1. A = "motor speed OK"

$= 0.3/20 + 0.6/30 + 0.8/40 + 1/50 + 0.7/60 + 0.4/70$

B = "motor voltage nominal"

$= 0.1/1 + 0.3/2 + 0.8/3 + 1/4 + 0.7/5 + 0.4/6 + 0.2/7$

The fuzzy inference rule is

If A, then B.

Now the fact is

$A' = 0.4/20 + 0.7/30 + 1/40 + 0.6/50 + 0.3/60 + 0.1/70$

Then the degree of fulfillment of the fact to the condition is

$r = h(A \cap A')$

$= h(0.3/20 + 0.6/30 + 0.8/40 + 0.6/50 + 0.3/60 + 0.1/70)$

$= 0.8$

Since it is a single condition, the firing strength is the degree of fulfillment.

So we have

$B' = \min(0.8, B)$

$= 0.1/1 + 0.3/2 + 0.8/3 + 0.8/4 + 0.7/5 + 0.4/6 + 0.2/7$

2. Consider the two triangular-shape fuzzy numbers A and B

$$A(x) = \begin{cases} 0 & for\ x \le -1\ and\ x > 3 \\ \frac{x+1}{2} & for\ -1 < x \le 1 \\ \frac{3-x}{2} & for\ 1 < x \le 3 \end{cases} \qquad B(x) = \begin{cases} 0 & for\ x \le 1\ and\ x > 5 \\ \frac{x-1}{2} & for\ 1 < x \le 3 \\ \frac{5-x}{2} & for\ 3 < x \le 5 \end{cases}$$

Determine the arithmetic operations of $(A/B)(x)$

Ans: From the chapter 5, we know that

α-cut-set is: $A_\alpha = [2\alpha - 1, 3 - 2\alpha]$

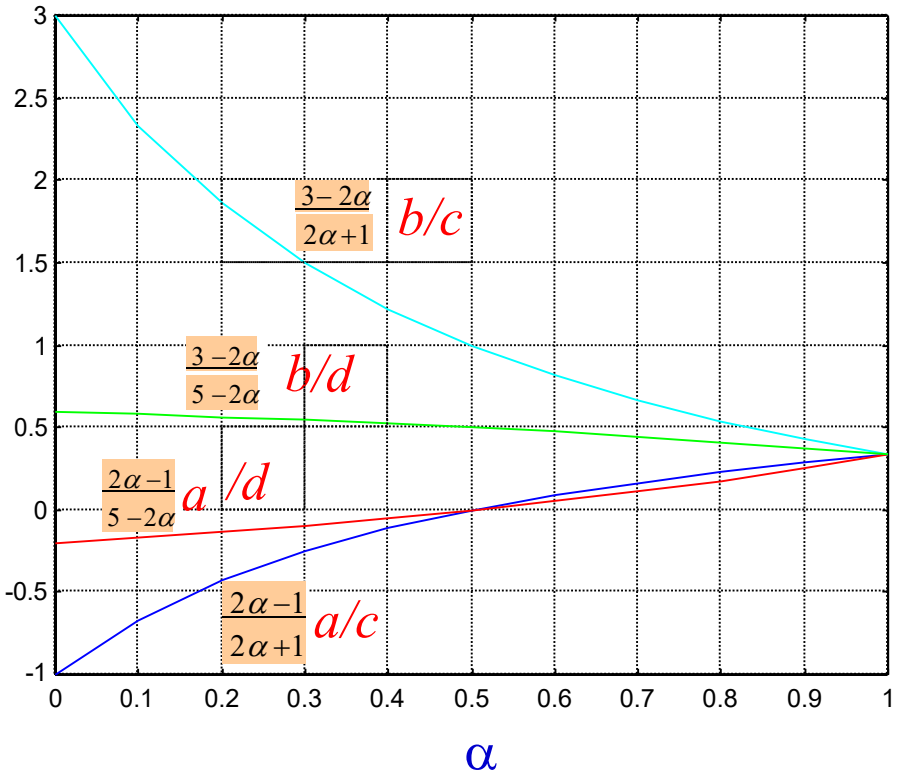
$B_\alpha = [2\alpha + 1, 5 - 2\alpha]$

Division: $[a, b] / [c, d] = [\min(a/c, a/d, b/c, b/d), \max(a/c, a/d, b/c, b/d)]$

$$(A/B)_\alpha = \begin{cases} \left[\dfrac{2\alpha-1}{2\alpha+1}, \dfrac{3-2\alpha}{2\alpha+1}\right] & \text{for } \alpha \in [0,0.5] \\ \left[\dfrac{2\alpha-1}{5-2\alpha}, \dfrac{3-2\alpha}{2\alpha+1}\right] & \text{for } \alpha \in [0.5,1] \end{cases}$$

