

# Stat 150 Homework # 3 Due February 20

## Problems:

**Q 1** On a game show there are two contestants. A contestant answers a series of questions until they make a mistake and then it becomes the other contestants turn. Contestant one answers questions correctly 70% of the time while contestant two answers correctly 80% of the time. Over the long run what proportion of questions are asked of contestant 1?

**Q 2** A standard 6 sided die is rolled repeatedly. Let  $S_n$  be the sum of the first  $n$  rolls. Over the long run what proportion of the time is  $S_n$  divisible by 3?

**Q 3** A Markov chain has five states  $0, 1, 2, 3, 4$ . The transitions are given as follows: when  $i \geq 1$  the next step chooses a uniform state in  $0, 1, \dots, i-1$ . When  $i = 0$  the next step is 4.

- Is the Markov chain irreducible? Aperiodic? Explain.
- What is the stationary distribution?

**Q 4** Last week we had the following Markov chain. Two bags each contain  $n$  balls. In total there are  $2n$  balls,  $n$  are green and  $n$  are red. After each unit of time a ball is selected from each bag and the two are swapped over (i.e the selected balls are put each put in the opposite bag). Let  $X_k$  be the number of green balls in the first bag after  $k$  steps.

Let  $T$  be the first time when all the balls in the first bag are the same colour. Find

$$\mathbb{P}[X_T = 0 \mid X_0 = i].$$

**Q 5** Let  $X_n$  be a finite state, irreducible and aperiodic Markov chain. If  $\pi$  is the stationary distribution explain why  $\pi_i > 0$  for all  $i$ .