optical effects such as out of focus blurring, or blurring due to camera motion.

The ultimate goal of image restoration is to reconstruct the original image from its degraded version. It is essentially an inverse problem, where we apply the inverse of the transformation that caused the degradation. The better we can model the degradation, the better we are able to find its inverse. In many cases, however, we will only have limited statistical knowledge of the degradation, and the inverse transform will be correspondingly ill-conditioned.

Some restoration techniques can be performed very successfully using neighbourhood operations, while others require the use of frequency domain processes. Image restoration remains one of the most important areas of image processing.

1. A model of the image degradation/restoration process

As figure.1 shows, the degradation process is modeled as a degradation function that, together with an additive noise term, operates on an input image $f(x, y)$ to produce a degraded image $g(x, y)$. Given $g(x, y)$, some knowledge about the degradation function H, and some knowledge about the additive noise term $\eta(x, y)$, the objective of restoration is to obtain an estimate $\hat{f}(x, y)$ of the original image. We want the estimate to be as close as possible to the original input image and, in general, the more we know about H and η, the closer $\hat{f}(x, y)$ will be to $f(x, y)$.

If H is a linear, spatially invariant process, the degraded image is given in the spatial domain by

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

where $h(x, y)$ is the spatial representation of the degradation function and, the symbol "*" indicates convolution. The convolution in the spatial domain is equal to multiplication in the frequency domain, so we may write the preceding model in an equivalent frequency domain representation:

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

where the terms in capital letters are the Fourier transforms of the corresponding terms in the convolution equation.

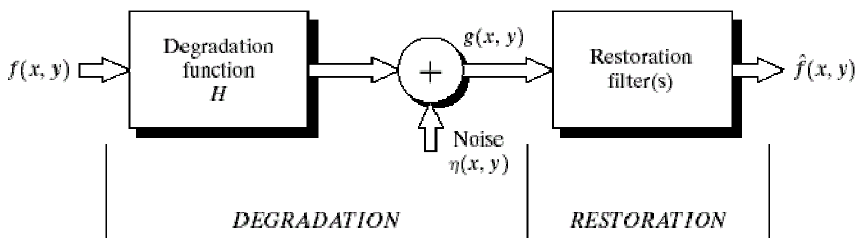

Figure.1 A model of the Image degradation / restoration process

Figure 2: Original image (left), degraded image (right).

2. Noise models

Noise is unwanted fluctuation in the pixel values of an image. The principal sources of noise in digital images arise during image acquisition (digitization) and/or transmission. The performance of imaging sensors is affected by a variety of factors, such as environmental conditions during image acquisition, and by the quality of the sensing elements themselves. For instance, in acquiring images with a CCD camera, light levels and sensor temperature arc major factors affecting the amount of noise in the resulting image. Images are corrupted during transmission principally due to interference in the channel used for transmission. For example, an image transmitted using a wireless network might be corrupted as a result of lightning or other atmospheric disturbance.

2.1 Properties of noise

We assume that noise is independent of spatial coordinates, and that it is uncorrelated with respect to the image itself (that is, there is no correlation between pixel values and the values of noise components). Although these assumptions are at least partially invalid in some applications (quantum-limited imaging, such as in X-ray and nuclear medicine imaging, is a good example), the complexities of dealing with spatially dependent and correlated noise are beyond the scope of our discussion.

3.2 Inverse Filtering

The simplest approach to restoration is direct inverse filtering. An inverse filter is a linear filter, where we compute an estimate, $\hat{F}(u, v)$, of the transform of the original image simply by dividing the transform of the degraded image, $G(u, v)$, by the degradation function.

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

And then obtain the corresponding estimate of the image by taking the inverse Fourier transform of $\hat{F}(u,v)$ [recall that $G(u,v)$ is the Fourier transform of the degraded image]. This approach is appropriately called inverse filtering. We can express our estimate as

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

This is an interesting expression. It tells us that even if we know the degradation function we cannot recover the undegraded image [the inverse Fourier transform of $F(u, v)$] exactly because $N(u, v)$ is a random function whose Fourier transform is not known. There is more bad news. If the degradation has zero or very small values, then the ratio $N(u, v)/H(u, v)$ could easily dominate the estimate $F(u,v)$. This, in fact, is frequently the case, as will be demonstrated shortly.

