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Programming Methodology

UNIT 17

Recursion

NUS National University of Singapore | School of Computing

Unit 17: Recursion

Objectives:

- Understand the nature of recursion
- Learn to write recursive functions
- Comparing recursive codes with iterative codes

Reference:

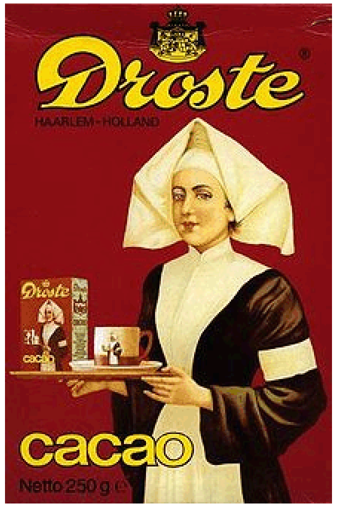
- Chapter 9: Recursion

Unit 17: Recursion

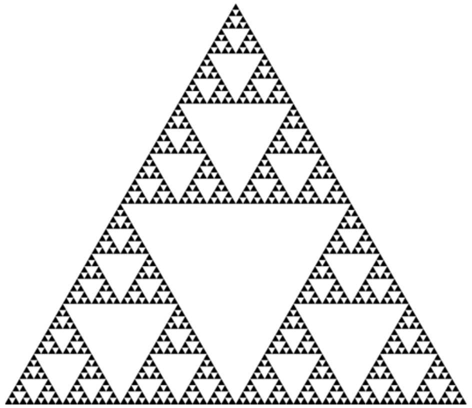
1. Introduction (1/3)

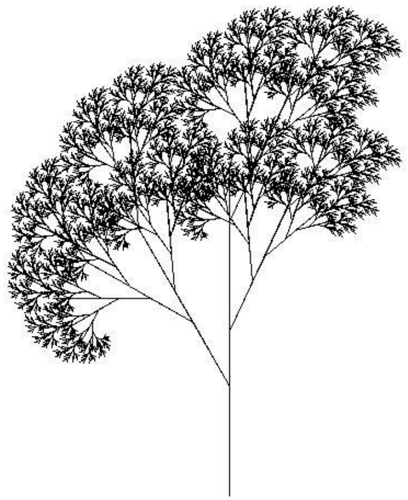
RECURSION **A central idea in CS**

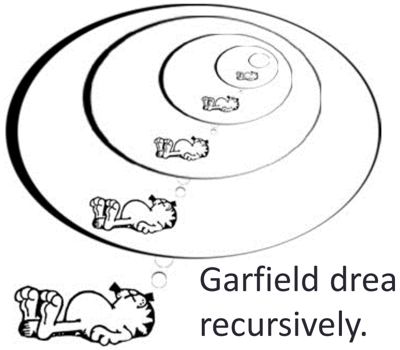
Some examples of recursion (inside and outside CS):



Droste effect


Sierpinski triangle


Recursive tree


Garfield dreaming recursively.

1. Introduction (2/3)

RECURSION **A central idea in CS.**

Definitions based on recursion:

Recursive definitions:

1. A person is a descendant of another if
   - the former is the latter's child, or
   - the former is one of the descendants of the latter's child.
2. A list of numbers is
   - a number, or
   - a number followed by a list of numbers.

Dictionary entry:
Recursion: See recursion.

Recursive acronyms:
1. GNU = GNU's Not Unix
2. PHP = PHP: Hypertext Preprocessor

To understand recursion, you must first understand recursion.

1. Introduction (3/3)

- There is NO new syntax needed for recursion.
- Recursion is a form of (algorithm) design; it is a problem-solving technique for divide-and-conquer paradigm
  - Very important paradigm – many CS problems solved using it
- Recursion is:

> **A method where**
> **the solution to a problem**
> **depends on**
> **solutions to smaller instances**
> **of the SAME problem.**

2. Two Simple Classic Examples

- From these two examples, you will see how a recursive algorithm works

Winding phase

Invoking/calling 'itself' to solve smaller or simpler instance(s) of a problem …

… and then building up the answer(s) of the simpler instance(s).

Unwinding phase

2.1 Demo #1: Factorial (1/3)

$n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$

Iterative code (version 1):

```
// Pre-cond: n >= 0
int factorial_iter1(int n) {
  int ans = 1, i;
  for (i=2; i<=n; i++) {
    ans *= i;
  }
  return ans;
}
```

Iterative code (version 2):

```
// Pre-cond: n >= 0
int factorial_iter2(int n) {
  int ans = 1;
  while (n > 1) {
    ans *= n;
    n--;
  }
  return ans;
}
```

Unit17_Factorial.c

2.1 Demo #1: Factorial (2/3)

$n! = n \times (n - 1) \times (n - 2) \times \ldots \times 2 \times 1$

Recurrence relation:
$n! = n \times (n - 1)!$
$0! = 1$

Doing it the recursive way?

```
// Pre-cond: n >= 0
int factorial(int n) {
  if (n == 0)
    return 1;
  else
    return n * factorial(n-1);
}
```

No loop!
But calling itself (recursively) brings out repetition.

Note: All the three versions work only for n < 13, due to the range of values permissible for type int. This is the limitation of the data type, not a limitation of the problem-solving model.

2.1 Demo #1: Factorial (3/3)

```
int f(int n) {
  if (n == 0)
    return 1;
  else
    return n * f(n-1);
}
```

- Trace factorial(3). For simplicity, we write f(3).

