EE4305 Fuzzy/Neural Systems for Intelligent Robotics

PART II: FUZZY SYSTEMS

Chapter 7: Multi-rule Fuzzy Inference

Topics to be Covered…

- Fuzzy sets and crisp sets
- Fuzzy operations, fuzzy relations, fuzzy compositions
- Extension principle, fuzzy numbers
- Approximate reasoning, fuzzy inference
- Multi-rule Fuzzy Inference
- Fuzzy knowledge based control (FKBC)

A typical fuzzy if-then rule:

If *E* is *NB* 'and' *E-dot* is *PB* then *U* is *NS*

1. Atomic fuzzy proposition

2. Compound fuzzy propositions

3. Fuzzy if-then statement

4. Multiple fuzzy if-then statements: To be covered in Chapter 7

Fuzzy Inference:

Rule: If X is A, then Y is B

Fact: X is A',

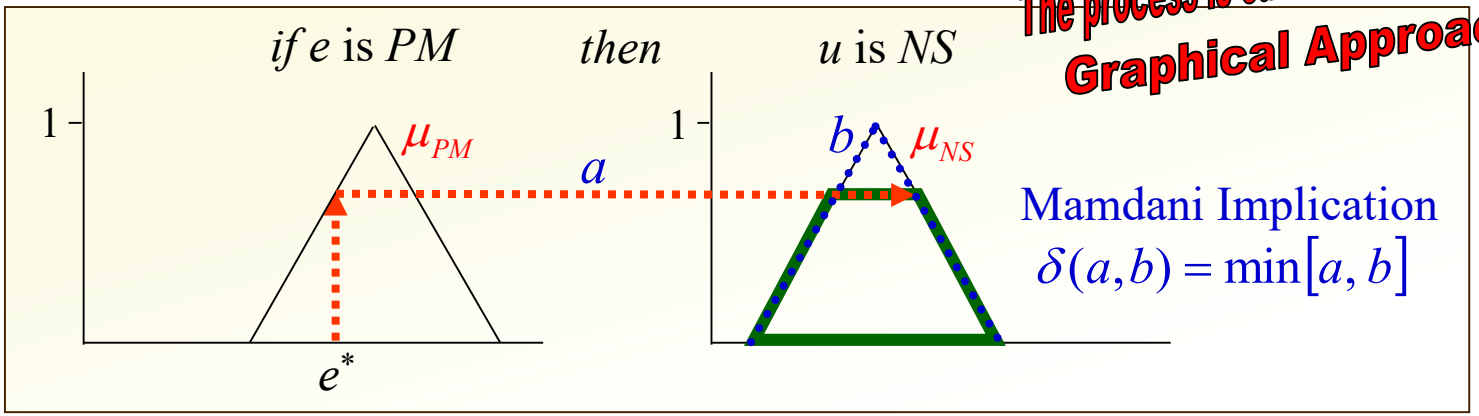
Conclusion: Y is B'

Step one: Evaluate the degree of fulfilment of the fact to the condition:

$$r(A') = h(A' \cap A)$$

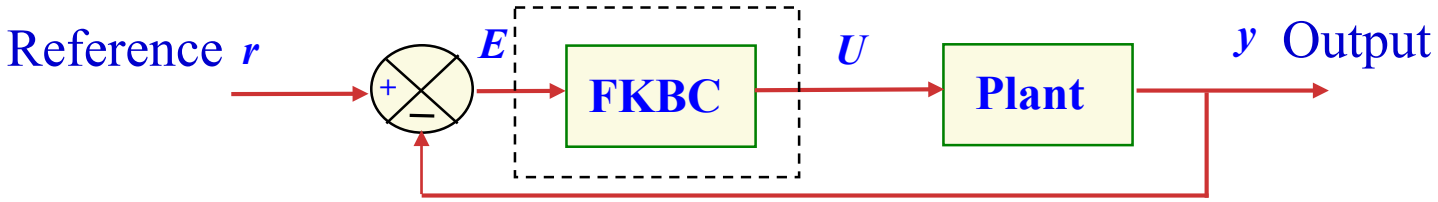
Step Two: Set the firing strength as $r(A')$, and draw the conclusion by truncating the membership function of B:

