# NATIONAL UNIVERSITY OF SINGAPORE

**EXAMINATION FOR**

(Semester I: 2015/2016)

## EE4305 INTRODUCTION TO FUZZY/NEURAL SYSTEMS

November/December 2015 - Time Allowed: 2 Hours

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INSTRUCTIONS TO CANDIDATES:

1. This examination paper contains **FOUR (4)** questions and comprises **FIVE (5)** pages.
2. Answer all FOUR questions.
3. All questions carry equal marks.
4. This is a closed book examination. But the candidate is permitted to bring into the examination hall a single A4 size formula sheet with formulae and related course material hand-written on both sides. The candidate may refer to this sheet during the examination.

Q.1 (a) Find the equilibrium of the fuzzy complement $c_\gamma(a) = \dfrac{\gamma^2(1-a)}{a+\gamma^2(1-a)}, \quad 1 > \gamma > 0$

(7 Marks)

(b) For a nonlinear system as shown in Fig. 1.1, describe how fuzzy logic can be used to represent the input-output mapping of the system.


Fig. 1.1 A nonlinear system

(8 Marks)

(c) Fig. 1.2 shows an electrical circuit. The current, $I$, resistance, $R$, and voltage, $V$, are presumed to be fuzzy variables. It can be obtained that $R_{eq} = R_1 + R_2$, and $V = I \cdot R_{eq}$. The membership functions for the resistances and current are,

$R_1 = 0.5/3 + 0.8/4 + 0.6/5$
$R_2 = 0.3/8 + 1/9 + 0.4/10$
$I = 0.8/1 + 1/2 + 0.5/3$

Using the extension principle, find the voltage $V$.

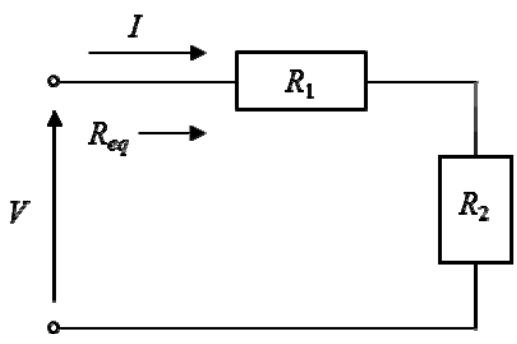


Fig. 1.2 An electrical circuit

(10 Marks)

Q.2 (a) Let $A$ be a fuzzy set defined in the universe of discourse $X$ = [0, 1]. Assume that the membership function of $A$, $\mu_A(x)$, is continuous on $x \in X$, and $\mu_{\bar{A}} = 1 - \mu_A$. Use an appropriate method to verify that $A \cap \bar{A} \neq \varnothing$.

(7 Marks)

(b) Discuss the advantages and disadvantages of genetic algorithms in the design and optimization of a fuzzy logic control system.

(8 Marks)

(c) The equations describing a motor system are given as,

$$x_1(k+1) = x_2(k) + x_1(k)$$
$$x_2(k+1) = 8.64V(k) + 0.36x_2(k)$$

The state variables are $x_1 = \theta$ and $x_2 = \dot{\theta}$, where $\theta$ = rotational angle, $V$ = motor control voltage, and $k$ is the iteration index. The membership functions for $x_1$ and $x_2$ are shown in Fig. 2.1 and the membership functions for $V$ are shown in Fig. 2.2. The rule-based system is given in Table 2.1 and the entries in the table are the control voltage, $V$. Using the initial conditions of $x_1 = 5$ and $x_2 = -75$, conduct two Mamdani fuzzy graphical inference simulation cycles for this system using the center-of-area defuzzification method.

Table 2.1

| $x_1$ \ $x_2$ | N | Z | P |
|---|---|---|---|
| N | NB | P | N |
| Z | P | Z | P |
| P | Z | Z | PB |

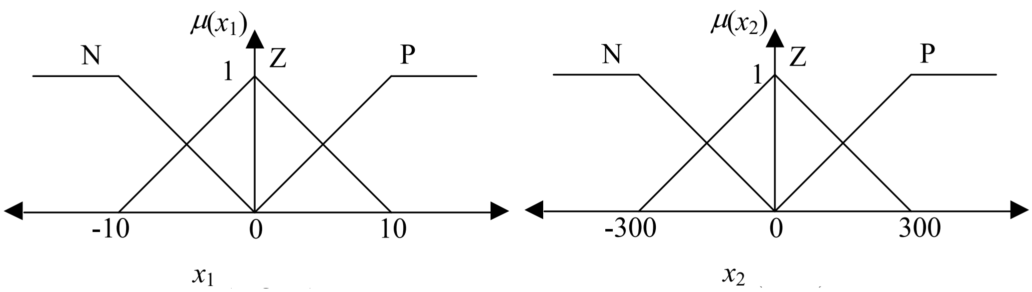


Fig. 2.1

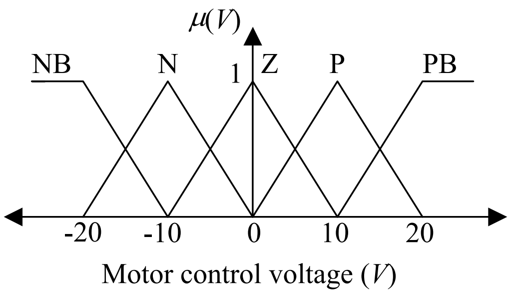


Fig. 2.2

(10 Marks)

