NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester I: 2016/2017)

EE4305 - INTRODUCTION TO FUZZY/NEURAL SYSTEMS

November/December 2016 - Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES:

1. This examination paper contains **FOUR (4)** questions and comprises **SIX (6)** pages.
2. Answer all FOUR questions.
3. All questions carry equal marks.
4. This is a closed book examination. But the candidate is permitted to bring into the examination hall a single A4 size formula sheet with formulae and related course material hand-written on both sides. The candidate may refer to this sheet during the examination.

Q.1 (a) Consider a fuzzy set,

$$A(x) = 10^{-x}$$

Use the extension principle to derive $f(A)$, where $f(x) = x^2$ for all $x \in X = [0, 10]$.

(5 Marks)

(b) Is B a fuzzy number? Justify your answer.

$$B(x) = \begin{cases} 1 & \text{for } 0 \le x \le 10 \\ 0 & \text{otherwise.} \end{cases}$$

(5 Marks)

(c) For a Mamdani fuzzy system with input x and output y, the membership functions are given as follows,

S = 1/0.5 + 0.6/0.6 + 0.2/0.7 + 0/0.75
M = 0/0.5 + 0.4/0.6 + 0.8/0.7 + 1/0.75 + 0.8/0.8 + 0.4/0.9 + 0/1
B = 0/0.75 + 0.2/0.8 + 0.6/0.9 + 1/1
VB = 1/40 + 0.6/28 + 0.2/20 + 0/18
VM = 0/40 + 0.4/28 + 0.8/20 + 1/18 + 0.8/16 + 0.4/12 + 0/10
VS = 0/18 + 0.2/16 + 0.6/12 + 1/10

(i) Based on the above membership functions for the input and output, develop fuzzy relations using the following three rules:

Rule 1: IF x is S, THEN y is VB
Rule 2: IF x is M, THEN y is VM
Rule 3: IF x is B, THEN y is VS

(ii) For crisp inputs x = 0.6, and x = 0.8, find the associated outputs, y, for this system using the max-min composition and center-of-area defuzzification method.

(15 Marks)

Q.2 (a) When gyros are calibrated for axis bias, they are matched with a temperature. Thus, we can have a relation of gyro bias (GB) vs. temperature (T). Suppose that we have fuzzy sets for a given gyro bias and a given temperature as follows,

GB = 0.2/3 + 0.4/4 + 1/5 + 0.4/6 + 0.2/7
T = 0.4/66 + 0.6/68 + 1/70 + 0.6/72 + 0.4/74

(i) Use a Mamdani implication to find the relation:

IF gyro is GB, THEN temperature is T

(ii) Suppose that we are given a new gyro bias (GB') as follows,

GB' = 0.6/3 + 1/4 + 0.6/5

Using max-min composition, find the new temperature associated with the new bias.

(10 Marks)

(b) Suppose that we want to design a fuzzy system to balance the inverted pendulum shown in Fig. 2.1, where m_c, m and g are the mass of the cart, mass of the pole and gravitational acceleration constant, respectively. Let the angle θ and its derivative $\dot{\theta}$ be the inputs to the fuzzy system and the force u applied to the cart be its output.

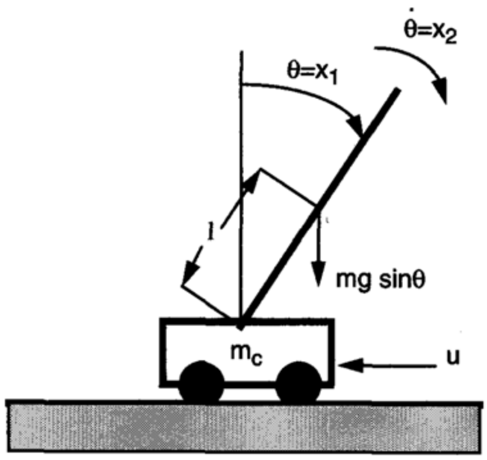

Fig. 2.1 An inverted pendulum system

Q.2 continues on Page 4

Design a few fuzzy IF-THEN rules based on common sense of how to balance the inverted pendulum. Based on these rules, apply an appropriate genetic algorithm to optimize the performance of the fuzzy system. At least the following should be included in your answer:

(i) An appropriate representation scheme for individuals/strings;
(ii) An appropriate fitness function;
(iii) An appropriate set of crossover and mutation;
(iv) An appropriate selection scheme.

