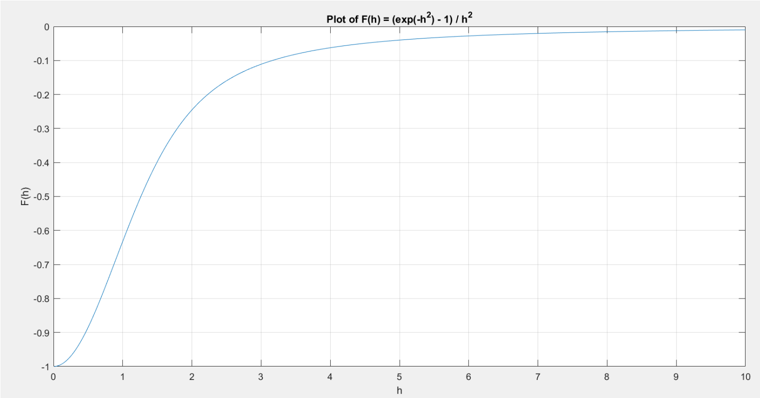

Figure A

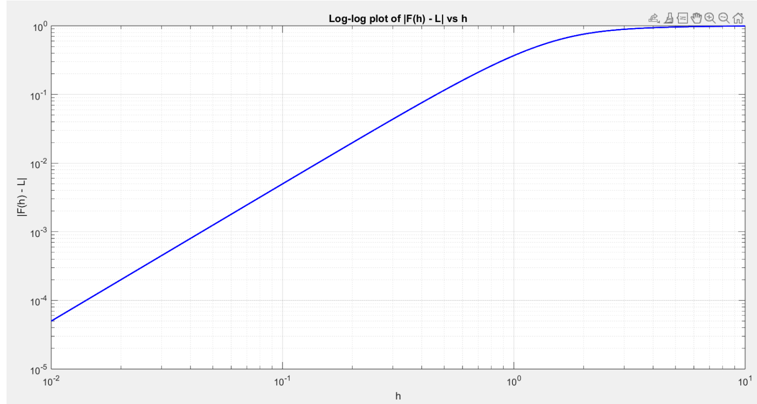

Figure B

a) From the above graph (Figure A), we can estimate the $\lim_{h \to 0} F(h)$ = -1. In the case that h is chosen too small, rounding errors may occur as e^{-h^2} in the numerator, ($e^{-h^2} - 1$), approaches 1 which when subtract by 1 can introduce rounding errors into the calculation.

b) To find the limit analytically, we can apply L'Hopital's rule where: $f(x) = e^{-h^2} - 1$ and $g(x) = h^2$

   i) This results in the expression: $\frac{-2he^{-h^2}}{2h}$ and can proceed to apply L'Hopital's again

   ii) Resulting in $L = \lim_{h \to 0} \frac{-4h^2e^{-h^2} - 2}{2}$, which evaluates to $L = \lim_{h \to 0} F(h) =- 1$

Using the power-law relationship, it is possible to find the slope of the linear region of this log-log plot (figure B) to find the p value, or at least an approximation. Based on my matlab code, this relation can be modeled as:

$$log(|F(h) - L|) = p \cdot log(h) + constant$$

The above is a linear equation that by using the matlab function, polyfit, an approximation of the slope (p value) can be found. For the sake of what we're trying to find, the constant is irrelevant. Polyfit generates an nth degree polynomial function that fits the data. To achieve this, an estimated linear region must be found, which is a set of point along the line that behave similar to that of h^p. I.e we are looking for where the log of the error, $log(|F(h) - L|)$, scales linearly. The main concern about this method is that if h is too large, it won't follow this rule, and if h is too small, round-off errors can occur. As such i have selected the range of $\left[1^{-2}, 1^{-1}\right]$. Using this method, we can approximate $p = 1.9989$. To verify this, I applied taylor series expansion to $F(h)$, where the first term is $\frac{2he^{-h^2} - e^{-h^2} - 1}{h^2}$. The rate of convergence of $F(h)$ can be modeled using the following:

$$F(h) = L + O(B_h) \text{ where } B_h = \frac{1}{h^p}.$$

Noting that the functioning convergence behaviour is related to the first term of its taylor series, $\frac{1}{h^2}$, and that $|F(h) - L| \leq \frac{1}{h^2}$, we can determine that the function converges to -1 at a rate of $O(\frac{1}{h^2})$, where $p = 2$.

Now that the analytical solution for p is similar to the machine calculated p, we can determine, by power-law relation, R.O.C and Taylor series expansion, $p \simeq 2$.