CS1010 Programming Methodology

http://www.comp.nus.edu.sg/~cs1010/

UNIT 3

# Algorithmic Problem Solving

NUS National University of Singapore | School of Computing

# Unit 3: Algorithmic Problem Solving

1. Computational Thinking: Coin Change
2. Euclid's Algorithm
3. Algorithmic Problem Solving Process
4. Algorithm
5. Control Structures
6. Examples of Pseudocodes

# Coin Change


## Problem statement

Given the above coin denominations: 1¢, 5¢, 10¢, 20¢, 50¢, and $1, assuming that you have unlimited supply of them, find the minimum number of coins needed for a given amount.

This is called **optimisation** problem in CS – finding the best among all possible solutions.

*Examples:*

- 375¢ → ? coins
- 543¢ → ? coins

*Question you may ask:*
Do I need to report how many coins of each denomination in my solution?


No, you don't have to. (But in the process of computing the solution you will somehow get to know how many coins of each denomination you need.)

# Contract of a Task

The Task Giver

The amount (in ¢) →

← The min. no. of coins

The Task Solver

**The Task Giver:**

- Provides the necessary inputs (arguments) to the Task Solver
- Does not provides unnecessary data to the Task Solver
- Does not care/need to know how the Task Solver solves the task

**The Task Solver:**

- Accepts the necessary inputs (arguments) from the Task Giver
- Returns the result to the Task Giver
- Does not provides extraneous data to the Task Giver or perform extraneous work

# Euclid's Algorithm (1/3)

- First historically documented algorithm by Greek mathematician Euclid in 300 B.C.
- Also known as Euclidean Algorithm

1. Let $A$ and $B$ be integers with $A > B \geq 0$.
2. If $B = 0$, then the GCD is $A$ and algorithm ends.
3. Otherwise, find $q$ and $r$ such that

$$A = q.B + r \quad \text{where } 0 \leq r < B$$

4. Replace $A$ by $B$, and $B$ by $r$. Go to step 2.

- $q$ is not important; $r$ is the one that matters.

- $r$ could be obtained by $A$ modulo $B$ (i.e. remainder of A / B)

- Assumption on $A > B$ unnecessary

- We will rewrite the algorithm

# Euclid’s Algorithm (2/3)

1. Let $A$ and $B$ be integers with $A > B \geq 0$.
2. If $B = 0$, then the GCD is $A$ and algorithm ends.
3. Otherwise, find $q$ and $r$ such that

   $$A = q.B + r \qquad \text{where } 0 \leq r < B$$

4. Replace $A$ by $B$, and $B$ by $r$. Go to step 2.

Rewritten in modern form

```
// Pre-cond: A and B are non-negative
// integers, but not both zeroes.

Algorithm GCD(A, B) {
    while (B > 0) {
        r ← A modulo B
        A ← B
        B ← r
    }
    return A
}
```

What the Task Giver must provide to this Task Solver

Details of Task Solver unknown to Task Giver

What this Task Solver returns to the Task Giver.

Pre-condition: What must be true for this Task Solver to work.

# Euclid's Algorithm (3/3)

You might have guess what this algorithm does from its name but let's trace it with some examples.

*Very important skill!*

```
// Pre-cond: A and B are non-negative
// integers, but not both zeroes.

Algorithm GCD(A, B) {
    while (B > 0) {
        r ← A modulo B
        A ← B
        B ← r
    }
    return A
}
```

So do you know what does the Euclid's Algorithm do now?

Let's trace GCD(12, 42)

