EE4305 Fuzzy/Neural Systems for Intelligent Robotics

PART II: FUZZY SYSTEMS

Chapter 6: Fuzzy Inference

Topics to be Covered…

- Fuzzy sets and crisp sets
- Fuzzy operations, fuzzy relations, fuzzy compositions
- Extension principle, fuzzy numbers
- Approximate reasoning, fuzzy inference
- Multi-rule Fuzzy Inference
- Fuzzy knowledge based control (FKBC)

In the last five chapters, we have learned the basic concepts and operations of fuzzy sets.

Now it is time to apply them to build the fuzzy inference rules.

Approximate Reasoning, Fuzzy Inference

- In traditional logic, main tools of reasoning are tautologies: e.g.,

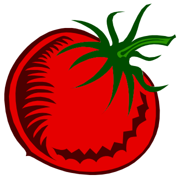
$$\frac{\begin{array}{c}\text{Rule : If } A \text{ then } B\\ \text{Fact : } A \text{ is true}\end{array}}{}$$

Rule : If A then B

Fact : A is true

Conclusion: B is true

- What about…



the tomato is *very* red,

if the tomato is red *then* the tomato is ripe,

→ the tomato is *very* ripe

Extensions: (1) Allow statements that are characterized by Fuzzy sets;

(2) Relax (slightly) the condition A

Rule : If X is A then Y is B

Fact: X is A'

Conclusion : Y is ...

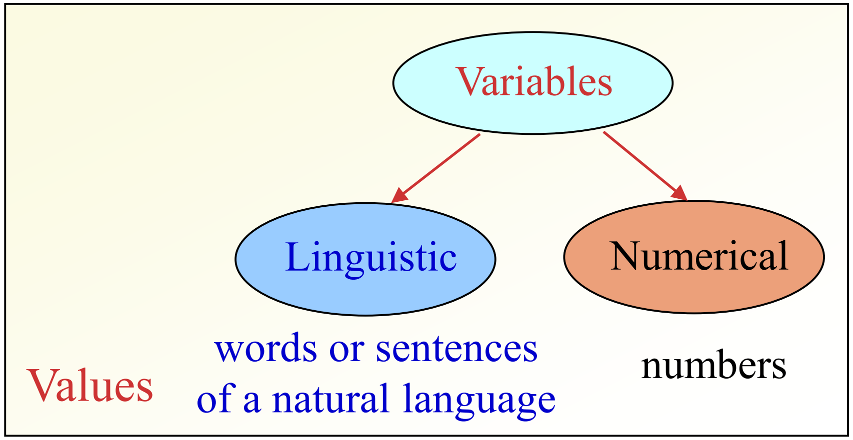
Approximate Reasoning ↔ Fuzzy Logic

When we try to convert the expert knowledge into fuzzy inference rules, we need to deal with human language.

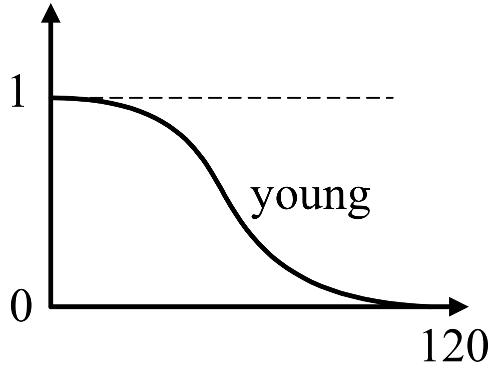
- Linguistic Variables

  Variable whose values are words or sentences in a natural language

  - *Age* → values are linguistic rather than numerical, i.e., *young*...



