$$M = \frac{1}{(2a+1)^2}\,(2a+1, 2a+1) \qquad \text{III.10}$$

Filtered averager weighted

It is based on the same concept of the mean filter but in this case it is uses 5 or 9 smoothing point etc...

•The 5 point smoothing mask:

$$M = \frac{1}{6} \times \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

•The 9 point smoothing mask:

$$M = \frac{1}{10} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

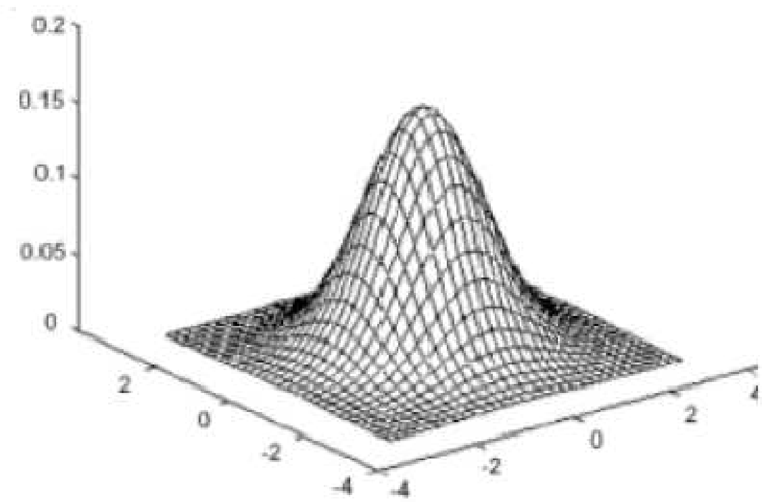
Filtered Gaussian

It is an optimal linear filter. The convolution mask of this filter is obtained by using the 2D Gaussian function of standard deviationσ :

The size of the mask (kernel) is usually odd square (N ×N)

The parameter σ is called the standard deviation, it determines the width of the Gaussian bell.

$$g(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



Example 2:

If for example σ= 0.8, N = 3; we have the following 3x3 filter:

$g(-1,-1)$	$g(0,-1)$	$g(1,-1)$
$g(-1,0)$	$g(0,0)$	$g(1,0)$
$g(-1,1)$	$g(0,1)$	$g(1,1)$

$$\simeq \frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

and σ = 1, N = 5 we have the following 5x5 filter:

$g(-2,-2)$	$g(-1,-2)$	$g(0,-2)$	$g(1,-2)$	$g(2,-2)$
$g(-2,-1)$	$g(-1,-1)$	$g(0,-1)$	$g(1,-1)$	$g(2,-1)$
$g(-2,0)$	$g(-1,0)$	$g(0,0)$	$g(1,0)$	$g(2,0)$
$g(-2,1)$	$g(-1,1)$	$g(0,1)$	$g(1,1)$	$g(2,1)$
$g(-2,2)$	$g(-1,2)$	$g(0,2)$	$g(1,2)$	$g(2,2)$

$$\simeq \frac{1}{300} \times \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 18 & 30 & 18 & 4 \\ 6 & 30 & 48 & 30 & 6 \\ 4 & 18 & 30 & 18 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

Nonlinear filters (Median filter)

sort these elements in ascending or descending order and finally take the median pixel which is in the middle of the data vector.

30	10	20
10	250	25
20	25	30

bruit

10 10 20 20 25 25 30 30 250

La médiane

30	10	20
10	**25**	25
20	25	30

Minimum and maximum filters (min / max)

The minimum and maximum filters order all the pixels in the neighborhood, and select either the smallest or the highest.

