Assignment 9

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Question Q. 1. Show that the 13-digit ISBN13 system (a) can detect all 1-errors, (b) but not all 2-errors come from transposition of digits.

Answer Q. 1.
Let such ISBN13 code be $n = n_1 n_2 n_3 \dots n_{13}$.
Let the error digit be n_i, which is mistakenly changed to m_i, where $n_i \neq m_i$
Let the new (error) value be T, and original (correct) value be S

Consider case 1, i is a odd number,
Then we have $T - S = 1 \times m_i - 1 \times n_i = m_i - n_i$
Since $0 \leq m_i, n_i \leq 9$ and $m_i \neq n_i$, we must have $m_i - n_i \not\equiv 0 \pmod{10}$

Consider case 2, i is a even number,
Then we have $T - S = 3 \times m_i - 3 \times n_i = 3\,(m_i - n_i)$
Since $m_i - n_i \not\equiv 0 \pmod{10}$ and $3 \perp 10$, we must have $3\,(m_i - n_i) \not\equiv 0 \pmod{10}$

Because all ISBN13 code with 1-digit error will not satsify the detection condition,
Therefore ISBN13 can detect all 1-digit errors.

Consider a transposition of 2-digits, n_i and n_{i+1},
Then we have $T - S = \pm\,(3n_i + n_{i+1} - n_i - 3n_{i+1}) = \pm 2\,(n_i - n_{i+1})$
Therfore, we have 1 example for it: 2770000000000 have the first and second digit swapped into 7270000000000.
The sum of first code is $2 + 7 \times 3 + 7 = 30$, which is divisible by 10.
The sum of second code is $7 + 2 \times 3 + 7 = 20$, which is also devisible by 10.
Therefore, it cannot check all 2-digit errors due to transposition.

Question Q. 2. Show that the codeword a cannot be a valid codeword.

Answer Q. 2.
According to the lecture notes, the minimum distance of Hamming (7,4) code is 3.

Therefore, for any valid code d, there does not exists a valid code that have a distance less than 3.
Hence a, which only differs from d by 2 bits, cannot be a valid one.