EE3731C Tutorial - Classical Signal 4

Department of Electrical and Computer Engineering

Questions 2 and 3 won't be covered in the exams.

1. Consider one cycle of the Spincycle transform illustrated in Figure 1. Suppose x is equal to [6 12 120 116]. What are the coefficients at location (a)-(j) in the spincycle, assuming we use the Haar transform, where $g = g_0 = \left[\frac{\sqrt{2}}{2} \ \frac{\sqrt{2}}{2}\right]$, $h = \left[-\frac{\sqrt{2}}{2} \ \frac{\sqrt{2}}{2}\right]$ and $h_0 = \left[\frac{\sqrt{2}}{2} \ -\frac{\sqrt{2}}{2}\right]$

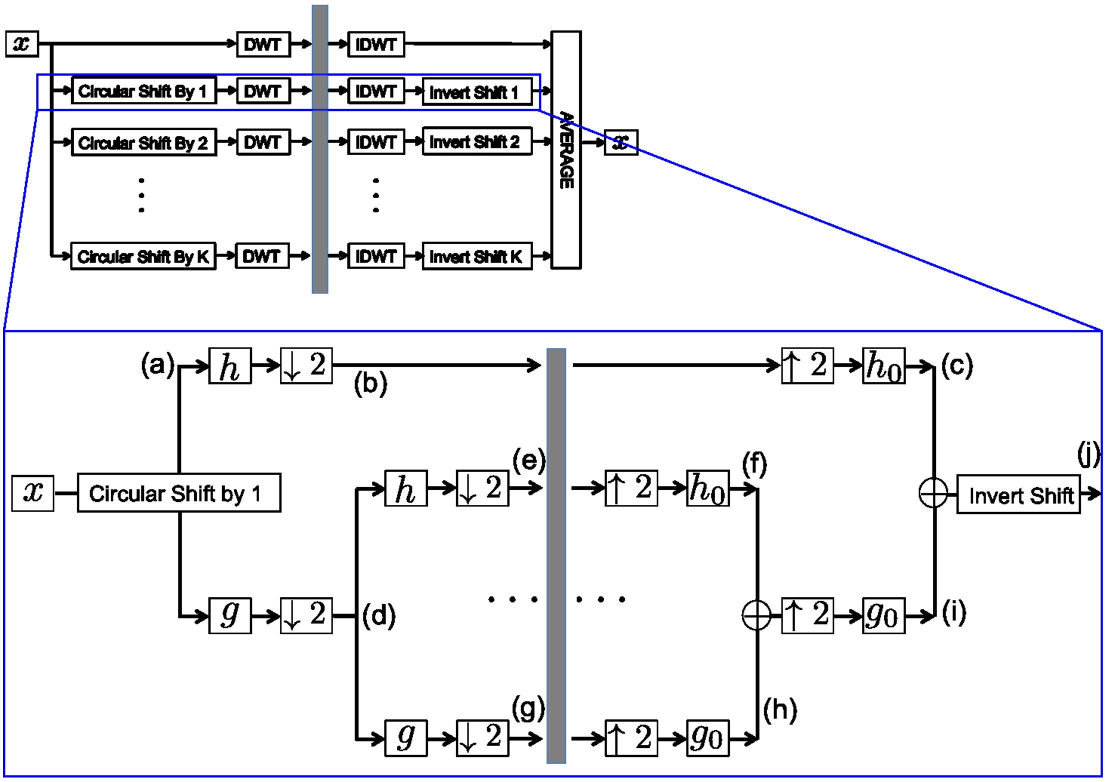

Figure 1: Zooming in on one cycle of Spincycle

2. Let $\phi(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$; $\phi_{s,u}(t) = 2^{-s/2}\phi(2^{-s}t - u)$, for $s, u \in \mathbb{Z}$. We can think of $\phi_{s,u}(t)$ as $\phi(t)$ stretched by scale 2^s and translated by $2^s u$. $\phi(t)$ is called the Haar scaling function and is intimately related to the Haar discrete wavelet transform we learned in lecture. We will explore this relationship in this tutorial and next lecture.

   (a) Show that for a fixed scale s: $\int_{-\infty}^{\infty} \phi_{s,u}(t)\phi_{s,u'}(t)dt = \delta(u-u')$ for all $u, u' \in \mathbb{Z}$, where $\delta(0) = 1$, and is 0 otherwise.

   (b) Let V_s = set of all functions that are linear combinations of $\phi_{s,u}$ (fixed s)

   $$V_s = \left\{ f(t) = \sum_{u \in \mathbb{Z}} \alpha_s[u]\phi_{s,u}(t), \text{where } \alpha_s[u] \in \mathbb{R} \right\}$$

   Show that

   $$\alpha_s[u'] = \int_{-\infty}^{\infty} f(t)\phi_{s,u'}(t)dt \text{ for all } u' \in \mathbb{Z}$$

   (c) Show that if $f(t) \in V_0$, then $f(2^{-s}t) \in V_s$ for all $s \in \mathbb{Z}$

   (d) Show that if $f(t) \in V_0$, then $f(t-u) \in V_0$ for all $u \in \mathbb{Z}$

   (e) Show that if $f(t) \in V_0$, then $f(t) \in V_{-1}$

3. Let $\psi(t) = \begin{cases} 1 & 0 \leq t < 1/2 \\ -1 & 1/2 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$; $\psi_{s,u}(t) = 2^{-s/2}\psi(2^{-s}t - u)$ for $s, u \in \mathbb{Z}$. We can think of $\psi_{s,u}(t)$ as $\psi(t)$ stretched by scale 2^s and translated by $2^s u$. $\psi(t)$ is called the Haar wavelet function (also called the Haar mother wavelet) and is intimately related to the Haar discrete wavelet transform we learned in lecture. We will explore this relationship in this tutorial and next lecture.

   (a) Show that $\int_{-\infty}^{\infty} \psi_{s,u}(t)\psi_{s',u'}(t)dt = \delta(s-s', u-u')$, where $\delta(0,0) = 1$ and is 0 otherwise.

   (b) Let W_s = all functions that are linear combinations of $\psi_{s,u}$ for a fixed scale s

   $$W_s = \left\{ f(t) = \sum_{u} \beta_s[u]\psi_{s,u}(t), \text{where } \beta_s[u] \in \mathbb{R} \right\}$$

   Show that

   $$\beta_s[u'] = \int_{-\infty}^{\infty} f(t)\phi_{s,u'}(t)dt$$

   (c) Show that $V_0 \cap W_0 = \{0\}$. In other words, the only function $f(t)$ contained in both V_0 and W_0 is $f(t) = 0$.

   (d) Let $V_1 \oplus W_1$ be the set of functions that are linear combinations of $\phi_{1,u}$ and $\psi_{1,u}$. Show that $V_0 = V_1 \oplus W_1$.