Name:

Math 114 diagnostic quiz (ungraded)

Tuesday, 20 January 2015

Please don't consult your textbook or notes. This is just to give me an idea of what concepts it's safe for me to use without explanation in class.

Math

1. Let G be a group. Define (as informally as you like): normal subgroup of G, index of a subgroup $H \leq G$.

2. Let F be a field, R a ring, and $\phi : F \to R$ a ring homomorphism. What can you say about the kernel, $\ker \phi$, and the image, $\phi(F)$?

3. Let F be a field and $p(X)$ an element of the polynomial ring $F[X]$. How does one construct a field extension $E \supset F$ such that $p(X)$ has a root in E, (a) if $p(X)$ is irreducible in $F[X]$, (b) if $p(X)$ is reducible in $F[X]$?

4. Let E be a field, $F \subset E$ a subfield. Define (as informally as you like): automorphism of E, characteristic of E, degree of E over F ($[E : F]$).

   What is the characteristic of F?

   What does it mean for an element $\alpha \in E$ to be algebraic over F?

5. What is a basis for $\mathbf{Q}[X]$ as a vector space over $\mathbf{Q}$?

6. Does $\mathbf{R}$ contain a subfield that is isomorphic to $\mathbf{Q}[X]/(X^3 - 2)$? (Why or why not?)

   How about a subfield isomorphic to $\mathbf{Q}[X]/(X^2 + 1)$? (Why or why not?)

   What subfields of $\mathbf{C}$ are isomorphic to $\mathbf{Q}[X]/(X^3 - 2)$?

7. What can you tell me about finite fields? Give an example of a finite field with 49 elements (or, if you can't, some other finite field).

Survey

1. Which of the following subjects would you most like to see after Galois theory? Circle one or more (or zero if you have no opinion); feel free to add to the list.

   module theory, advanced linear algebra, commutative algebra, homological algebra, noncommutative algebra

2. Are there any topics from introductory algebra (groups and rings) that you find especially difficult?

3. If your name is frequently mispronounced, please tell me how to pronounce it.

4. What's your major? What areas of math are you most interested in? Any other remarks?