  - *Age*: refers to [0, 120] as the Universe of Discourse
  - Value: *young* can be represented by some fuzzy set



- (Def) Notion of a linguistic variable: $< X, T_X, \chi, M_X >$

X : name of the linguistic variable, e.g., *age*, *error*…

T_X: the set of *linguistic values* (Term set)

– For *age*, T_X = {*old*, *very old*, *not so old*, *very young*}

– An arbitrary element of this set is denoted by T_X, e.g., *old*

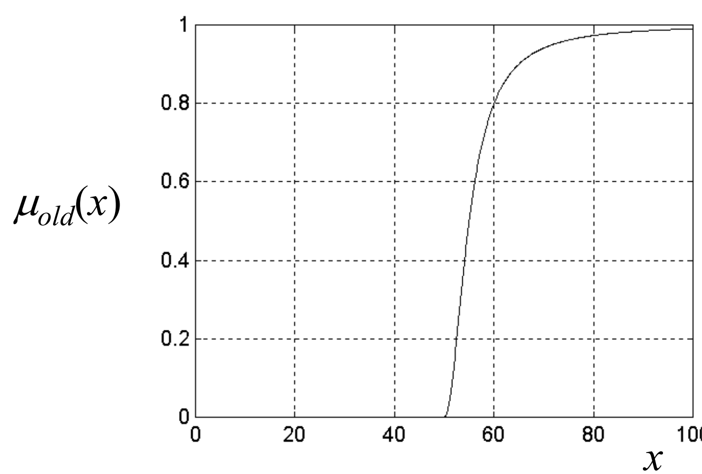
χ : the actual physical domain over which the X takes its crisp values

– E.g., For *age*, it can be the interval [0, 120]

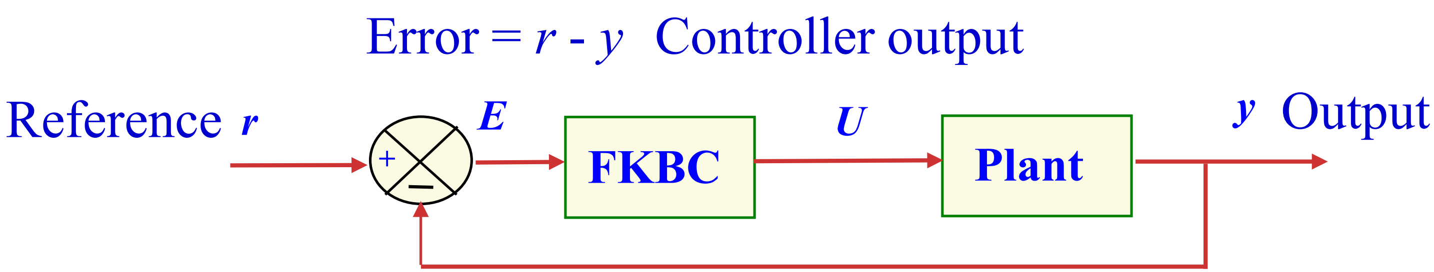
$M_X : T_X \rightarrow \mu_{TX}$ a *membership function* to give each T_X with a meaning

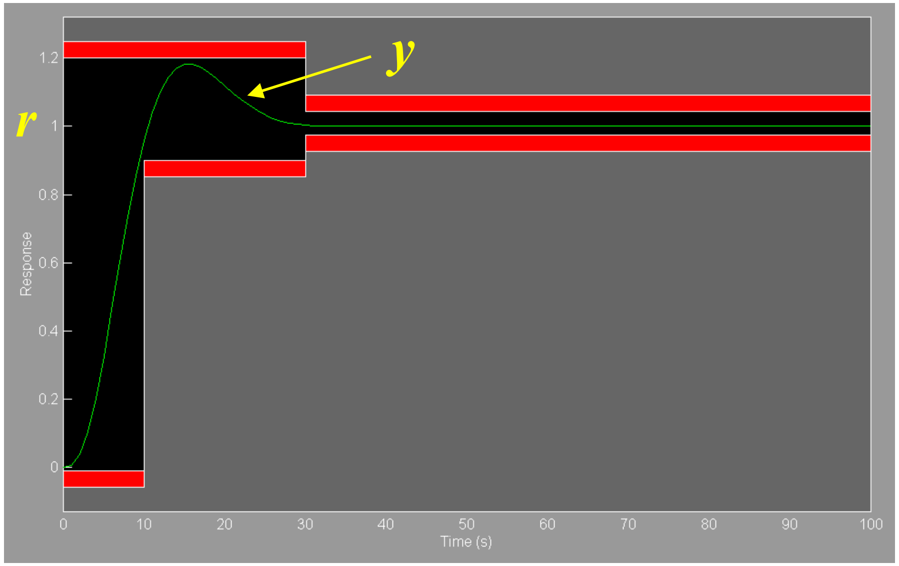
Example:

