# HW 11
# Math 125a, Fall 2014

1. Show that there is a structure $\mathcal{M}$ interpreting the language with $0$, $1$, $+$, $\times$ and $<$. satisfying the following conditions.

   - $\mathcal{M}$ is elementarily equivalent to the real numbers with the natural interpretations of the operations.
   - $\mathcal{M}$ has an "infinitesimal" element $a$ such that $0 < a$ and for each $n$, $a < 1/(\underbrace{1 + \cdots + 1}_{n \text{ times}})$.

2. For $\mathcal{M}$ as in the previous problem, show that the sum of two infinitesimal elements of $\mathcal{M}$ is infinitesimal.

3. Use the Compactness Theorem for first order logic to give another solution to this problem from HW 4: Let $T$ be an infinite set of finite binary sequences such that for every sequence $s$, if $s$ is in $T$ then every initial segment of $s$ is in $T$. Show that there is an infinite binary sequence $P$ such that every finite initial segment of $P$ is an element of $T$.

4. Suppose that $\prec$ is a partial ordering of $\mathbb{N}$. Use the Compactness Theorem for first order logic to show that there is a total ordering $\prec^*$ of $\mathbb{N}$ such that for all $n$ and $m$ in $\mathbb{N}$, if $n \prec m$ then $n \prec^* m$.