Problem Set 7, Math 172 Spring '15

This problem set is due Tuesday, March 10th, 2015 at **the beginning of class**. All class guide rules apply. "VLW" refers to Van Lint and Wilson, our textbook. **All questions, when graded, are worth an equal number of points unless stated otherwise, though not all questions would end up being graded.**

Warning: to combat both the idea that "optional" problems should just be ignored and the idea that you should do all "non-optional" problems, I will no longer mark hard/unsolved problems as "optional." This warning will self-destruct after this problem set, but the message stays.

---

1. True or false (give proofs):
   (a) $n^2 = O(n^2 \log(n))$.
   (b) $n^2 = o(n^2 \log(n))$.
   (c) $n^2 + 5n = n^2(1 + o(1)) = O(n^2)$.
   (d) $\binom{n}{k} \leq (en/k)^k$.
   (e) $(n/k)^k \leq \binom{n}{k}$.

2. Classify all possible pairs of (possibly distinct) fair 6-sided dice with integers on them (it is possible to have a fair die with multiple copies of an integer, such as a die with sides $(1, 1, 2, 5, 4, -8)$. All 6 **sides** must have equal probability, but the 1 ends up being twice as likely) such that if we roll them and record their sum, the resulting distribution is indistinguishable from rolling a pair of fair normal dice. (as a silly example, rolling a pair of 1-sided dice with sides 1 and looking at the sum is indistinguishable from rolling die A and die B, where A has a single side with value 2 and B has a single side with value 0. In both situations you can only get 2, with probability 1).

3. Count L_n, the number of ways to put n distinguishable flags onto indistinguishable poles. As an example, if the flag set is [4], we can have one pole with the 3 flag alone, and the other pole with 2 on top of the 1 on top of the 4 flag. You must use the exponential formula somewhere.

4. We have n kinds of objects, and we want to count the number of ways to select a k-list/tuple of the objects. Find the number of ways to do this if we have:
   (a) Just 1 object of each kind.
   (b) Infinite objects of each kind.
   (c) Just l objects of each kind.

5. Consider a graph G, two of its vertices being v and w. Suppose there exists two paths of lengths (number of edges) l_1 and l_2 between v and w. Now suppose l_1 is odd and l_2 is even. Show that G contains an odd cycle (defined as C_n where n is odd) as a subgraph.

You: "This is so easy!"

Problem: "These two paths aren't necessarily disjoint."

You: "Oh."

---

6. Let a *codeword* be a n-tuple of elements in $\mathbf{Z}_2^n$ (so a string of 0's and 1's). Call a codeword *even* if it has an even number of 1's. Find the fraction of codewords that are even for every n, and find the limit of this fraction as $n \to \infty$.

7. How much time did you spend on this problem set? What comments do you have of the problems? (difficulty, type, enjoyment, fairness, etc.)