Assignment 2

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Answer Q. 1.

For eigenvalues of A, we have

$$|\mathbf{A} - \lambda \mathbf{I}| = 0 \tag{1}$$

with

$$\begin{bmatrix} a & b \\ b & -a \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$
$$= \begin{bmatrix} a-\lambda & b \\ b & -a-\lambda \end{bmatrix}$$

hence

$$\begin{vmatrix} a-\lambda & b \\ b & -a-\lambda \end{vmatrix} = 0$$
$$-\left(\alpha+\lambda\right)\left(\alpha-\lambda\right) - b^2 = 0 - a^2 + \lambda^2 - b^2 \qquad = 0$$
$$\lambda^2 = a^2 + b^2$$
$$\lambda = \pm\sqrt{a^2+b^2}$$

For eigenvalue $\sqrt{a^2+b^2}$ and eigenvector v_1 of $\mathbf{A}$, we have

$$\left(\mathbf{A} - \sqrt{a^2+b^2}\mathbf{I}\right)\mathbf{v}_1 = 0$$
$$\begin{bmatrix} a+\sqrt{a^2+b^2} & b \\ b & -a+\sqrt{a^2+b^2} \end{bmatrix}\mathbf{v}_1 = 0$$

Hence we have

$$\left[\begin{array}{cc|c} a+\sqrt{a^2+b^2} & b & 0 \\ b & -a+\sqrt{a^2+b^2} & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & \frac{b}{a+\sqrt{a^2+b^2}} & 0 \\ 1 & \frac{-a+\sqrt{a^2+b^2}}{b} & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 0 & \frac{b}{a+\sqrt{a^2+b^2}} - \frac{-a+\sqrt{a^2+b^2}}{b} & 0 \\ 2 & \frac{b}{a+\sqrt{a^2+b^2}} + \frac{-a+\sqrt{a^2+b^2}}{b} & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 0 & \frac{b\left(\sqrt{a^2+b^2}-a\right)}{\left(a+\sqrt{a^2+b^2}\right)\left(\sqrt{a^2+b^2}-a\right)} - \frac{-a+\sqrt{a^2+b^2}}{b} & 0 \\ 2 & \frac{b\left(\sqrt{a^2+b^2}-a\right)}{\left(a+\sqrt{a^2+b^2}\right)\left(\sqrt{a^2+b^2}-a\right)} + \frac{-a+\sqrt{a^2+b^2}}{b} & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 0 & \frac{\sqrt{a^2+b^2}-a}{b} - \frac{-a+\sqrt{a^2+b^2}}{b} & 0 \\ 2 & \frac{\sqrt{a^2+b^2}-a}{b} + \frac{-a+\sqrt{a^2+b^2}}{b} & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & \frac{a+\sqrt{a^2+b^2}}{b} & 0 \\ 0 & 0 & 0 \end{array}\right]$$

which can be reduced to