# ps4_problem2

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## 0.1 IDS/ACM/CS 158: Fundamentals of Statistical Learning

### 0.1.1 PS4, Problem 2b: Decision Boundary of the Bayes Classifier

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Notes: Please use python 3.6

You are required to properly comment and organize your code.

- Helper functions (add/remove part label according to the specific question requirements)

[1]:

```python
import numpy as np
import matplotlib.pyplot as plt
```

- Part b

[2]:

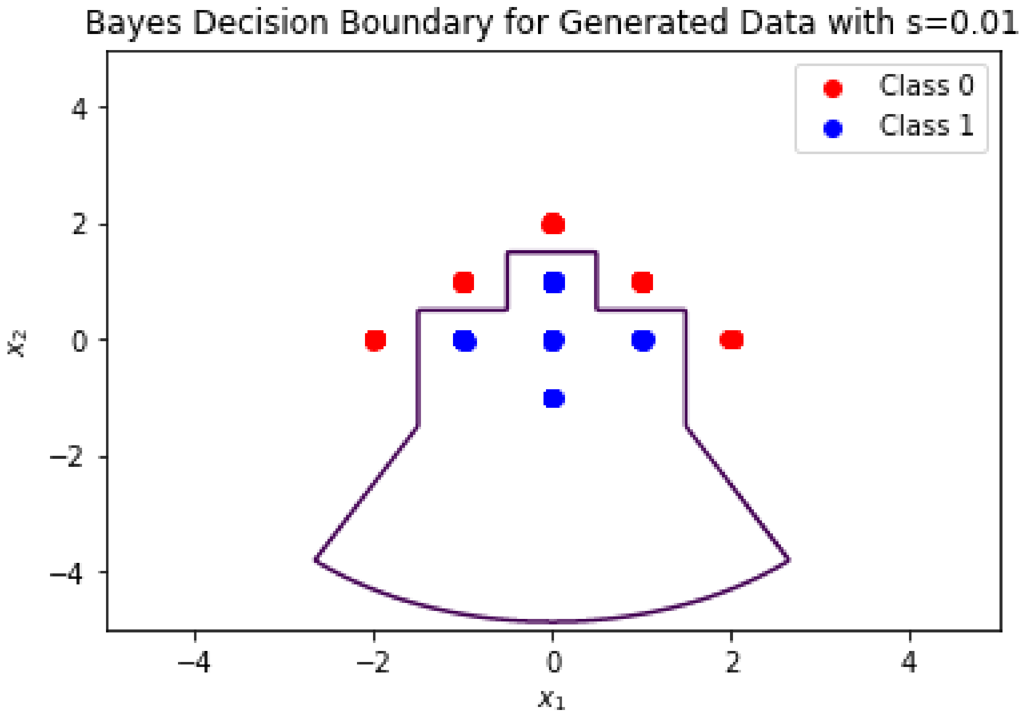
```python
# define our constants
p = 2
n = 100
us = [
    [-2, 0],
    [-1, 1],
    [0, 2],
    [1, 1],
    [2, 0]
]

vs = [
    [0, 1],
    [-1, 0],
    [0, 0],
    [1, 0],
    [0, -1]
]

ss = [.01, .1, 1]
```

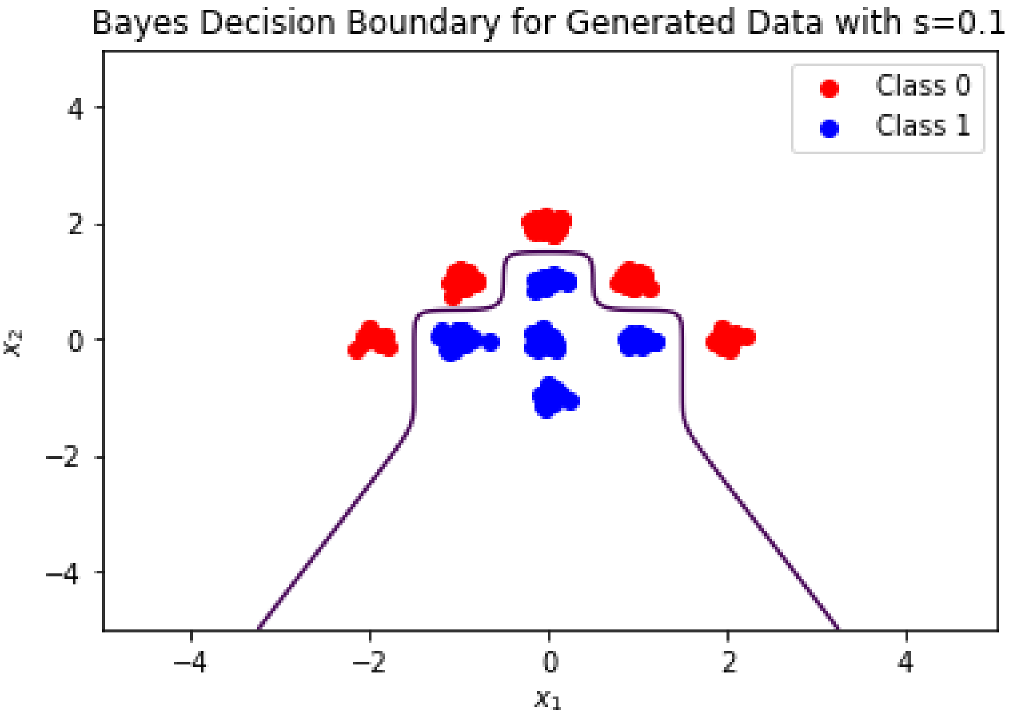
[3]:

