Chapter 1-1

1. Show $cO = O$

$cO = c(O + O) = cO + cO$

Adding the additive inverse $-cO$ to both sides, we get $cO = 0$

2. Let $c \neq 0$ and $v \in V$. Show $cv = O \implies v = O$

Note that $cv = cv_1 + \cdots + cv_n$

Since $c \neq 0$ the only way $cv = 0$ is for each $v_i = 0$. Thus $v = O$

3. What is the additivte identity in the vector space of functions

$0(x0$ is the additive identity

4. Show for $v, w \in V$, $v + w = O \implies w = -v$

We have $v + w = (v_1 + w_1) + \cdots + (v_n + w_n) = O$

Thus $v_i + w_i = 0$ for each $i \implies v_i = -w_i$

But then this implies that $w = -v$

5. Show for $v, w \in V$, $v + w = v \implies w = O$

We have $v + w = (v_1 + w_1) + \cdots + (v_n + w_n) = v_1 + \cdots + v_n$

But then this implies $w_i = 0$

Thus $w = O$

6.: Let $A_1, A_2 \in R^n$. Show that the set of all vectors $B \in R^n$ such that B is perpendicular to both A_1, A_2 is a subspace

- Clearly $O \in B$ since $A_1 \cdot O = A_2 \cdot O = 0$
- Let $X, Y \in B$. Then $A_1(X + Y) = A_1X + A_1Y = 0$ and $A_2(X + Y) = A_2X + A_2Y = 0$
- Let $c \in R$. Then $A_1(cX) = cA_1X = 0$ and $A_2(cX) = cA_2X = 0$

8. Show that the following sets of elements in R^2 form subspaces

- (x, y) such that $x = y$
  - Clearly $O \in U$ since $(0, 0) \implies 0 = 0$
  - For $A, B \in U$ we see that $A + B = (a + b, a + b) \implies a + b = a + b$
  - For $c \in R$, we see that $cA = (ca, ca) \implies ca = ca$
- (x, y) such that $x - y = 0$
  - Clearly $O \in U$ since $(0, 0) \implies 0 - 0 = 0$
  - For $A, B \in U$ we see that $A + B = (a_1 + b_1, a_2 + b_2) \implies a_1 + b_1 - a_2 - b_2 = 0$
  - For $c \in R$, we see that $cA = (ca_1, ca_2) \implies ca_1 - ca_2 = c(a_1 - a_2) = 0$
- (x, y) such that $x + 4y = 0$
  - Clearly $O \in U$ since $(0, 0) \implies 0 - 4 * 0 = 0$
  - For $A, B \in U$ we see that $A + B = (a_1 + b_1, a_2 + b_2) \implies (a_1 + b_1) + 4 * (a_2 + b_2) = 0$
  - For $c \in R$, we see that $cA = (ca_1, ca_2) \implies ca_1 + 4ca_2 = 0$

10. If U, W are subspaces of V show that $U \cap W$ and $U + W$ are subspaces

- $U \cap W$
  - Clearly $O \in U$ and $O \in W \implies O \in U \cap W$
  - For $v, w \in U \cap W$, we have that $u + w \in U$ and $u + w \in W$ since they are both subspaces. Thus $u + w \in U \cap W$
  - For $c \in K$, we have $cv \in U$ and $cv \in W$ since they are both subspaces. Thus $cv \in U \cap W$
- $U + W$
  - Clearly $O \in U + W$ by taking $O \in U$ and $O \in W$
  - For $v, w \in U + W$, we have $v + w = (v_U + w_U) + (v_W + w_W)$ which belong to their respective subspaces

- For $c \in K$, we have $cv = cv_U + cv_W$ which belong to their respective subspaces