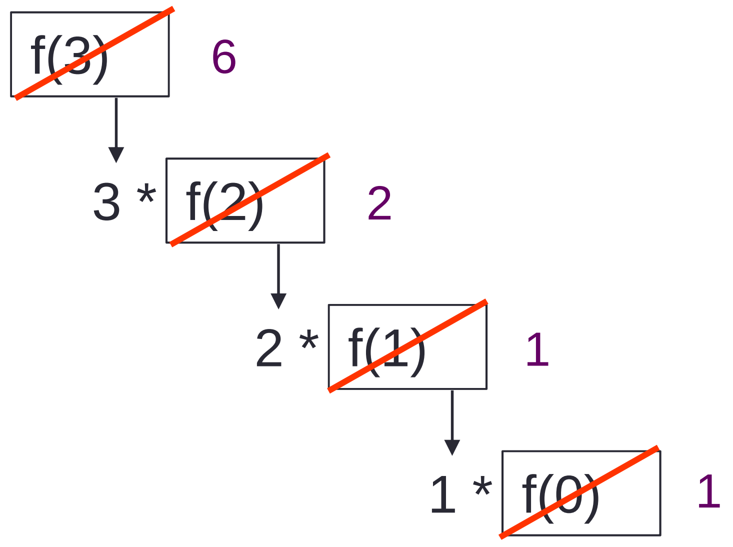
Winding:

```
f(3): Since 3 ≠ 0, call 3 * f(2)
    f(2): Since 2 ≠ 0, call 2 * f(1)
        f(1): Since 1 ≠ 0, call 1 * f(0)
            f(0): Since 0 == 0, …
```

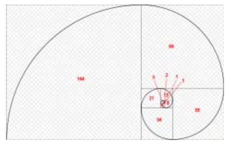
Unwinding:

```
            f(0): Return 1
        f(1): Return 1 * f(0) = 1 * 1 = 1
    f(2): Return 2 * f(1) = 2 * 1 = 2
f(3): Return 3 * f(2) = 3 * 2 = 6
```

Trace tree:



2.2 Demo #2: Fibonacci (1/4)

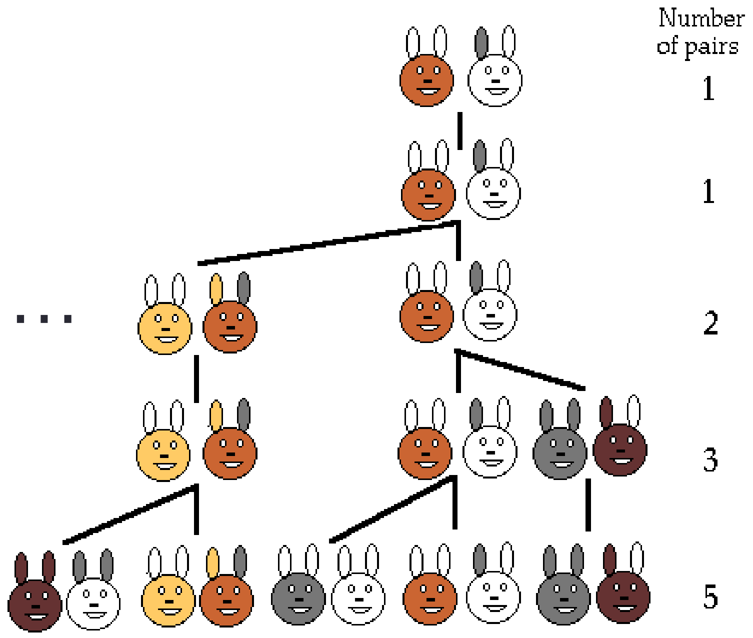


- The Fibonacci series models the rabbit population each time they mate:

  1, 1, 2, 3, 5, 8, 13, 21, …

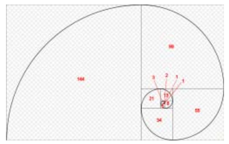
- The modern version is:

  0, 1, 1, 2, 3, 5, 8, 13, 21, …



- Fibonacci numbers are found in nature (sea-shells, sunflowers, etc)
  - http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat.html

2.2 Demo #2: Fibonacci (2/4)



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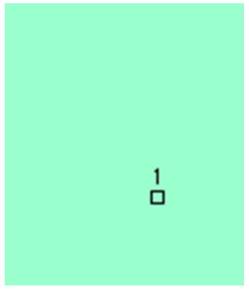
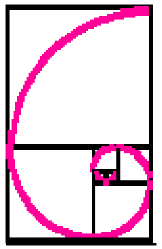
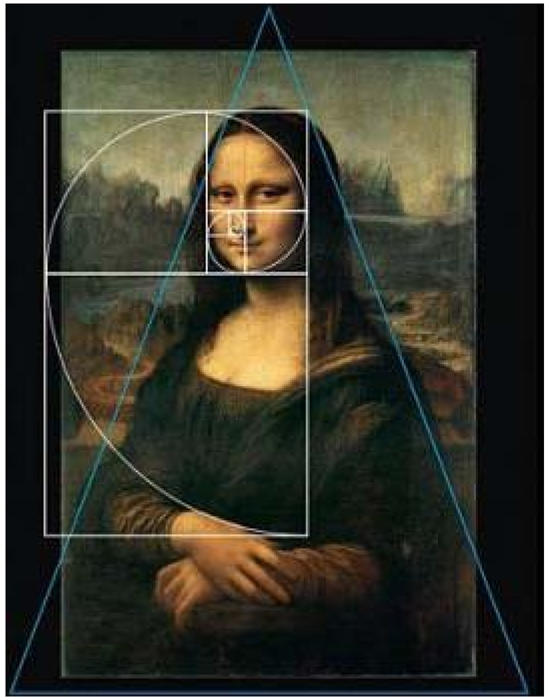






