math.berkeley.edu/~yanzhang/classes/math172.s15

responsible for what we learn in lecture and what he tells us we need to know

look at class participation guide

1/22/2015

Chapter 13 |

Maps (functions) $f: \overset{\text{domain}}{X} \to \overset{\text{codomain}}{Y}$

$$x \to f(x)$$

$$\text{range} = \{y \in Y \mid \exists\, x \in X \text{ with } f(x) = y\}$$

<u>Let's count maps</u>

Count maps $[n] \to [k]$

k^n for each $x \in [n]$, there are k choices

(product principle)

Count injective maps $[n] \to [k]$

$k(k-1)(k-2)\dots(k-n+1) =: (k)_{(n)}$ "k permute n"

side note: $k(k+1)(k+2)\dots(k+n-1) =: (k)^{(n)}$

<u>Balls and Bins</u> |

n balls   k bins

① ② ... ⓝ $\sqcup_1\ \sqcup_2 \dots \sqcup_k$

$\Updownarrow$

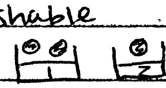
function $f(1) = 1$

$f(2) = 3$

$f(3) = 1$

n balls indistinguishable

● ● ... ●



$\Updownarrow$

● ● || ● ● || ●

k-1 "dividers"   stars & bars

$\Rightarrow \binom{n+k-1}{k-1}$

same as NY walking problem from last time

$$\left(\!\binom{k}{n}\!\right) := \binom{n+k-1}{k-1} = \binom{n+k-1}{n}$$

$$\left(\!\binom{x}{y}\!\right) := \binom{x+y-1}{y}$$

n balls k bins where answer is $\binom{k}{n}$

"injective" maps

not well-defined for indistinguishable balls + bins

$N = [n]$

$K = [k]$

a map $f: N \to K$ with N indistinguishable?

Define $f, g: N \to K$ to be equivalent if $\exists$ bijection $u: N \to N$ s.t.

$f(u(a)) = g(a)$ $\forall a \in N$.

When we say we're counting maps with N indistinguishable

we're counting equivalence classes of this relation

Bogart appendix A.2

a relation on S and T is a subset $R \subseteq S \times T$.

example $\leq$ is a relation on $\mathbb{N}$ and $\mathbb{N}$.

$a \leq b \iff (a,b) \in R$

An equivalence relation on S is a relation R on $S \times S$

1) symmetric $(a,b) \in R \to (b,a) \in R$

2) reflexive $(a,a) \in R$ $\forall a \in S$

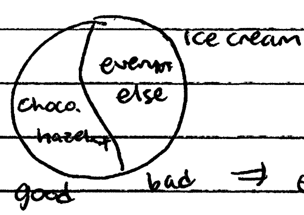
3) transitive $(a,b), (b,c) \in R \to (a,c) \in R$

$\Updownarrow$

equivalence relations $\iff$ $S = S_1 \cup S_2 \dots S_n$

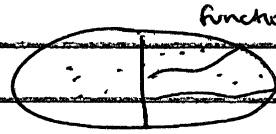
disjoint union

$S_1, \dots, S_n$ is a partition, the sets $S_1, \dots, S_n$ are equivalence classes



$\Rightarrow$ equivalence classes

functions $N \to K$



count # parts

n indistinguishable balls, k bins

$\Updownarrow$

equivalence classes of maps

$N \to K$ w/ N indistinguishable K distinguishable

<u>Twelvefold Way</u> | Gian-Carlo Rota

N	K	all	inj	sur.
D	D	k^n	$(k)_n$	$k!\,S(n,k)$
I	D	$\left(\!\binom{k}{n}\!\right)$	$\binom{k}{n}$	$\binom{k}{n-k}$
D	I	$\sum_{i=0}^{k} S(n,i)$	1 if $n \le k$, 0 otherwise	$S(n,k)$
I	I	$\sum_{i=0}^{k} p_i(n)$	1 if $n \le k$, 0 otherwise	$p_k(n)$

Bogart, Chapter 3, "Twentyfold Way"

<u>Surjections</u> $D \to I$

$N \to K$

do small cases

$S(1,1)=1$	(1)
$S(2,1)=1$	(12)
$S(2,2)=1$	(1)(2)
$S(3,1)=1$	(123)
$S(3,2)=3$	(12)(3) (13)(2) (23)(1)
$S(3,3)=1$	(1)(2)(3)

$\uparrow$

Stirling numbers (2nd kind)

$I \to I$

$f: N \to K$

$p_k(n)$	$p_1(5)=1$	5
$\uparrow$	$p_2(5)=2$	4+1, 3+2
<u>partition</u>	$p_3(5)=2$	2+2+1, 3+1+1
<u>numbers</u>	$p_4(5)=1$	2+1+1+1
	$p_5(5)=1$	1+1+1+1+1

- maps, well-defined notions of distinguishable / indistinguishable
- equivalence relations / partitions
- Twelvefold way