Chapter III
Filtering and Preprocessing

Dr. Iman Elawady

INTRODUCTION

Generally, the noise in image considered as an additive random field which can be caused from the acquisition device (magnetic influences, etc.) or from the scene itself (stray light …)

filtration or local smoothing consists of reducing or eliminating the noise generated from the acquisition device by checking each pixel intensity values and its neighborhood. This process aims to improve the image as much as possible so that the following processing is optimal in terms of computation time and quality.

Models of noise

additive noise:
$g(x, y) = f(x, y) + b(x, y)$

multiplicative noise:
$g(x, y) = f(x, y) \times b(x, y)$

Convolutional noise :
$g(x, y) = f(x, y) * b(x, y)$

We will see how to remove the additive noise, which represents the most common noise.

Characterizationof the noise

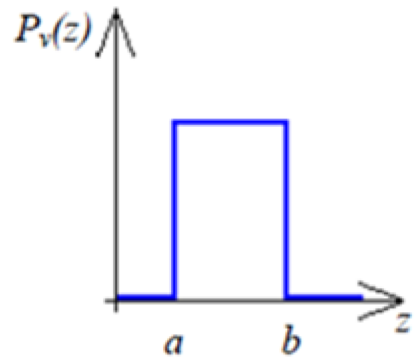
We will characterize a noise using statistics
(Noise Probability Density Function (PDF))

Original Image

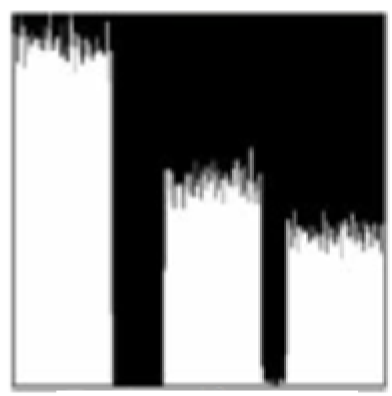
Uniform noise

It varies between 2 values a and b

$$p_v(z)=\begin{cases}1/(b-a) & \text{If } \quad a\le z\le b \\ 0 & \text{else}\end{cases} \qquad \text{III.1}$$



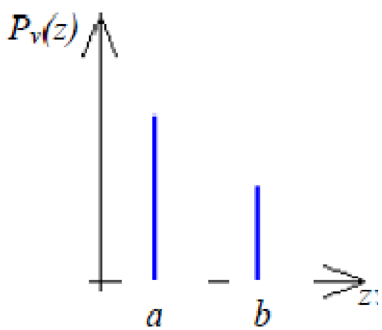
noisy image

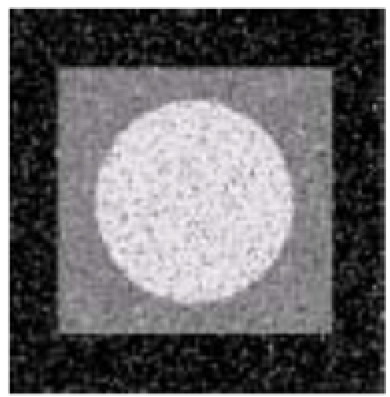


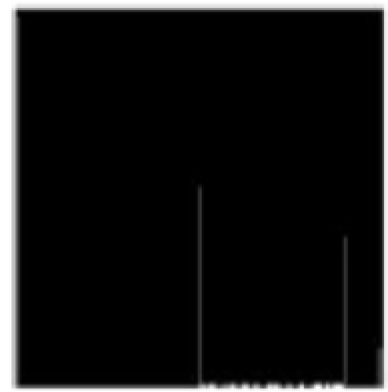
Uniform noise

Impulse noise (Salt and pepper)

$$P_v(z) = \begin{cases} P_a & \text{If } z = a \\ P_b & \text{If } z = b \\ 0 & \text{else} \end{cases}$$



