Assignment 3

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Question 1a. Write down the payoff matrix of this game

Table 1: Payoff matrix

	F	U
A	$(2, 2)$	$(3, 1)$
R	$(4, 0)$	$(0, 4)$

Question 1b. Determine if there is any dominant strategy equilibrium and pure Nash equilibrium for this game when the game is played once only.

Proof. There are none dominant strategy equilibrium and pure Nash equilibrium

If (F, A) is chosen, then Ellen will choose U for greater benefit.
If (U, A) is chosen, then Toni will choose R for greater benefit.
If (F, R) is chosen, then Toni will choose A for greater benefit.
If (U, R) is chosen, then Ellen will choose F for greater benefit.
$\therefore$ What ever combination of choice if chosen, one of the players would still like to change there strategy.
$\therefore$ No pure Nash equilibrium exists.
$\because$ Every dominant strategy equilibrium must be a pure Nash equilibrium.
$\therefore$ No dominant strategy equilibrium.

$\square$

Question 1c. Find one mixed (non-pure) Nash equilibrium for this game when it is played many times.

Table 2: Payoff matrix with probability

	F(p)	U(1-p)
A(q)	$(2,2)$	$(3,1)$
R(1-q)	$(4,0)$	$(0,4)$

Proof. The deduction is given below.
Let p be the probability of F is chosen, and q be the probability of A is chosen.
Let E_E and E_T be expected payoff of Ellen and Toni respectively.

$$\begin{aligned}
E_E(p,q) &= 2pq + 3(1-p)q + 4(p)(1-q) + 0(1-p)(1-q) \\
E_E(1,q) &= 2q + 4(1-q) \\
E_E(0,q) &= 3q \\
E_E(p,q) &= 2pq + 3(1-p)q + 4(p)(1-q) + 0(1-p)(1-q) \\
&= 3q(1-p) + p(2q + 4(1-q)) \\
&= (1-p)E_E(0,q) + pE_E(1,q) \\
E_T(p,q) &= 2pq + 1(1-p)q + 0p(1-q) + 4(1-p)(1-q) \\
E_T(p,1) &= 2p(1) + 1(1-p)(1) + 4(1-p)(1-1) \\
&= 2p + (1-p) \\
E_T(p,0) &= 4(1-p)(1-0) \\
&= 4(1-p) \\
E_T(p,q) &= q(2p + (1-p)) + 4(1-p)(1-q) \\
&= qE_T(p,1) + (1-q)E_T(p,0)
\end{aligned}$$

Hence we have the following equations

$$E_E(p*, q*) \geq E_E(p, q*) \tag{1}$$
$$E_T(p*, q*) \geq E_T(p*, q) \tag{2}$$
$$E_E(p,q) = p(E_E(1,q) - E_E(0,q)) + E_E(0,q) \tag{3}$$
$$E_T(p,q) = q(E_T(p,1) - E_T(p,0)) + E_T(p,0) \tag{4}$$

Substituting (3) to (1)

$$\begin{aligned}
p*(E_E(1,q*) - E_E(0,q*)) + E_E(0,q*) &\geq p(E_E(1,q*) - E_E(0,q*)) + E_E(0,q*) \\
(E_E(1,q*) - E_E(0,q*)) &= 0 \\
E_E(1,q*) &= E_E(0,q*) \\
2q* + 4(1-q*) &= 3q* \\
4 - 2q* &= 3q* \\
q* &= 0.8
\end{aligned}$$

Substituting (4) to (2)

$$\begin{aligned}
q*(E_T(p*,1)-E_T(p*,0))+E_T(p*,0) &\geq q(E_T(p*,1)-E_T(p*,0))+E_T(p*,0)\\
(E_T(p*,1)-E_T(p*,0)) &= 0\\
E_T(p*,1) &= E_T(p*,0)\\
2p*+(1-p*) &= 4(1-p*)\\
2p*+1-p* &= 4-4p*\\
5p* &= 3\\
p* &= 0.6
\end{aligned}$$

Verifying combination of $(0.6, 0.8)$

$$\begin{aligned}
E_E(0.6,q) &= 2(0.6)q+3(1-0.6)q+4(0.6)(1-q)\\
&= 1.2q+3+1.2q+2.4-2.4q\\
&= 2.4
\end{aligned}$$

Hence Ellen has no incentive to change his position when $p = 0.6$

$$\begin{aligned}
E_T(p,0.8) &= 2p(0.8)+(1-p)(0.8)+4(1-p)(1-0.8)\\
&= 1.6p+0.8-0.8p+0.8-0.8p\\
&= 1.6
\end{aligned}$$

Hence Toni has no incentive to change his position when $q = 0.8$
Hence $(0.6, 0.8)$ is a mixed Nash equilibrium.

□

Question 2a. Write down the payoff function for each player.

Proof. Below gives the payoff functions □

Table 3: Payoff functions

	Player 1	Player 2
Won	$v_1(B)-b_1$	$v_2(B)-b_2$
Lost	$v_1(M)-4+b_2$	$v_2(M)-4+b_1$

Question 2b. What is the best strategy for player 1 if player 1 knows player 2 is going to submit the bid $b_2 = 1$?

Proof. Below provides the deduction
For $b_2 = 1$,

if player 1 wins, his payoff is $v_1(B) - b_1$, where $b_1 > 1$
If he loses, his payoff is $v_1(M) - 4 + 1 = v_1(M) - 3$

$$\begin{aligned}
&\because v_1(B) > v_1(M) \\
&\therefore \text{There must exists some } b_1 > 1 \text{ that} \\
&\quad v_1(B) - b_1 > v_1(M) - 3
\end{aligned}$$

Therefore, player 1 should make a minimum winning bid.
It is given that for $b_1 = b_2$, player 1 wins.
Hence he should make a bid of $b_1 = 1$ □

Question 2c. For this auction game, is it possible to have a Nash equilibrium of the form (k, k) where $0 \leq k \leq 2$?

Proof. Below discuss such possibility
For a outcome of (k, k),

$$b_1 = b_2 = k \tag{5}$$

Hence player 1 wins with a payoff of

$$v_1(B) - k \tag{6}$$

and player 2 loses with a payoff of

$$v_2(M) - 4 + k \tag{7}$$

However, for player 2, if he/she make a slightly higher bid of $k + \Delta k$, where $\Delta k > 0$ Then he wins with a payoff of

$$v_2(B) - k - \Delta k \tag{8}$$

It is given that

$$v_2(B) > v_2(M) \tag{9}$$

and for $0 \leq k \leq 2$

$$-k \geq -4 + k \tag{10}$$

hence there must exists some Δk that

$$v_2(B) - k - \Delta k > v_2(M) - 4 + k \tag{11}$$

Therefore, for any k where $0 \leq k \leq 2$, player 2 would make a sightly higher bid $k + \Delta k$ to reduce his/her payoff.
Hence no Nash equilibrium exists. □