**k-Permutation**: arrangement of $k$ elments from a set of $n$ elements $\quad P(n,k) = \dfrac{n!}{(n-k)!}$

**Permutation With Repetition**: Can permute each object type $a_i!$ times $\quad \dbinom{n}{a_1, a_2, \ldots, a_k} = \dfrac{n!}{a_1! \cdots a_k!}$

**Combination**: Total number of ways to create a $k$-element subset of $[n]$ $\quad \dbinom{n}{k} = \dfrac{P(n,k)}{k!}$

- This comes from being able to permute the $k$-subset $k!$ ways

**Binomial Theorem**: $(x+y)^n = \displaystyle\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$

**Multinomial Theorem**: $(x_1 + \cdots + x_k)^n = \displaystyle\sum_{\substack{a_1+\cdots+a_k=n \\ a_1,\ldots,a_k \geq 0}} \binom{n}{a_1, a_2, \ldots, a_k} x_1^{a_1} \cdots x_k^{a_k}$

**Pigeon Hole Principle**: If $n$ pigeons are placed into $k$ holes, then at least one hole has at least $\lceil \dfrac{n}{k} \rceil$ (round up)

**Weak Composition**: Ordered $k$-tuple $(a_1, \ldots, a_k)$ such that $a_i \geq 0$ and $\displaystyle\sum_{i=1}^{k} a_i = n$ $\quad \dbinom{n+k-1}{k-1}$

**Compositions**: Ordered $k$-tuple $(a_1, \ldots, a_k)$ such that $a_i \geq 1$ and $\displaystyle\sum_{i=1}^{k} a_i = n$ $\quad \dbinom{n-1}{k-1}$

**Partition of [n]**: $\{A_1, \ldots, A_k\}$ such that blocks are pairwise disjoint and $\displaystyle\bigcup_{i=1}^{k} A_i = X$ $\quad S(n,k) = S(n-1,k-1) + kS(n-1,k)$

**Bell's Number**: Total number of partitions of $[n]$ into any sized blocks $\quad B(n) = \displaystyle\sum_{k=1}^{n} S(n,k) = \sum_{i=1}^{n} \binom{n-1}{i-1} B(n-i)$

**Partition of n**: $(a_1, \ldots, a_k)$ such that $a_1 \geq \cdots \geq a_k$ and $\displaystyle\sum_{i=1}^{k} a_i = n$ $\quad$ total: $p(n)$ $\quad$ $k$-parts: $p_k(n) = p_{k-1}(n-1) + p_k(n-k)$

- Represented using **Ferrers Diagram**: partial rectangular grid with $k$ rows, each with $a_i$ squares (conjugate is also valid)

**Twelvefold Way Counting**

- $n$ labelled balls into $k$ labelled bins: $\quad k^n \quad k!S(n,k) \quad P(n,k)$
- $n$ unlabelled balls into $k$ labelled bins: $\dbinom{n+k-1}{k-1} \quad \dbinom{n-1}{k-1} \quad \dbinom{k}{n}$
- $n$ labelled balls into $k$ unlabelled bins: $\sum_{i=1}^{k} S(n,i) \quad S(n,k) \quad 1$
- $n$ unlabelled balls into $k$ unlabelled bins: $\sum_{i=1}^{k} p_i(n) \quad p_k(n) \quad 1$

**Inclusion-Exclusion Principle**: $\left|\bigcup_{i=1}^{n} A_i\right| = |X| - \displaystyle\sum_{I \subseteq [n]} (-1)^{|I|} |A_I| = \left|\bigcap_{i=1}^{n} \bar{A}_i\right| = |X| - \sum_{I \subseteq [n]} (-1)^{|I|} |A_I|$

**OGF**: $F(x) = \sum_{n=0}^{\infty} a_n x^n \qquad \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \qquad \sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$