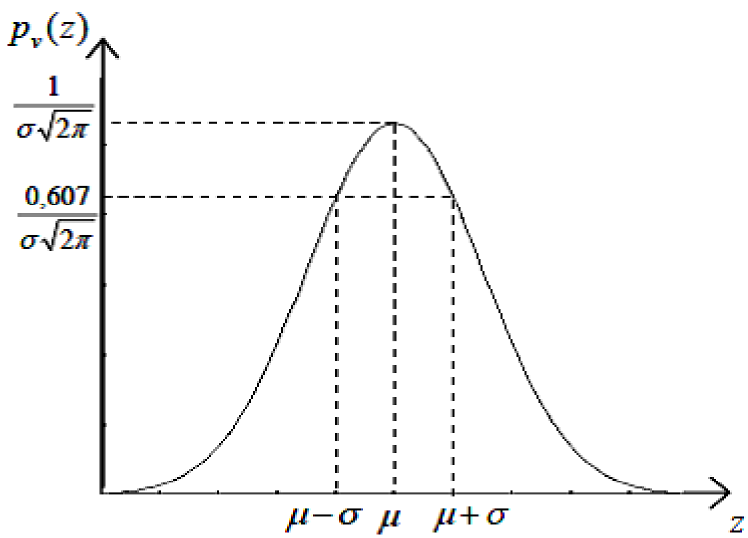

noisy image

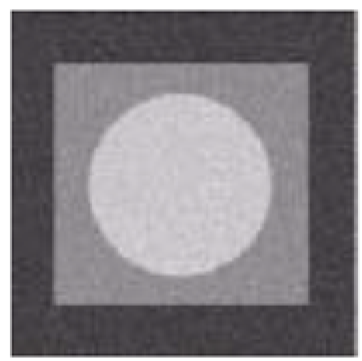

Salt and pepper

Gaussian noise

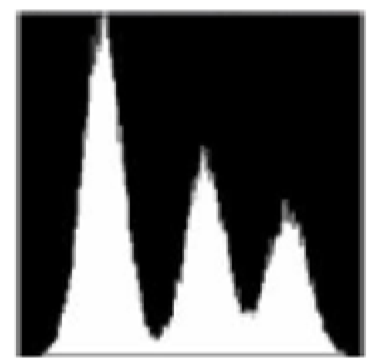
The probability density of Gaussian noise is a function of the mean μ and the deviationtypeσ.

$$P_v(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{(z-\mu)^2}{2\sigma^2}}$$





