EE3731C Tutorial - Statistical Signal 3

Department of Electrical and Computer Engineering

1. Let $x_1, \cdots, x_N$ be independent samples from a Gaussian distribution. The Gaussian distribution has a known mean of μ and an unknown variance σ^2. Find the ML estimate of σ^2.

2. Give examples of Markov chain where the conditions of the Fundamental Theorem of Markov Chain does not hold.

3. Let x_n be a random walk defined as

$$x_0 = 0$$
$$x_N = x_0 + \sum_{n=1}^{N} z_n,$$

   where z_n is an i.i.d. process with $p(z_n = -1) = p(z_n = 1) = 1/2$. Define the absolute value random process $y_n = |x_n|$. Find $p\left(\max_{1 \leq n \leq 20} y_n = 10 | y_{20} = 0\right)$.

4. Let x_n be a random walk defined as

$$p(x_0) = \begin{cases} \frac{1}{5} & x_0 \in \{-2, -1, 0, 1, 2\} \\ 0 & \text{otherwise} \end{cases}$$
$$x_N = x_0 + \sum_{n=1}^{N} z_n,$$

   where z_n is an i.i.d. process with $p(z_n = -1) = p(z_n = 1) = 1/2$. What is $p(x_0 | x_{11} = 2)$?

5. Let x_n be a random walk defined as

$$x_0 = 0$$
$$x_N = x_0 + \sum_{n=1}^{N} z_n,$$

   where $z_n, n \geq 0$ is a discrete time white Gaussian noise process, i.e.., $z_1, z_2, \cdots$ are i.i.d $\mathcal{N}(0, 1)$. Given $x_1 = 4, x_2 = 2$ and $0 \leq x_3 \leq 4$, find the MMSE estimate of x_4