EE3731C Tutorial - Pattern Recognition 1

Department of Electrical and Computer Engineering

1. Find eigenvalues and eigenvectors for matrix $A = \begin{pmatrix} 5 & 3 \\ 1 & 3 \end{pmatrix}$

2. For matrix $A = \begin{pmatrix} 2 & a \\ a & 2 \end{pmatrix}$, calculate eigenvalues of A and specify the values of a for which A is a valid covariance matrix

3. Consider the following set of data:

$$\begin{pmatrix} -2 \\ 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

   (a) What is the first principal axis?

   (b) What is the first principal component of the datapoint $x = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$?

   (c) What is the reconstruction error from representing $x = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ using just the first principal component?

4. Let u_i and v_i be $d_1 \times 1$ and $d_2 \times 1$ column vectors respectively. In general, $(u_1, \cdots, u_N) \begin{pmatrix} v_1^T \\ \vdots \\ v_N^T \end{pmatrix}$ is $d_1 \times d_2$ matrix, which is given by $\sum_{i=1}^{N} u_i v_i^T$. This might not be obvious, so let's verify a simple example. Let's consider $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 5 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. Verify that the usual matrix multiplication gives the same results as

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} (1\ \ 2\ \ 3) + \begin{pmatrix} 2 \\ 5 \end{pmatrix} (4\ \ 5\ \ 6) + \begin{pmatrix} 1 \\ 4 \end{pmatrix} (7\ \ 8\ \ 9)$$