k-Permutation: arrangement of k elments from a set of n elements $P(n,k) = \dfrac{n!}{(n-k)!}$

Permutation With Repetition: Can permute each object type $a_i!$ times $\dbinom{n}{a_1, a_2, \ldots, a_k} = \dfrac{n!}{a_1! \cdots a_k!}$

Combination: Total number of ways to create a k-element subset of $[n]$ $\dbinom{n}{k} = \dfrac{P(n,k)}{k!}$

- This comes from being able to permute the k-subset $k!$ ways

Binomial Theorem: $(x+y)^n = \displaystyle\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$

Multinomial Theorem: $(x_1 + \cdots + x_k)^n = \displaystyle\sum_{\substack{a_1+\cdots+a_k=n \\ a_1,\ldots,a_k \geq 0}} \binom{n}{a_1, a_2, \ldots, a_k} x_1^{a_1} \cdots x_k^{a_k}$

Pigeon Hole Principle: If n pigeons are placed into k holes, then at least one hole has at least $\lceil \dfrac{n}{k} \rceil$ (round up)

Weak Composition: Ordered k-tuple $(a_1, \ldots, a_k)$ such that $a_i \geq 0$ and $\displaystyle\sum_{i=1}^{k} a_i = n$ $\dbinom{n+k-1}{k-1}$

Compositions: Ordered k-tuple $(a_1, \ldots, a_k)$ such that $a_i \geq 1$ and $\displaystyle\sum_{i=1}^{k} a_i = n$ $\dbinom{n-1}{k-1}$

Partition of [n]: $\{A_1, \ldots, A_k\}$ such that blocks are pairwise disjoint and $\displaystyle\bigcup_{i=1}^{k} A_i = X$ $S(n,k) = S(n-1,k-1) + kS(n-1,k)$

Bell's Number: Total number of partitions of $[n]$ into any sized blocks $B(n) = \displaystyle\sum_{k=1}^{n} S(n,k) = \sum_{i=1}^{n} \binom{n-1}{i-1} B(n-i)$

Partition of n: $(a_1, \ldots, a_k)$ such that $a_1 \geq \cdots \geq a_k$ and $\displaystyle\sum_{i=1}^{k} a_i = n$ total: $p(n)$ k-parts: $p_k(n) = p_{k-1}(n-1) + p_k(n-k)$

- Represented using **Ferrers Diagram**: partial rectangular grid with k rows, each with a_i squares (conjugate is also valid)

Twelvefold Way Counting

- n labelled balls into k labelled bins: k^n $k!S(n,k)$ $P(n,k)$
- n unlabelled balls into k labelled bins: $\dbinom{n+k-1}{k-1}$ $\dbinom{n-1}{k-1}$ $\dbinom{k}{n}$
- n labelled balls into k unlabelled bins: $\sum_{i=1}^{k} S(n,i)$ $S(n,k)$ 1
- n unlabelled balls into k unlabelled bins: $\sum_{i=1}^{k} p_i(n)$ $p_k(n)$ 1

Inclusion-Exclusion Principle: $\left| \bigcup_{i=1}^{n} A_i \right| = |X| - \displaystyle\sum_{I \subseteq [n]} (-1)^{|I|} |A_I| = \left| \bigcap_{i=1}^{n} \bar{A}_i \right| = |X| - \sum_{I \subseteq [n]} (-1)^{|I|} |A_I|$

OGF: $F(x) = \sum_{n=0}^{\infty} a_n x^n$ $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$

Power Series Formulas: $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ $\qquad (1+x)^a = \sum_{n=0}^{\infty} \binom{a}{n} x^n$ $\qquad \sum_{n=1}^{\infty} n x^{n-1} = \left(\sum_{n=0}^{\infty} x^n\right)' = \frac{1}{(1-x)^2}$

OGF: $\sum_{n=0}^{\infty} a_n x^n$ $\qquad (AB)(x) = \sum_{n=0}^{\infty} \left(\sum_{i=0}^{n} a_i b_{n-i}\right) x^n$ $\qquad$ **EGF**: $\sum_{n=0}^{\infty} a_n \frac{x^n}{n!}$ $\qquad (AB)(x) = \sum_{n=0}^{\infty} \left(\sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}\right) \frac{x^n}{n!}$