2.2 Demo #2: Fibonacci (3/4)

0, 1, 1, 2, 3, 5, 8, 13, 21, …

Unit17_Fibonacci.c

Iterative code:

```
// Pre-cond: n >= 0
int fib_iter(int n) {
  int prev1 = 1,
      prev2 = 0, sum;

  if (n < 2)
    return n;
  for (; n>1; n--) {
    sum = prev1 + prev2;
    prev2 = prev1;
    prev1 = sum;
  }
  return sum;
}
```

Recursive code:

```
// Pre-cond: n >= 0
int fib(int n) {
  if (n < 2)
    return n;
  else
    return fib(n-1) + fib(n-2);
}
```

Recurrence relation:

$f_n = f_{n-1} + f_{n-2} \quad n \geq 2$

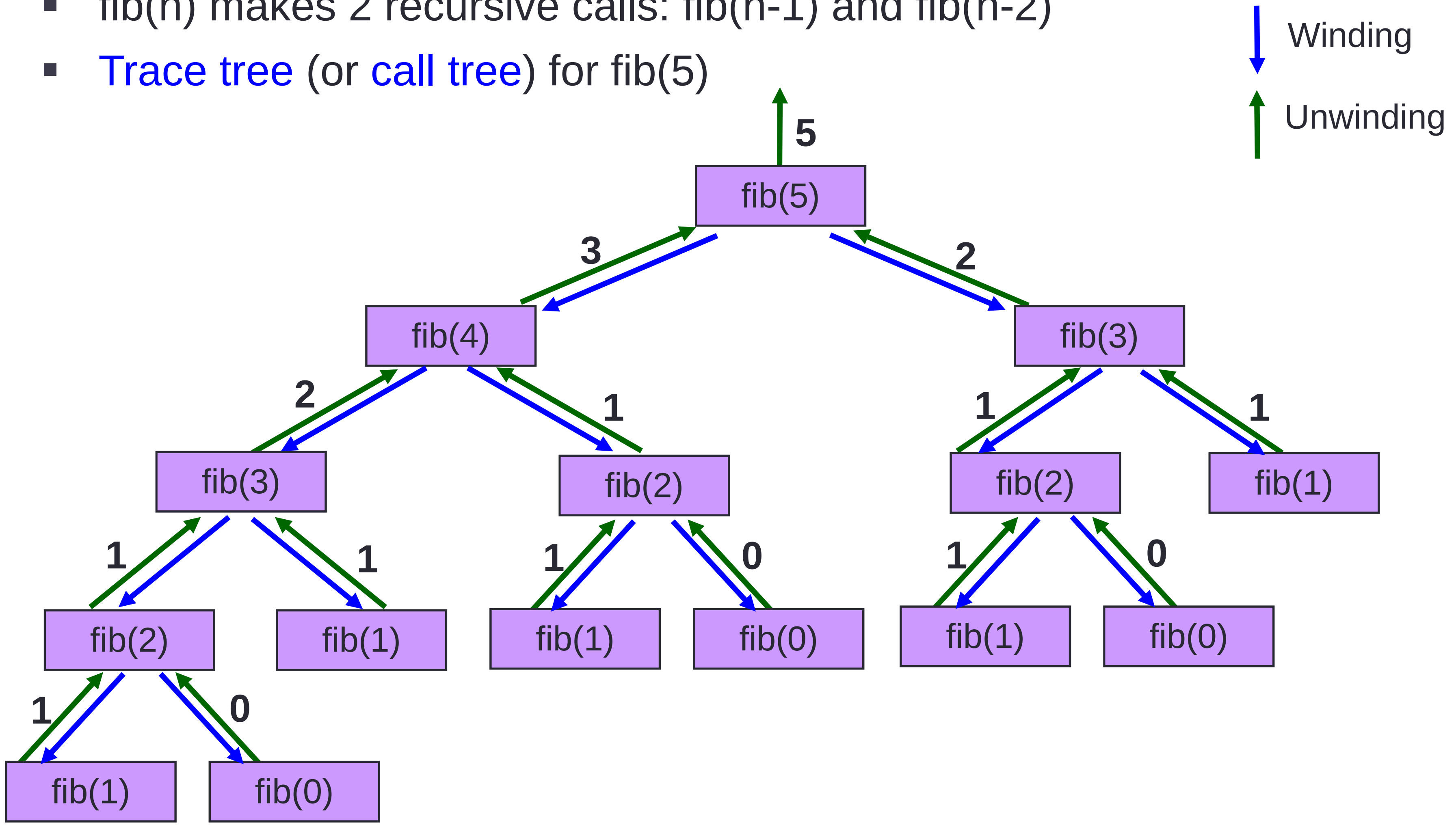
$f_0 = 0$

$f_1 = 1$

2.2 Fibonacci (4/4)

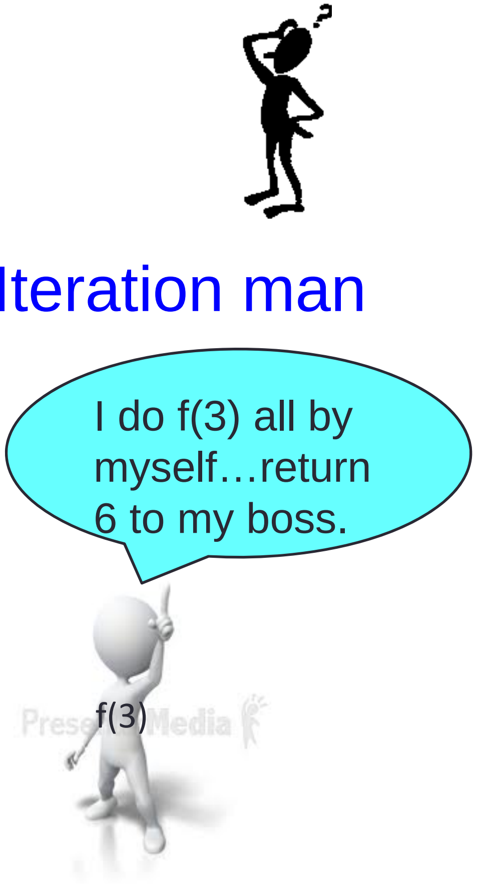
```
int fib(int n) {
   if (n < 2)
      return n;
   else
      return fib(n-1) + fib(n-2);
}
```

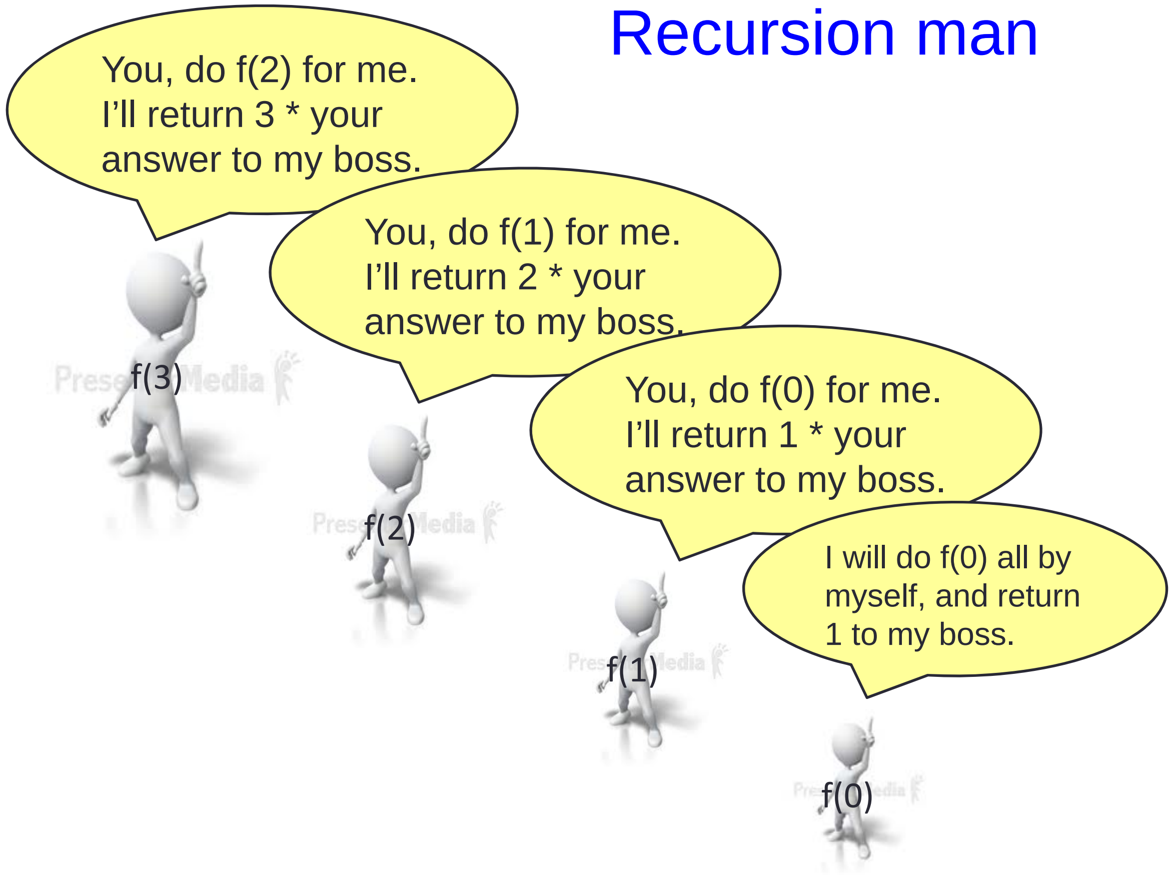
- fib(n) makes 2 recursive calls: fib(n-1) and fib(n-2)
- Trace tree (or call tree) for fib(5)



3. Gist of Recursion (1/6)

Iteration vs Recursion: How to compute factorial(3)?




Recursion man
You, do f(2) for me. I'll return 3 * your answer to my boss.
f(3)
You, do f(1) for me. I'll return 2 * your answer to my boss.
f(2)
You, do f(0) for me. I'll return 1 * your answer to my boss.
f(1)
I will do f(0) all by myself, and return 1 to my boss.
f(0)

3. Gist of Recursion (2/6)

- Problems that lend themselves to a recursive solution have the following characteristics:
  - One or more simple cases (also called base cases or anchor cases) of the problem have a straightforward, non-recursive solution
  - The other cases can be redefined in terms of problems that are smaller, i.e. closer to the simple cases
  - By applying this redefinition process every time the recursive function is called, eventually the problem is reduced entirely to simple cases, which are relatively easy to solve
  - The solutions of the smaller problems are combined to obtain the solution of the original problem

