NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester I : 2016/2017)

EE3731C – SIGNAL PROCESSING METHODS

Dec 2016 – Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** printed pages.
2. All questions are compulsory. Answer **ALL** questions.
3. This is a **CLOSED BOOK** examination. One A4-size formula sheet is allowed.
4. Programmable calculators are not allowed.

Q1 (25 marks). Subquestions (a), (b) and (c) can be answered independently.

(a) Perform PCA on the following set of data: $\begin{pmatrix}3\\3\end{pmatrix}\begin{pmatrix}-1\\-1\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix}\begin{pmatrix}2\\0\end{pmatrix}$. What is the first principal axis? What is the first principal component of the datapoint = $\begin{pmatrix}0\\1\end{pmatrix}$?

(13 marks)

(b) Consider training data $x_1 = \begin{pmatrix}1\\0\end{pmatrix}$, $x_2 = \begin{pmatrix}0\\1\end{pmatrix}$, $x_3 = \begin{pmatrix}-1\\0\end{pmatrix}$, $x_4 = \begin{pmatrix}0\\-1\end{pmatrix}$ with corresponding class labels $y_1 = 0,\ y_2 = 0,\ y_3 = 1,\ y_4 = 1$. What is the 3-NN estimate of the class label posterior probabilities of datapoints $x_5 = \begin{pmatrix}0\\0.5\end{pmatrix}$ and $x_6 = \begin{pmatrix}-1\\-1\end{pmatrix}$, where the distance metric used is the Euclidean distance? What are the MAP classifications of data points x_5 and x_6?

(6 marks)

(c) Consider data samples x_1, x_2, x_3, x_4 to be 1, 3, 4, 10. Using the Gaussian Parzen's window: $\frac{1}{\sqrt{2\pi h^2}} e^{-\frac{x^2}{2h^2}}$, what is the Parzen's window estimate of $p_h(x)$ at $x = 2$ and $x = 5$ for $h = 1$?

(6 marks)

Q2 (25 marks). Subquestions (a), (b), and (c) can be answered independently.

(a) Consider the following system T. Given an input $x[n]$, the output is $y[n] = T(x[n]) = ax[n] + b$. Is the system stable? Is the system causal? Is the system linear? Is the system time invariant? Is the system memoryless? Please explain your reasoning for full credits.

(5 mark)

(b) We wish to use Kaiser window method to design a FIR filter with generalized linear phase that meets the following specifications:

$$0.9 < \left|H\left(e^{jw}\right)\right| < 1.1, \qquad 0 \le |w| \le 0.2\pi$$
$$\left|H\left(e^{jw}\right)\right| < 0.06, \qquad 0.3\pi \le |w| \le 0.475\pi$$
$$1.9 < \left|H\left(e^{jw}\right)\right| < 2.1, \qquad 0.525\pi \le |w| \le \pi$$

The Kaiser window will be applied to the ideal impulse response associated with the ideal frequency response given by

$$H_I\left(e^{jw}\right) = \begin{cases} e^{-jwM/2}, & 0 \le |w| \le 0.25\pi \\ 0, & 0.25\pi \le |w| \le 0.5\pi \\ 2e^{-jwM/2}, & 0.5\pi \le |w| \le \pi \end{cases}$$

Find the Kaiser window parameters β and M so that the resulting windowed filter satisfies the above criteria. <u>Justify your choice of δ and Δw</u>

(12 marks)

(c) Consider two discrete time sequences $x_1[n]$ and $x_2[n]$, which are non-zero for $0 \le n \le N-1$. In class, we have learned that we can zero pad both sequences, and then exploit the fast Fourier transform to efficiently perform discrete convolution between x_1 and x_2. What is the minimum number of zeros that must be added to each sequence for this fast convolution procedure to work. Please explain your reasoning for full credits.

(8 marks)

Q3 (25 marks). Subquestions (a) and (b) can be answered independently. Subquestion (c) depends on subquestions (a) and (b), but you might be able to answer it without fully answering subquestions (a) and (b).

(a) Consider a biased coin where q is the probability of getting a head when the coin is tossed. The coin is tossed N times independently (q is fixed for all the coin tosses), resulting in N_1 heads and N_0 tails. What is the ML estimate of q? Please show your steps.

(10 marks)

(b) Consider a biased coin where q is the probability of getting a head when the coin is tossed. The coin is tossed N times independently (q is fixed for all the coin tosses), resulting in N_1 heads and N_0 tails. Suppose we assume the following prior distribution on q, such that the probability distribution function of q is given by $p(q) = \begin{cases} 2(1-q) & 0 \leq q \leq 1 \\ 0 & \text{otherwise} \end{cases}$. What is the MAP estimate of q? Please show your steps.

(12 marks)

(c) Why is the MAP estimate smaller than the ML estimate?

(3 marks)

Q4 (25 marks). Subquestions (a), (b) and (c) can be answered independently.

Let y have the distribution $p(y = 0) = 1/4$ and $p(y = 1) = 3/4$. Let

$$p(x|y = 0) = \begin{cases} 3/4 & x = 1 \\ 1/4 & x = 2 \end{cases}$$

and

$$p(x|y = 1) = \begin{cases} 1/4 & x = 1 \\ 3/4 & x = 2 \end{cases}$$

Suppose Mary plays the following game with John everyday from day $n = 1$ to $n = \infty$. On day n, Mary independently sampled (x_n, y_n) from the above distribution $p(x, y)$. Given y_n, John has to guess the value of x_n. Suppose John guesses $\hat{x}_n$ on the n-th day, then John has to pay Mary $(x_n - \hat{x}_n)^2$ dollar.

(a) Suppose John employs the optimal strategy to minimize payment to Mary. What should John guess on days when Mary tells John that $y = 0$?

(6 marks)

(b) Suppose John employs the optimal strategy to minimize payment to Mary. What should John guess on days when Mary tells John that $y = 1$?

(6 marks)

(c) Suppose John employs the MAP strategy. How much does John have to pay Mary each day (on average)?

(13 marks)

END OF PAPER