For each pixel I (i, j) we have

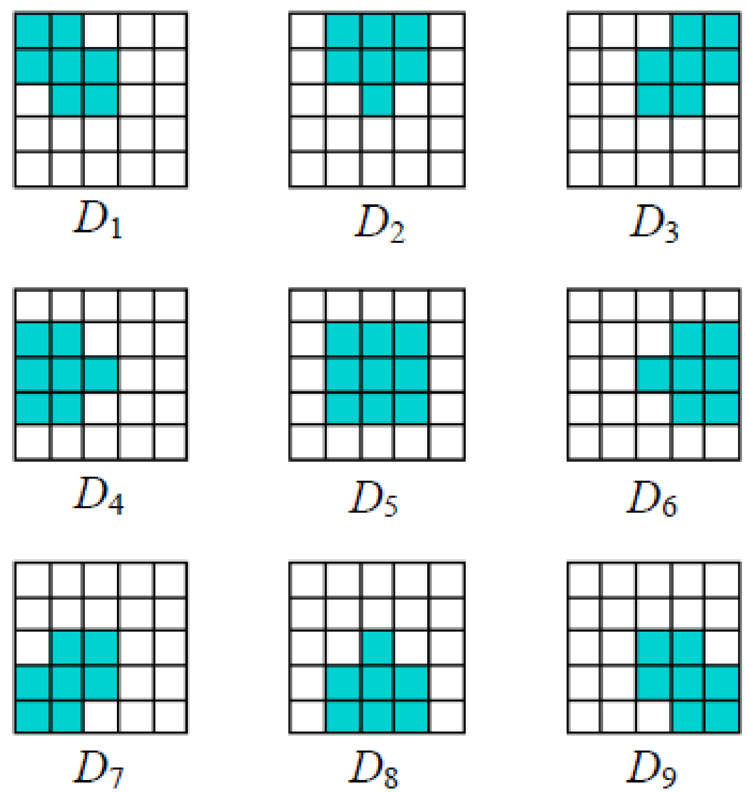
$$I'(x,y) = \begin{cases} I(x,y) & if \quad i_{min} \leq I(x,y) \leq i_{max} \\ i_{min} & if \quad I(x,y) < i_{min} \\ i_{max} & if \quad I(x,y) > i_{max} \end{cases} \qquad (\text{III}.20)$$

Filter Nagao

This filter is also called a neighborhood selection filter. We examine the set of neighbors contained in a 5x5 square centered on (x, y):

For each domain D_i (x, y), we calculate the average avg_i (x, y) and the variance var_i (x, y)

The result of the filter Nagao consists in replacing the value of the central pixel by the average $avg_i(x, y)$ of the domain $D_i(x, y)$ which presents the smallest variance $var_i(x, y)$.



FREQUENTIAL RESPONSE of FILTER

The frequency response of filter is given by:

$$H(\omega_1,\omega_2)=\sum_{n_1}\sum_{n_2} h(n_1,n_2)\,e^{-j(\omega_1 n_1+\omega_2 n_2)} \qquad \text{III.13}$$

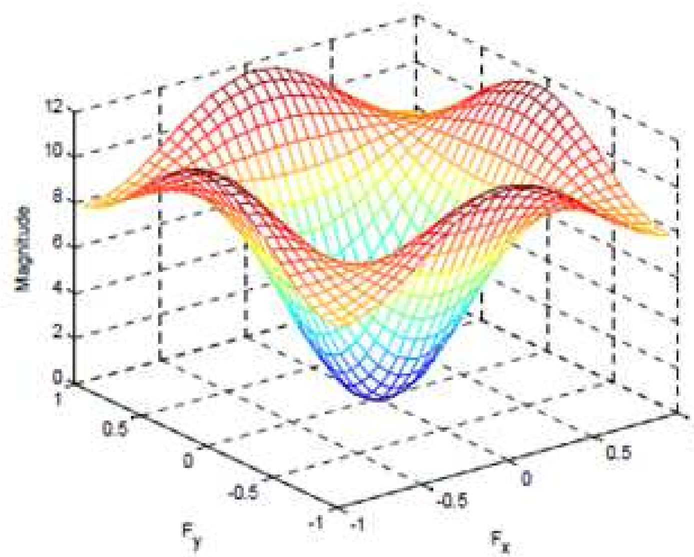
Or ω1 and ω2 vary from –π to + π. The variables ω1 and ω2 respectively represent the row frequency and the column frequency.