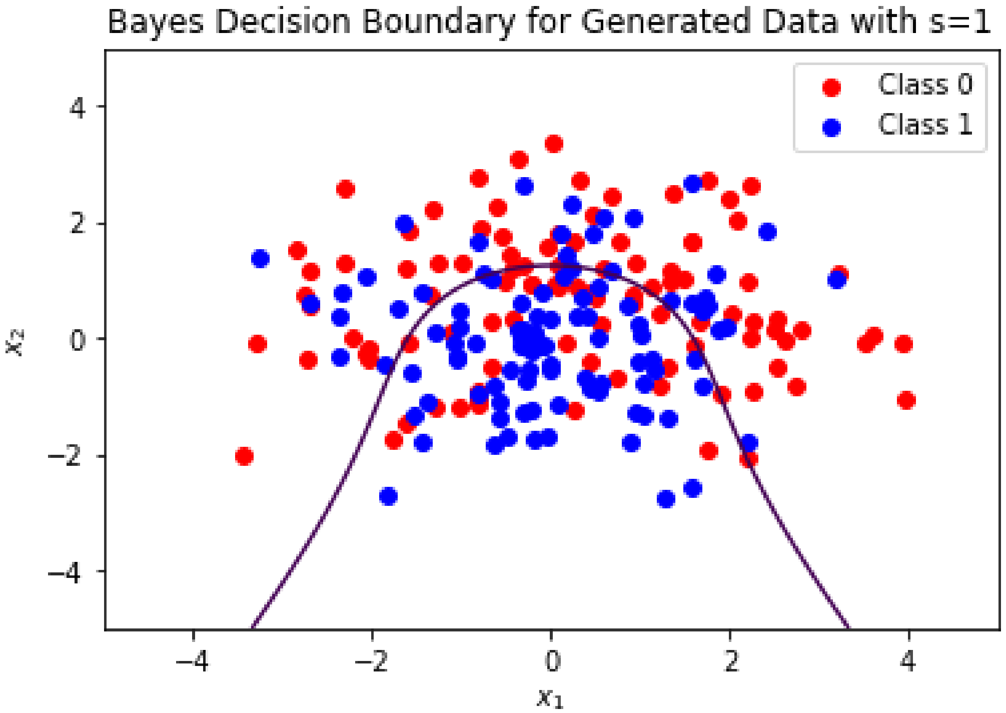
```python
def boundary(x, s, us, vs):
    """
    x - candidate point for boundary point
    s - scalar for variance
    us - list of mean vectors for class 0
    vs - list of mean vectors for class 1

    returns the derived quantity from part 2a
    """
    class_1_sum = 0
    class_0_sum = 0

    for i in range(len(us)):
        class_0_sum += np.exp(-1/(2*s) * ((x[0]-us[i][0])**2 + (x[1]-us[i][1])**2))
        class_1_sum += np.exp(-1/(2*s) * ((x[0]-vs[i][0])**2 + (x[1]-vs[i][1])**2))

    return class_1_sum - class_0_sum
```

[4]:

```python
for s in ss:
    class_0 = []
    class_1 = []

    for i in range(n):
        # for each point we randomly pick a mean vector
        rand_u = us[np.random.choice(len(us))]
        rand_v = vs[np.random.choice(len(vs))]

        # generate random point with randomly seleted mean and variance sI_p
        class_0.append(np.random.normal(rand_u, s))
        class_1.append(np.random.normal(rand_v, s))

    # get the points for the boundary
    class_0 = np.array(class_0)
    class_1 = np.array(class_1)

    x_array = np.arange(-5, 5.01, 0.01)
    y_array = np.arange(-5, 5.01, 0.01)
    X,Y = np.meshgrid(x_array, y_array)
    Z = np.ndarray((1001, 1001))

    for i in range(1001):
        for j in range(1001):
            Z[i][j] = boundary([X[i][j], Y[i][j]], s, us, vs)

    plt.contour(X, Y, Z, [0])
    plt.scatter(class_0[:,0], class_0[:,1], c='red', label='Class 0')
    plt.scatter(class_1[:,0], class_1[:,1], c='blue', label='Class 1')
    plt.xlabel('$x_1$')
    plt.ylabel('$x_2$')
    plt.title('Bayes Decision Boundary for Generated Data with s={}'.format(s))
    plt.legend()
    plt.show()
```




Bayes Decision Boundary for Generated Data with s=0.1
Class 0
Class 1
4
2
0
−2
−4
$x_2$
−4
−2
0
2
4
$x_1$


Bayes Decision Boundary for Generated Data with s=1
Class 0
Class 1
4
2
0
−2
−4
$x_2$
−4
−2
0
2
4
$x_1$