One approach to get around the zero or small-value problem is to limit the filter frequencies to values near the origin. Where $H(0, 0)$ is equal to the average value of $h(x, y)$ and that this is usually the highest value of $H(u, v)$ in the frequency domain. Thus, by limiting the analysis to frequencies near the origin, we reduce the probability of encountering zero values.

Figure 4: degraded image (left), restored image (right).

3.3 Minimum Mean Square Error (Wiener) Filtering

The Wiener filter is a linear spatially invariant filter, in which is chosen such that it minimizes the mean-squared error (MSE) between the ideal and the restored image. This criterion attempts to make the difference between the ideal image and the restored one, the remaining restoration error - as small as possible. This error measure is given by.

$$e^2 = E[(f - \hat{f})^2]$$

Where $E\{\bullet\}$ is the expected value of the argument. It is assumed that the noise and the image are uncorrelated; that one or the other has zero mean, and that the gray levels in the estimate are a linear function of the levels in the degraded image. Based on these conditions, the minimum of the error function is given in the frequency domain by the expression.

$$
\begin{aligned}
\hat{F}(u,v) &= \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)\left|H(u,v)\right|^2 + S_\eta(u,v)} \right] G(u,v) \\
&= \left[\frac{H^*(u,v)}{\left|H(u,v)\right|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v) \\
&= \left[\frac{1}{H(u,v)} \, \frac{\left|H(u,v)\right|^2}{\left|H(u,v)\right|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)
\end{aligned}
$$

$H(u, v)$ = degradation function

$H^*(u, v)$ = complex conjugate of $H(u, v)$

$| H(u, v) |^2 = H^*(u, v) H(u, v)$

$S_\eta(u, v) = | N(u, v) |^2$ = power spectrum of the noise

$S_f(u, v) = | F(u, v) |^2$ = power spectrum of the undegraded image

As before, $H(u, v)$ is the transform of the degradation function and $G(u, v)$ is the transform of the degraded image. The restored image in the spatial domain is given by the inverse Fourier transform of the frequency-domain estimate $F(u, v)$.

Figure 5: degraded image (left), restored image (right).

Conclusion

Image restoration remains one of the most important areas of image processing. We have some conclusions and remarks about the filters which we used in our work:

A median filter is an example of a non-linear spatial filter, it give good result for removal of salt & pepper noise, bat it make the image blur.

In the inverse filter If the degradation has zero or very small values, then the ratio $N(u, v)/H(u, v)$ could easily dominate the estimate $F(u,v)$.

The Wiener filter give us the good result, if the noise is zero, then the noise power spectrum vanishes and the Wiener filter reduces to the inverse filter.

References

Rafael Gonzalez and Richard E. Woods. Digital Image Processing. Addison-Wesley, second edition, 2002.

Geoff Dougherty, Digital Image Processing for Medical Applications, Addison Cambridge, 2009.

Alasdair McAndrew, An Introduction to Digital Image Processing with Matlab, Victoria University of Technology, 2004.

Al Bovik, Handbook of image and video processing, Academic Press,2000.

MATLAB program

```
close all
clear all
clc
I = imread('spine.tif');
G = rgb2gray(I);
D = double(G)/255.0;
figure(1), imshow(D), title('original image')
%--------------------------------- degredation
[m n] = size(D);
h = fspecial('motion',28,15);
H = fft2(h,m,n);
Ih = ifft2(H.*fft2(D));
G = imnoise(Ih,'salt & pepper', 0.01);

figure(2), imshow(G), title('degraded image')
%--------------------------------- median filter
IFM = medfilt2(G);
figure(3), imshow(IFM), title('undegraded image by median filter')
%--------------------------------- inverse filter
IFI = ifft2((abs(H)> 0.45).*fft2(G)./H);
figure(4), imshow(IFI), title('undegraded image by inverse filter')
%--------------------------------- winer filter
FFTG = fft2(G);
k = 0.05;
IFW = FFTG.*(abs(H).^2./(abs(H).^2+k)./H);
IFWINER = ifft2(IFW);
figure(5), imshow(IFWINER), title('undegraded image by wiener filter')
```

K = wiener2(mn,[5 5]);

K = filter2(fspecial('average',3),mn)/255;

L = medfilt2(J,[3 3]);