Problem Set 11, Math 172 Spring '15

This problem set is due Tuesday, January 27th, 2015 at **the beginning of class**. All class guide rules apply. "VLW" refers to Van Lint and Wilson, our textbook. **All questions, when graded, are worth an equal number of points unless stated otherwise, though not all questions would end up being graded.**

1. (**You will not receive a passing grade for this course without turning in this problem**) Acknowledge (**in your own words, through signature**) that you have:
   (a) read and understood the Class Guide and will hold yourself responsible to it;
   (b) understood that if you have any unresolved questions/complaints about the class policy, that you will tell me ASAP so we can resolve it, and if you are silent now then I assume we have agreed on class policy **(If you have an issue, DON'T put it on this homework, since it requires speedy grader turnaround. Instead, bring it up on Piazza or class so we can settle it.)**.

2. There is a chess variant where we keep the position of all 16 of the pawns, but **each** player shuffles the 8 pieces behind his/her pawns randomly. How many possible starting positions of the game are there? See `http://en.wikipedia.org/wiki/Chess_piece` for a diagram of a "normal" setup of the board. (Optional: if you could choose how to shuffle your pieces, how would you place them?)

3. (VLW 13A) On a circular array with n positions, we wish to place the integers $[r]$, **in clockwise order** (but possibly leaving blanks in the array) such that consecutive integers of $[r]$, including the pair $(r, 1)$, are not in adjacent positions in the array. Arrangments obtained by roation are considered the same. In how many ways can this be done? (Evil twin of VLW 13A, optional; points only for heroism) Now, suppose $r = n$ and we were to lift the "in clockwise order" restraint (so the numbers go wherever) such that consecutive integers of $[n]$, including the pair $(n, 1)$, are not in adjacent positions. What is the answer now?

4. Read and understand Appendix A.2 in Bogart on equivalence relations. Do Bogart problem 356 (on page 157): If we have an equivalence relation that divides a set with k elements up into equivalence classes, each of size m, what is the number n of equivalence classes? Explain why. Sketch a reason (no complete proof is needed, but you need to show you understand) why this result is equivalent to the "quotient principle" or "symmetry" as we discussed in class.

5. Suppose you have n ranks and k suits (so a normal deck has $n = 13$ and $k = 4$), how many ways are there to make "three of a kind" with a **set** of 5 playing cards? What about "two pair"? Which one is more valuable? (for definitions in a normal deck, look at `http://en.wikipedia.org/wiki/List_of_poker_hands`.

6. Extend the "Twelvefold way" to *increasing* functions from $[n]$ to $[k]$, where we define a function f to be *increasing* if for all $x < y, x \in [n], y \in [n]$, we have $f(x) < f(y)$. (so you should count 3 things: all such functions, all such injective functions, and all

such surjective functions) Then do the same for *nondecreasing* functions f, defined by those where for all $x < y, x \in [n], y \in [n]$, we have $f(x) \leq f(y)$. (Optional: comment on the answers)

7. (VLW 13C, variation) Give a solution to the following question: How many pairs (A_1, A_2) of subsets of $\{1, 2, \ldots, n\}$ are there such that $A_1 \cap A_2 = \emptyset$? (Optional: find two solutions, one involving binomial coefficients, and one should be one-line long)

8. How much time did you spend on this problem set? What comments do you have of the problems? (difficulty, type, enjoyment, fairness, etc.)

9. Successfully make at least one question/statement about something **you care about** (for example: question about lecture material, asking someone to borrow notes because you missed class, homework question help, answering a question from someone else, introducing yourself and indicating something you would like to eventually learn in this class, class policy questions / suggestions, etc.) on Piazza by 11:59PM on **Monday, 1/26/15**. You do not have to copy this text to the physical problem set you turn in; it will be graded by us looking on Piazza. Acknowledge (**through signature**) that you have gotten your Piazza account to work, know that you are responsible for what goes on there (for example, errata for homework), and understand how to post / view / reply. (**To the grader: disgard any Piazza action where it is obvious that the person did not care about the assignment, such as finding a random combinatorics question from the internet at 11:59:59 PM or simply answering "no" to a question without any explanation**)