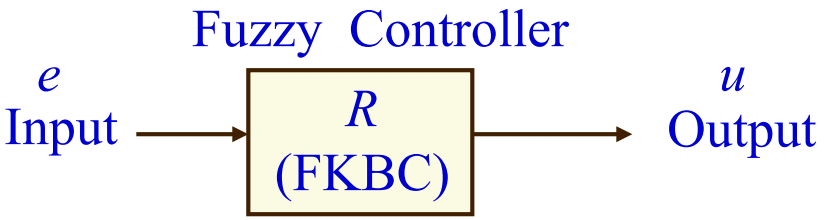
$$B'(y) = min(r(A'), B(y))$$



Firing of rules, Tipping problem…

- Inference in Fuzzy Controller (*first visit*)





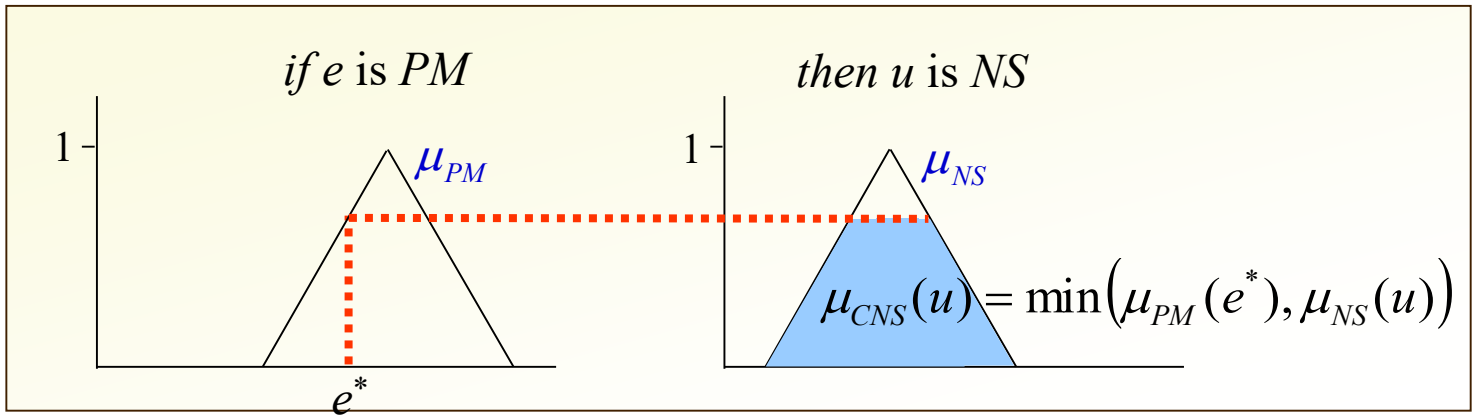
R : a set of fuzzy if-then rules

: Fuzzy knowledge Base

Problem: Given R (or a set of if-then rules) and e, find u.

♦ Multi-rule Fuzzy Inference

The result of fuzzy inference for one rule is a truncated fuzzy set:



The general Schema of multi-rule fuzzy inference has the form:

Rule 1 : If $\mathcal{X}$ is A_1, then $\mathcal{Y}$ is B_1
Rule 2 : If $\mathcal{X}$ is A_2, then $\mathcal{Y}$ is B_2
. . . . . .
Rule n : If $\mathcal{X}$ is A_n, then $\mathcal{Y}$ is B_n
Fact : $\mathcal{X}$ is A'

Conclusion : $\mathcal{Y}$ is B'

The most common way to determine B' is the "method of interpolation".

