

EE3731C Tutorial - Statistical Signal I
Department of Electrical and Computer Engineering

1. Consider random variables x and y with joint distribution

$$p(x, y) = \begin{cases} x + y & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Verify that $p(x, y)$ is a valid probability distribution
 - (b) Find $p(x)$, $p(y)$ and $p(x|y)$
 - (c) Are x and y independent? Explain.
 - (d) Find $E(x)$ and $E(y)$
 - (e) Find $Var(x)$ and $Var(y)$
 - (f) Find $Cov(x, y)$
 - (g) Find $E(x^2|y = 0.5)$
2. Show that $E[(x - a)^2]$ is minimised when $a = E(x) = \mu_x$
3. Suppose c is a non-random constant. Show that $Var(cx) = c^2 Var(x)$
4. Suppose $E(x) = 5, E(x^2) = 30, E(y) = 3, E(xy) = 4$
- (a) Find $E(2x + 3y)$
 - (b) Find $Var(2x)$.
 - (c) Find $Cov(x, y)$.
 - (d) Are x and y independent? Explain.
5. We learned in class that if two random variables x and y are independent, then $Cov(x, y) = 0$. The converse is not true. Give an example where $Cov(x, y) = 0$, but x and y are not independent.