3. Gist of Recursion (3/6)

- To write a recursive function:
  - Identify the base case(s) of the relation
  - Identify the recurrence relation

```
// Pre-cond: n >= 0
int factorial(int n) {
   if (n == 0)
      return 1;

   else
      return n * factorial(n-1);
}
```

```
// Pre-cond: n >= 0
int fib(int n) {
   if (n < 2)
      return n;

   else
      return fib(n-1) + fib(n-2);
}
```

3. Gist of Recursion (4/6)

- Always check for base case(s) first
  - What if you omit base case(s)?
- Do not write redundant base cases

```
int factorial(int n) {
  if (n == 0)
    return 1;
  else if (n == 1)
    return 1;
  else if (n == 2)
    return 2;
  else if (n == 3)
    return 6;
  else
    return n * factorial(n-1);
}
```

redundant

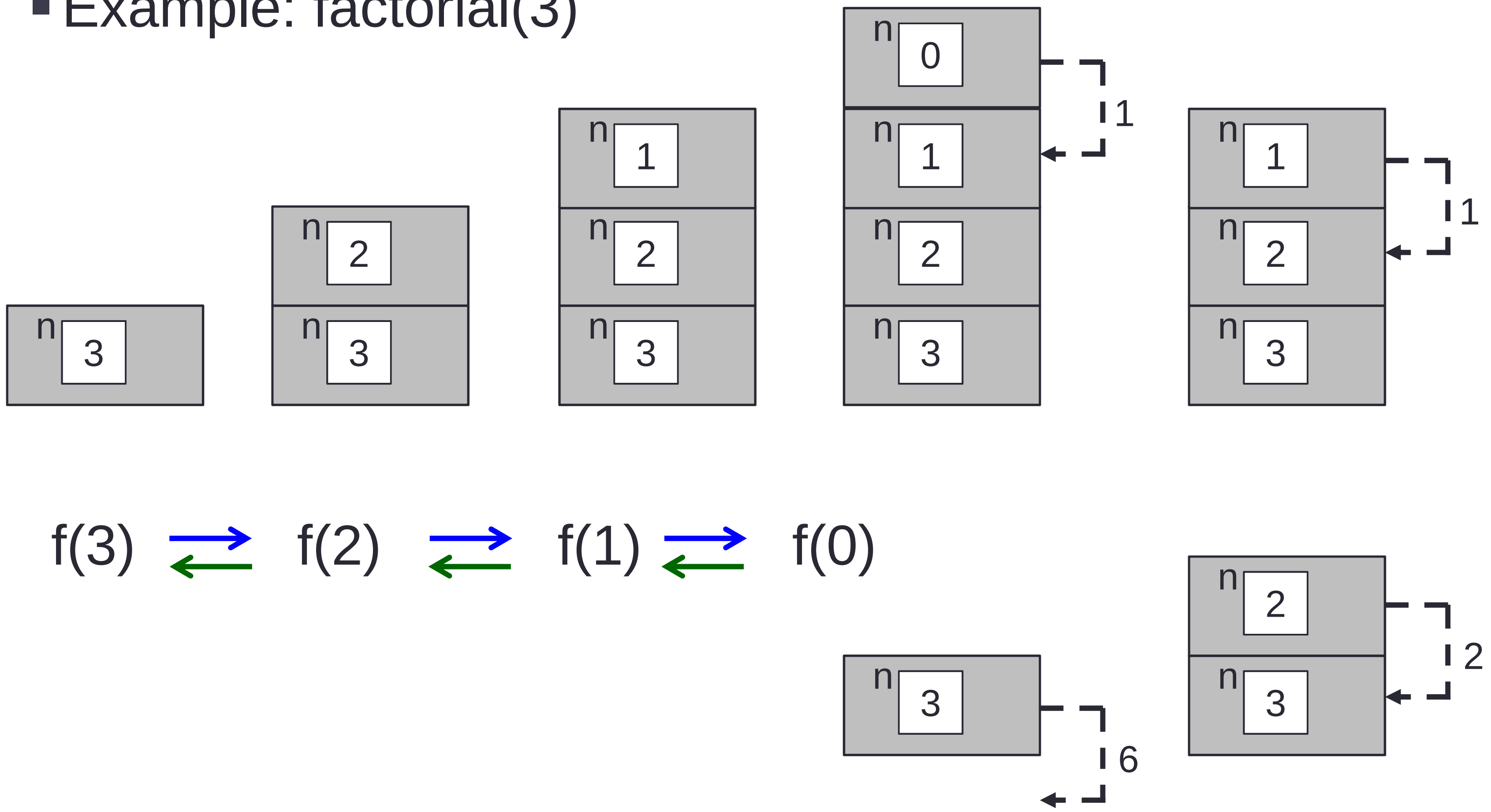
3. Gist of Recursion (5/6)

- When a function is called, an activation record (or frame) is created by the system.
- Each activation record stores the local parameters and variables of the function and its return address.
- Such records reside in the memory called stack.
  - Stack is also known as LIFO (last-in-first-out) structure
- A recursive function can potentially create many activation records
  - **Winding**: each recursive call creates a separate record
  - **Unwinding**: each return to the caller erases its associated record

```
int f(int n) {
  if (n == 0) return 1;
  else return n * f(n-1);
}
```

3. Gist of Recursion (6/6)

- Example: factorial(3)



4. Exercise #1: Greatest Common Divisor

- The recurrence relation for Greatest Common Divisor (GCD) of two non-negative integers *a* and *b*, not both zero, is given below:

$$\mathrm{GCD}(a,b) = \begin{cases} a, & b = 0 \\ \mathrm{GCD}(b, a\%b), & \text{otherwise} \end{cases}$$

- Write a function int gcd(int a, int b) to compute the GCD of *a* and *b*. Skeleton program Unit17_GCD.c is given.