Step 1. Calculate the degree of fulfilment, $r_k(A')$, for the given fact and the condition of each *if-then* rule k

$$r_k(A') = h(A' \cap A_k)$$

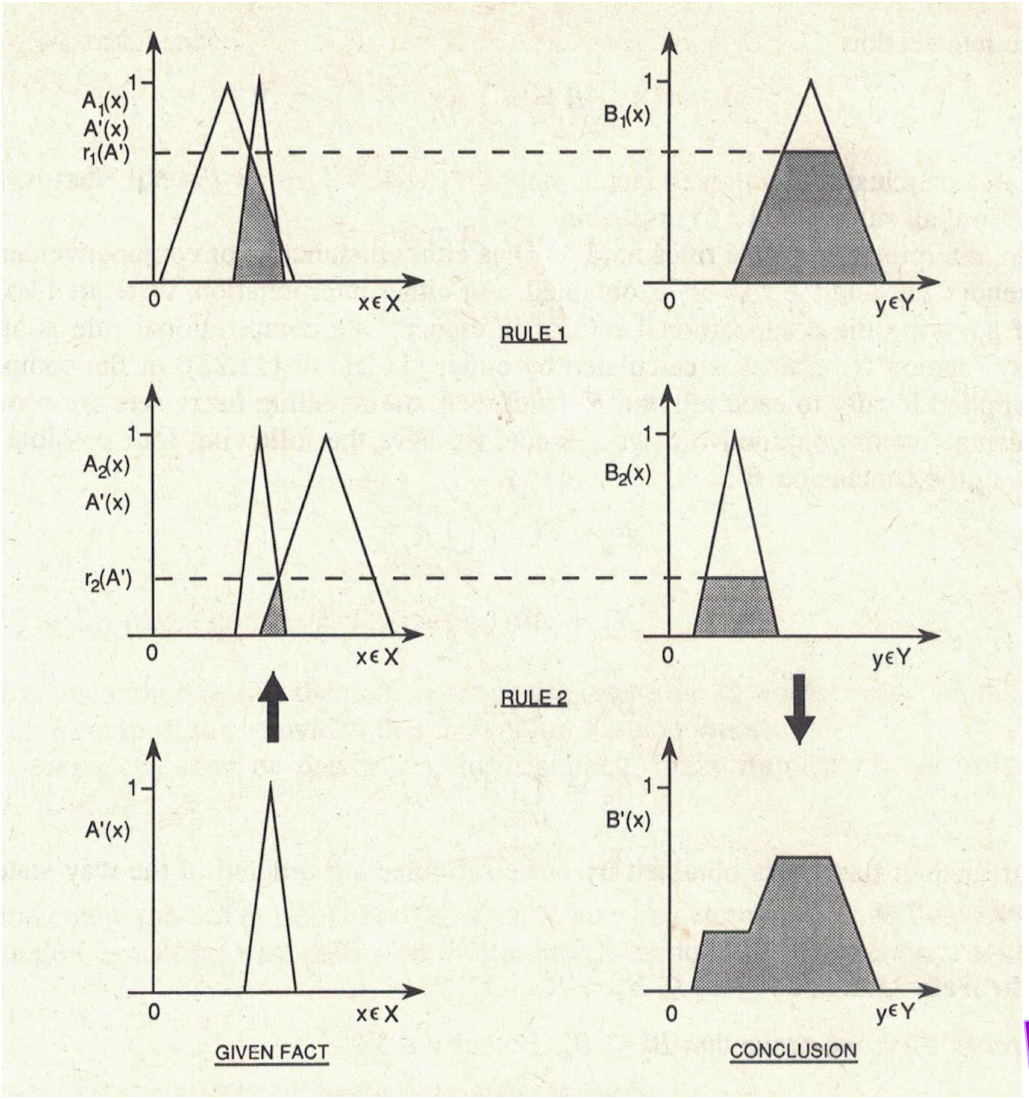
Step 2. Calculate conclusion of each *if-then* rule k, by truncating the B_k with the firing strength $r_k(A')$,

$$B_k'(y) = min(r_k(A'), B_k(y))$$

Step 3. Combine all individual conclusion by 'union':