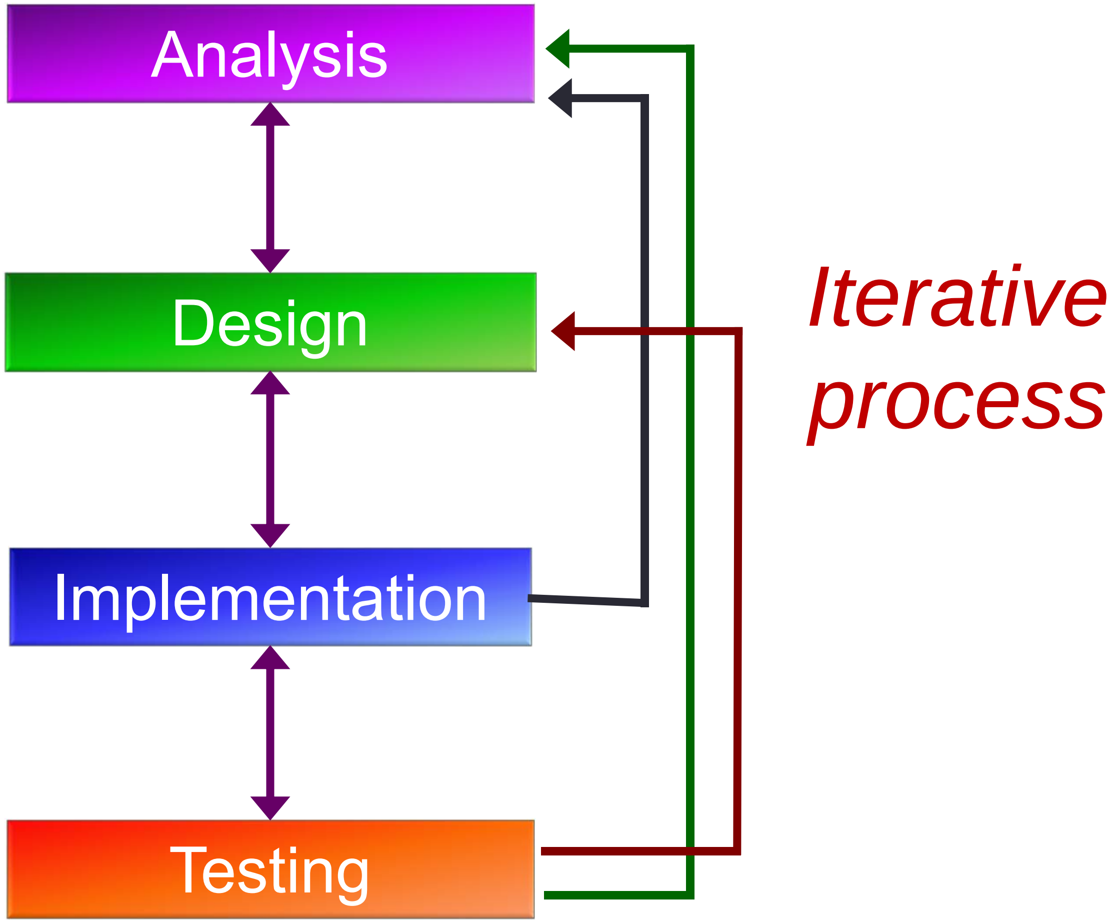
| (B > 0)? | r | A | B |
|---|---|---|---|
| | | 12 | 42 |
| true | 12 | 42 | 12 |
| true | 6 | 12 | 6 |
| true | 0 | 6 | 0 |
| false | | | |