$$\mu_{old}(x) = \begin{cases} 0 & x \in [0,50] \\ \left(1 + \left(\dfrac{x-50}{5}\right)^{-2}\right)^{-1} & x \in [50,100] \end{cases}$$



A Closed-loop Control System





- Example: Error

$< E, T_E, \varepsilon, M_E >$

E: error

e.g., T_E = {NB, NM, NS, ZO, PS, PM, PB} $= T_U$

e.g., T_E = {NB}

e.g., ε = [-6, 6]

$M_E : T_E \rightarrow \mu_{TE}$

Control output (input to the system)

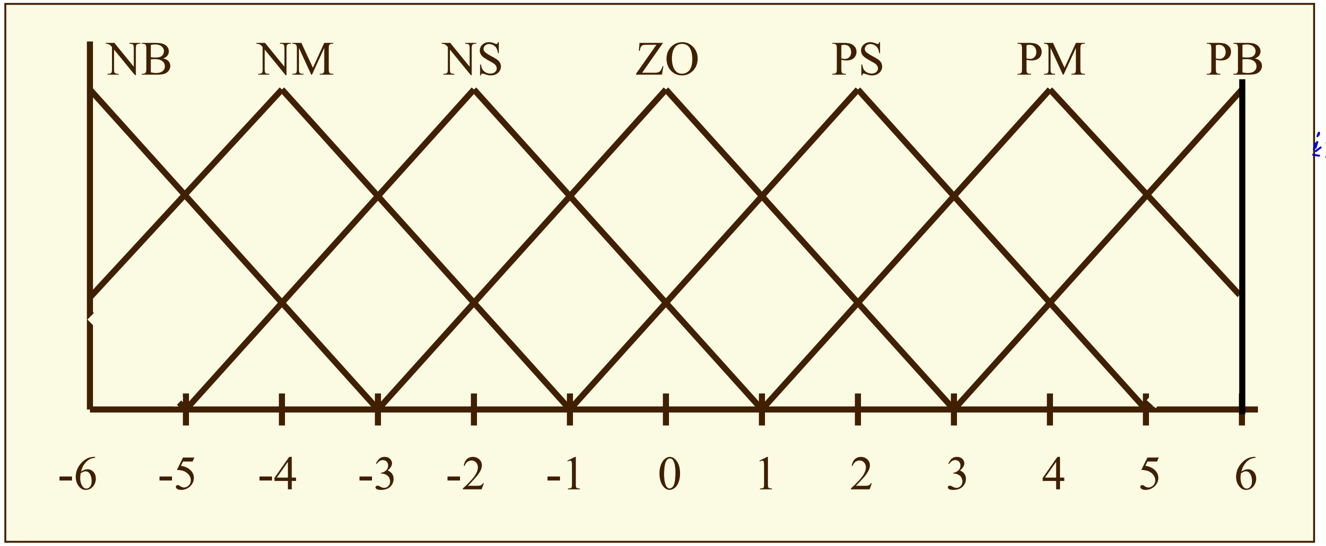
$< U, T_U, \mathcal{U}, M_U >$

U: control output

e.g., T_U = {ZO}

$= \mathcal{U}$

$M_U : T_U \rightarrow \mu_{TU}$



The shape for each membership function here is only an example

A typical fuzzy if-then rule:

If $\underline{E \text{ is } NB}$ 'and' $\underline{E\text{-}dot \text{ is } PB}$ then $\underline{U \text{ is } NS}$

