EE3731C Tutorial - Classical Signal 3

Department of Electrical and Computer Engineering

1. (a) $H_{lp}^0 = e^{-j\omega M/2} H_{lp}$. Using the property that $\mathcal{F}(x[n-n_0]) = e^{-j\omega n_0} X(e^{j\omega})$. Therefore

$$h_{lp}^0[n] = h_{lp}[n - M/2] = \frac{\sin \omega_c (n - M/2)}{\pi(n - M/2)}$$

(b) $H_{hp}^0 = e^{-j\omega M/2} - H_{lp}^0$. Note that if $M/2$ is an integer, then $\mathcal{F}^{-1}(e^{-j\omega M/2}) = \delta(n - M/2)$. However, this does not make sense when $M/2$ is not an integer. More generally, for $M/2 \in \mathbb{R}$,

$$\begin{aligned}
\mathcal{F}^{-1}(e^{-j\omega M/2}) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-j\omega M/2} e^{j\omega n} d\omega \\
&= \frac{1}{2\pi} \frac{1}{j(n - M/2)} \left[e^{j\omega(n-M/2)} \right]_{-\pi}^{\pi} \\
&= \frac{\sin \pi(n - M/2)}{\pi(n - M/2)} \quad \text{since } \sin x = \frac{e^{jx} - e^{-jx}}{2j}
\end{aligned}$$

Therefore

$$h_{hp}^0[n] = \frac{\sin \pi(n - M/2)}{\pi(n - M/2)} - \frac{\sin \omega_c (n - M/2)}{\pi(n - M/2)}$$

2. From the filter requirements, the allowed ringing is $\delta = 0.021$, and the stopband frequency is $\omega_s = 0.35\pi$, and passband frequency is $\omega_p = 0.5\pi$. The ideal frequency cutoff is $\omega_c = \frac{0.35\pi + 0.5\pi}{2} = 0.425\pi$. The transition width is $\omega_p - \omega_s = 0.15\pi$. Let's plug in the Kaiser window formula:

$$\begin{aligned}
A &= -20 \log_{10} \delta = -20 \log_{10} 0.021 = 33.5556 \\
\beta &= 0.5842(A - 21)^{0.4} + 0.07886(A - 21) = 2.5974 \\
M &= \frac{A - 8}{2.285 \triangle\omega} = 23.73 \approx 24 \ \text{ (in general we round up)}
\end{aligned}$$

In the previous question, we show that for the ideal highpass filter $H_{hp} = \begin{cases} 0, & |\omega| < \omega_c \\ e^{-j\omega M/2}, & \omega_c < |\omega| \leq \pi \end{cases}$, the corresponding impulse response is $h_{hp}[n] = \frac{\sin \pi(n-M/2)}{\pi(n-M/2)} - \frac{\sin \omega_c(n-M/2)}{\pi(n-M/2)}$. Therefore the impulse response of the resulting windowed highpass filter is

$$h[n] = \begin{cases} \left(\frac{\sin \pi(n-M/2)}{\pi(n-M/2)} - \frac{\sin \omega_c(n-M/2)}{\pi(n-M/2)} \right) \frac{I_0\left[\beta\left(1 - \left[\frac{n-M/2}{M/2}\right]^2\right)^{1/2}\right]}{I_0(\beta)}, & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases},$$

where $\beta = 2.5974$ and $M = 24$.

Remarks: The above is sufficient for tests/exams for this class. However, if we plot the frequency response of the resulting highpass filter, it turns out the ringing exceeds the specified δ by a little. To reduce ringing, we can increase M. However, if we increase M to 25, then the filter becomes a type 2 FIR filter, which will have a frequency response of 0 at $\omega = \pi$. This is not acceptable for a highpass filter because we would like $|H(e^{j\pi})| = 1$ at $\omega = \pi$. Therefore, if we want to meet the ringing requirements, we have to increase M to 26.

3. This is a bandpass filter with transition bands from $0.25\pi \le |\omega| \le 0.35\pi$ and $0.6\pi \le |\omega| \le 0.65\pi$. Therefore the transition widths correspond to $0.35\pi - 0.25\pi = 0.1\pi$ and $0.65\pi - 0.6\pi = 0.05\pi$. Therefore we will consider the more stringent transition width $\triangle\omega = 0.05\pi$. The stopband ringing criteria is 0.01, while the passband ringing criteria is 0.05. Therefore we adopt the more stringent criterion of $\delta = 0.01$.