Result returned: **?**

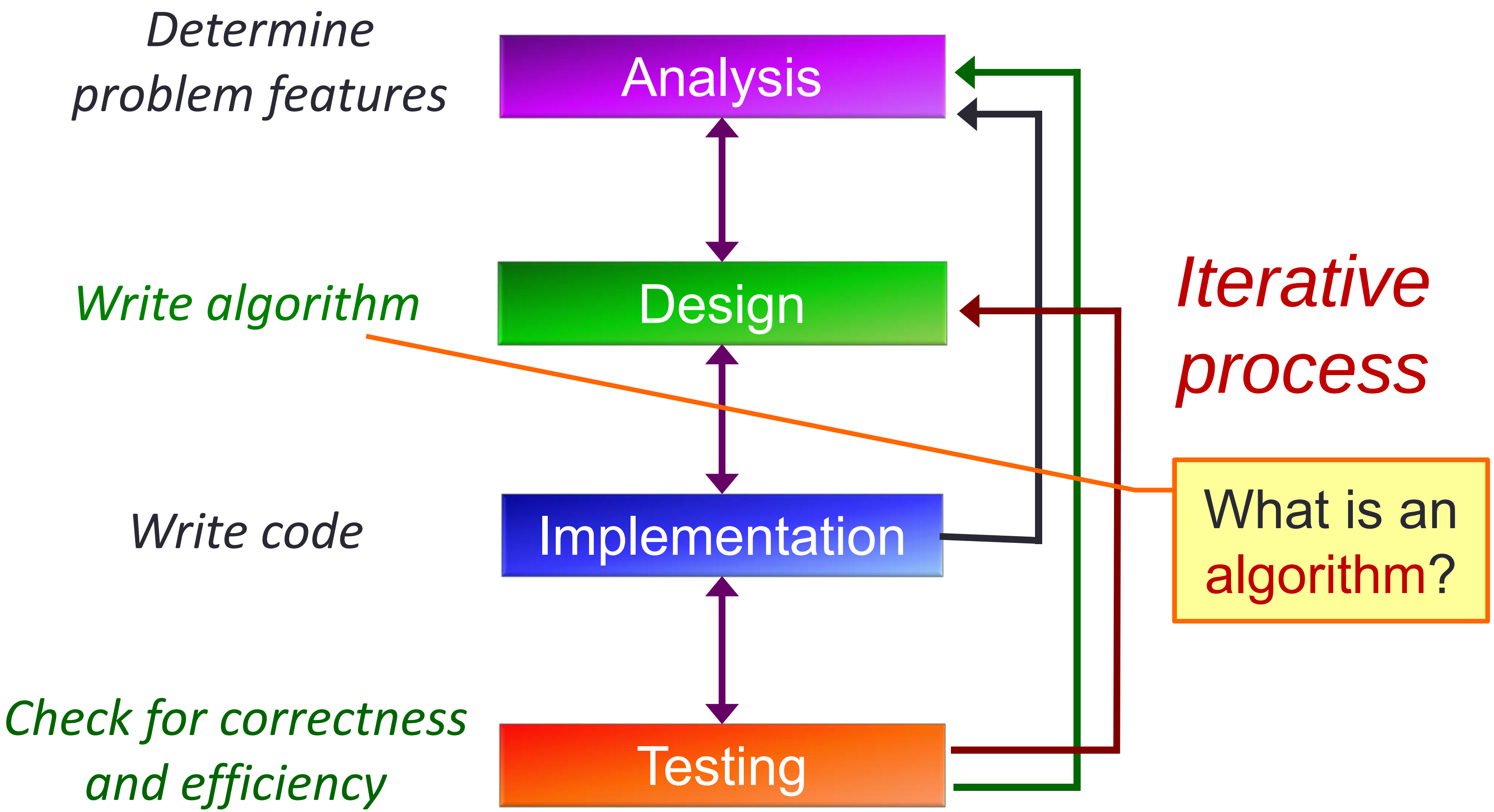
# Polya’s Problem Solving Process

We (roughly)map Polya’s steps to algorithmic problem solving.

1. Understand the Problem
2. Make a Plan
3. Do the Plan
4. Look back



# Algorithmic Problem Solving



# Algorithm (1/3)

- An **algorithm** is a well-defined computational procedure consisting of *a set of instructions*, that takes some value or set of values as *input*, and produces some value or set of values as *output.*

