EE4305 Fuzzy/Neural Systems for Intelligent Robotics

PROBLEM SOLVING 2

1. Consider a function approximation problem with a single input variable, x, and a single output variable, y. Suppose the training data of (x, y) pairs are: (0, 1.5), (1, 2.5), (2, 3.6) and (3, 4.2), and we wish to use a single-layer perceptron for this problem, i.e., $y = w \cdot x - b$. Assume that the error function is given as, $E = \sum_{j=1}^{4} [d(j) - y(j)]^2$, where d is the desired output. Determine the optimal values for the weight, w, and bias, b.

2. Consider the cost function

$$E(\mathbf{w}) = \frac{1}{2}\sigma^2 - \mathbf{r}_{\mathbf{x}d}^T \mathbf{w} + \frac{1}{2}\mathbf{w}^T \mathbf{R}_\mathbf{x} \mathbf{w}$$

where σ^2 are some constants, and

$$\mathbf{r}_{\mathbf{x}d} = \begin{bmatrix} 0.8182 \\ 0.354 \end{bmatrix}$$

$$\mathbf{R}_\mathbf{x} = \begin{bmatrix} 1 & 0.8182 \\ 0.8182 & 1 \end{bmatrix}$$

(a) Find the optimum value $\mathbf{w}^*$ for which E($\mathbf{w}$) reaches its minimum value.

(b) Use the method of steepest descent to compute $\mathbf{w}^*$ for the following two values of learning-rate parameter:

(i) $\eta = 0.3$;
(ii) $\eta = 1.0$

For each case, plot the trajectory traced by the evolution of the weight vector $\mathbf{w}(n)$ in the W-plane. Start from $\mathbf{w}(0) = [0\ 0]^T$.