CMSC420 Advanced Data Structures

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Contents

1 Lists

```
init() => initializes list
get(i) => returns element at index i
set(i, x) => sets ith element to x
length() => returns number of elements in the list
insert(i, x) => insert x prior to element a_{i} (shifts indices after)
delete(i) => deletes ith element (shift indices after)
```

Sequential Allocation (Array): when array is full, increase its size but a constant factor (e.g. 2). Amortized array operations still O(1)

Linked Allocation (Linked List)

Stack(push, pop): on on end of the list
Queue(enqueue, dequeue): insert at tail (end) and remove from head (start)
Deque(combo stack and queue): can isnert and remove from either ends of list
Multilist: multiple lists combined 1 aggregate structure (e.g. ArrayList)

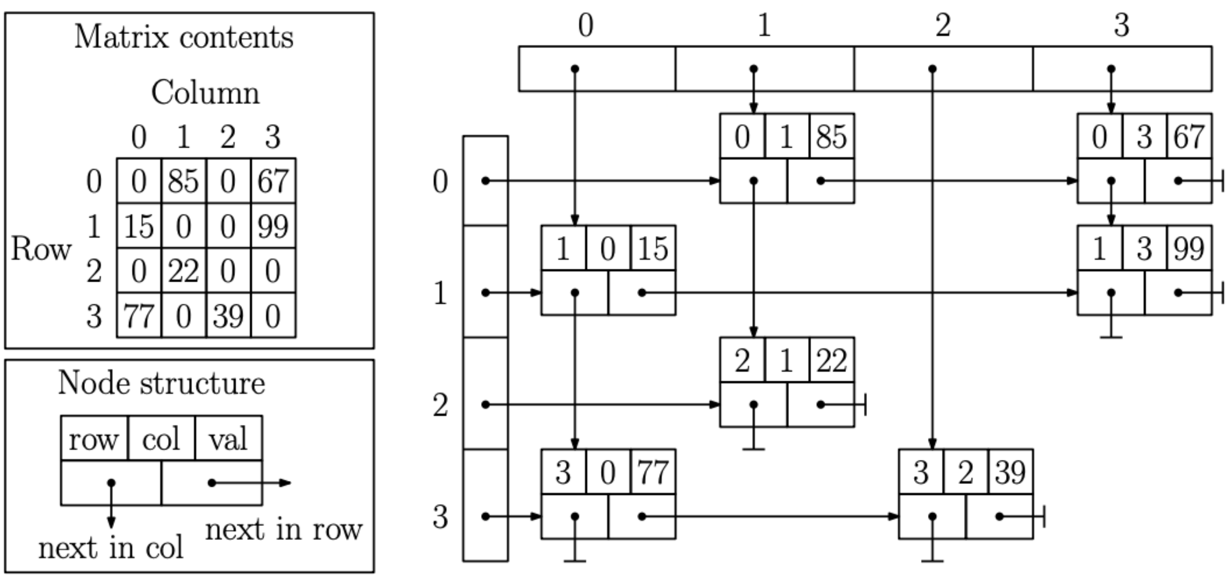


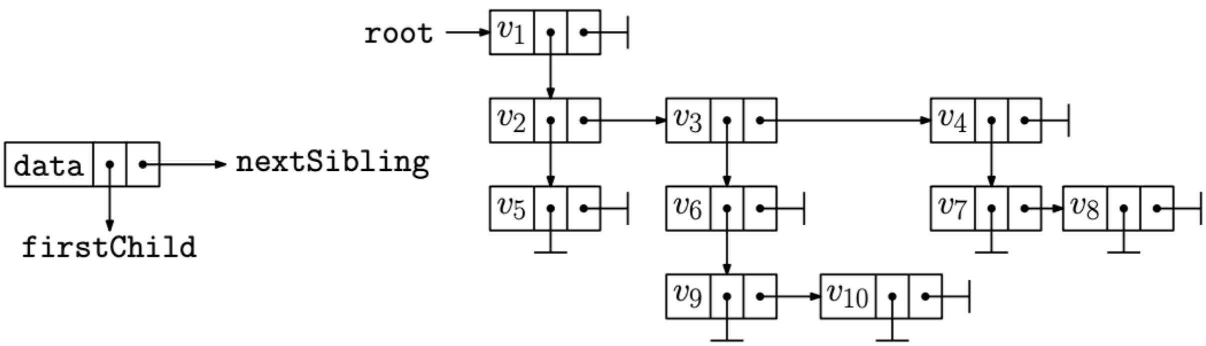
Fig. 2: Sparse matrix representation using a multilist structure.

