# The expert amateur

Getting really good at becoming not too bad

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## How sick are you?

You're feeling sick and so you go to the doctor, who runs a barrage of tests. It turns out that you test positive for a very very rare disease that affects 0.1% of the population.

In shock, you ask the doctor how likely is it that you have the disease. The doctor says that the test is fairly good — the test correctly identifies 99% of those who have the disease and incorrectly identifies 1% of those who do not have the disease.

**Q1: How likely is it that you have the disease?**

Being concerned, you go to another doctor, who also runs a barrage of tests (using another lab). It turns out you also test positive for the same very very rare disease.

**Q2: How likely is it that you have the disease now?**

# Bayes Theorem

$$P(+, D) = P(+|D)P(D) = P(D|+)P(+)$$

$$P(D|+) = \frac{Pr(+|D)P(D)}{P(+)} \Rightarrow P(D|+) = \frac{Pr(+|D)P(D)}{P(+|D)P(D) + P(+|ND)P(ND)}$$

$$Q1 : P(D|+) = \frac{0.99 \times 0.001}{0.99 \times 0.001 + 0.01 \times 0.999} = 0.090$$

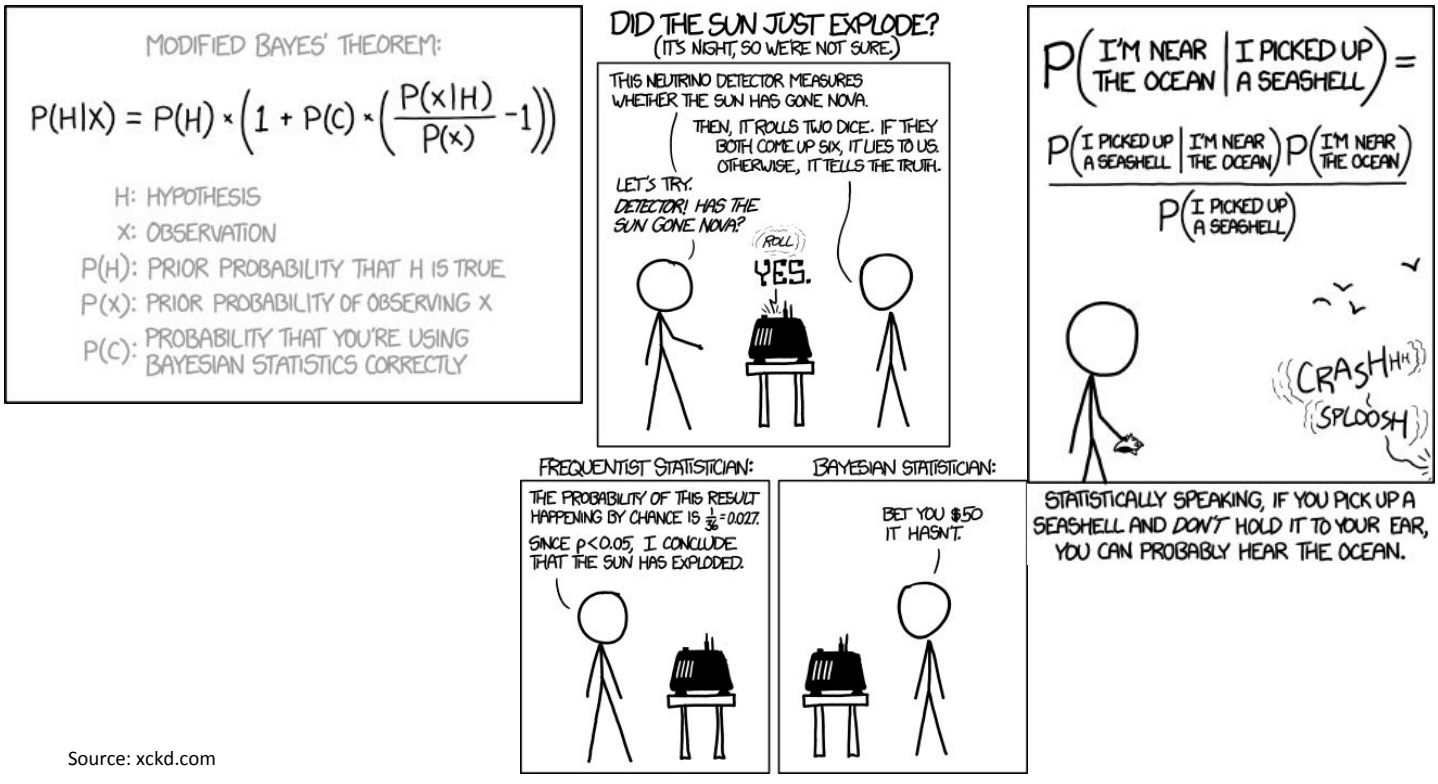
$$Q2 : P(D|+) = \frac{0.99 \times 0.09}{0.99 \times 0.09 + 0.01 \times 0.91} = 0.907$$

# Bayes Theorem makes sense

- Imagine 1000 people. Based on the given statistics, 1 person has the disease on average. This person will get identified as sick with very high probability.
- The test will also identify as sick 10 people who do not have the disease.
- So you are part of a group of 11 people, of which only one has the disease — so the likelihood you have the disease is 1 in 11 —> 9%
- After the second positive test, the likelihood of you having the disease goes up to 91% which also makes sense. It is unlikely that both labs are wrong.
- However, note that the probability is still not as high as the probability that the test is right given the disease.

# History of Bayes Theorem (from WikiPedia)

- Bayes' theorem was named after Thomas Bayes (1701–1761), who studied how to compute a distribution for the probability parameter of a binomial distribution (in modern terminology).
- Bayes's unpublished manuscript was significantly edited by Richard Price before it was posthumously read at the Royal Society. Price edited[8] Bayes's major work "An Essay towards solving a Problem in the Doctrine of Chances" (1763), which appeared in *Philosophical Transactions*,[9] and contains Bayes' theorem. Price wrote an introduction to the paper which provides some of the philosophical basis of Bayesian statistics. In 1765, he was elected a Fellow of the Royal Society in recognition of his work on the legacy of Bayes.[10][11]
- The French mathematician Pierre-Simon Laplace reproduced and extended Bayes's results in 1774, apparently unaware of Bayes's work.[note 1][12] The Bayesian interpretation of probability was developed mainly by Laplace.[13]

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