1. Atomic fuzzy proposition

2. Compound fuzzy propositions

3. Fuzzy if-then statement

4. Multiple fuzzy if-then statements: To be covered in Chapter 7

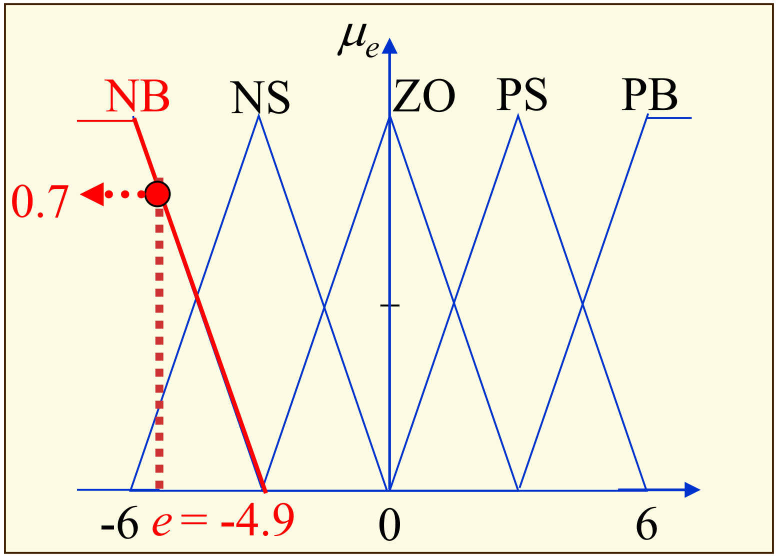
- **Fuzzy Propositions**
  - (Def) Atomic fuzzy proposition:

    "Error has the value negative-big" $\leftrightarrow$ *E* is *NB*

Fuzzy set

It is defined by a membership function μ_{NB} on the physical domain of, e.g., $\varepsilon = [-6, 6]$ for the physical variable 'error';

Degree of membership of –4.9 in μ_{NB} can be found, e.g., $\mu_{NB}(-4.9) = 0.7$



what about NS?

- (Def) Compound fuzzy propositions:

  E.g., *X* is *A* ‘and’ *X* is *B*, *X* is *A* ‘or’ *X* is *B*

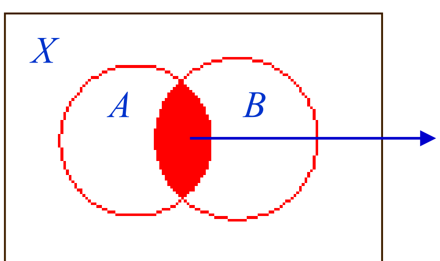
  How to interpret such statements?

- Conjunction ($\cap$): and

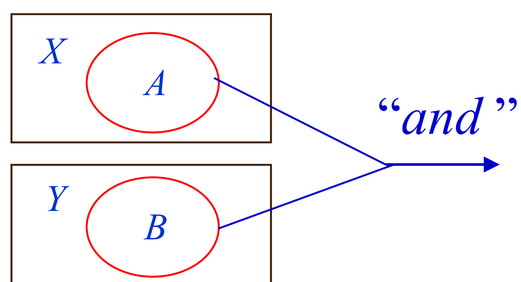
Symbolic	Meaning
X is *A*	μ_A
X is *B*	μ_B
$\therefore$ *X* is *A* ‘and’ *B*	$\therefore\ \mu_{A \cap B}$

- Disjunction ($\cup$): or

Symbolic	Meaning
X is *A*	μ_A
X is *B*	μ_B
$\therefore$ *X* is *A* ‘or’ *B*	$\therefore\ \mu_{A \cup B}$



“*and*” = $\cap$



$\mu(a, b)$
$= \min\{\mu(a), \mu(b)\} / (a, b)$

- Conjunction (and)/Disjunction(or) over different space variables:

E.g.,

E is NB with $\int_{\varepsilon} \mu_{NB}(e)/e$; $\dot{E}$ is PS with $\int_{\dot{\varepsilon}} \mu_{PS}(\dot{e})/\dot{e}$