$$B'(y) = \bigcup_k B_k'(y) = max[B_1'(y), B_2'(y), \ldots, B_n'(y)]$$

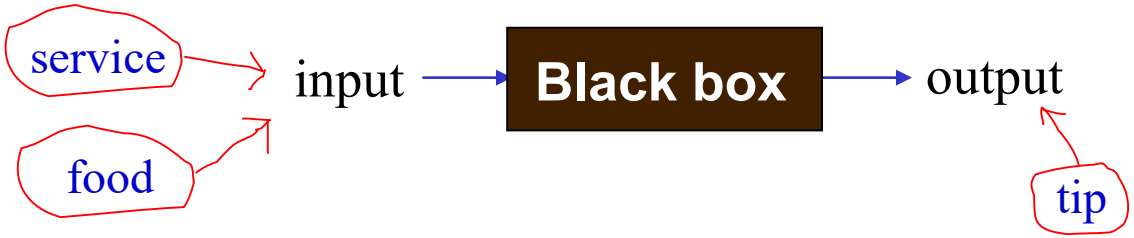
Illustration of the method of interpolation for multi-rule inference:



Deffuzzification method will be learnt in the next chapter

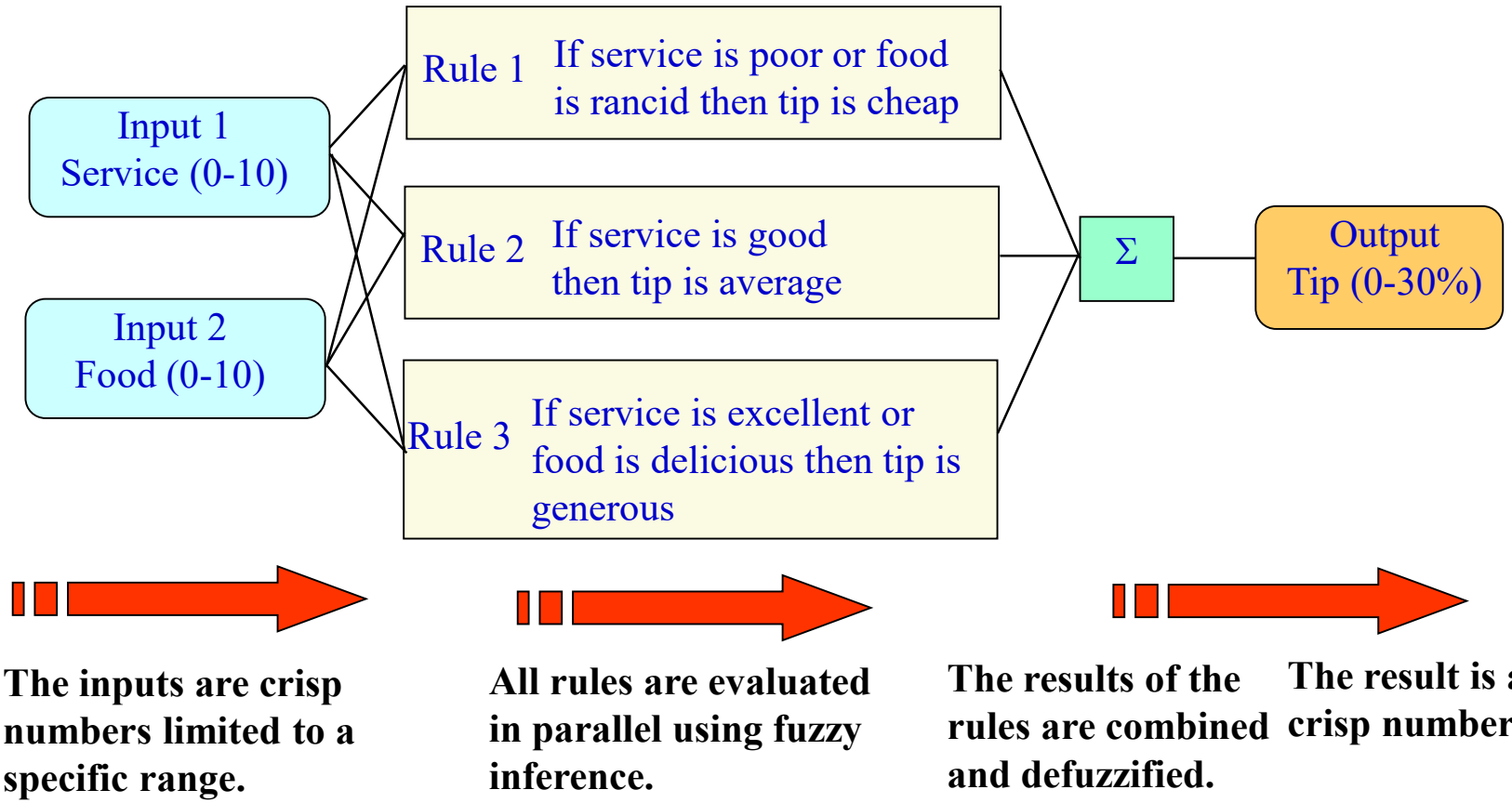
A Fuzzy Inference Example : The Basic Tipping Problem