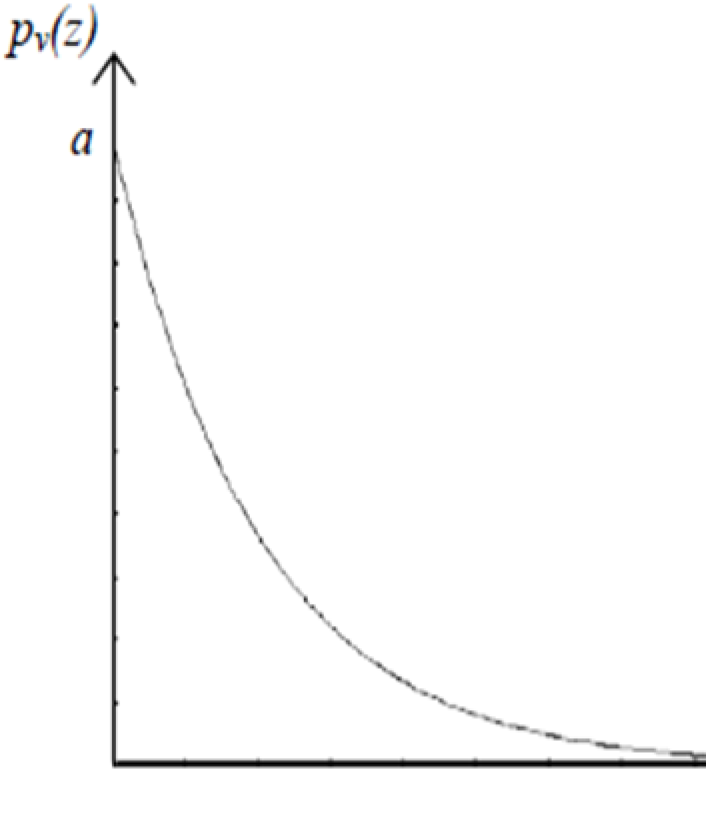
noisy image

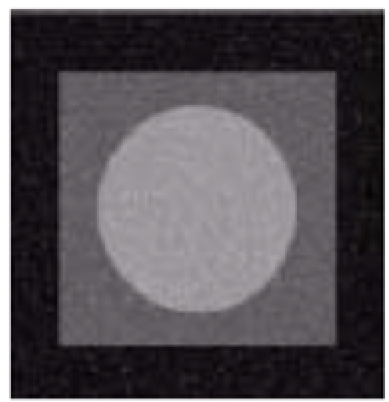


Gaussian noise

Exponential noise

$$p_v(z)=\begin{cases} ae^{-|az|} & \text{if} \quad z \geq 0 \\ 0 & \text{if not} \end{cases} \qquad \text{III.4}$$

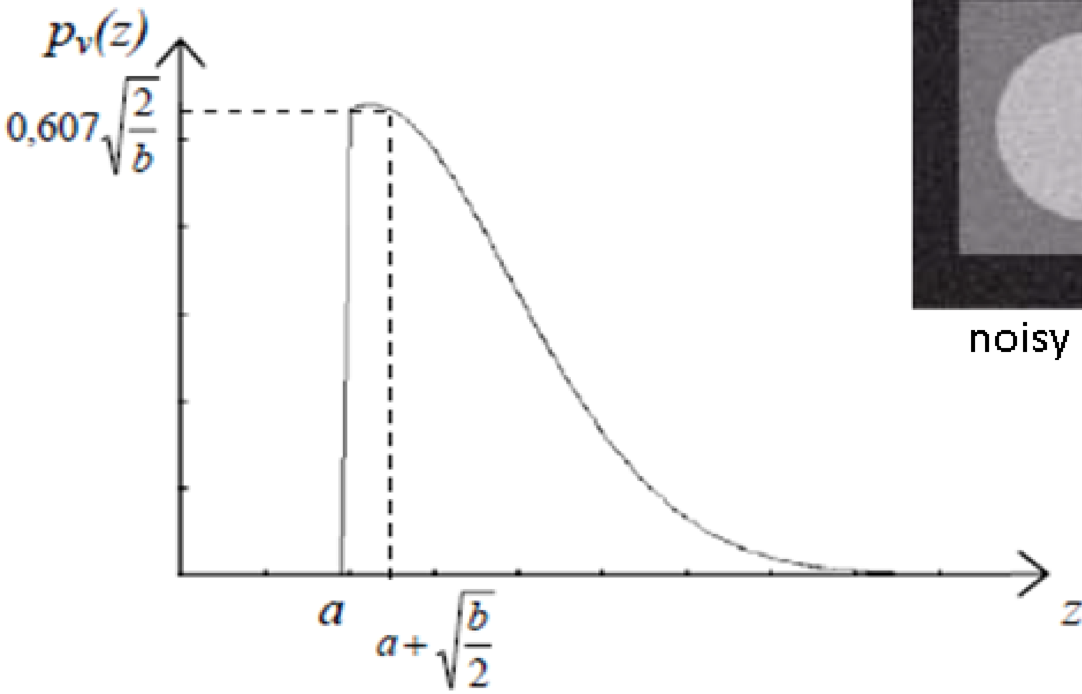


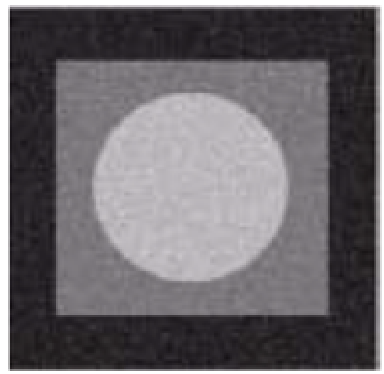

noisy image


Exponential noise

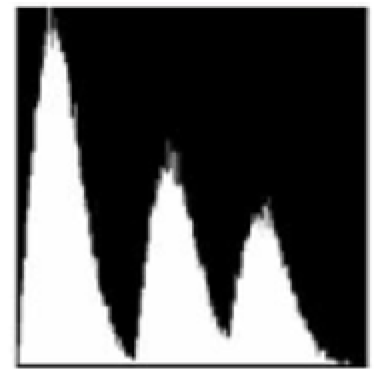
Rayleigh noise

$$p_v(z)=\begin{cases}\frac{2}{b}(z-a)e^{-\frac{(z-a)^2}{b}} & \text{If} \quad z \geq a \\ 0 & \text{If not}\end{cases} \qquad \text{III.5}$$