Note that the linguistic variables e and $\dot{e}$ are defined on *different* spaces ε and $\dot{\varepsilon}$. Then,

E is NB and $\dot{E}$ is PS $\Rightarrow$ $\mu(e,\dot{e}) = \int_{\varepsilon\times\dot{\varepsilon}} min[\mu_{NB}(e), \mu_{PS}(\dot{e})]/(e,\dot{e})$

E is NB or $\dot{E}$ is PS $\Rightarrow$ $\mu(e,\dot{e}) = \int_{\varepsilon\times\dot{\varepsilon}} max[\mu_{NB}(e), \mu_{PS}(\dot{e})]/(e,\dot{e})$

- Fuzzy If-then Statements

A fuzzy conditional or a fuzzy if-then production rule is symbolically expressed as *if* < fuzzy proposition > *then* <fuzzy proposition >

E.g. "If $\underline{E \text{ is } NB \text{ `and' } \dot{E} \text{ is } PB}$ then $\underline{U \text{ is } NS}$"

"Knowledge = organized collection of propositions"

“If $\underline{E \text{ is } NB}$ then $\underline{U \text{ is } NS}$”

– fuzzy proposition
– input
– antecedent

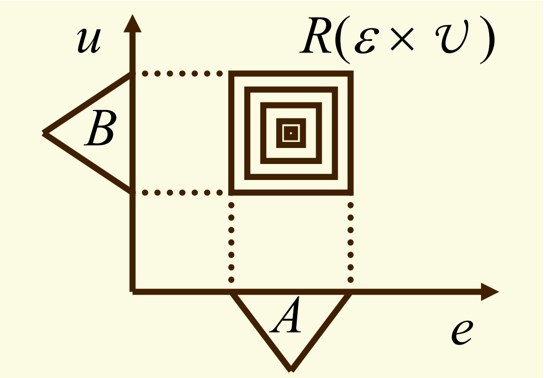
– fuzzy proposition
– output
– consequent

♦ How to describe fuzzy if-then rule ?, e.g., if E is A then U is B

if E is A then U is B

$\Updownarrow$

$\Rightarrow$ fuzzy relation $R(\varepsilon \times \mathcal{U})$



Implication is just a special type of relation!

$A(X) \longrightarrow \boxed{R(X,Y)} \longrightarrow B(Y)$???

If A then B, with a fuzzy relation $R(X,Y)$

E.g., R: John is *some what taller* than Albert

$A(X)$: John is *rather tall*

$\Rightarrow$ Albert is *medium*

Once the relation can be quantified, then it is very easy to draw conclusion by Fuzzy Composition (covered in chapter 4)

Fuzz set	o	Fuzzy relation	=	Fuzz set
$1 \times n$		$n \times m$		$1 \times m$
$A(X)$		$R(X,Y)$		$B(Y) = A \circ R$

There are various ways to define the fuzzy implication relation.

♦ Type of Fuzzy Implications (Representing the meaning of if-then rules)

