**Question 1: (See Apendix 1)**

1. Immigration Canada has the choice between whether to administer a $100 test to income immigrants. If they decide not to test the immigrants, they are either admitted or rejected. However, if they are admitted, the sick immigrants would cost them $100,000 per person that the national health care system will cover.

If:

P(Test (-) | No Disease) = 1 then P(Test (+) | No Disease) = 0

P(Test (-) | Disease) = 0.2 then P(Test (+) | Disease) = 0.8

Law of total probabilities gives:

=P(Test (-)) = P(Test (-) | Disease) \* P(Disease) + P(Test (-) | No Disease) \* P(No Disease)

=0.2 \* 0.1 + 1 \* 0.9

= 0.92

=P(Disease | Test (-)) = P(Disease and Test (-)) | P(Test (-)) = P (Test (-) | Disease) \* P(Disease)

=0.2 \* 0.1 / 0.92

=0.022

1. We assume that if the immigrant does not have the disease, he will automatically be admitted in the country. Therefore, the expected monetary value of an immigrant without administering the test is:

= P(Disease) \* (Cost of Sickness) + P(No Disease) \* (Contribution to Economy)

= 0.1 \* (-100,000) + 0.9 \* 10,000

= -1,000

The expected value of the test or EVSI:

=P(Test (+)) \* (Cost of Test) + (Cost of Rejection) + [P(Test (-)) \* P(Disease | Test (-)) \* (Cost of Sickness)] + [P(Test (-)) \* (Chance you are healthy) \* (Contribution)]

= 0.08 \* (-100) + 0 + (0.92 \* 0.022 \* (-100,000)) + (0.92 \* 0.978 \* 9,900)

= 6,875.6

1. Hence, the expected value of perfect information:

(EVPI) = Difference between EVSI – EVPI

= 6,873,6 – (-1,000)

= 7,873,6.

Therefore, Immigration Canada should use the test to maximize its outcome out of the immigrants and avoid unnecessary costs.

**Question 2 :**

Operations Research Probable Outcomes:

10% of getting A

40% of getting B

50% of getting C

Statistics:

70% of getting B

25% of getting C

5% of getting D

Patty’s Preferences Indicate: (~) is used to indicate he is indifferent

Getting a C ~ 0.25A + 0.75D

Getting a B ~ 0.70A + 0.30D

We can replacing the letters C and B with the corresponding equalities:

Operations Research:

10% of getting A

40% of getting (0.70A + 0.30D)

50% of getting (0.25A + 0.75D)

Statistics:

70% of getting (0.70A + 0.30D)

25% of getting (0.25A + 0.75D)

5% of getting D

Expected Utility of Patty’s final grade under Operations Research course:

=0.10A+0.28A+0.12D+0.125A+0.375D

=0.5050A + 0.4950D

Expected Utility of Patty’s final grade under Stats course:

=0.49A+0.21D+0.0625A+0.1875D+0.05D

=0.5525A + 0.4475D

We can assume that the Utility of getting the best outcome (A) is 1, and getting the worst outcome (D) is 0.

Therefore:

0.5525A + 0.4475D > 0.5050A + 0.4950D

0.5525(1) + 0.4475(0) > 0.5050(1) + 0.4950(0)

0.5525 > 0.5050

We would choose to take the Statistics course.

**Question 3:**

1. For Decision Problem 1, if the decision maker prefers A over B that means:

A: [U($1Million)] **>** B: [0.1 U ($5 Million) + 0.89 U($1 Million) + 0.01 U ($0)]

Hence: 0.11 U ($1million) **>** 0.1 U ($5 Million) + 0.01 U ($0)

If we move everything to the LHS:

0.11 U ($1million) **-** 0.1 U ($5 Million) + 0.01 U ($0) > 0

For Decision Problem 2, we compare:

C: [0.11 U ($1 million) + 0.89 U($0)] **against** D: [0.1 U ($5 Million) + 0.9 U ($0)]

Hence, if we difference C and D: (C – D)

0.11 U ($1 million) **–** (0.1 U ($5 Million) + 0.9 U ($0) - 0.89 U($0))

= 0.11 U ($1 million) **-** 0.1 U ($5 Million) - 0.01 U ($0)

From chossing A over B, we know 0.11 U ($1 million) **-** 0.1 U ($5 Million) - 0.01 U ($0) > 0. Therefore, C – D > 0, C>D

If the decision maker prefers A over B in Decision Problem 1, the descision maker should also prefer C over D in Decision Problem 2 according to Expected Utilities.