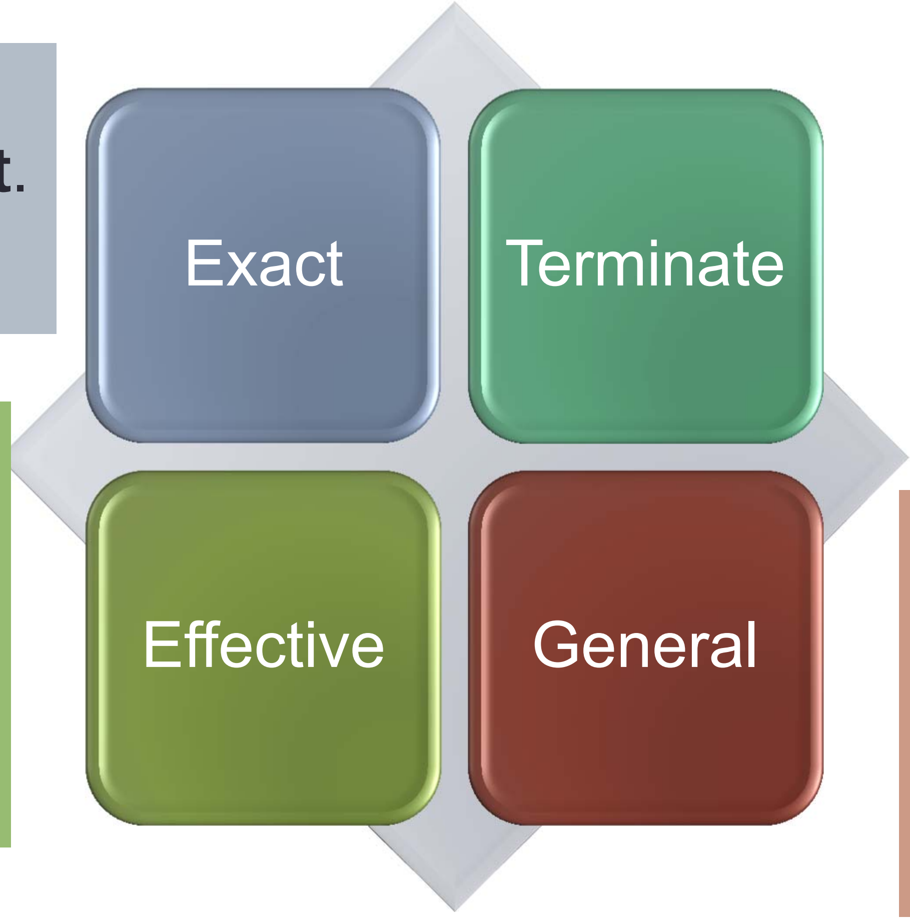

'Algorithm' stems from 'Algoritmi', the Latin form of al-Khwārizmī, a Persian mathematician, astronomer and geographer.
Source: http://en.wikipedia.org/wiki/Algorithm

# Algorithm (2/3)

- An **algorithm** has these properties:

Each step must be **exact**. (Or it will not be precise.)

The algorithm must **terminate**. (Or no solution will be obtained.)



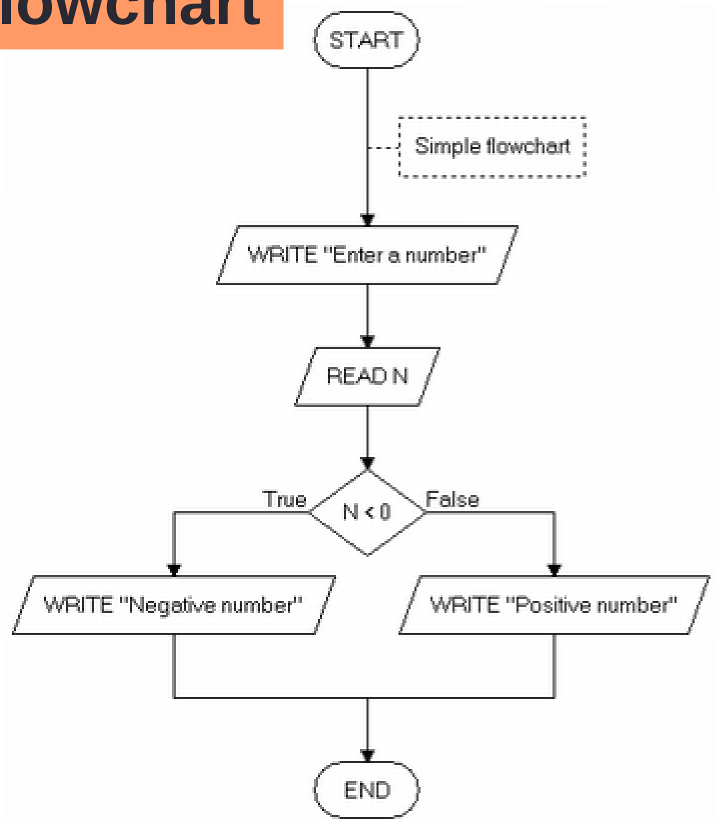

The algorithm must be **effective**. (i.e. it must solve the problem.)

The algorithm must be **general**. (Within the constraints of the system/language.)

# Algorithm (3/3)

- Ways of representing an algorithm:

## Flowchart




## Pseudocode

PSEUDOCODE

```
set total to zero

get list of numbers

loop through each number in the list
   add each number to total
end loop

if number more than zero
   print "it's positive" message
else
   print "it's zero or less" message
end if
```

lynda.com

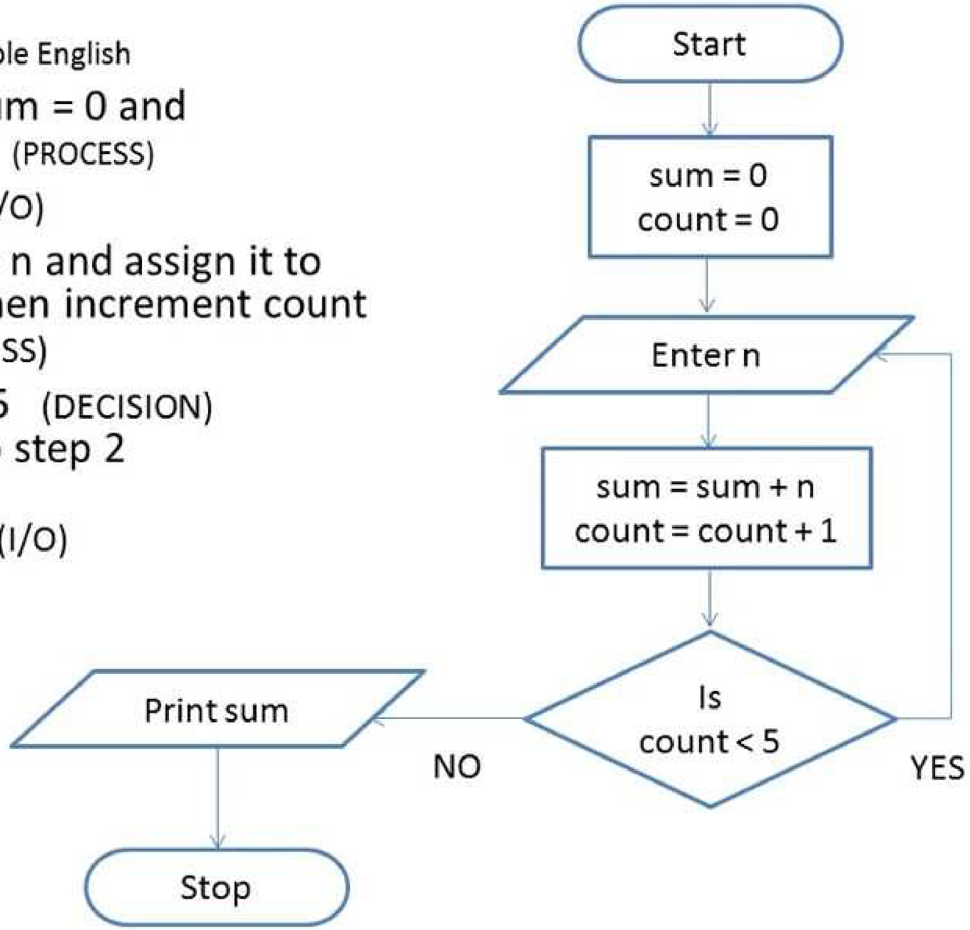
# Algorithm: Example #1

## Find the sum of 5 numbers

Algorithm in simple English

1. Initialize sum = 0 and count = 0 (PROCESS)
2. Enter n (I/O)
3. Find sum + n and assign it to sum and then increment count by 1 (PROCESS)
4. Is count < 5 (DECISION)
   if YES go to step 2
   else
   Print sum (I/O)

