1/20/2015

Intro

1) 13587 - how many factors?

2) dating site BerkeleyLove.com, how to represent?

3) Towers of Hanoi

Combinatorics

- enumerative / algebraic
  - counting
  - "how many?"
- extremal / structural
  - how do phenomena appear
  - relationships between objects

How to learn it?

- no ε - δ
  - good and bad
- problem-solving
- build a zoo (like a toolbox)
  - care about objects

Zoo

lists = ordered collection of elements

$$L = (s_1, \ldots, s_n)$$

$$(1,2,3) \neq (2,1,3)$$

sets = collection of elements

$$S = \{s_1, \ldots, s_n\}$$

You can ask is x in S?

$\forall x,\ x \in S$ or $x \notin S$

$$\{1,2,3\} = \{2,1,3\}$$

$[n] = \{1, 2, \ldots, n\}$

$|S|$ = # elements in set

$|[n]| = n$

$B = \{$rye, wheat, beerbread$\}$

$F = \{$cheese, ham, tofurkey, beets, PB&J$\}$

$|B| = 3 \quad |F| = 5$

How many possible sandwiches?

$S_{\text{sandwiches}} \simeq \{(x,y) \mid x \in B, y \in F\}$

$\longleftrightarrow$

bijection

Theme: to count S, give a bijection $S \to S'$, count S'

bijection: map $\underset{\text{set}}{f : X} \to \underset{\text{set}}{Y}$

f is one-to-one (injection) and onto (surjection)

Injection: $\forall x \neq y, x,y \in X, f(x) \neq f(y)$

Surjection: $\forall y \in Y, \exists x \in X, f(x) = y$

product principle

$S, T \Rightarrow S \times T$ (cartesian product)

$S \times T = \{(x,y) \mid x \in S, y \in T\}$

$|S \times T| = |S| \times |T|$ (a homomorphism)

$S = \{\text{salad}\}$

meals are defined as sandwich or salad

How many meals are there?

Sum principle

$S, T \Rightarrow S \cup T = \{x \mid x \in S \text{ or } T\}$

$|S \cup T| = |S| + |T|$ if S, T are disjoint $\Leftrightarrow S \cap T = \emptyset$

$|S \cup T| = |S| + |T| - |S \cap T|$: $\overset{\cap}{\{x \mid x \in S \text{ and } x \in T\}}$

meals $\simeq$ sandwich or salad

$\simeq$ (bread and filling) or salad

Sum/product $\Rightarrow (3 \times 5) + 1 = 16$

roller coaster problem

n compartments, 2 people in each, they can switch seats

pairs don't switch, how many configurations? 2^n

product principle

prob $\simeq S_1 \times S_2, \ldots \times S_n$

$\{\text{Alice Bob}, \text{Bob Alice}\}$

How many subsets of $[n]$ are there? 2^n

prob $\simeq S_1 \times \dots S_n$

$\quad\quad \parallel$

$\quad\quad \begin{Bmatrix} \text{1 is in set} \\ \text{1 is not in set} \end{Bmatrix}$

how many lists of size k, using different elements of [n] are there?

$S_1 \times S_2 \times \dots \times S_k$

$S_1 = \begin{Bmatrix} \text{1 is the first element} \\ \text{2 is the first element} \\ \vdots \\ \text{n is the first element} \end{Bmatrix}$ $\quad S_2 = \begin{Bmatrix} \text{1st unused element so far is 2nd elet.} \\ \text{2nd} \quad\quad\quad \text{2nd elet.} \\ \vdots \\ \text{n-1} \quad\quad\quad \text{2nd elet.} \end{Bmatrix}$

$|S| = n \cdot (n-1) \dots (n-k+1)$

$\quad = \frac{n!}{(n-k)!}$

permutation

= lists of size n from [n] (distinct)

n! permutations

- how many sets of size k of [n] are there?

$\underbrace{S_{\text{lists of size } k}}_{\frac{n!}{(n-k)!}} \simeq \underbrace{S_{\text{sets of size } k}}_{\parallel ?} \times \underbrace{S_{\text{ordering } k \text{ elements}}}_{k!}$ ← quotient principle / symmetry

$? = \frac{n!}{(n-k)!\,k!} = \binom{n}{k}$ comes from product princ.

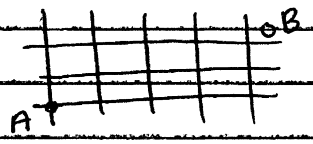
New strat to count X, if each $x \in X$ corresponds to exactly s elements of Y, then $|X| = \frac{|Y|}{s}$

- how many ways of seating n people around a round table are there (can rotate entire table). ↙ X

use symmetry. count $X' = \{\text{all seatings no rotation}\} \Rightarrow |X'| = n!$

but each $x \in X$ corresponds to n rotations so answer is $\frac{n!}{n} = (n-1)!$

- You are in NY



how many non-stupid ways to get to your friend are there?

non stupid paths $\simeq$ paths $A \to B$ only up or right

$\simeq$ paths with 3 ups and 3 rights

$\simeq$ orderings of 3 0's and 3 1's

$\binom{6}{3} \Leftarrow \simeq$ choice of 3 of 6 coordinates of $(c_1, c_2, \dots, c_6)$ to be 0, others are 1s.

math. berkeley. edu / ~ yanzhang / classes / math172 s15

responsible for what we learn in lecture and what he tells us we need to know

look at class participation guide