– **K**leene-**D**ienes Implication

Implication operator

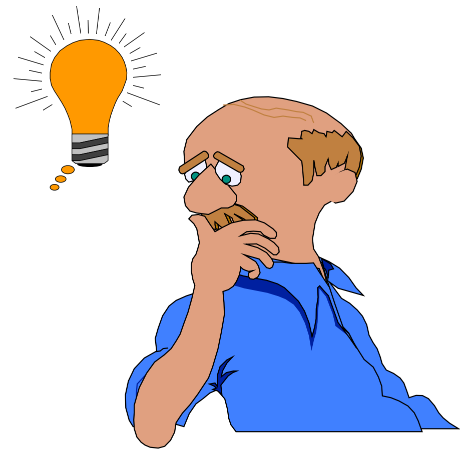
$$\delta(a,b)=\max[1-a,\,b]$$

– Lukasiewicz Implication

$$\delta(a,b)=\min[1,\,1-a+b]$$

– Zadeh Implication

$$\delta(a,b)=\max[\min(a,b),\,1-a]$$

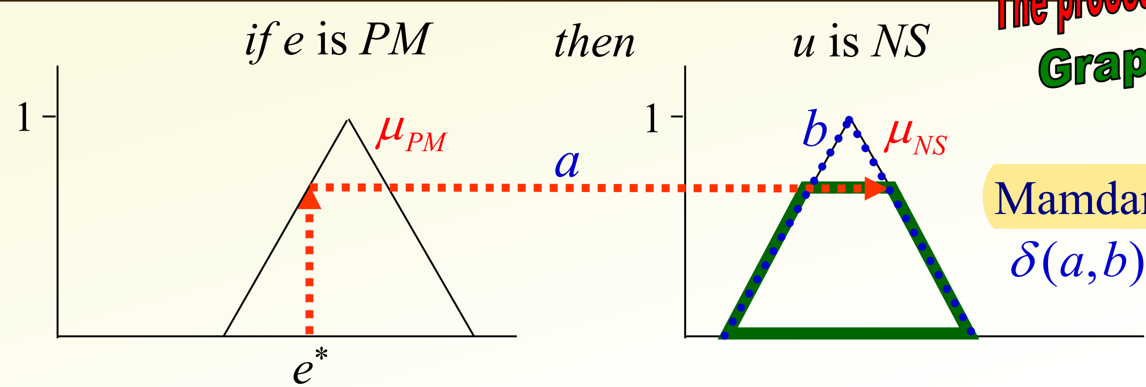


– Mamdani Implication

$$\delta(a,b)=\min[a,\,b]$$

The process is called Fuzzy Inference

Graphical Approach!



Mamdani Implication

$$\delta(a,b)=\min[a,\,b]$$

Now we are ready to answer the question posed at the beginning:

$$\underline{\text{Rule : If } X \text{ is } A \text{ then } Y \text{ is } B}$$

$$\text{Fact: } X \text{ is } A'$$

$$\text{Conclusion : } Y \text{ is } B'$$

With Mamdani Implication, the procedure is very easy to understand, even without doing the fuzzy composition.

To simplify the notation, we will use A(x) to denote the membership function for a fuzzy set A.

In the above inference rule, A and B are fuzzy sets, whose membership functions, $A(x)$ and $B(y)$, are known.

Give the fact, which is also described by a fuzzy set, $A'(x)$.

How to draw the conclusion, which is described by the fuzzy set $B'(y)$?

Rule : If X is A then Y is B

Fact: X is A'

Conclusion : Y is B'

If the fact matches the condition perfectly, i. e. $A'(x)$=A(x), then what is the conclusion B'(y)?

Of course, B'(y)=$B(y)$.

If the fact does not match the condition at all, i. e. $A' \cap A = \varnothing$, then what is the conclusion B'(y)?

Of course, B'(y)= $\varnothing$.

If the fact partially matches the condition, $A' \cap A \neq \varnothing$, what do we expect?

We can only draw a conclusion partially, i.e. B'(y)$\subset$$B(y)$.

In order to compute the conclusion exactly, we need to measure the degree to which the fact matches the condition of the rule.