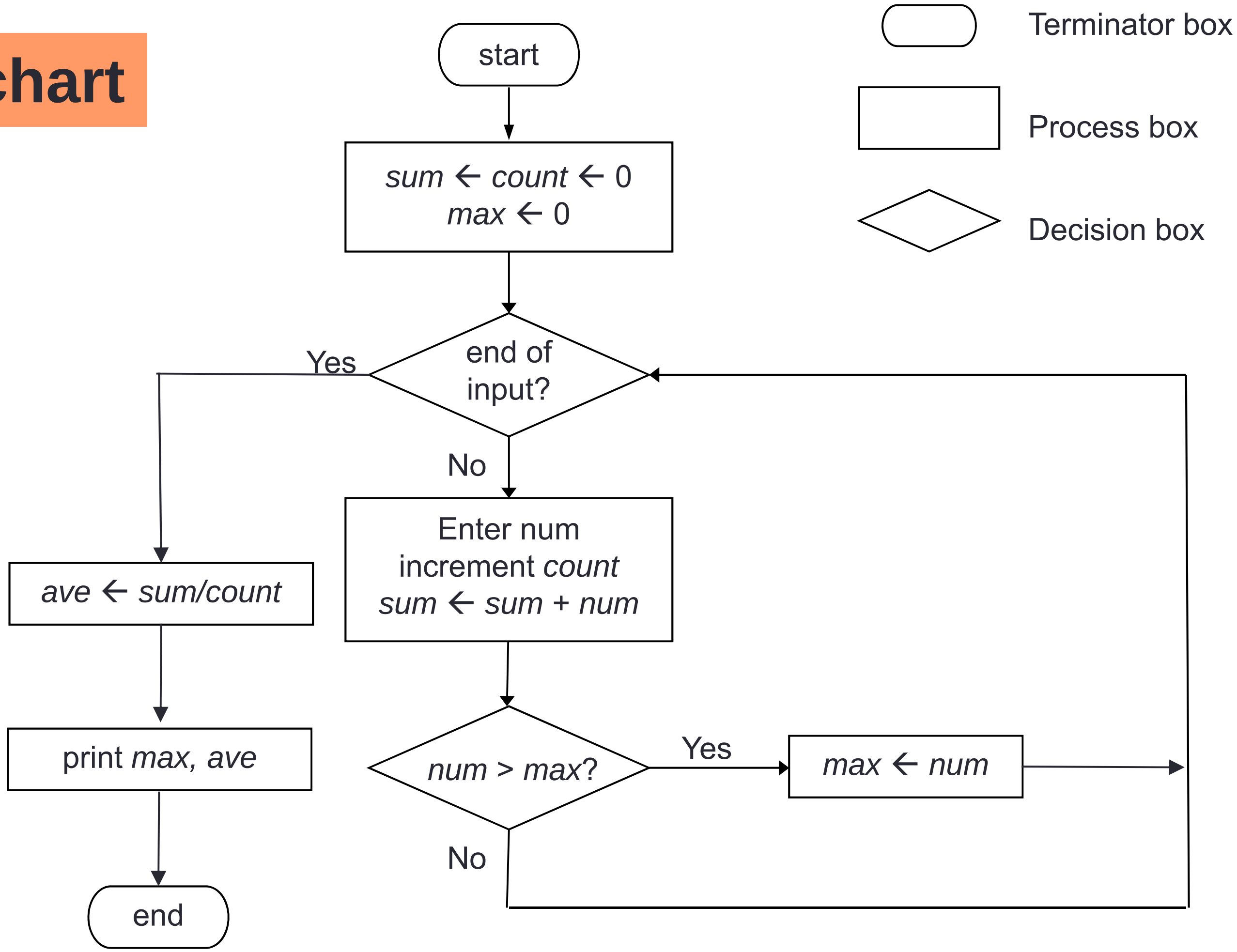
## Flowchart



# Algorithm: Example #2 (1/2)

- Find maximum and average of a list of numbers:

**Flowchart**



# Algorithm: Example #2 (2/2)

- Find maximum and average of a list of numbers:

**Pseudocode**

The need to **initialise** variables.

```
sum ← count ← 0          // sum = sum of numbers
                         // count = how many numbers are entered?
max ← 0                  // max to hold the largest value eventually
for each num entered,
    count ← count + 1
    sum ← sum + num
    if num > max
        then max ← num
ave ← sum / count

print max, ave
```

The need to **indent**.

Are there any errors in this algorithm?

# Algorithm: Pseudocode

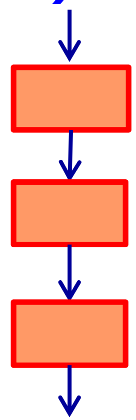
- We will write algorithms in pseudocode instead of flowchart as the former is more succinct
- However, unlike programming languages, there are no standard rules on how pseudocodes should look like
- General guidelines:
  - Every step must be unambiguous, so that anybody is able to hand trace the pseudocode and follow the logic flow
  - Use a combination of English (but keep it succinct) and commonly understood notations (such as ← for assignment in our previous example)
  - Use indentation to show the control structures

# Control Structures (1/2)

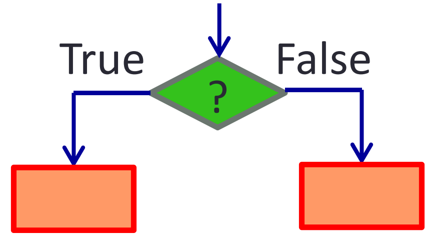
- An algorithm is a set of instructions, which are followed sequentially by default.
- However, sometimes we need to change the default sequential flow.
- We study 3 control structures.