4. Exercise #2: Tracing

- Given the following 2 recursive functions, trace mystery1(3902) and mystery2(3902) using the trace tree method.

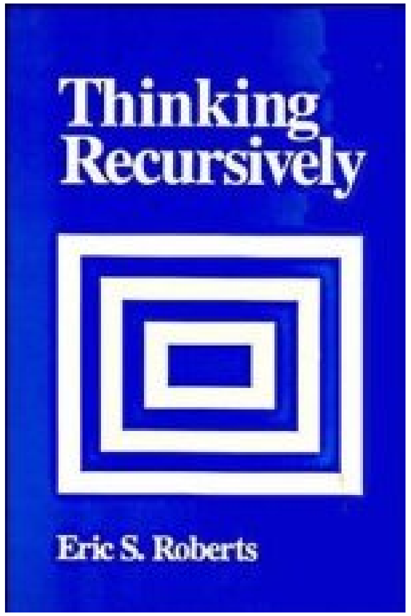
```c
void mystery1(int n) {
   if (n>0) {
      printf("%d", n%10);
      mystery1(n/10);
   }
}
```

```c
void mystery2(int n) {
   if (n>0) {
      mystery2(n/10);
      printf("%d", n%10);
   }
}
```

The order of statements does matter!

5. Thinking Recursively

- It is apparent that to do recursion you need to think “recursively”:
  - Breaking a problem into simpler problems that have identical form
- Is there only one way of breaking a problem into simpler problems?



5.1 Think: Sum of Squares (1/5)

- Given 2 positive integers x and y, where $x \leq y$, compute

  sumSq(x,y) = $x^2 + (x+1)^2 + \ldots + (y-1)^2 + y^2$

- For example

  sumSq(5,10) = $5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 = 355$

- How do you break this problem into smaller problems?
- How many ways can it be done?
- We are going to show 3 versions
- See Unit17_SumSquares.c

5.1 Think: Sum of Squares (2/5)

- Version 1: 'going up'

```c
int sumSq1(int x, int y) {
   if (x == y) return x * x;
   else return x * x + sumSq1(x+1, y);
}
```

- Version 2: 'going down'

```c
int sumSq2(int x, int y) {
   if (x == y) return y * y;
   else return y * y + sumSq2(x, y-1);
}
```

5.1 Think: Sum of Squares (3/5)

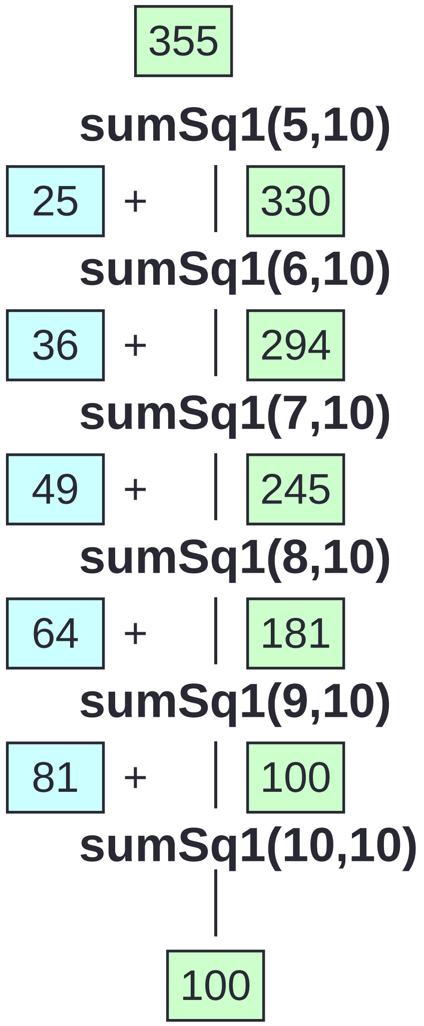
- Version 3: 'combining two half-solutions'

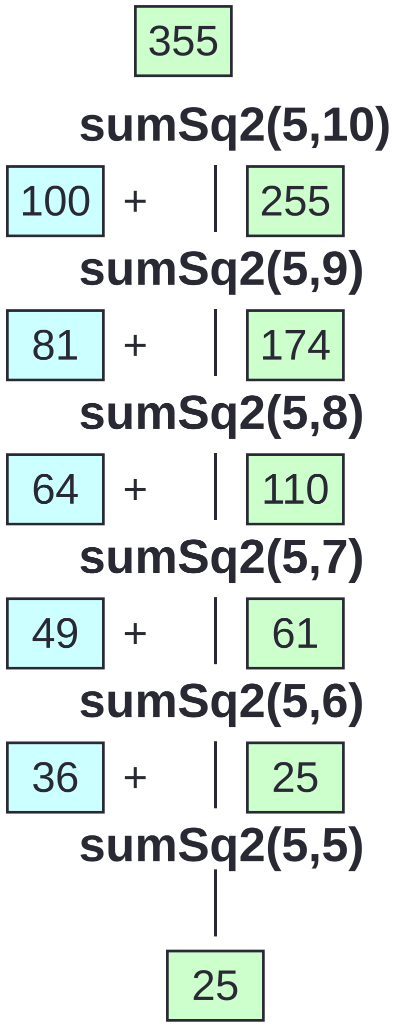
```
int sumSq3(int x, int y) {
   int mid; // middle value

   if (x == y)
      return x * x;
   else {
      mid = (x + y)/2;
      return sumSq3(x, mid) + sumSq3(mid+1, y);
   }
}
```

