**Problem 1 (See Appendix A):**

Stage: time t= 1, …, 3

State: St – stock at the beginning of the state

Decision: at = number of units produced in the stage t

Dynamics

* + *Dt* = demand in the stage t
  + *i* = inventory cost per product
  + *ct =* production cost per product in period t
  + *yt* = Binary variable. If there is at>0, it has value 1, otherwise 0

ft(St) = minimize the total costs incurred by the company while also satisfying demand

Recursion:

***The optimal way of producing is in the first stage to produce 200 radios, in the second stage 600 radios.***

**Problem 2:**

Stage: Beginning of year t=1,…,6  
State: Beginning of year t=1,…,6  
Decision: Keep the car or sell the car  
State Dynamics: Minimum net cost incurred = Cost of purchasing car at t and operating until x

* represent the operating cost incurred during the i year.
* represent the value of ith year old car

Therefore:

+…+

Hence,

Not able to sell after year six implies that:

Therefore, if the car kept for a year and sold at year 6,

If car bought at t=4 and kept all the way,

If in year 3, and kept all the way,

If the car is purchased at year 3, trade it in at year 6 since it has min value

If in year 2, and kept all the way,

If the car is purchased in year 2, trade it in at year 4 since it has the min value

If in year 1, and kept all the way,

If the car is purchased in year 1, trade it in at year 3,4 or 6 since it has the same min value

If in year 0, and kept all the way,

If the car is purchased in year 0, trade it in at year 2 since it has the minimum value

***A new car which is purchased in year 0 should be traded in year 2 and the new car purchased in year 2 should be replaced in year 4, finally the car bought in year 4 should be replaced at 6. This would mean a minimum cost of $14440.***

**Problem 3:**

Stage: time t=1, …, 20

State: plant is build in location i in time t

Decision: bi -plant is build (1), otherwise 0

Dynamics:

* + Ki = production of i plant in kWh
  + Dt = demand for energy in kWh
  + Et, Et + 1 amount of energy in kWh available in the state t, t+1, …

Recursion:

* Ci= building cost in i location
* Hi = operation cost of i plant

**Problem 4:**

We have a total of $600 to invest and we have 3 different types of spares at 3 different prices for 3 systems:

* $100 to add a spare for system I
* $300 to add a spare for system 2
* $200 to add a spare for system 3

We can only add two spares to each system.

Stage: investment option t=1,2,3

State: how much money we have left

Decision: where should we allocate the resource Ut

State dynamics: Xt+1=Xt - Ut

Xt =current amount of money

Ut= how much we invest

Value Function: maximize the probability for which the computer will work properly given that we can only spend $600 on the spares

Ft(Xt) = max (probability of a computer that works properly)

Ft(Xt) = max Ft+1(Xt-Ut)

F3(200>X) = 0.7

F3 (300>X>=200) = 0.9

F3 (600>X>=400) = 0.98

F2(200>X) = 0.6 \* F3 (200>X) = 0.42

F2 (300>X>=200) = 0.6 \* F3 (300>X>=200) = 0.54

F2 (400) = max (0.85 \* 0.7, 0.8 \* 0.98) = 0.595

F2 (500) = max (0.85 \* 0.9, 0.6 \* 0.98) = 0.765

F2 (600) = max (0.95 \* 0.7, 0.85 \* 0.9, 0.6 \*0.98) = max (0.665, 0.765, 0.588) = 0.765

F1(600) = max (0.85 \*0.765, 0.9 \* 0.765, 0.95 \* 0.595) = max (0.65025, 0.6885, 0.565) = 0.6885

***Solution = we should allocate one spare for system 1, one spare for system 2 and one spare for system 3.***

**Problem 5:**

Stage: t = Year

State: dt = Firm’s asset position at the start of year t

Decision: Xt = The amount to invest in year t

State dynamics: dt+1 = dt + P\*Xt - (1-P)\*Xt + yq\*q

d0 = 10,000

Let ft (dt) = maximum expected asset at the end of year t

ft (dt) = max{ dt + P\*Xt - (1-P)\*Xt + yq\*q + ft+1 (dt+1)}

Xt=0 when dt≤0，otherwise 0≤Xt≤dt

When t=10, we can use the recursive function to calculate backwards untill t=1 and find the maximization result.

**Problem 6:**

Stage: The number of periods t 1…100

State: Current stock of inventory St

Decision: The number of units to produce Xt

State Dynamics: St + 1 = St + Xt – dt

V(St) = Minimize expected costs of the project while maximizing the total expected profit by the company

V(St) =max(min((St + Xt) \* 20, dt \* 20) – max([(St + Xt) – dt] \* 1, [dt - (St + Xt)] \* 10) – ct \* Xt + Vt + 1E(max((St + Xt) – dt), 0)))

* + *dt* = demand in the stage t
  + *Ct =* production cost per product in period t

Explanations:

Revenue Condition: min((St + Xt) \* 20, dt \* 20)

The company can either sell the amount of units the company has for the period or only sell the amount demanded. Therefore, the minimum value is taken between the two options which indicates total sales for the period.

Shortage or Surplus Condition: max([(St + Xt) – dt] \* 1, [dt - (St + Xt)] \* 10)

The company will either have a surplus or a shortage within the period. A shortage occurs when the units produced in the period is less than the demand which is indicated as the second option in the max function. A surplus occurs when you have excessive inventory in the period which is specified in the first part of the max function. In every period, one of these costs will be positive while the other negative. Therefore, we take the highest cost which is the cost the company incurs at period t.

Variable Cost Condition: ct \* Xt

The company will have a production cost every period depending on the units it produces. The total productions cost is the product between the number of units produced and the cost to produce one unit.

Recursive Call Condition: Vt + 1E(max((St + Xt) – dt), 0))

When we move to the next stage, the state, which is the amount of inventory owned by the company, the St must be adjusted for the next period. We can either have a surplus which is indicated in the first part of the max function, or we can have sold out and have nothing.