# Control Structures (2/2)

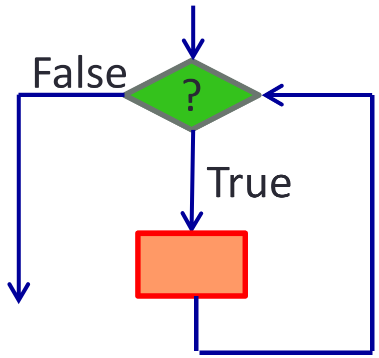
**Sequence**
- Default



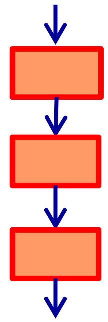
**Selection**
- Also called **branching**




**Repetition**
- Also called **loop**



# Control Structures: **Sequence (1/2)**

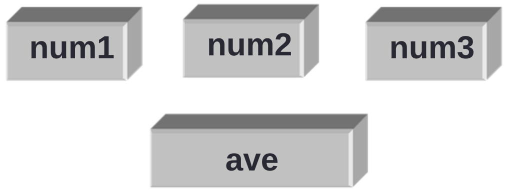


- Task: Compute the average of three integers

***A possible algorithm:***

> enter values for *num1*, *num2*, *num3*
> *ave* ← ( *num1* + *num2* + *num3* ) / 3
> print *ave*

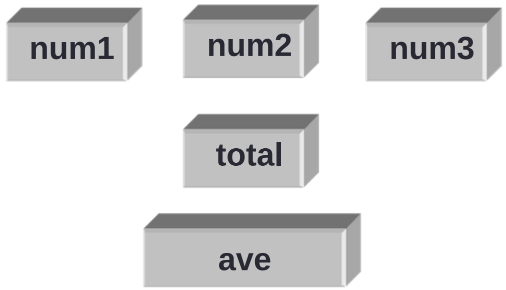
***Variables used:***




***Another possible algorithm:***

> enter values for *num1*, *num2*, *num3*
> *total* ← ( *num1* + *num2* + *num3* )
> *ave* ← *total* / 3
> print *ave*

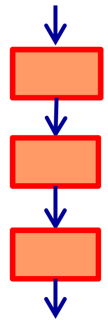
***Variables used:***





Each box represents a variable.
**Important concepts**: Each variable has a unique name and contains a value.

# Control Structures: **Sequence (2/2)**



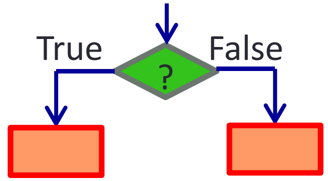
- Task: Compute the average of three integers
- How the program might look like

Unit3_prog1.c

```
// This program computes the average of 3 integers
#include <stdio.h>


int main(void) {
   int num1, num2, num3;
   float ave;

   printf("Enter 3 integers: ");
   scanf("%d %d %d", &num1, &num2, &num3);

   ave = (num1 + num2 + num3) / 3.0;
   printf("Average = %.2f\n", ave);

   return 0;
}
```

Don’t worry about the C syntax; we will discuss it next week. For now, just to show you how the algorithm is translated into the code. The logic remains the same, but you need to write the code according to the rules of the programming language.

# Control Structures: **Selection (1/3)**




- Task: Arrange two integers in ascending order (sort)

***Algorithm A:***

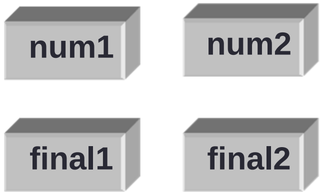
enter values for *num1*, *num2*

***// Assign smaller number into final1,***
***// and larger number into final2***
if (*num1* < *num2*)
then *final1* ← *num1*
*final2* ← *num2*
else *final1* ← *num2*
*final2* ← *num1*

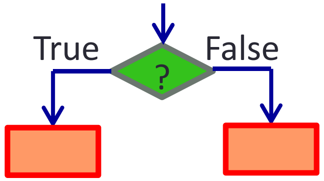
***// Transfer values in final1, final2 back to num1, num2***
*num1* ← *final1*
*num2* ← *final2*

***// Display sorted integers***
print *num1*, *num2*

***Variables used:***



# Control Structures: **Selection (2/3)**




- Task: Arrange two integers in ascending order (sort)