- Given a number between 0 and 10 that represents the quality of service and quality of food respectively at a restaurant (where 10 is excellent), what should the tip be?



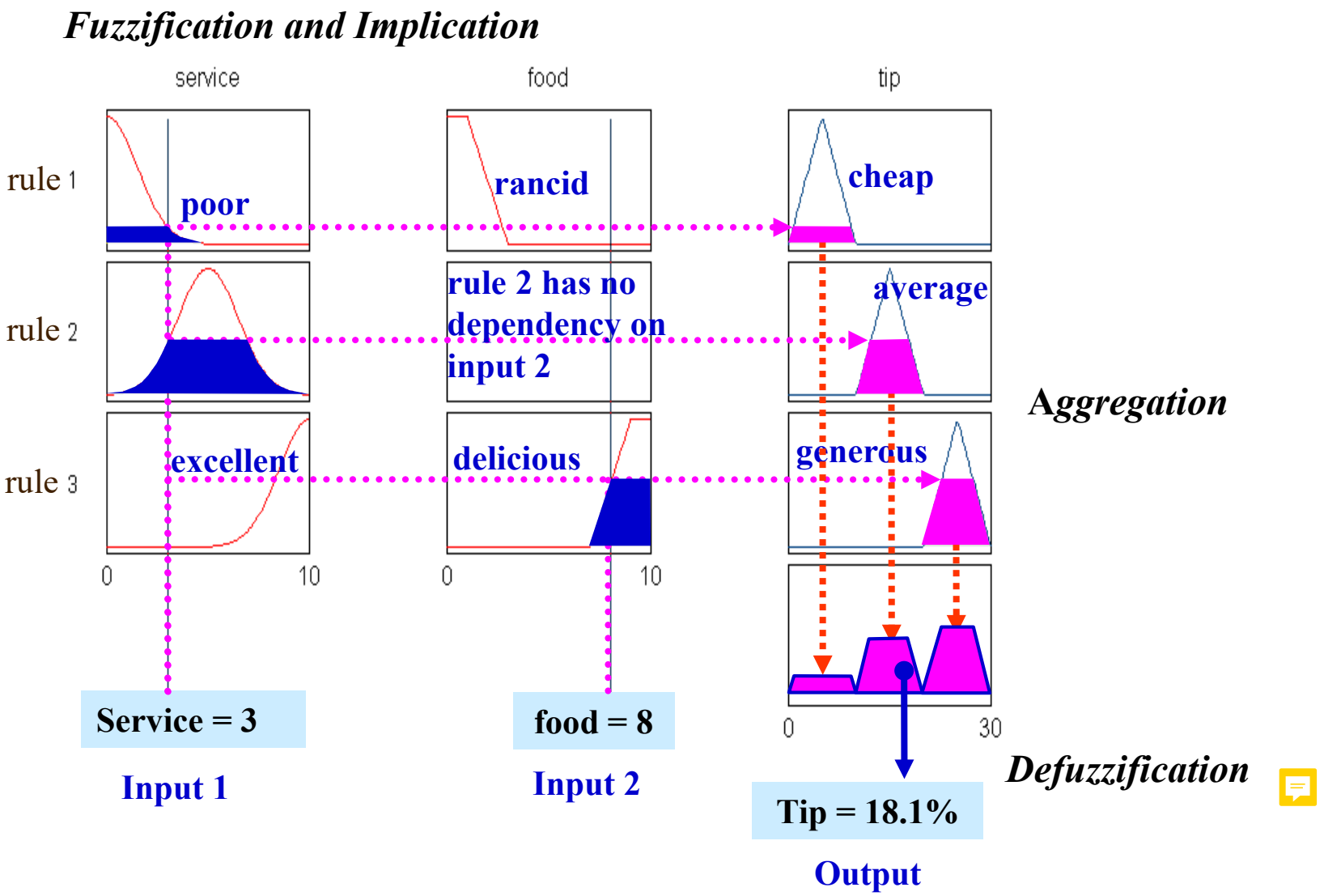
Using fuzzy approach, a rule-based for this problem is defined:

	Service	Food	Service/Food
1.	*if service is poor then tip is cheap*	*If food is rancid then tip is cheap*	*If service is poor or the food is rancid then tip is cheap*
2.	*if service is good then tip is average*		*if service is good then tip is average*
3.	*if service is excellent then tip is generous*	*If food is delicious then tip is generous*	*If service is excellent or food is delicious then tip is generous*



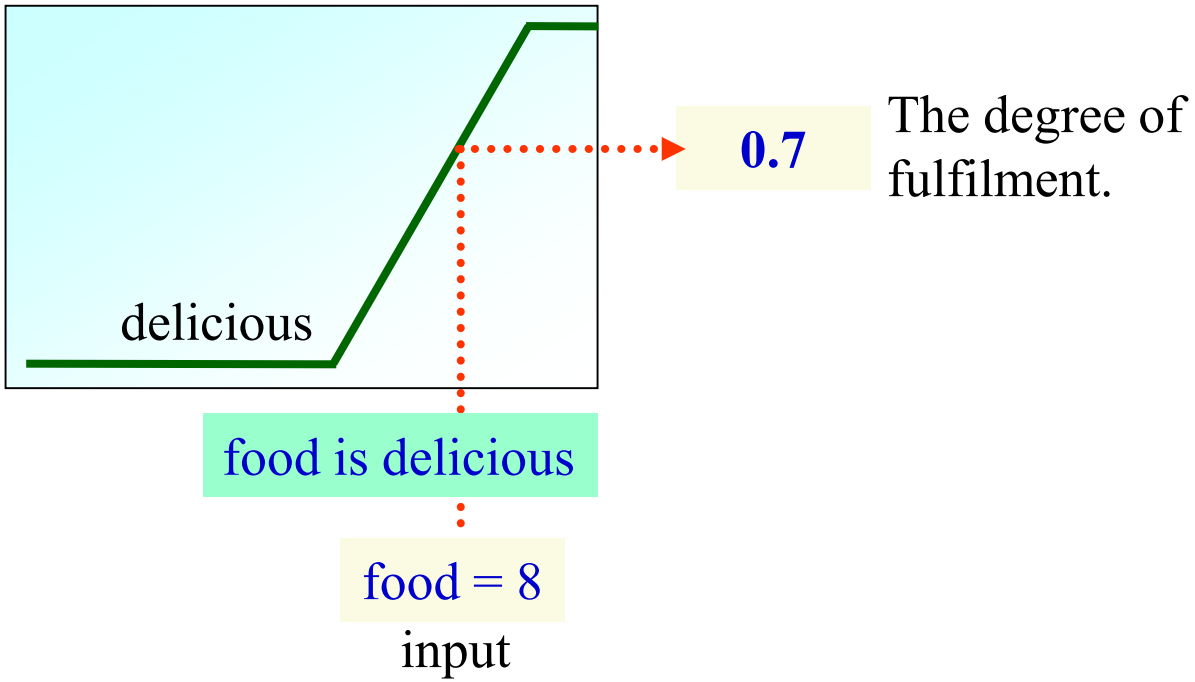
The point of all fuzzy logic systems is to map an input space to an output space. The primary tool for doing this is a list of *if-then* statements. All rules are fired in parallel, and the final conclusion is the combination of all.