Rule : If X is A then Y is B

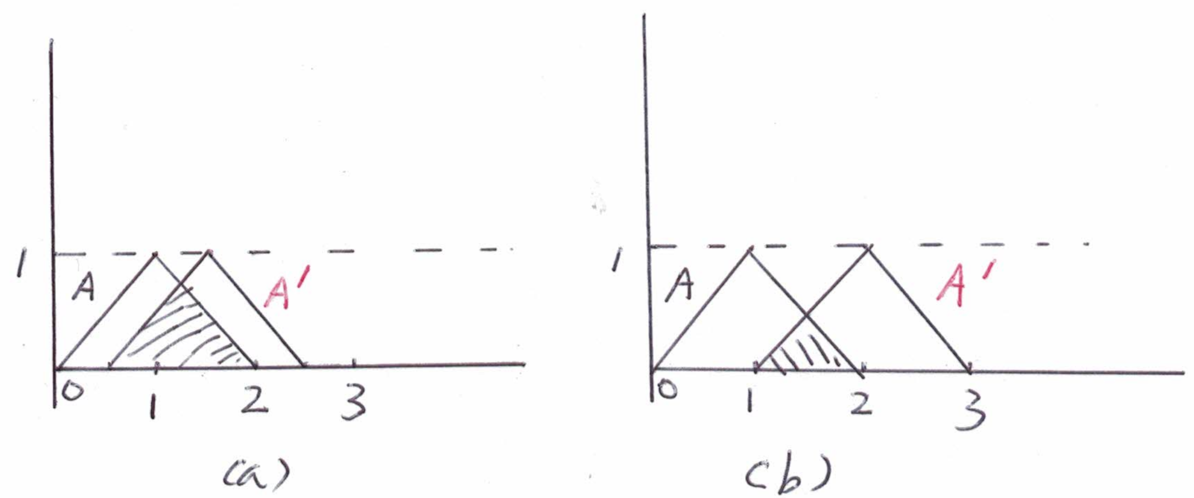
Fact: X is A'

Conclusion : Y is B'

We need to measure the degree to which the fact matches the condition of the rule.

We need to obtain a measure from the intersection of the fact and the condition $A' \cap A$.

Let's compare the two cases:



Visually we can see that case (a) is a better match of the condition. But how to quantify this?

It seems that we can use the shaded area of the membership function of the intersection.

Rule : If X is A then Y is B

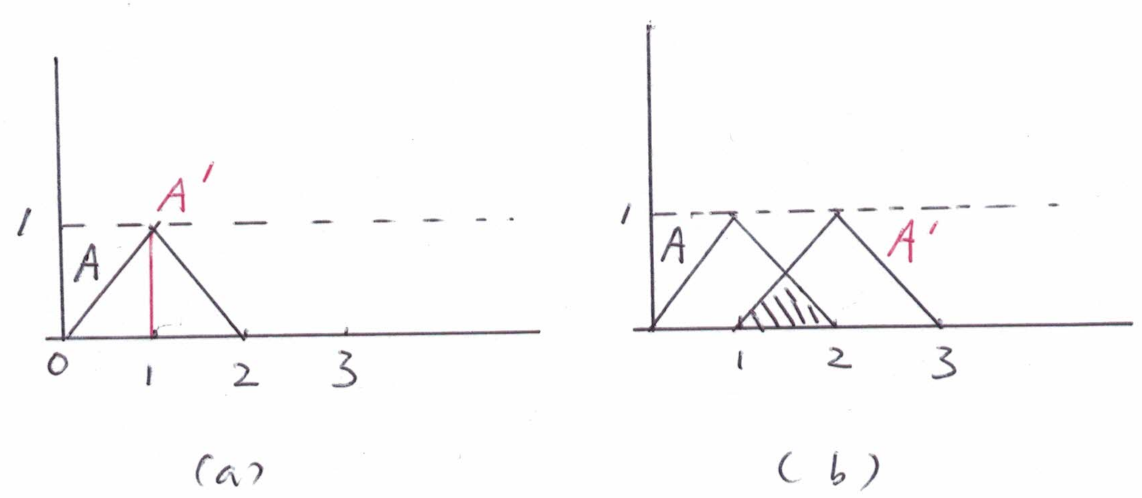
Fact: X is A'

Conclusion : Y is B'

Let's consider another interesting case where the fact is just one crisp number.

The condition: "if the temperature is around 1 °C"

The fact: the temperature is exactly 1°C.



Does this fact fulfill the condition completely?

Certainly! Hence, a better measure of the degree to which the fact matches the condition of the rule, is simply the height of the intersection!

Rule : If X is A then Y is B

Fact: X is A'

Conclusion : Y is B'

♦ (Def) Degree of Fulfilment of Fuzzy Condition:

The degree of fulfilment of the given fact to the condition of *if-then* rule is defined to be the height of the intersection of the fact and condition.

$$r(A') = h(A' \cap A)$$

Once we know the degree of fulfilment of the fact to the condition, we then know how to draw the conclusion.