5.1 Think: Sum of Squares (4/5)

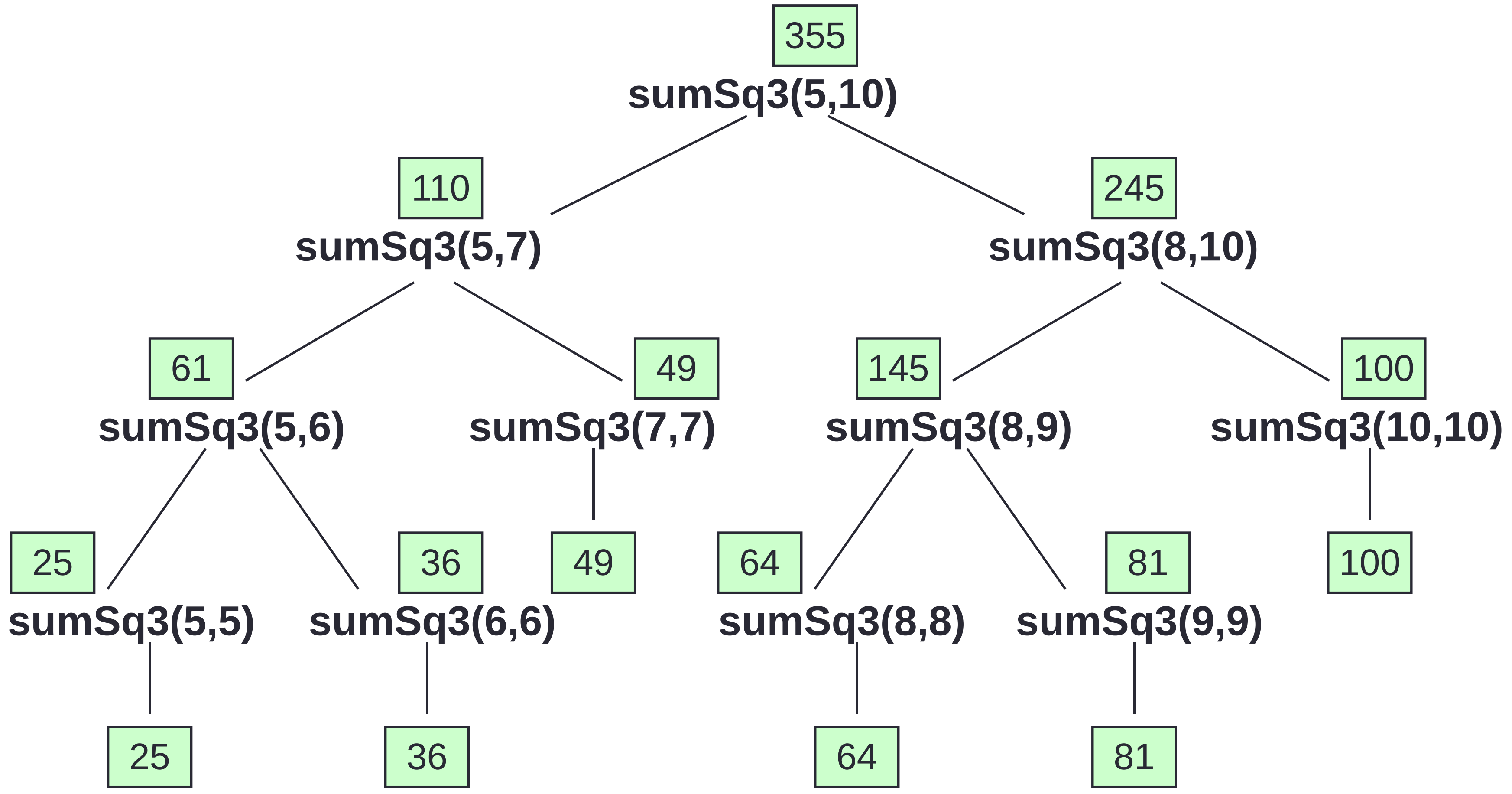
- Trace trees





5.1 Think: Sum of Squares (5/5)

- Trace tree



5.2 Demo #3: Counting Occurrences (1/4)

- Given an array

  int list[ ] = { 9, -2, 1, 7, 3, 9, -5, 7, 2, 1, 7, -2, 0, 8, -3 }

- We want

  countValue(7, list, 15)

  to return 3 (the number of times 7 appears in the 15 elements of list.

5.2 Demo #3: Counting Occurrences (2/4)

Unit17_CountValue.c

Iterative code:

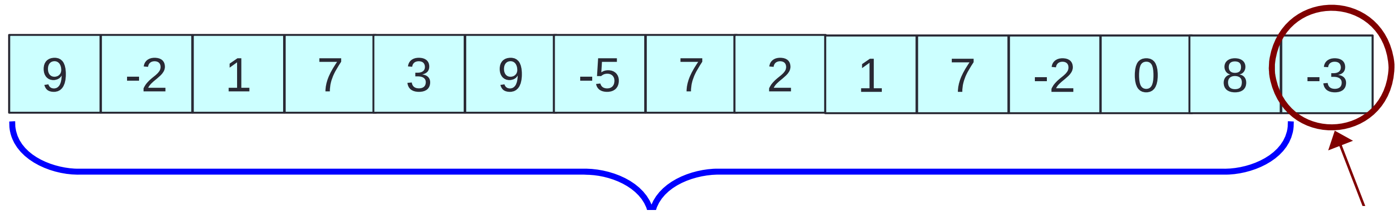
```c
int countValue_iter(int value, int arr[], int size)
{
   int count = 0, i;

   for (i=0; i<size; i++)
      if (value == arr[i])
         count++;

   return count;
}
```

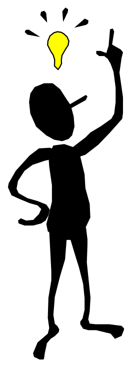
5.2 Demo #3: Counting Occurrences (3/4)

- To get countValue(7, list, 15) to return 3.
- Recursive thinking goes…



… and get someone to count the 7 in this smaller problem, …

If I handle the last element myself, …



… then, depending on whether the last element is 7 or not, my answer is either his answer or his answer plus 1!