1. The decision maker have a strong tendency to avoid risk and seize a larger probability of receiving money, while the amount doesn’t matter as much. The utility of money increases very slowly after 1 million whichs makes the decision maker unwilling to take any more risk.

**Question 4**

U1(0) = 0.9966 (Assume it equals 1 for simplicity in future calculations)

U1(10) = 0

U2(0) = 1

U2(10) = 0

We also assume:

K1 + K2 + K3 = 1

Given in the question:

U(0, 10) ~ 0.4U(0, 0) + 0.6U(10, 10)

U(10, 0) ~ 0.5U(0, 0) + 0.5U(10, 10)

U(0, 0) = 0.4[K1 + K2 + K3]

U(10, 10) = 0

U(0, 10) = K1

U(10, 0) = K2

Therefore:

K1 = 0.4U[K1 + K2 + K3]

K2 = 0.5U[K1 + K2 + K3]

After isolating the equations, we know the values of all the K values.

K1 = 0.4

K2 = 0.5

K3 = 0.1

The final utility equation:

U(X1, X2) = 0.4U(X1) + 0.5U(X2) + 0.1U(X1)U(X2)

We can use the previous equation calculate the next three questions.

X1 = shortage of blood

X2 = surplus of blood

U(Ordering 28 pints) = 0.5(U(0, 3)) + 0.5(U(7, 0))

= 0.5[0.4U(0) + 0.5U(3) + 0.1U(0)U(3) + 0.4U(7) + 0.5U(0) + 0.1U(7)U(0)]

= 0.5[0.4 + 0.455 + 0.091 + 0.081167 + 0.5 + 0.0203]

= 0.7737

U(Ordering 30 pints) = 0.5(U(0, 5)) + 0.5(U(5, 0))

= 0.5[0.4U(0) + 0.5U(5) + 0.1U(0)U(5) + 0.4U(5) + 0.5U(0) + 0.1U(5)U(0)]

= 0.5[0.4 + 0.375 + 0.075 + 0.1505 + 0.5 + 0.03763]

= 0.769065

U(Ordering 32 pints) = 0.5(U(0, 7)) + 0.5(U(3, 0))

= 0.5[0.4U(0) + 0.5U(7) + 0.1U(0)U(7) + 0.4U(3) + 0.5U(0) + 0.1U(3)U(0)]

= 0.5[0.4 + 0.255 + 0.051 + 0.2351908 + 0.5 + 0.058798]

=0.7499

Therefore, Ordering 28 pints yields the greatest amount of utility.

**Question 5 (See Appendix 2 & 3)**

The 4 car brands picked for this exercise are as follows: **Honda**, **Ford**, **Skoda** and **Ferrari**. They will be compared based on their *economy, practicality, safety and reliability* and *driving pleasure.* The most complex step in identifying the appropriate car to purchase was developing the pairwise comparisons. In the beginning, we had difficulty creating pairwise comparisons that did not contain any inconsistencies. However, through careful analysis, we created four matrixes that were all significant at the 10% confidence interval. We found the most economical car to be Honda, Skoda was the most practical, Ford was the most reliable, and Ferrari was the most satisfying to drive.

The process to identify the pairwise comparisons was done as a team. Although we had divergent opinions on some aspects of the comparisons, we turned to appraisals and expert opinions on the internet for a definitive answer. The team also made sure that personal biases were not affecting our decision-making process by clearly identifying our reasons for creating the comparison. Finally, we tested for inconsistency and fixed any discrepancies to ensure that all the values were significant.

After completing all five matrixes, we concluded that the best car for our needs was Honda. Honda is the preferable choice because it contains high economy and practically scores. Our preferences also weighed economy and practicality as our primary concerns; therefore, it became apparent that Honda was the appropriate choice. Ferrari was the lowest on our list because its only strong score was driving pleasure; however, driving pleasure was not a major concern.