$A_\alpha = [2\alpha - 1, 3 - 2\alpha]$

$B_\alpha = [2\alpha + 1, 5 - 2\alpha]$



(i) $\frac{2\alpha-1}{2\alpha+1} = x$ for $\alpha \in [0, 0.5]$

$2\alpha - 1 = 2x\alpha + x$

$\alpha = \frac{x+1}{2-2x}$ for $x \in [-1, 0]$

(ii) $\frac{3-2\alpha}{2\alpha+1} = x$ for $\alpha \in [0, 0.5]$

$3 - 2\alpha = 2x\alpha + x$

$\alpha = \frac{3-x}{2x+2}$ for $x \in [1, 3]$

(iii) $\frac{2\alpha-1}{5-2\alpha} = x$ for $\alpha \in [0.5, 1]$

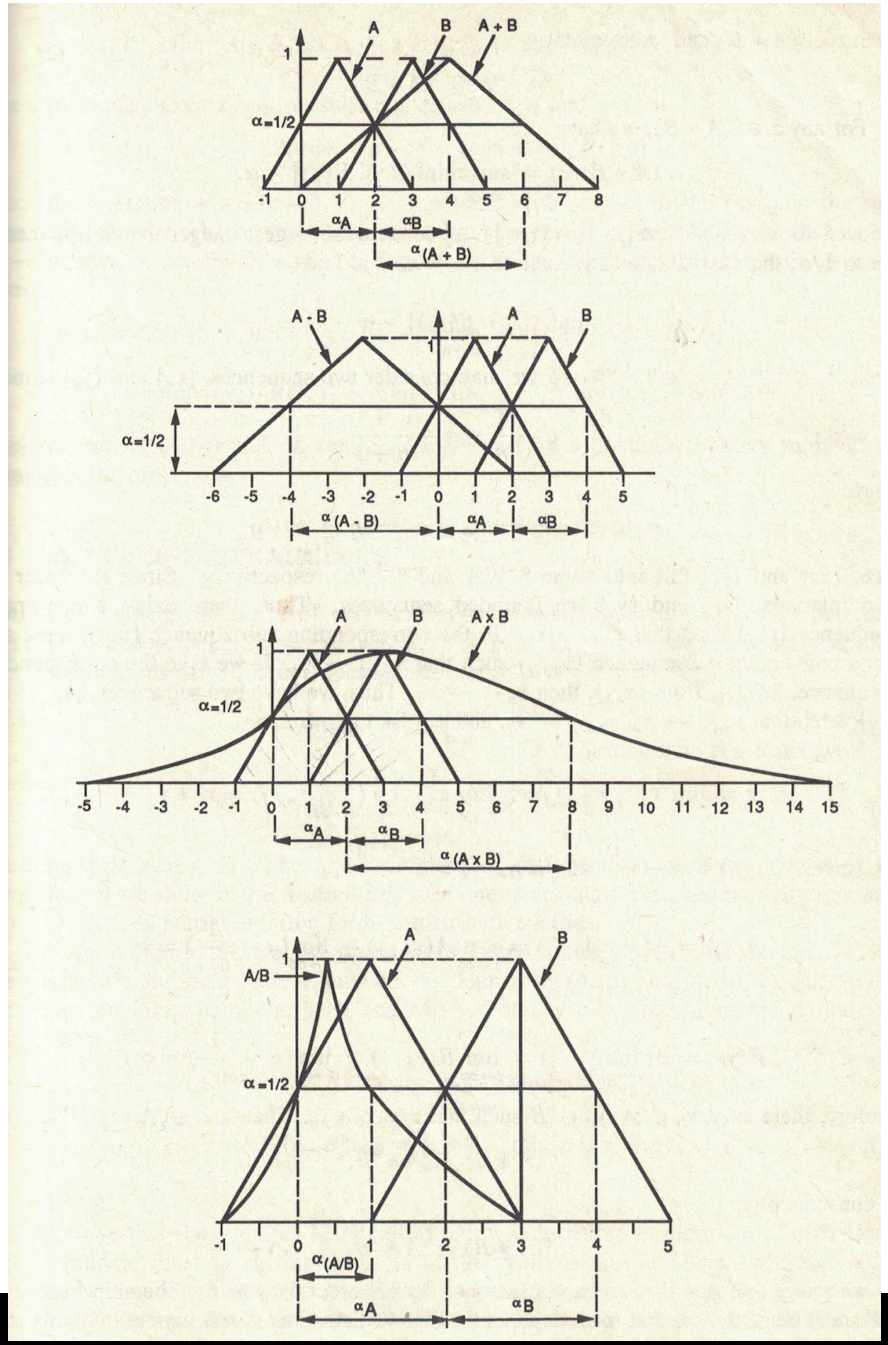
$2\alpha - 1 = 5x - 2x\alpha$

$\alpha = \frac{5x+1}{2x+2}$ for $x \in [0, \frac{1}{3}]$

(iv) $\frac{3-2\alpha}{2\alpha+1} = x$ for $\alpha \in [0.5, 1]$

$\alpha = \frac{3-x}{2x+2}$ for $x \in [\frac{1}{3}, 1]$

$$(A/B)(x) = \begin{cases} 0 & \text{otherwise} \\ \dfrac{x+1}{2-2x} & \text{for } -1 \le x < 0 \\ \dfrac{5x+1}{2x+2} & \text{for } 0 \le x < \dfrac{1}{3} \\ \dfrac{3-x}{2x+2} & \text{for } \dfrac{1}{3} \le x < 3 \end{cases}$$



A=1, B=3

A+B=1+3=4, with triangular membership (covered in chapter 5)

A-B=1-3=-2 , with triangular membership

A·B=1·3=3, with nonlinear membership

A/B=1/3, with nonlinear membership (for this problem)

3. Consider the following fuzzy rule

if X is P then Y is Q

The fuzzy sets P and Q is given by

