Assignment 5

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Q 1a. Model this as a simple game.

A 1a. This game can be modeled as following

$$[13; 10, 5, 2, 5] \tag{1}$$

with voter A, B, C, D as 1, 2, 3, 4 respectively

Q 1b. Does the simple game defined in a) has property M ? Explain your answer.

A 1b. Such game has property M

$\because$ For $\{1,2\}$, which is a winning collation, all $\{1,2\} \subset S$ $(\{1,2,3,4\},\{1,2,4\},\{1,2,3\})$ is a winning collation
and
For $\{1,4\}$, which is a winning collation, all $\{1,4\} \subset S$ $(\{1,2,3,4\},\{1,2,4\},\{1,3,4\})$ is a winning collation
and
For $\{1,3\}$, which is a losing collation, all $S \subset \{1,3\}$ $(\{1\},\{3\})$ is a losing collation
$\therefore$ by the definition of property M, such game has property M

Q 1c. Are B and D symmetric players for the simple game defined in a)? Explain your answer.

A 1c. B and D are symmetric players because they have equal values of 5.

Q 1d. Find the Shapley-Shubikindex of each voter.

A 1d.

There are totally $4! = 24$ colations
For player 1,
player 1 is pivotal when N_σ^i is $\{2\}, \{4\}, \{2,3\}, \{2,4\}$ and $\{3,4\}$
Hence number of permutation that player 1 is pivotal is given by

$$2 \times 1! \times 2! + 3 \times 2! \times 1! = 10 \tag{2}$$

Hence SSI(1) $= \frac{10}{24} = \frac{5}{12}$ For player 2,
player 2 is pivotal when N_σ^i is $\{1\}, \{1,3\}$ and $\{3,4\}$
Hence number of permutation that player 2 is pivotal is given by

$$1 \times 1! \times 2! + 2 \times 2! \times 1! = 6 \tag{3}$$

Since player B and D are symmetric players, both SSI(2) and SSI(4) is $\frac{6}{24} = \frac{1}{4}$
For player 3,
player 3 is pivotal when N_σ^i is $\{2,4\}$
Hence number of permutation that player 3 is pivotal is given by

$$1 \times 1! \times 2! = 2 \tag{4}$$

Hence SSI(3) $= \frac{2}{24} = \frac{1}{12}$

Q 2a. Show that a small state (non-permanent member) is a pivotal player for a permutation of the 15 states when seven small states come last in the permutation.

A 2a.

When 7 small states come last,
The first $15 - 7 = 8$ players must includes all five big states, hence there are no veto situations.
And the 9th player, which is the 1st one of the 7 small states, becomes a pivotal player because her she has the choice of wheather become the 9th player to a yes vote, which passes the motion.

Q 2b. Show that a permutation of the 15 states in which a small state (non-permanent member) is a pivotal player can occur only if seven small states come last in the permutation.

A 2b.

For a small state i to be a pivotal member in σ,
The 5 big state must be in front of i, since passage requires the agreement of 5 big states.
Also, to statisfy the condition of passage of 9 votes, another 3 small state must be in front of i as well, so that i becomes a pivotal player as mentioned in (a).
Therefore, the remaining last $15 - 5 - 3 = 7$ members must be small states. Hence a small state (non-permanent member) is a pivotal player can occur only if seven small states come last in the permutation.

Q 2c. Show that the SSI of any nonpermanent member is 0.00186.

A 2c.

From (b), a small state (non-permanent member) is a pivotal player can occur only if seven small states come last in the permutation.
Therefore, the first 8 players of such permutation σ must include 5 big states and 3 of 10 small states.
And the last 7 players of σ must include 7 of 10 small states.
Hence number of permutations statisfying such requirements is given by

$$8! \times {}_{10}\mathrm{P}_7 = 2.4385536 \times 10^{10} \tag{5}$$

Hence such SSI is given by $2.4385536 \times 10^{10} \div 15! = 0.0186$

Q 2d. Find the SSI of any permanent member.

A 2d. Such SSI is given by $\frac{1-0.0186\times 10}{5} = 0.163$