noisy image



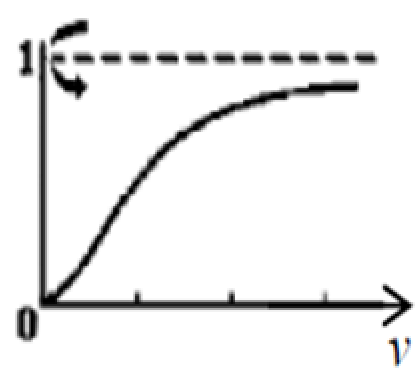
Rayleigh noise

Spatial filtering

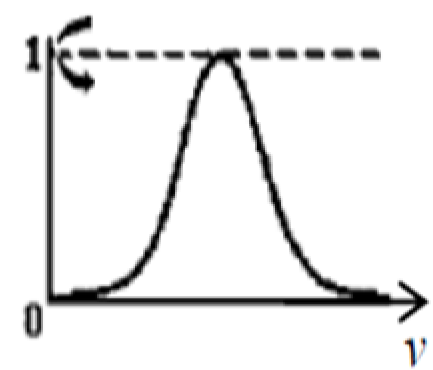
Spatial filtering is essentially a convolution (2D) operation. f is the image to filter and g the spatial filter (mask)

low pass filter

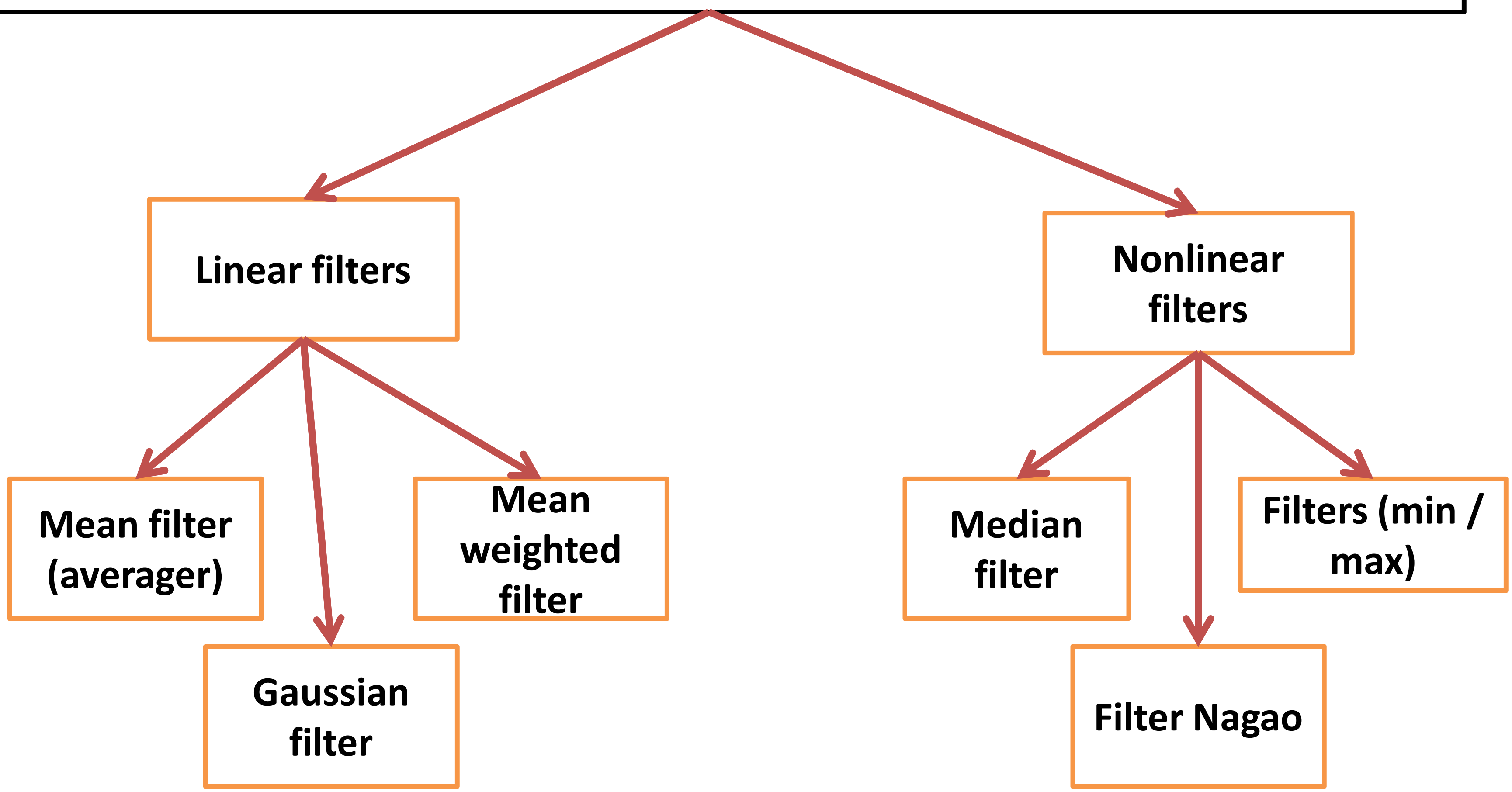
High pass filter



baseband filter



SPATIAL FILTERING



Linear filters

A first class of approach is based on information redundancy. The new value of the pixel is calculated by averaging the values of it's neighborhood.

the filtered image g is obtained by the convolution equation according to the following expression:

$$g(x,y) = (f * M)(x,y) = \sum_{i=x_1}^{x_2} \sum_{j=y_1}^{y_2} f(x-i, y-j) M(i,j) \qquad \text{III.6}$$

The convolution kernel (or mask) of the M filter has a compact support included in [x1, x2] x [y1, y2]:

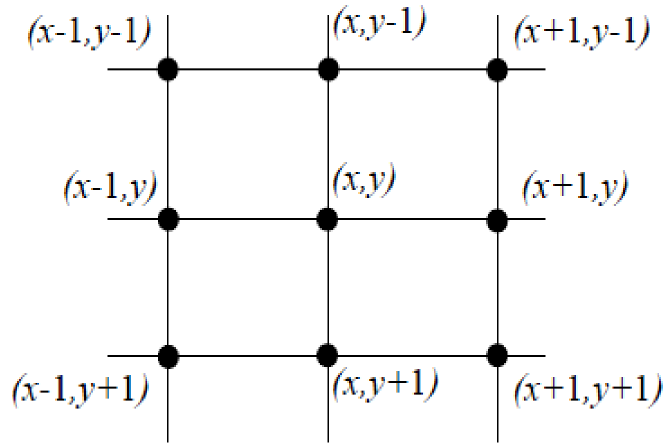
Generally the filter is of dimensions *di* odd and it is symmetrical. In this case

$$[x_1, x_2] = \left[-\frac{d_1 - 1}{2}, \frac{d_1 - 1}{2}\right] \quad et \quad [y_1, y_2] = \left[-\frac{d_2 - 1}{2}, \frac{d_2 - 1}{2}\right]$$

$$(f * M)(x, y) = \sum_{i=-(d_1-1)/2}^{(d_1-1)/2} \sum_{j=-(d_2-1)/2}^{(d_2-1)/2} f(x+i, y+j) M(i, j) \qquad \text{III.7}$$

m_1	m_2	m_3	$\longleftarrow$ y-1
m_4	m_5	m_6	$\longleftarrow$ y
m_7	m_8	m_9	$\longleftarrow$ y+1
$\uparrow$ x-1	$\uparrow$ x	$\uparrow$ x+1	

here $d1 = d2 = d = 3$.



$$
\begin{aligned}
g(x,y) = {} & m_1 f(x-1,y-1) + m_2 f(x,y-1) + m_3 f(x+1,y-1) \\
& + m_4 f(x-1,y) + m_5 f(x,y) + m_6 f(x+1,y) \\
& + m_7 f(x-1,y+1) + m_8 f(x,y+1) + m_9 f(x+1,y+1)
\end{aligned}
\qquad \text{III.8}
$$