Sparse Matrix: create 2n linked lists for each row and col
Each entry stores a row index, col index, value, next row ptr, and next col ptr

2 Trees

Free Tree: connected, undirected graph with no cycles (like MST)
Root Tree: each non-leaf node has ≥ 1 children and a single parent (except root)
    Aborescence = out-tree   Anti-arborescence = in-tree
    Depth = max # of edges of path from root to a node

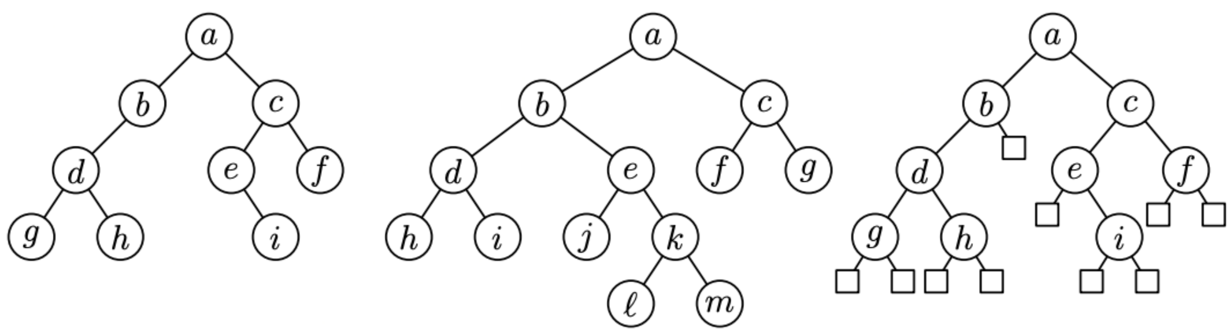
One way to represent tree is to have a pointer to first child and then a pointer to next sibling



(a)   (b)

Fig. 3: Standard (binary) representation of rooted trees.

Binary Tree: rooted, ordered tree where each non-leaf node has 2 possible children (left, right)
    Full Tree: All nodes either have 0 children or 2 children
    Can make full binary tree by extending tree by adding external nodes to replace all empty subtrees



(a)   (b)   (c)

Fig. 4: Binary trees: (a) standard definition, (b) full binary tree, (c) extended binary tree.

```
class BinaryTreeNode<E> {
  private E entry;
  private BinaryTreeNode<E> left;
  private BinaryTreeNode<E> right;
  ...
}
```

In-order traversal: left, root, right
Pre-order traversal: root, left, right
Post-order traversal: left, right, root

If there are n internal nodes in an extended tree, there are n+1 external nodes

Proof by induction: Extended tree binary tree with n internal nodes has n+1 external nodes has 2n+1 total nodes

Let x(n) = number of external nodes given n internal nodes and prove x(n) = n + 1

Base Case x(0) = 1 a tree with no internal nodes has 1 external node

IH: Assume x(i) = i + 1 for all i $\leq$ n - 1

IS: let n_L and n_R be the number of nodes in Left and Right subtrees

x(n) = $(n_L + 1) + (n_R + 1) = (1 + n_L + n_R) + 1$ = n + 1 external nodes

so n + 1 (external) + n (internal) = 2n + 1

Moreover, about 1/2 of nodes of extended Binary Tree are leaf nodes

Threaded Binary Tree: Give null pointers information about where to traverse next

If left-child = null then stores reference to node's inorder predecessor

If right-child = null then stores references to node's inorder successor

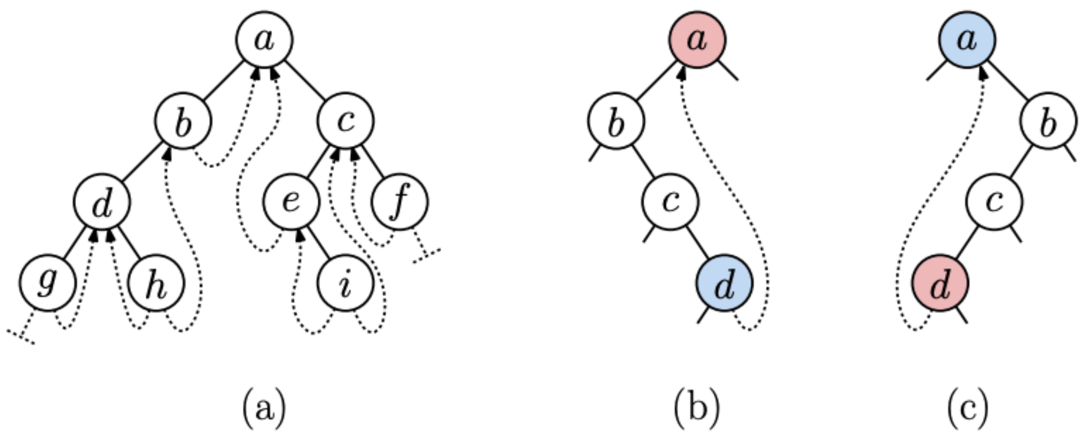


Fig. 6: A Threaded Tree.