Assignment 9

Michael Lee
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University Number 3035569110
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Question Q. 1. Show that the 13-digit ISBN13 system (a) can detect all 1-errors, (b) but not all 2-errors come from transposition of digits.

Answer Q. 1.
Let such ISBN13 code be $n = n_1 n_2 n_3 \ldots n_{13}$.
Let the error digit be n_i, which is mistakenly changed to m_i, where $n_i \neq m_i$
Let the new (error) value be T, and original (correct) value be S

Consider case 1, i is a odd number,
Then we have $T - S = 1 \times m_i - 1 \times n_i = m_i - n_i$
Since $0 \leq m_i, n_i \leq 9$ and $m_i \neq n_i$, we must have $m_i - n_i \not\equiv 0 \pmod{10}$

Consider case 2, i is a even number,
Then we have $T - S = 3 \times m_i - 3 \times n_i = 3\,(m_i - n_i)$
Since $m_i - n_i \not\equiv 0 \pmod{10}$ and $3 \perp 10$, we must have $3\,(m_i - n_i) \not\equiv 0 \pmod{10}$

Because all ISBN13 code with 1-digit error will not satsify the detection condition,
Therefore ISBN13 can detect all 1-digit errors.

Consider a transposition of 2-digits, n_i and n_{i+1}, where
Consider case 1, i is a odd number,
Then we have $T - S = \pm\,(3n_i + n_{i+1} - n_i - 3n_{i+1} = \pm 2\,(n_i - n_{i+1})$

$$x^2 + y^2 = 1$$
$$y = \sqrt{1 - x^2}$$

Question Q. 2. Heuristic Reason for Benfords Law

Answer Q. 2.
Considering that
$$100d(1 + r\%)^{f(d)} = 100(d + 1) \tag{1}$$

For d is a interger between 1 to 9.
We have

$$d(1+r\%)^{f(d)} = (d+1) \tag{2}$$

$$(1+r\%)^{f(d)} = 1+1/d \tag{3}$$

$$f(d) = \log(1+1/d)/\log(1+r\%) \tag{4}$$

The total time F for all digit changes is given by

$$F = \sum_{n=d}^{9} f(d) \tag{5}$$

$$= \left(\sum_{n=d}^{9} \log(1+1/d)\right) / \log(1+r\%) \tag{6}$$

$$= \log 10/\log(1+r\%) \tag{7}$$

$$= 1/\log(1+r\%) \tag{8}$$

Hence

$$P(\text{first digit} = d) = f(d)/F = \log(1+1/d) \tag{9}$$