EE4305 Fuzzy/Neural Systems for Intelligent Robotics

SOLUTION FOR PB 2

1 The error function is $E = \sum_{j=1}^{4} [d(j) - y(j)]^2$

where $y(j) = w \cdot x(j) - b$

Hence, $E = (1.5 + b)^2 + (2.5 - w + b)^2 + (3.6 - 2w + b)^2 + (4.2 - 3w + b)^2$

$$\frac{\partial E}{\partial w} = 2(2.5 - w + b) + 4(3.6 - 2w + b) + 6(4.2 - 3w + b) = 0$$
$$\frac{\partial E}{\partial b} = 2(1.5 + b) + 2(2.5 - w + b) + 2(3.6 - 2w + b) + 2(4.2 - 3w + b) = 0$$

Simplifying the two equations, we have

$$44.6 + 12b - 28w = 0$$
$$23.6 + 8b - 12w = 0$$

Solving the two equations, we have w = 0.92 and b = -1.57

2. (a) Given the cost function:

$$E(\mathbf{w}) = \frac{1}{2}\sigma^2 - \mathbf{r}_{\mathbf{x}d}^T \mathbf{w} + \frac{1}{2}\mathbf{w}^T \mathbf{R}_{\mathbf{x}} \mathbf{w}$$

Gradient of E wrt **w** yields:

$$\nabla E = -\mathbf{r}_{\mathbf{x}d} + \mathbf{R}_{\mathbf{x}} \mathbf{w}$$

Optimum value of **w** is defined by $\nabla E = \mathbf{0}$

$$-\mathbf{r}_{\mathbf{x}d} + \mathbf{R}_{\mathbf{x}} \mathbf{w} = \mathbf{0}$$

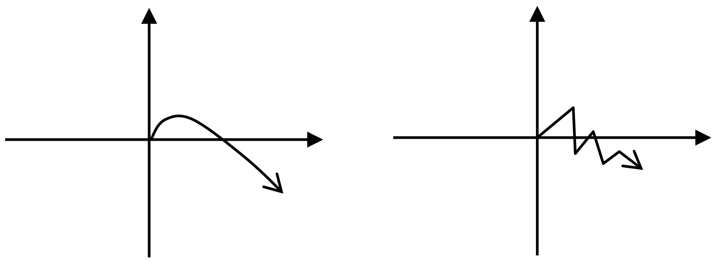
That is, $\mathbf{w}^* = \mathbf{R}_{\mathbf{x}}^{-1} \mathbf{r}_{\mathbf{x}d} = \begin{bmatrix} 1.599 \\ -0.954 \end{bmatrix}$

(b) Using the method of steepest descent, we may compute $\mathbf{w}^*$ by apply the recursion:

$$\begin{aligned}\mathbf{w}(n+1) &= \mathbf{w}(n) - \eta\nabla E \\ &= \mathbf{w}(n) - \eta(-\mathbf{r}_{\mathbf{x}d} + \mathbf{R}_{\mathbf{x}}\mathbf{w}(n))\end{aligned} \quad \text{-------(1)}$$

Choose initial condition: $\mathbf{w}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Use Eq (1) to compute the trajectory of $\mathbf{w}(n)$ for increasing n.



η=0.3 η=1.0