- There are a few distinct parts to the process:



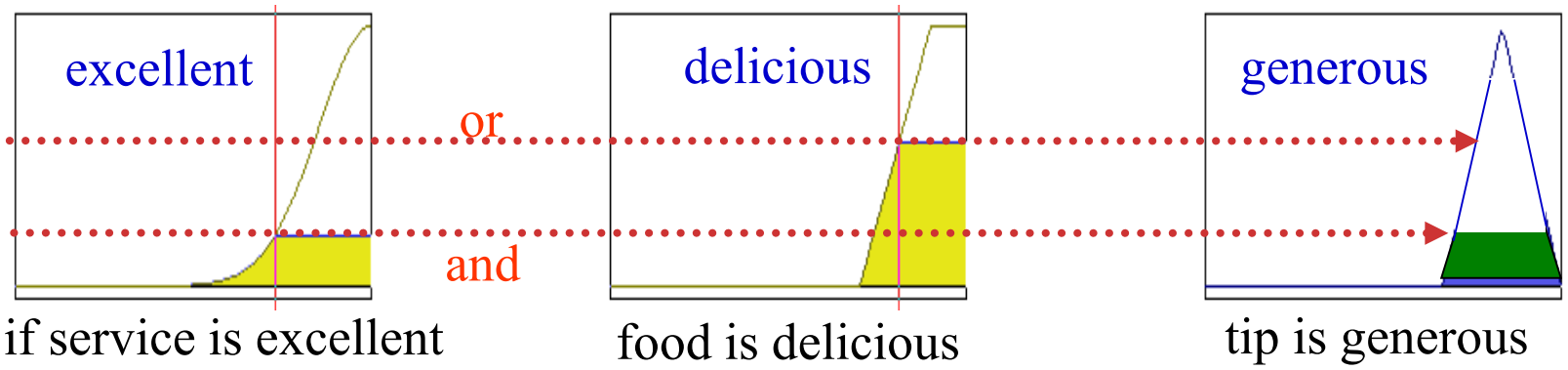
♦ **Step 1: Compute the degree of fulfilment of inputs to the conditions**

Since the input is a crisp set with one score. The intersection of this input with the condition is the *membership functions* associated with the input variables which determine the degree of fulfilment.