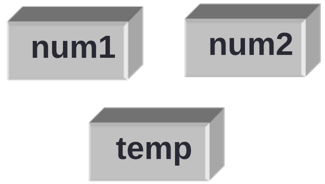
***Algorithm B:***

enter values for *num1*, *num2*

**// Swap the values in the variables if necessary**
if (*num2* < *num1*)
then *temp* ← *num1*
*num1* ← *num2*
*num2* ← *temp*

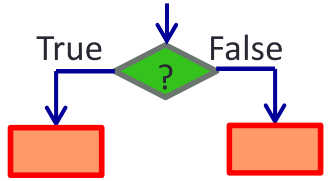
**// Display sorted integers**
print *num1*, *num2*

***Variables used:***




Compare Algorithm A with Algorithm B.

# Control Structures: **Selection (3/3)**

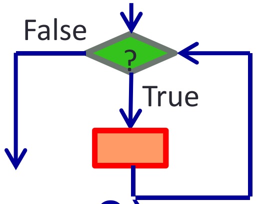



- How the program might look like for Algorithm B

Unit3_prog2.c

```c
// This program arranges 2 integers in ascending order
#include <stdio.h>

int main(void) {
   int num1, num2, temp;

   printf("Enter 2 integers: ");
   scanf("%d %d", &num1, &num2);

   if (num2 < num1) {
      temp = num1; num1 = num2; num2 = temp;
   }
   printf("Sorted: num1 = %d, num2 = %d\n", num1, num2);

   return 0;
}
```

# Control Structures: **Repetition (1/3)**




- Task: Find sum of positive integers up to *n* (assume *n*>0)

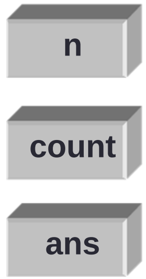
***Algorithm:***

enter value for *n*

**// Initialise a counter *count* to 1, and *ans* to 0**
*count* ← 1
*ans* ← 0
while (*count* ≤ *n*) do
    *ans* ← *ans* + *count*    **// add *count* to *ans***
    *count* ← *count* + 1    **// increase *count* by 1**

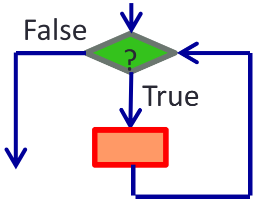
**// Display answer**
print *ans*

*Variables used:*




Initialisation is very important!

# Control Structures: **Repetition (2/3)**




- Important to trace pseudocode to check its correctness

***Algorithm:***

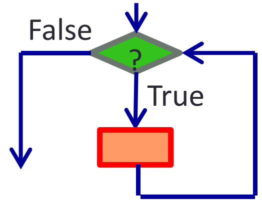
enter value for *n*
*count* ← 1
*ans* ← 0
while (*count* ≤ *n*) do
  *ans* ← *ans* + *count*
  *count* ← *count* + 1
**// Display answer**
print *ans*

Assume user enters **3** for *n*.

| (*count* <= *n*)? | *count* | *ans* |
|---|---|---|
| | 1 | 0 |
| true | 2 | 1 |
| true | 3 | 3 |
| true | 4 | 6 |
| false | | |

Output: **6**

# Control Structures: **Repetition (3/3)**




- How the program might look like

Unit3_prog3.c

```
// Computes sum of positive integers up to n
#include <stdio.h>

int main(void) {
   int n; // upper limit
   int count = 1, ans = 0; // initialisation

   printf("Enter n: ");
   scanf("%d", &n);

   while (count <= n) {
      ans += count;
      count++;
   }
   printf("Sum = %d\n", ans);

   return 0;
}
```

# Coin Change


## Problem statement

Given the above coin denominations: 1¢, 5¢, 10¢, 20¢, 50¢, and $1, assuming that you have unlimited supply of them, find the minimum number of coins needed for a given amount.

- Can you write out the pseudocode of your algorithm?
- What must the Task Giver send to the Task Solver?
- What must the Task Solver return to the Task Giver?

```
Algorithm CoinChange(amt) {

      . . .

      return coins
}
```



Knowing the algorithmic problem solving process

Knowing the properties of an algorithm

Knowing the three control structures

Knowing how to write algorithms in pseudocode

Knowing how to trace algorithms to verify their correctness

# End of File

## Algorithm of Success

```
while(noSuccess)
{
        tryAgain();

        if(Dead)
            break;
}
```