Stat 150 Homework # 2 Due February 13

Problems:

Q 1 Can a reversible chain be periodic (demonstrate with an explanation or example)?

Q 2 Consider the probability transition matrix

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0.1 & 0.9 \\ 0.2 & 0.2 & 0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0.9 & 0 & 0 & 0 & 0 & 0.1 & 0 \end{pmatrix}$$

What are the classes of communicating states? Which are recurrent? Which states are aperiodic?

Q 3 If states i and j communicate and i is aperiodic show that j must also be aperiodic.

Q 4 A biased coin lands heads with probability 2/3 and tails with probability 1/3. We toss the coin repeatedly and let U_n be the indicator of the event that there are an even number of heads in the first n coin tosses.

- What is the transition matrix of U_n.
- What is its stationary distribution.
- After n coin tosses, what is the probability of an even number of heads.

Q 5 Two bags each contain n balls. In total there are $2n$ balls, n are green and n are red. After each unit of time a ball is selected from each bag and the two are swapped over (i.e the selected balls are put each put in the opposite bag). Let X_n be the number of green balls in the first bag.

- What is the transition matrix of X_n.
- Find the stationary distribution.
- Is the Markov chain reversible?

Q 6 For $0 < p < 1$ let X_n be an auto-regressive process,

$$X_{n+1} = pX_n + W_{n+1}$$

where W_n are IID $N(0, 1)$ random variables. What is the stationary distribution of the Markov chain? What happens if $p > 1$?