Stat 133 | Midterm Project
Fall 2014 | Due Oct. 30^{th}, 2014

Simulation of a Birth and Assassination Process

Consider the birth and assassination problem:

- The head of the family is born at time 0, and is assassinated at a random time X, which has an exponential distribution with rate κ.
- While alive, the head has children according to a Poisson process. That is, the time between births follows an exponential distribution with rate λ.
- As soon as they are born, the children produce offspring, according to the Poisson process with inner-arrival times of the births following the exponential(λ) distribution. And their children produce offspring according to this same distribution, and so on.
- When the head is assassinated, then his children become vulnerable to assassination. The time between when the parent is assassinated and a child is assassinated follows the exponential(κ) distribution, and is independent of the siblings' assassination times.
- The process continues in this manner, where the offspring are protected from assassination until their parent is assassinated. Then they are subject to assassination (at a random exponential time), and while alive, they continue to produce offspring (and their offspring produce offspring, . . .) at rate λ.

This process was proposed and studied by Professor Aldous ('The Birth-and-Assassination Process', Aldous and Krebs, *Statistics & Probability Letters*, 1990, pp 427-430). For this assignment, you are to write functions that simulate this stochastic process and carry out a study of the behavior of this process for different values of λ and κ.

The skeletons of the functions to perform the simulation are in the file `BAexperiment.R`. The inputs to the simulation will be: λ, the birth rate; κ, the assassination rate; $maxGen$ the maximum number of generations to simulate (of course the family could die out before reaching this maximum); and $maxTime$ the maximum amount of time to observe the simulation (again, the family could die out before this time is reached).

The data for one experiment will be returned as a list with one element per generation. Each element in the list will be a data frame, with one row for each person born in that generation. The data frame will have four variables: `parentID` the identifier for the parent; `id`, the identifier for the child (a number from 1 to n, where n is the total number of children born in that generation); *birth*, the birth date; and *death*, the assassination date.

Some information that you may find useful about the probability distributions:

- When we observe a process on a fixed interval, say (0, t) where the inner-arrival times follow an exponential(λ), the number "hits" in the interval will be random number. The distribution of this number follows the Poisson($\lambda * t$) distribution. A hit in our case is the birth of a child.
- Given the number of hits (births), the location of these hits (i.e., the birth dates) follows the uniform distribution on $(0, t)$.

Once you are able to generate one family tree, you will conduct an experiment to study how the parameters λ and κ affect the size and longevity of the family. For example, what is the likelihood that the family dies out? What is the typical number of generations and the total number of people in the family?

The file `MonteCarloBA.R` will help you organize your simulation study. You will choose a set of parameter values to study, for each (λ, κ) pair, you will generate many families at random, you will summarize each family tree, compute summary statistics for the families generated with the same λ and κ, and then compare the summary statistics across λs and κs.

After you have these statistics, make two to four plots of the results and write a brief (one-page) report of your findings. Use the report template to create an HTML report. Use the `reportTemplate.html` file as a template for the report.
If you are working as a pair, you should also profile your code as mentioned in the file `BAexperiment.R` and discuss improvements as part of your report. If you are working by yourself you don't need to do this part.

TURN IN to BSpace:
The two code files: `BAexperiment.R` and `MonteCarloBA.R`, with your code.
Also, turn in your image files (use png files) and html file, i.e., `reportTemplate.html`.