In order to keep the average of l'image f, the sum of the elements of the filter is normalizedée To 1 :

$$\sum_i m_i = 1$$

Separable filters

A 2D filter is separable if it is possible to decompose the convolutional kernel h_{2D} in two 1D filters applied successively horizontally then vertically (or vice versa)

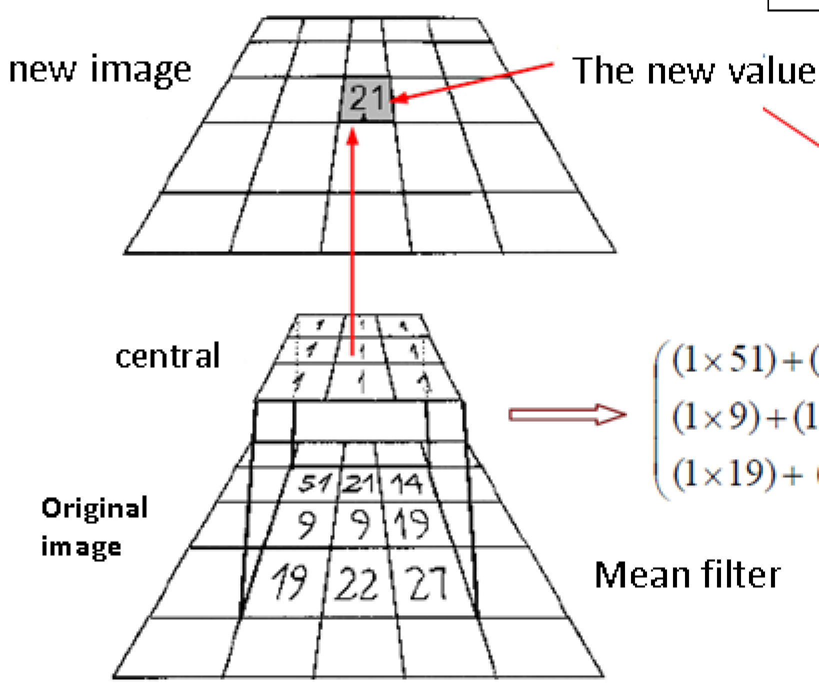
$$
\begin{array}{|c|c|c|} \hline a & b & c \\ \hline \end{array}
\otimes
\begin{array}{|c|} \hline \alpha \\ \hline \beta \\ \hline \gamma \\ \hline \end{array}
=
\begin{array}{|c|c|c|} \hline a\alpha & b\alpha & c\alpha \\ \hline a\beta & b\beta & c\beta \\ \hline a\gamma & b\gamma & c\gamma \\ \hline \end{array}
=
\begin{array}{|c|} \hline \alpha \\ \hline \beta \\ \hline \gamma \\ \hline \end{array}
\times
\begin{array}{|c|c|c|} \hline a & b & c \\ \hline \end{array}
$$

Mean filter

This filter calculates the average of gray levels located in a square window. Then, it replaces the value of the central pixel by this average.

$$\frac{1}{9}\times\begin{array}{|c|c|c|}\hline 1&1&1\\\hline 1&1&1\\\hline 1&1&1\\\hline\end{array} \qquad \frac{1}{25}\times\begin{array}{|c|c|c|c|c|}\hline 1&1&1&1&1\\\hline 1&1&1&1&1\\\hline 1&1&1&1&1\\\hline 1&1&1&1&1\\\hline 1&1&1&1&1\\\hline\end{array}$$

Example



$$\begin{pmatrix}(1\times 51)+(1\times 21)+(1\times 14)+\\ (1\times 9)+(1\times 9)+(1\times 19)+\\ (1\times 19)+(1\times 22)+(1\times 27)\end{pmatrix}/9=21$$

$$M = \frac{1}{(2a+1)^2}\,(2a+1, 2a+1) \qquad \text{III.10}$$

Filtered averager weighted

It is based on the same concept of the mean filter but in this case it is uses 5 or 9 smoothing point etc...

•The 5 point smoothing mask:

$$M = \frac{1}{6} \times \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

•The 9 point smoothing mask:

$$M = \frac{1}{10} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$