**Part 1: Big-O, Big-Ω, and Big-Θ Analysis and Proofs**

Michael Krakovsky

1) The tightest upper bound (Big-O) of the expression is c\*O(n2).

To find c:

T(n) =

Remove the negative values: T(n) =

Convert n into n2: T(n) =

Add components, get c: T(n) =

Therefore, the tightest upper bound is 4 \* O(n2).

2)

Assume f(n) such that T(n) O(f(n)) & O(f(n)) is a proper subset of O(n2)

c \* f(n) O(n2)

c \* f(n)

c \* f(n)

We know n2 O(n2), thus since c \* f(n) , for some n0. Therefore, we have a contradiction since O(f(n)) O(n2) and O(f(n)) O(n2)

Therefore, O(n2) is T(n) is the highest upper bound.

3)

(n)

(n)

Therefore, c = 9

4)

Suppose we are past some n, let us take the max of the two bounds. Both will be non-negative numbers and let c1, c2 = 0.5, 1.

.5(max(f(n), g(n)) + max(f(n), g(n) + max(f(n), g(n))

.5(max(f(n), g(n)) + max(f(n), g(n) + max(f(n), g(n))

5)

Prove is O(2n)

(n )

Therefore, = c \* O( where c = 2

6)

Suppose that the proposition is true.

Therefore, there will be a n1 n0 where all the values of f(n) are less than . However, this cannot be true. Since , the function inevitably surpasses regardless of the value of c. While c is a constant and will never grow itself, will grow with an increasing n value thus disproving that all values passed some n0 will be less than .