♦ Step 2. Fuzzy inference

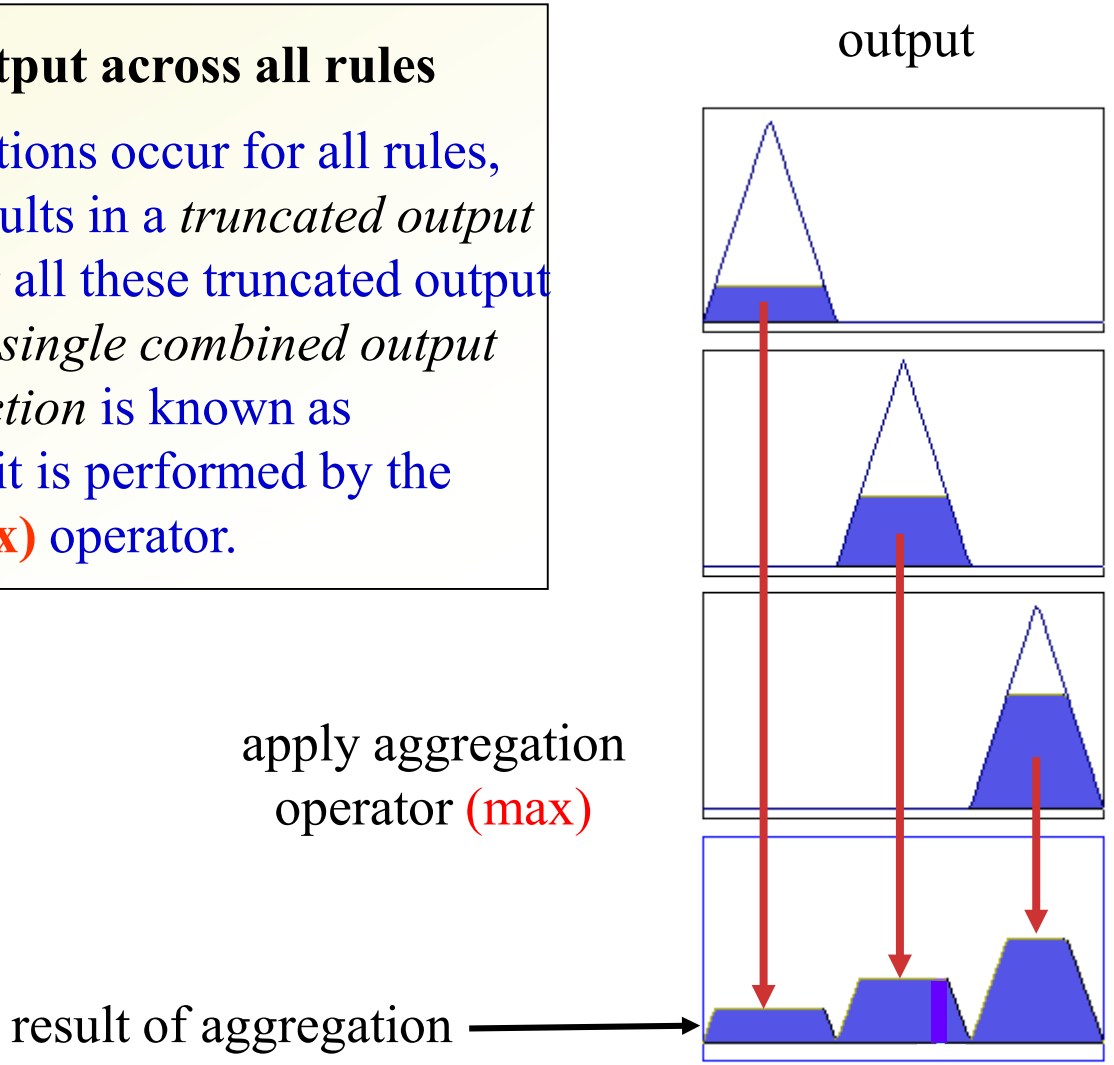
Apply fuzzy operator: If the condition is made up of multiple statements joined by connectives (**'and' = min**) or (**'or' = max**), then the fuzzy operator resolves the multiple conditions into a number between 0 and 1, which serves as the firing strength for each rule.



Apply implication operator: The implication method is defined as the shaping of the conclusion. The simple way is to truncate the membership function by the firing strength.
Implication occurs for each rule.

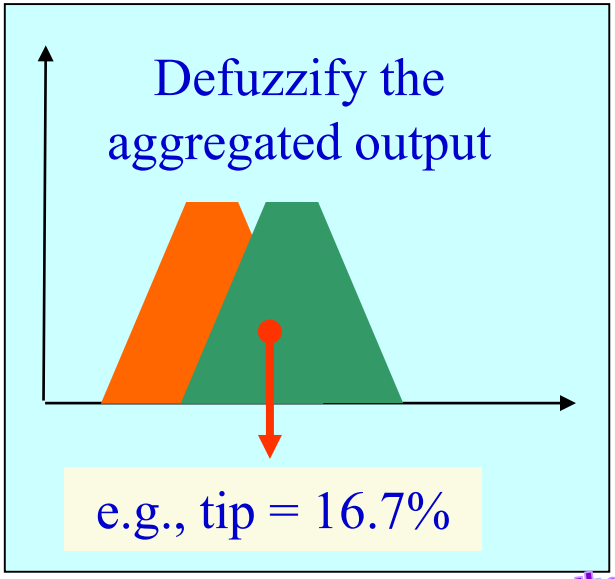
3. Aggregate output across all rules

The above operations occur for all rules, and each rule results in a *truncated output fuzzy set*. Joining all these truncated output fuzzy sets into *a single combined output membership function* is known as aggregation and it is performed by the aggregation **(max)** operator.



- **Step 4. Defuzzify the aggregated output fuzzy set**

The aggregated membership function needs to be reduced to *a crisp value*. The *defuzzification method* returns this value given from the sometimes oddly shaped aggregate.



Deffuzzification method will be learnt in the next chapter