   (a) Let's plug in the Kaiser window formula:

$$\begin{aligned} A &= -20\log_{10}\delta = -20\log_{10} 0.01 = 40 \\ \beta &= 0.5842(A-21)^{0.4} + 0.07886(A-21) = 3.3953 \\ M &= \frac{A-8}{2.285\triangle\omega} = 89 \approx 90 \quad \text{(in general we round up)} \end{aligned}$$

   Therefore $M = 90$.

   (b) Delay of filter is $M/2 = 45$.

   (c) The first cutoff frequency corresponds to $\frac{0.25\pi+0.35\pi}{2} = 0.3\pi$, while the second cutoff frequency corresponds to $\frac{0.6\pi+0.65\pi}{2} = 0.625\pi$. The ideal frequency response corresponds to

$$H_I(e^{j\omega}) = \begin{cases} 0, & |\omega| < 0.3\pi \\ e^{-j\omega M/2}, & 0.3\pi \le |\omega| \le 0.625\pi \\ 0, & |\omega| > 0.625\pi \end{cases}$$

   This is equal to the difference of two lowpass filters $H_I(e^{j\omega}) = H_{lp}^1(e^{j\omega}) - H_{lp}^2(e^{j\omega})$, where

$$H_{lp}^1(e^j\omega) = \begin{cases} e^{-j\omega M/2}, & |\omega| \le 0.625\pi \\ 0, & \text{otherwise} \end{cases}$$

$$H_{lp}^2(e^j\omega) = \begin{cases} e^{-j\omega M/2}, & |\omega| \le 0.3\pi \\ 0, & \text{otherwise} \end{cases}$$

   Since the ideal lowpass filter with linear phase has impulse response $h_{lp}[n] = \frac{\sin\omega_c(n-M/2)}{\pi(n-M/2)}$, the impulse response of the ideal bandpass filter with the above frequency response is:

$$h_I[n] = \frac{\sin 0.625\pi(n-45)}{\pi(n-45)} - \frac{\sin 0.3\pi(n-45)}{\pi(n-45)}$$

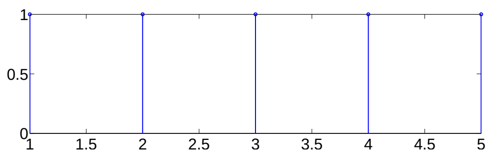
4. (a) In matlab:
   » x = ones(1, 5);
   » stem(x);
   See Figure 1a

   (b) In matlab:
   » y = ones(1, 5);
   » stem(x);
   See Figure 1b

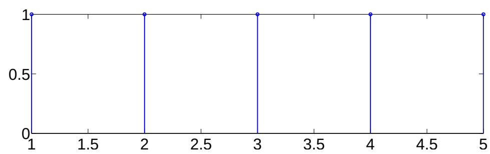
   (c) In matlab:
   » stem(conv(x, y));
   The results look like a triangle (Figure 1c)

   (d) In matlab:
   » stem(ifft(fft(x).*fft(y)));
   The results look like a rectangle (Figure 1d)

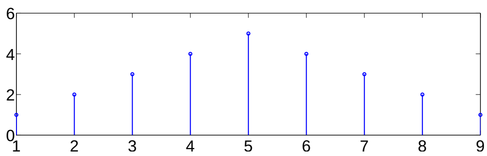
   (e) In matlab:
   » xpad = [x 0 0 0 0]; ypad = [y 0 0 0 0];
   » stem(ifft(fft(xpad).*fft(ypad)));
   See Figure 1e



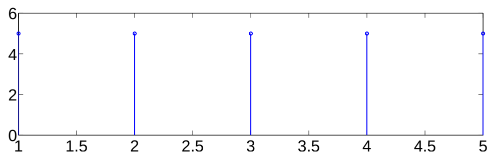
(a) x



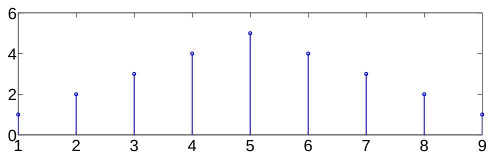
(b) y



(c) convolution of x and y using conv.m



(d) circular convolution of x and y using FFT



(e) convolution of x and y using FFT

Figure 1: Illustration of convolution