2/5/15

Suppose we pick a simple graph w/ n vertices and n-1 edges uniformly on $V(G) = [n]$. What's the asymptotic probability we get a tree?

- choosing n-1 of the $\binom{n}{2}$ edges
- there are n^{n-2} trees
- asymptotic ⇒ take n as $n \to \infty$

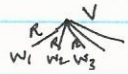
$$\frac{n^{n-2}}{\binom{\binom{n}{2}}{n-1}} \approx \binom{n^2}{n} \approx \frac{(n^2)!}{n!(n^2-n)!} \approx \frac{(n^2)^n}{n!} \approx \frac{n^n}{e^n} \qquad \approx \frac{n^{n-2}}{n^n e^n} = \frac{1}{n^2 e^n}, \quad e^n \gg n^2$$

$\frac{1}{n^2e^n}$ small as $n \to \infty$

∴ very unlikely to get a tree

In K_6, if we color all edges w/ 2 colors, then there is a copy of K_3 ($= C_3$) which is all of 1 color.

Proof:



by pigeonhole, $\exists \geq 3$ red edges (v, w_i)

look at $(w_1, w_2), (w_2, w_3), (w_1, w_3)$. If any red then done. If all blue then you have a blue triangle so done.

3.3 | $\exists\, N(q_1, \dots, q_s)$, $q_i \geq 2$, (for all $(q_1, \dots, q_s)$) such that if $N \geq N(q_1, \dots, q_s)$ if all edges of K_N are colored with colors $C_1, \dots, C_s$, then $\exists i$ such that there is a K_{q_i}, all edges of color C_i.

$N(2,2) = 2$ $\qquad N(2,3) = 3$

$N(3,3) = 6$

N(p,p) |

K_n, For a p-subset $S \subseteq [n]$, how many colorings has all edges in that $K_{|S|}$ the same?

$$2^{\binom{n}{2} - \binom{p}{2}} \cdot 2$$

Analysis tangent: In an alternating series $a_1 - a_2 + a_3 - a_4 + \dots$ if $|a_i| > |a_{i+1}|$

If you're before a subtraction you're overestimating, if before addition you're underestimating

Inclusion-Exclusion looks like this

"union bound" $= |E_1 \cup E_2 \cup E_3 \dots| \leq |E_1| + |E_2| + |E_3|$

Can we bound the union of all such "bad" situations?

bad colorings $\leq \binom{n}{p} 2^{\binom{n}{2} - \binom{p}{2}} \cdot 2$ $\quad$ want this to be $< 2^{\binom{n}{2}}$ b/c then $\exists$ at least 1 good coloring

Hi!
-Robbie

If $n < 2^{p/2}$, then $\binom{n}{p} 2^{\binom{n}{2} - \binom{p}{2}} \cdot 2 < 2^{\binom{n}{2}}$

$\Rightarrow N(p,p) \geq 2^{p/2}$

$$\frac{\binom{n}{p} 2^{\binom{n}{2} - \binom{p}{2} + 1}}{2^{\binom{n}{2}}}$$ want to be < 1

$$= \frac{2\binom{n}{p}}{2^{\binom{p}{2}}} \approx \frac{2n^p}{p!\,2^{\binom{p}{2}}}$$ if $n < 2^{\frac{p}{2}}$ then this is $< \frac{2 \cdot 2^{\frac{p^2}{2}}}{p!\,2^{\binom{p}{2}}} < 1$

$43 \leq N(5,5) \leq 49$

$103 \leq N(6,6) \leq 163$

$N(3,4) = 9$

4.1| Turan's Theorem

$\exists\, M(n,p)$ if G has $\geq M(n,p)$ edges ($|V(G)| = n$), G has K_p as a a subgraph. (he gives formula for $M(n,p)$)

Subgraph: G' of G $\quad V(G') \subseteq V(G)$

$\qquad E(G') \subseteq E(G)$

Induced subgraph: $V(G') \subseteq V(G)$

$\qquad E(G') = \{ e = (v,w) \in E(G),\ v,w \in V(G') \}$

 Subgraph  Induced subgraph 

What should $M(n,3)$ be?


$(n-1)$

 Complete bipartite graph K_{m_1, m_2}

$\frac{n}{2}$ $\frac{n}{2}$ $\approx \frac{n^2}{4}$ edges

- Ramsey Theory
  - philosophy
  - $N(q_1, \ldots, q_s)$
  - $N(p,p)$
- Turan's Theorem
  - triangles