$$P = 0.2/x_1 + 0.6/x_2 + 1/x_3 + 0.6/x_4 + 0.2/x_5$$

$$Q = 0.5/y_1 + 0.7/y_2 + 1/y_3$$

For a fuzzy input of $A = 1/x_1 + 0.7/x_2 + 0.5/x_3 + 0.3/x_4 + 0.1/x_5$, apply the rule of fuzzy inference to find the output fuzzy set of the system.

Solution:

Now the fact is

$A = 1/x_1 + 0.7/x_2 + 0.5/x_3 + 0.3/x_4 + 0.1/x_5$

Then the degree of fulfillment of the fact A to the condition P is

$r = h(A \cap P)$

$= h(0.2/x_1 + 0.6/x_2 + 0.5/x_3 + 0.3/x_4 + 0.1/x_5)$

$= 0.6$

Since it is a single condition, the firing strength is the degree of fulfillment.

So we have the output (conclusion)

$B = \min(0.6, Q)$

$= 0.5/y_1 + 0.6/y_2 + 0.6/y_3$

4. A FKBC system contains the following two rules: if x is A_1 and y is B_1 then z is C_1

if x is A_2 or y is B_2 then z is C_2

the fuzzy sets A_1, A_2, B_1, B_2, C_1 and C_2 are defined by

$$A_1 = 1/x_1 + 0.9/x_2 + 0.1/x_3$$
$$B_1 = 1/y_1 + 0.4/y_2 + 0.2/y_3$$
$$C_1 = 0.5/z_1 + 1/z_2 + 0.2/z_3$$
$$A_2 = 0.4/x_1 + 1/x_2 + 0.3/x_3$$
$$B_2 = 0.2/y_1 + 0.9/y_2 + 1/y_3$$
$$C_2 = 1/z_1 + 0.5/z_2 + 0.1/z_3$$

Given the input fuzzy set $A' = 0.3/x_1 + 0.6/x_2 + 1/x_3$ and $B' = 0.2/y_1 + 1/y_2 + 0.4/y_3$ determine the output fuzzy set F of the system.

Ans:

Let's check the first rule,

$$A_1 \cap A' = 0.3/x_1 + 0.6/x_2 + 0.1/x_3$$

Degree of fulfilment *of* A' *to* A_1 *is* 0.6

$$B_1 \cap B' = 0.2/y_1 + 0.4/y_2 + 0.2/y_3$$

Degree of fulfilment *of* B' *to* B_1 *is* 0.4

Since both conditions need to be satisfied, the firing strength is

$s=\min(0.6, 0.4)=0.4$

The output of rule 1 is

$C'_1 = \min[0.4, C_1] = 0.4/z_1 + 0.4/z_2 + 0.2/z_3$

For the second rule, let's check

$A_2 \cap A' = 0.3/x_1 + 0.6/x_2 + 0.3/x_3$

Degree of fulfilment *of* A' *to* A_2 *is* 0.6

$B_2 \cap B' = 0.2/y_1 + 0.9/y_2 + 0.4/y_3$

Degree of fulfilment of B' *to* B_2 *is* 0.9

Since only one of the conditions needs to be satisfied, the firing strength is

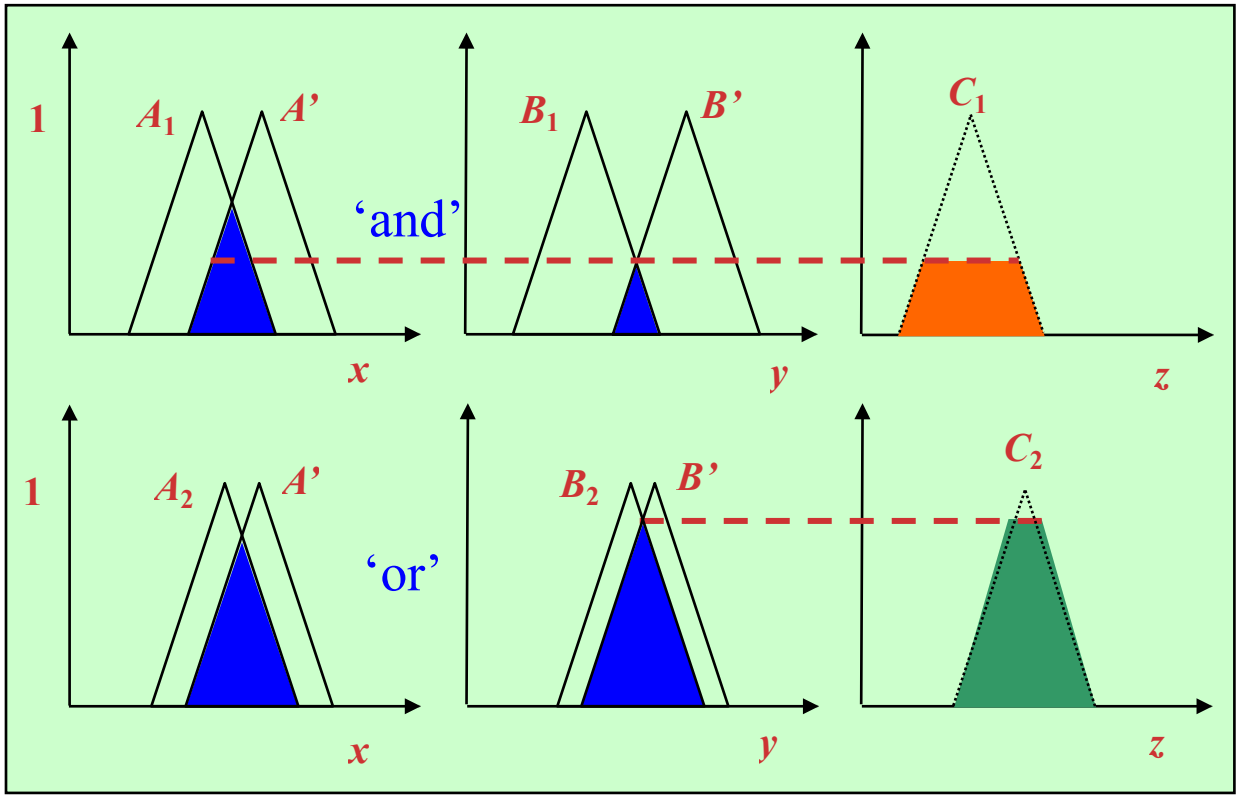
$s=\max(0.6, 0.9)=0.9$

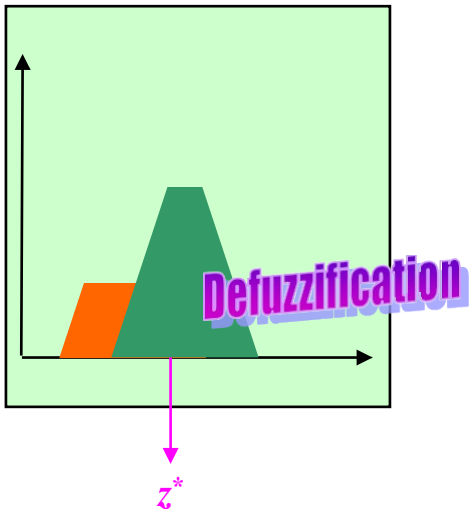
The output of rule 2 is

$C'_2 = \min[0.9, C_2] = 0.9/z_1 + 0.5/z_2 + 0.1/z_3$

So the overall output is the union of the two.

$F = C'_1 \cup C'_2 = 0.9/z_1 + 0.5/z_2 + 0.2/z_3$


1
A_1
A'
'and'
B_1
B'
C_1
x
y
z
1
A_2
A'
'or'
B_2
B'
C_2
x
y
z


Defuzzification
z^*