EE3731C Tutorial - Statistical Signal 2

Department of Electrical and Computer Engineering

1. Suppose random variables x_1 and x_3 are conditionally independent given x_2. Show that $p(x_1, x_2, x_3) = p(x_1)p(x_2|x_1)p(x_3|x_2) = p(x_3)p(x_2|x_3)p(x_1|x_2)$

2. A coin has a random bias q with pdf $p(q) = 2q$ for $0 \leq q \leq 1$ (0 otherwise), where q is the probability of the coin turning up head. q is random and unknown, but assumed to be fixed for the duration of the following random experiment: the coin is flipped 10 times independently, resulting in 9 heads.

   (a) Compute ML estimate of q

   (b) Compute MAP estimate of q

   (c) Compute MMSE estimate of q. Why is MMSE estimate smaller than MAP?

3. Consider random variables x and y with joint distribution

$$p(x, y) = \begin{cases} x + y & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   Suppose I play the following game with a student from day $n = 0$ to ∞. On day n, I sample $(x_n, y_n) \sim p(x, y)$. Given y_n, the student has to guess the value of x_n. Let the student's guess be $\hat{x}_n$. The student has to pay me $(x_n - \hat{x}_n)^2$ dollars.

   (a) To minimize the penalty (on average), should the student use ML, MAP or MMSE estimate?

   (b) Using the best strategy from (a), how much does the student pay me on average each day?

   (c) Suppose instead of the student paying me $(x_n - \hat{x}_n)^2$ dollars, I pay the student $\delta(x_n - \hat{x}_n)$ dollars. What is the optimal strategy the student should use to maximize the reward on average? How much would I pay the student on average each day?