Example :

Let us take as an example the filter h1 with:

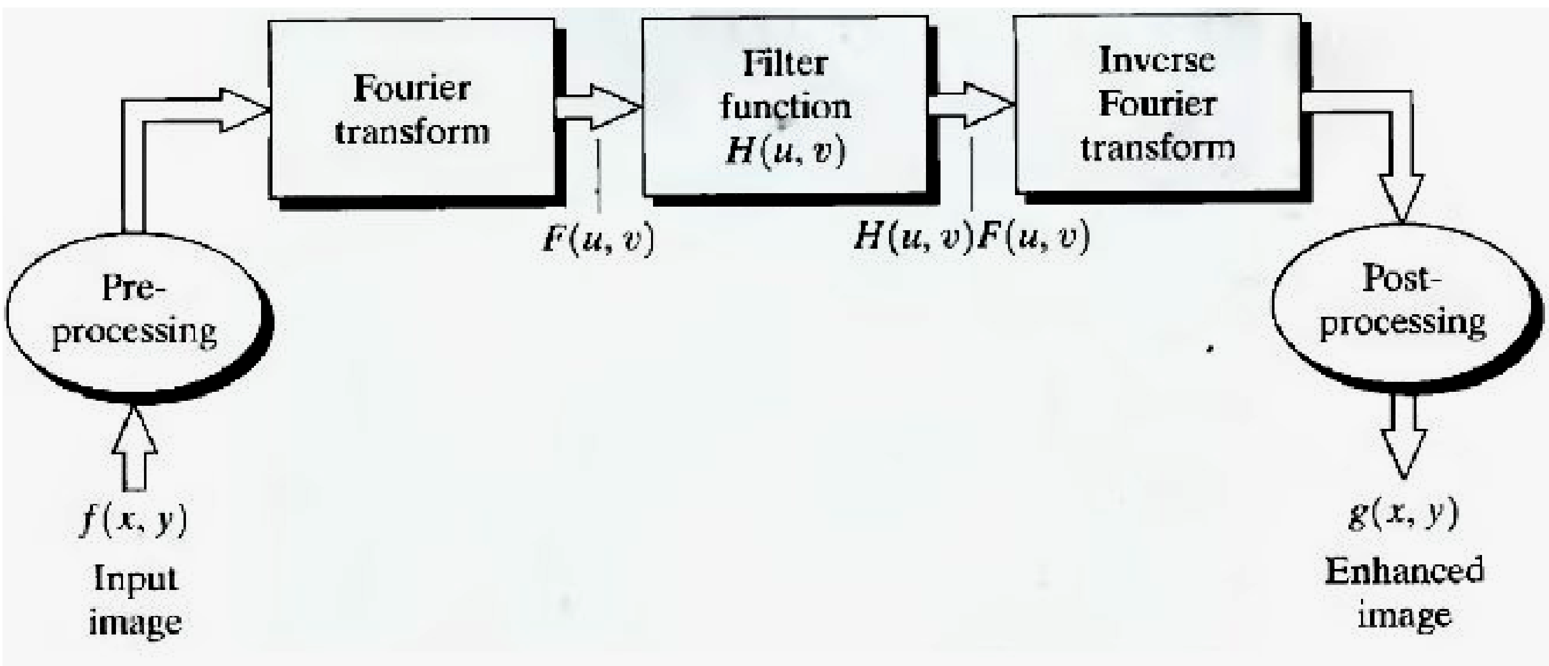
$$h_1=\begin{pmatrix} -1-1-1 \\ -1+8-1 \\ -1-1-1 \end{pmatrix}$$

$$H_1(\omega_1,\omega_2)=9-(2\cos\omega_1+1)(2\cos\omega_2+1)$$



Filtering in frequency domain

The principle of frequency filtering of an image is simple: take the *TF* of the image to be filtered, multiply the spectrum obtained by the transfer function of the filter, then take the *TF* inverse to produce the filtered image.



EVALUATION PARAMETERS

- **Root mean square error (MSE):**

The simplest parameter of the image quality measurement is the MSE. The large value of MSE means that the image is of low quality. The MSE is defined as follows:

$$MSE = \frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{N} \left(I(i,j) - \hat{I}(i,j) \right)^2$$

- **Peak signal-to-noise ratio (PSNR)**

The low value of the PSNR means that the image is low quality. PSNR is defined as follows:

$$PSNR = 10log_{10}\left(\frac{(2^n - 1)^2}{MSE}\right)$$