Problem Set 4, Math 172 Spring '15

This problem set is due Tuesday, February 17th, 2015 at **the beginning of class**. All class guide rules apply. "VLW" refers to Van Lint and Wilson, our textbook. **All questions, when graded, are worth an equal number of points unless stated otherwise, though not all questions would end up being graded.**

1. (VLW 5A, i) Show that a finite *regular* (meaning each vertex has the same positive degree) bipartite graph has a perfect matching.

2. (VLW 33B) Find the chromatic polynomial χ_G for the n-cycle C_n and the n-wheel W_n. The graph W_n is defined as the graph obtained from C_n by adding a new vertex v and joining it to all vertices of C_n.

3. Find the number of ways to place 3 rooks on the 8×8 chessboard such that no two rooks attack each other (meaning no two rooks in are in the same row or column).

4. (optional) Tell me why I shouldn't submit my 4-color theorem proof to the *Annals of Mathematics.*

5. Let G be a simple graph with 10 vertices and 26 edges. Show that G has at least 5 triangles.

6. What is
$$\sum_{k=0}^{n} k\binom{n}{k}?$$
Even if you solve this problem, try to find a one-line proof that captures the intuition of the problem.

7. How much time did you spend on this problem set? What comments do you have of the problems? (difficulty, type, enjoyment, fairness, etc.)