NATIONAL UNIVERSITY OF SINGAPORE

EE3731C – SIGNAL PROCESSING METHODS

(Semester 1 : AY2018/2019)

Time Allowed : 2.5 Hours

INSTRUCTIONS TO STUDENTS

1. Please write only your Student Number. Do not write your name.
2. This assessment paper contains **FOUR** questions and comprises **FIVE** printed pages.
3. Students are required to answer **ALL** questions.
4. Students should write the answers for each question on a new page.
5. This is a **CLOSED BOOK** assessment.
6. One A4-size formula sheet is allowed.
7. Electronic calculators are allowed but programmable calculators are not allowed.

Q.1 (25 marks). Sub-questions (a), (b) and (c) can be answered independently.

(a) Perform PCA on the following set of data: $\binom{4}{4}\ \binom{1}{3}\ \binom{3}{1}\ \binom{0}{0}$. What is the first principal axis? What is the first principal component of the data point $x = \binom{-2}{2}$?

(13 marks)

(b) Consider training data $x_1 = \binom{1}{0}, x_2 = \binom{0}{-1.5}, x_3 = \binom{-1}{0}, x_4 = \binom{1}{1}$ with corresponding class labels $y_1 = 0, y_2 = 0, y_3 = 1, y_4 = 1$. What is the 3-NN estimate of the class label posterior probability of the data point $x_5 = \binom{0}{0}$, where the distance metric used is the Euclidean distance? What is the MAP classification of the data point x_5? Repeat the above with the Manhattan distance metric.

(6 marks)

(c) Consider data samples x_1, x_2, x_3, x_4 to be $0, 2, 3, 6$. Using the Gaussian Parzen's window: $\frac{1}{\sqrt{2\pi h^2}} e^{-\frac{x^2}{2h^2}}$, what is the Parzen's window estimate of $p_h(x)$ at (i) $x = 1$ and (ii) $x = 4$ for $h = 1$?

(6 marks)

Q.2 (25 marks). Sub-questions (a), (b) and (c) can be answered independently.

Let y have the distribution $p(y=0)=1/4$ and $p(y=1)=3/4$. Let

$$p(x|y=0)=\begin{cases}7/8 & x=1\\ 1/8 & x=2\end{cases}$$

and

$$p(x|y=1)=\begin{cases}1/3 & x=1\\ 2/3 & x=2\end{cases}$$

Suppose Mary plays the following game with John every day from day $n=1$ to $n=\infty$. On day n, Mary independently sampled (x_n, y_n) from the above distribution $p(x,y)$. Given y_n, John has to guess the value of x_n. Suppose John guesses $\hat{x}_n$ on the n-th day, then John has to pay Mary $(x_n-\hat{x}_n)^2$ dollar.

(a) Suppose John employs the optimal strategy to minimize payment to Mary. What would John guess on days when Mary tells John that $y=0$?

(6 marks)

(b) Suppose John employs the optimal strategy to minimize payment to Mary. What would John guess on days when Mary tells John that $y=1$?

(6 marks)

(c) Suppose John employs the MAP strategy. How much does John have to pay Mary each day (on average)?

(13 marks)

Q.3 (25 marks). Sub-questions (a) and (b) can be answered independently.

(a) Suppose we want to sample $x \in \{1, 2, 3, 4\}$,

$$
\begin{aligned}
p(x = 1) &= 0.3 \\
p(x = 2) &= 0.2 \\
p(x = 3) &= 0.1 \\
p(x = 4) &= 0.4
\end{aligned}
$$

with the Metropolis algorithm. Suppose the proposal distribution is

$$
\begin{aligned}
q(x' = x - 1|x) = q(x' = x|x) = q(x' = x + 1|x) = 1/3 \quad & \text{for } x \in \{2,3\} \\
q(x' = 1|x) = 2/3; q(x' = 2|x) = 1/3 \quad & \text{for } x = 1 \\
q(x' = 3|x) = 1/3; q(x' = 4|x) = 2/3 \quad & \text{for } x = 4
\end{aligned}
$$

What is the equivalent Markov chain transition matrix T in this Metropolis algorithm? Please show your steps.

(18 marks)

(b) Consider an exponential distribution $p(x) = \lambda e^{-\lambda x}$. Suppose we observe N independent samples from the exponential distribution: $D = \{x_1, \cdots, x_N\}$. What is the maximum likelihood (ML) estimate of λ? Show your steps to get full credit.

(7 marks)

Q.4 (25 marks). Sub-questions (a), (b) and (c) can be answered independently.

(a) We wish to use the Kaiser window method to design a FIR filter with generalized linear phase that meets the following specifications:

$$1.95 < \left|H\left(e^{j\omega}\right)\right| < 2.05, \quad 0 \le |\omega| \le 0.15\pi$$
$$\left|H\left(e^{j\omega}\right)\right| < 0.1, \quad 0.25\pi \le |\omega| \le \pi$$

The Kaiser window will be applied to the ideal impulse response associated with the ideal frequency response given by

$$H_I(e^{j\omega}) = \begin{cases} 2e^{-j\omega M/2}, & 0 \le |\omega| \le 0.2\pi \\ 0, & 0.2\pi \le |\omega| \le \pi \end{cases}$$

Find the Kaiser window parameters β and M so that the resulting windowed filter satisfies the above criteria. <u>Justify your choice of δ and $\Delta\omega$.</u>

(8 marks)

(b) An LTI system has impulse response $h[n] = (1/4)^n u[n]$. Use DTFT to find the output of this system when the input is $x[n] = (1/2)^n u[n-3]$. You can use the fact that the DTFT of $a^n u[n]$ is $\frac{1}{1-ae^{-jw}}$. You can also use the identity that $\frac{1}{(1-ae^{-jw})(1-be^{-jw})} = \frac{a/(a-b)}{1-ae^{-jw}} - \frac{b/(a-b)}{1-be^{-jw}}$.

(12 marks)

(c) Consider the following system T. Given an input $x[n]$, the output is $y[n] = T(x[n]) = \sin(x[n])$. Is the system (i) stable, (ii) causal, (iii) linear, (iv) time invariant? Please explain your reasoning for full credits.

(5 marks)

END OF PAPER