Stat 150 Homework # 4 Due May 6

Problems:

Q 1 Let X be a geometric random variable with success probability p (i.e. $\mathbb{P}[X = k] = (1-p)^k p$ for $k = 0, 1, \ldots$). Find the probability generating function for X (see Monday's class). Let Y have distribution $Bin(X, q)$. Find the probability generating function for Y and its distribution.

Q 2

Calculate the probability generating function of a Poisson with mean λ. Using generating functions show that if X_1, X_2 are independent Poisson random variables with means λ_1, λ_2 then $X_1 + X_2$ is Poisson with mean $\lambda_1 + \lambda_2$.

Q 3 Let X_n be a Markov chain on $\{0, 1 \ldots, m\}$ with transition matrix

$$P_{i,i+1} = 1 - \frac{i}{m}, \quad p_{i,i-1} = \frac{i}{m}.$$

Show by induction (or otherwise) that

$$\mathbb{E}[X_n - \frac{m}{2} \mid X_0 = i] = (i - \frac{m}{2})(1 - \frac{2}{m})^n.$$

Q 4 Let Q be any Markov chain and μ a probability distribution. We'll construct a new Markov chain whose transitions are as follows: from a state x choose a new state y according to the the transition matrix Q. With probability

$$A_{xy} = \min\{1, \frac{\mu_y Q_{yx}}{\mu_x Q_{xy}}\}$$

accept the move and move to y. Otherwise reject it and stay at x.

(a) Write a formula for the transition matrix P.

(b) Show that μ is the stationary distribution of P

(c) On a graph G let μ be the uniform distribution and let Q be the transition matrix for the random walk on G. Describe the transitions of the new Markov chain P in this case.