5.2 Demo #3: Counting Occurrences (4/4)

Recursive code:

Unit17_CountValue.c

```
int countValue(int value, int arr[], int size) {
   if (size == 0)
      return 0;
   else
      return (value == arr[size-1]) +
               countValue(value, arr, size-1);
}
```

Note: The second return statement is equivalent to the following (why?):

```
   if (value == arr[size-1])
      return 1 + countValue(value, arr, size-1);
   else
      return countValue(value, arr, size-1);
```

6. Auxiliary Function (1/3)

- Sometimes, auxiliary functions are needed to implement recursion. Eg: Refer to Demo #3 Counting Occurrences.
- If the function handles the first element instead of the last, it could be re-written as follows:

```
int countValue(int value, int arr[],
               int start, int size) {
  if (start == size)
    return 0;
  else
    return (value == arr[start]) +
             countValue(value, arr, start+1, size);
}
```

6. Auxiliary Function (2/3)

- However, doing so means that the calling function has to change the call from:

```
countValue(value, list, ARRAY_SIZE)
```

to:

```
countValue(value, list, 0, ARRAY_SIZE)
```

- The additional parameter 0 seems like a redundant data from the caller's point of view.

6. Auxiliary Function (3/3)

- Solution: Keep the calling part as:

```
countValue(value, list, ARRAY_SIZE)
```

- Rename the original countValue() function to countValue_recur(). The recursive call inside should also be similarly renamed.
- Add a new function countValue() to act as a driver function, as follows:

```
int countValue(int value, int arr[], int size) {
  return countValue_recur(value, arr, 0, size);
}
```

- See program Unit17_CountValue_Auxiliary.c

7. Exercise #3: Sum Digits

- Write a recursive function int sum_digits(int *n*) that sums up the digits in *n*, assuming that *n* is a non-negative integer.
- Skeleton program Unit17_SumDigits.c is given.
- This exercise is mounted on CodeCrunch.
- Sample runs:

```
Enter a non-negative integer: 6543
Sum of its digits = 18
```

```
Enter a non-negative integer: 3708329
Sum of its digits = 32
```

7. Exercise #4: Sum Array

- Write a program Unit17_SumArray.c to read data into an integer array with at most 10 elements, and sum up all values in the array, using a recursive function.
- This exercise is mounted on CodeCrunch.
- Sample runs:

```
Enter number of elements: 6
Enter 6 values: 4 3 -2 0 1 3
Array read: 4 3 -2 0 1 3
Sum = 9
```

```
Enter number of elements: 8
Enter 8 values: 11 25 56 8 12 7 31 16
Array read: 11 25 56 8 12 7 31 16
Sum = 166
```

8. Types of Recursion

- Besides direct recursion (function A calls itself), there could be mutual or indirect recursion (we do not cover these in CS1010)
  - Examples: Function A calls function B, which calls function A; or function X calls function Y, which calls function Z, which calls function X.
- Note that it is <u>not typical</u> to write a recursive main() function.
- One type of recursion is known as tail recursion.
  - Not covered in CS1010

9. Tracing Recursive Codes

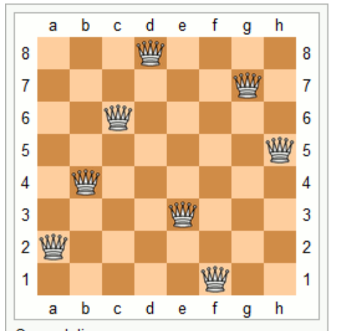
- Beginners usually rely on tracing to understand the sequence of recursive calls and the passing back of results.
- However, tracing a recursive code is tedious, especially for non-tail-recursive codes. The trace tree could be huge (example: fibonacci).
- If you find that tracing is needed to aid your understanding, start tracing with small problem sizes, then gradually see the relationship between the successive calls.
- Students should grow out of tracing habit and understand recursion by examining the relationship between the problem and its immediate subproblem(s).

10. Recursion versus Iteration (1/2)

- Iteration can be more efficient
  - Replaces function calls with looping
  - Less memory is used (no activation record for each call)
- Some good compilers are able to transform a tail-recursion code into an iterative code.
- General guideline: If a problem can be done easily with iteration, then do it with iteration.
  - For example, Fibonacci can be coded with iteration or recursion, but the recursive version is <u>very</u> inefficient (large call tree due to duplicate computations), so use iteration instead.

10. Recursion versus Iteration (2/2)

- Many problems are more naturally solved with recursion, which can provide elegant solution.
  - Towers of Hanoi
  - Mergesort (to be covered in CS1020)
  - The N Queens problem



One solution.

- Conclusion: choice depends on problem and the solution context. In general, use recursion if …
  - A recursive solution is natural and easy to understand.
  - A recursive solution does not result in excessive duplicate computation.
  - The equivalent iterative solution is too complex.

11. Towers Of Hanoi

- In a separate Powerpoint file.

Summary

- In this unit, you have learned about
  - Recursion as a design strategy
  - The components of a recursive code
  - Differences between Recursion and Iteration

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