# SOLUTION FOR PB1

(1)

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

so,

$$\varphi'(v) = \frac{d\varphi(v)}{dv}$$

$$= \frac{1}{(1+v^2)^{\frac{1}{2}}} - \frac{v^2}{(1+v^2)^{\frac{3}{2}}}$$

$$= \frac{1}{(1+v^2)^{\frac{3}{2}}}$$

$$= \left(\frac{\varphi(v)}{v}\right)^3$$

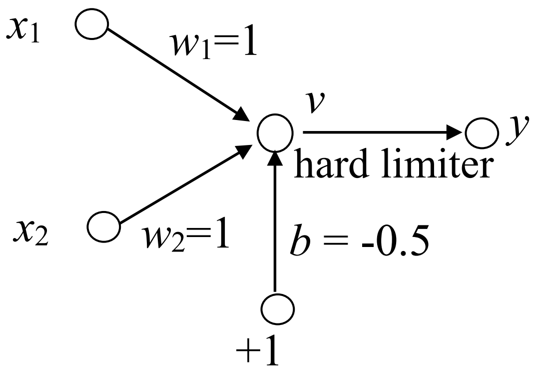
$$\varphi'(0) = 1$$

(2) OR Operation

Table: Truth table

| Inputs | | Output |
|---|---|---|
| $x_1$ | $x_2$ | $y$ |
| 1 | 1 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 0 | 0 |

The OR operation may be realized using the perceptron:




The hard limiter input is $v = w_1x_1 + w_2x_2 + b = x_1 + x_2 - 0.5$

If $x_1 = x_2 = 1$, then $v = 1.5$, and $y = 1$
If $x_1 = 0$, and $x_2 = 1$, then $v = 0.5$, and $y = 1$
If $x_1 = 1$, and $x_2 = 0$, then $v = 0.5$ and $y = 1$
If $x_1 = x_2 = 0$, then $v = -0.5$ and $y = 0$

These conditions agree with the truth table.

(3) Diff. $E(w)$ wrt $w$:

$$\frac{dE(w)}{dw} = -r_{xd} + r_x w$$

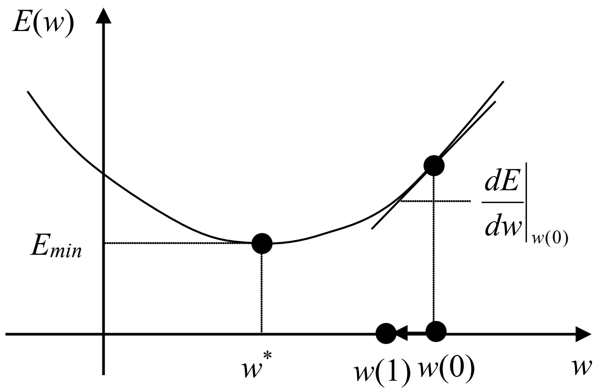
Optimal value of $w$ correspond to $\frac{dE(w)}{dw} = 0$

=> $w^* = r_{xd}/r_x$ (Check $\frac{d^2E(w)}{dw^2} = r_x > 0 \Rightarrow Min$ pt )

Using the method of steepest descent, recursive equation to compute the optimal value $w^*$:

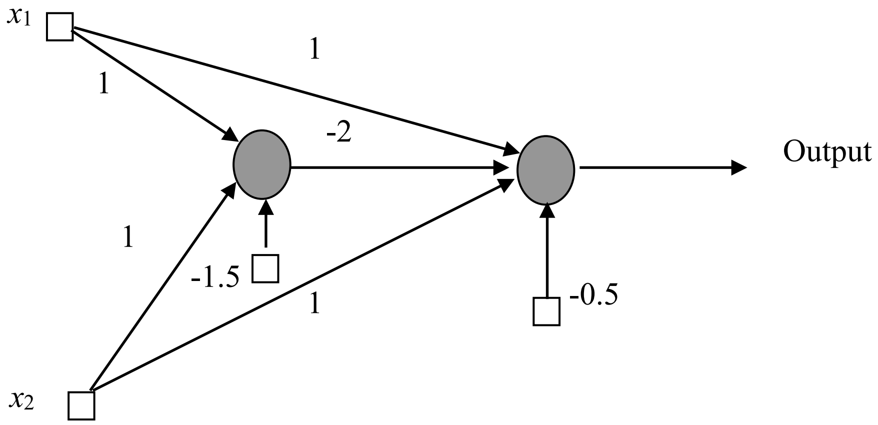
$$\begin{aligned} w(n+1) &= w(n) - \eta \frac{dE(w)}{dw} \\ &= w(n) - \eta(r_x w(n) - r_{xd}) \end{aligned}$$

where $\eta$ is the learning rate parameter.



(4)

Fig. 1




Assume each neuron is represented by a McCulloch-Pitts model.

Let $x_i = \begin{cases} 1\ if\_input\_bit = 1 \\ 0\ if\_input\_bit = 0 \end{cases}$

Induced local field of neuron 1 is $v_1 = x_1 + x_2 - 1.5$ =>

| | | | | |
|---|---|---|---|---|
| $x_1$ | 0 | 0 | 1 | 1 |
| $x_2$ | 0 | 1 | 0 | 1 |
| $v_1$ | -1.5 | -0.5 | -0.5 | 0.5 |
| $y_1$ | 0 | 0 | 0 | 1 |

The induced local field of neuron 2 is: $v_2 = x_1 + x_2 - 2y_1 - 0.5$ =>

| | | | | |
|---|---|---|---|---|
| $x_1$ | 0 | 0 | 1 | 1 |
| $x_2$ | 0 | 1 | 0 | 1 |
| $y_1$ | 0 | 0 | 0 | 1 |
| $v_2$ | -0.5 | 0.5 | 0.5 | -0.5 |
| $y_2$ | 0 | 1 | 1 | 0 |

=> XOR gate