Rule : If X is A then Y is B

Fact: X is A'

Conclusion : Y is B'

♦ (Def) Firing Strength of Fuzzy Rule

Firing strength is a number in [0,1] which measures the degree to which the rule is fired.

Let the firing strength be s, the conclusion is simply

$$B'(y) = min(s, B(y))$$

It means that the membership function B(y) is truncated by the firing strength.

If the firing strength is 1, the rule will be fully fired, then $B'(y) = B(y)$.

If the firing strength is 0, the rule will not be fired, then $B'(y) = \varnothing$.

If there is only one condition in the rule, then the firing strength is simply the degree of fulfilment of the fact to the condition.

Rule : If X is A then Y is B

Fact: X is A'

Conclusion : Y is B'

Step one: Evaluate the degree of fulfilment of the fact to the condition:

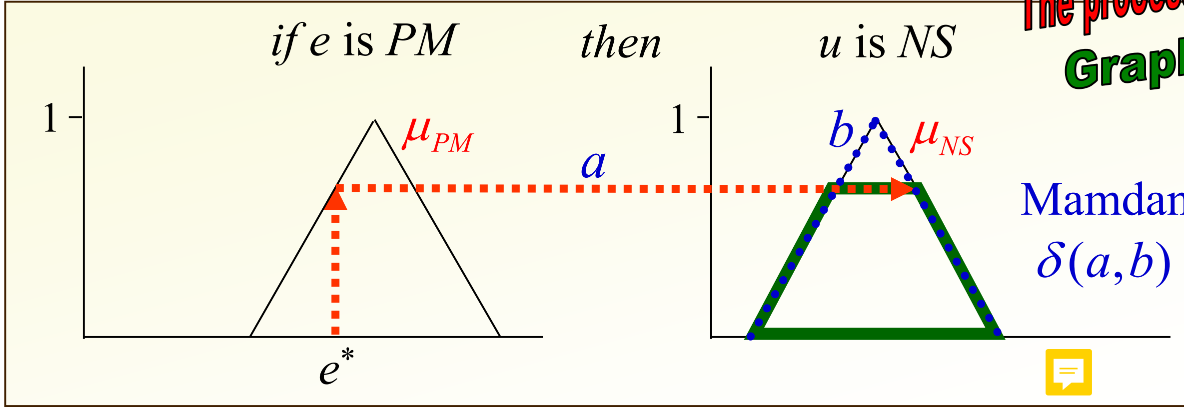
$$r(A') = h(A' \cap A)$$

Step Two: Set the firing strength as $r(A')$, and draw the conclusion:

$$B'(y) = min(r(A'), B(y))$$

The process is called Fuzzy Inference

Graphical Approach!



Mamdani Implication

$$\delta(a,b) = \min[a, b]$$

Rule : If X is A AND Y is B, *then z is C*

Fact: X is A', Y *is* B'

Conclusion : Z is C'

Step one: Evaluate the degree of fulfilment of the facts to the conditions:

$$r(A') = h(A' \cap A)$$

$$r(B') = h(B' \cap B)$$

Step Two: Compute the firing strength.

Since the two conditions have to be satisfied at the same time, i.e. "AND". The firing strength is obviously the minimum.

$$s = min(r(A'), r(B'))$$

Step Three: Draw the conclusion, i.e. truncate the membership function by the firing strength.

$$\mathrm{C}'(z) = min(s, C(z))$$

Rule : If X is A OR Y is B, *then z is C*

Fact: X is A', Y *is* B'

Conclusion : Z is C'

Step one: Evaluate the degree of fulfilment of the facts to the conditions:

$$r(A') = h(A' \cap A)$$

$$r(B') = h(B' \cap B)$$

Step Two: Compute the firing strength.

Since only one condition needs to be satisfied, i.e. "OR". The firing strength is obviously the maximum.

$$s = max(r(A'), r(B'))$$

Step Three: Draw the conclusion , i.e. truncate the membership function by the firing strength.

$$C'(z) = min(s, C(z))$$