Math 114: Notes on past lectures

This is a list of definitions and theorems covered in lecture so far, along with comments where necessary. All references are to Dummit and Foote unless otherwise noted.

On exams you will be responsible for all of these items except those marked as optional. You will not be responsible for definitions and theorems from Dummit and Foote that are not listed below, except those that are needed for definitions and theorems on the list to make sense (e.g. to understand what the degree of a field extension is, you need to know what a vector space is).

You don't need to memorize the proofs of any of the theorems, but have a general idea of how they go. Also know examples and non-examples of each concept covered; these have been given in lecture, on homework, and in Dummit and Foote.

Class 02, 01/22

Topics covered:

- field, field homomorphism, subfield (§7.1, §7.3)
- characteristic of a field, prime subfield of a field (§13.1, pp. 510-511)
- field extension, degree of field extension (§13.1, p. 512)
- subfield generated by a subset, simple extension, primitive element (§13.1, p. 517)
- automorphism (of a field), group of automorphisms ($\mathrm{Aut}(E)$, $\mathrm{Aut}(E/F)$), fixed subfield (§14.1, pp. 558-560)

Class 03, 01/27

Topics covered:

- Theorems 3 and 4, §13.1 (construction and degree of extension containing a root of a given polynomial)
- algebraic element, transcendental element, minimal polynomial of algebraic element, degree of algebraic element (§13.2, p. 520)
- Proposition 11, §13.2 (extension generated by an algebraic element)
- algebraic extension (§13.2, p. 520)
- Proposition 12, Corollary 13, §13.2 (finite extensions are algebraic)

Class 04, 01/29

Topics covered:

- Lemma 16, Theorem 17, Corollaries 18 and 19, §13.2 (finite extensions are those generated by finitely many algebraic elements; algebraic elements form a subfield)
- the algebraic closure of a field in an extension (the subfield defined in Corollary 19)
- Theorem 20, §13.2 (algebraic extensions of algebraic extensions are algebraic)
- splitting field (§13.4, p. 536)
- Theorem 25, Proposition 26, §13.4 (existence and maximum degree of splitting field)

Class 05, 02/03

Topics covered:

- Theorem 27, Corollary 28, §13.4 (uniqueness of splitting fields)
- an algebraic closure of a field, algebraically closed field (§13.4, p. 543)
- Proposition 29, §13.4 (any algebraic closure of a field is algebraically closed)

Class 06, 02/05

Topics covered:

- Propositions 30 and 31, §13.4 (existence and uniqueness of algebraic closures)
- Zorn's lemma (used in proving Propositions 30 and 31) (Appendix I, p. 909)
- separable polynomial, inseparable polynomial (§13.5, p. 546)
- separable extension (§13.5, p. 551)
- derivative of a polynomial (p. 546)
- Proposition 33, Corollary 34, §13.5 (detecting separability via the derivative; irreducible polynomials in characteristic 0 are separable)