Examples can be used to illustrate the design, e.g., crossover, mutation, etc. You should justify the design choices.

(15 Marks)

Q.3 (a) A neuron with a sigmoid activation function, $\varphi(v) = \dfrac{1}{1+\exp(-2v)}$, has two input weights, $w_1 = 0.5$ and $w_2 = 0.75$. For an input $X = (x_1, x_2) = (-1, 0.5)$, find the value of the bias weight such that the neuron output y is 0.8.

(10 Marks)

(b) Consider a two category classification problem in two input variables, x_1 and x_2. Suppose that C_1, the region corresponding to the first category (e.g., class 1), is shown in Fig. 3.1 and is given by,

$$C_1 = \{(x_1, x_2) : |x_1| < 1 \text{ and } |x_2| < 1\},$$

and the other category (e.g., class 0) is the remaining space. Suppose that we decide to use only McCulloch-Pitts neurons with hard limiter activation function. Is it possible to solve this problem exactly using a multi-layer perceptron (MLP)? If so, give the full details of a solution.

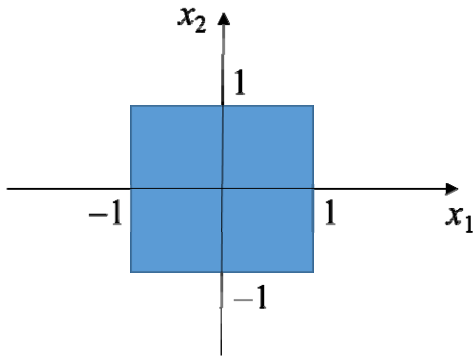

Fig. 3.1 A two category classification problem

(10 Marks)

(c) Discuss on how back-propagation (BP) learning in a multi-layer perceptron can be accelerated.

(5 Marks)

Q.4 (a) State the universal approximation theorem and discuss its implications.

(10 Marks)

(b) A function approximation problem with a training input pattern $\boldsymbol{x} = [0.2, 0.4, 0.7]^T$ is presented to a multilayer perceptron (MLP) as shown in Fig. 4.1, and the corresponding target output pattern is given as $\boldsymbol{d} = [1, 0]^T$. Assume that the learning rate parameter $\eta = 1.2$, the initial values for all biases are 0.01, all the values of input to hidden weights are –0.5, and all the hidden to output weights are 0.25.

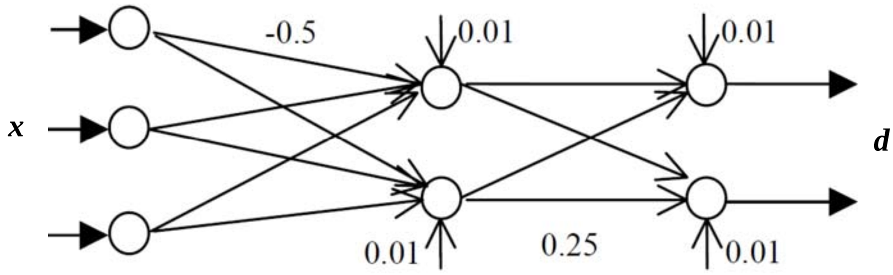

Fig. 4.1 A multilayer perceptron (MLP)

Assume that linear neurons, $\varphi(v) = v$, are used in the input layer, and sigmoidal neurons, $\varphi(v) = \dfrac{1}{1+\exp(-v)}$, are used in the hidden and output layers. Train the MLP network for one iteration (forward-pass and backward-pass) using the back-propagation (BP) algorithm.

(15 Marks)

END OF PAPER