Q.3 The multi-layer neural network shown in Fig. 3.1 has two inputs and one output. The network has two neurons in a hidden layer. The network is to be trained with backpropagation algorithm. Each neuron has a sigmoid activation function: $\varphi(v) = \frac{1}{1+e^{-v}}$. Assume that the biases to the neurons is +1 and the learning rate is 1.

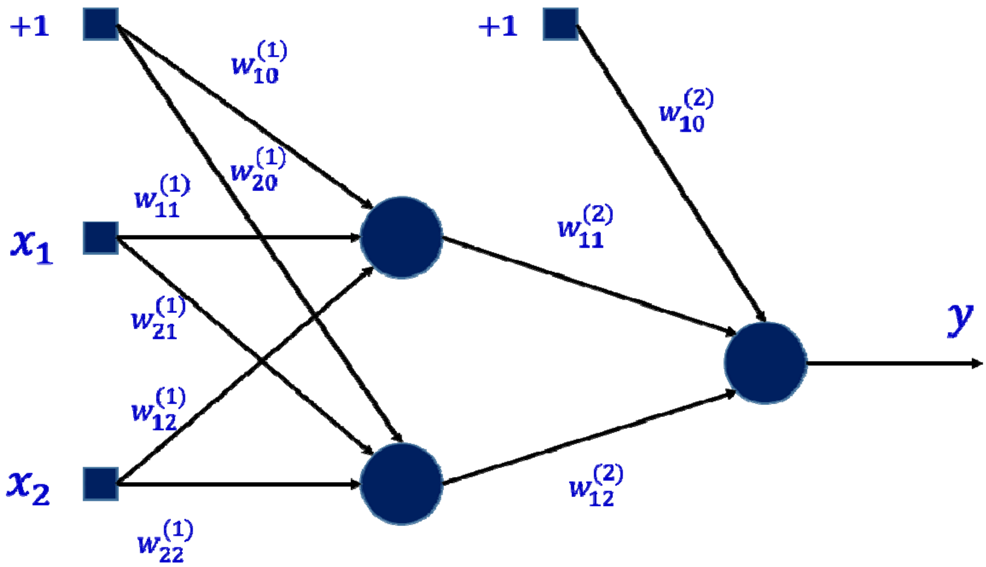


Fig. 3.1

The network has the following initial weights:

$$\left\{w_{10}^{(1)}, w_{20}^{(1)}, w_{10}^{(2)}\right\} = \{0.15, -0.05, 0.1\}$$

$$\left\{w_{11}^{(1)}, w_{12}^{(1)}, w_{11}^{(2)}\right\} = \{0.1, 0.3, 0.2\}$$

$$\left\{w_{21}^{(1)}, w_{22}^{(1)}, w_{12}^{(2)}\right\} = \{-0.2, -0.25, -0.3\}$$

Perform the training with the following training vectors:

| $x_1$ | $x_2$ | $y$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |

(i) Determine the modified weights, after one iteration the backpropagation algorithm with the first training sample.

(15 Marks)

(ii) Using the weights from the first iteration, determine the modified weights after one iteration of backpropagation algorithm with the second training sample.

(10 Marks)

Q.4 (a) A single-layer perceptron has two input neurons, biases and binary threshold function as activation function. The perceptron divides the space into two regions with a straight line. Analytically calculate a set of weights for such a perceptron so that the following set P of the 6 patterns of the form $(x_1, x_2, d)$ with $\varepsilon \ll 1$, is correctly classified. The inputs to the perceptron are $x_1$ and $x_2$, and the desired output is $d$.

$$P = \{(0, 0, -1); (2, -1, 1); (6 + \varepsilon, 3 - \varepsilon, 1); (6 - \varepsilon, 3 + \varepsilon, -1); (0, -2 - \varepsilon, 1); (0 - \varepsilon, -2, -1)\}$$

(12 Marks)

(b) A multi-layer perceptron has hidden units and output units. Why is such a kind of perceptron more powerful than a single-layer perceptron?

(5 Marks)

(c) Why are SOMs interesting for researchers who study biological nervous systems?

(8 Marks)

**END OF PAPER**