EE4704 Image Processing and Analysis

Semester 1, 2020/2021

Tutorial Set B

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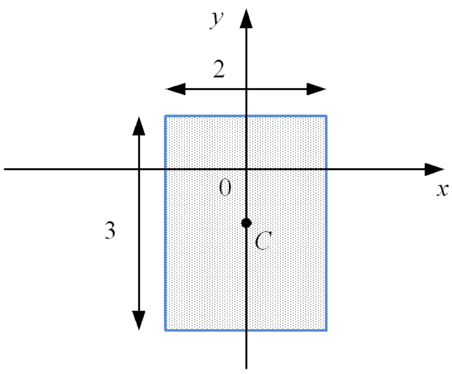
1. (a) Verify the (continuous) Fourier transform pairs

   (i) $\delta(x, y) \leftrightarrow 1$

   (ii) $1 \leftrightarrow \delta(u, v)$

   (b) What is the Fourier transform of $\delta(x - 0.5, y + 0.2)$? Obtain the expression for $\phi(u, v)$ and sketch $\phi(u, 0)$.

2. A continuous image $f(x, y)$ consists of a light rectangle on a dark background. The sides of the rectangle are parallel to the coordinate axes, and the intensities of the rectangle and background are 50 and 10, respectively. The centroid of the rectangle is at $C(0, -1)$. Express the image function, $f(x, y)$ $(-\infty < x, y < \infty)$ as the sum of $f_1(x, y)$ and $f_2(x, y)$, where $f_1(x, y)$ is a constant and $f_2(x, y)$ is a rectangle function. Obtain the *Fourier spectrum* of $f(x, y)$.



3. Let $f(x, y) = \cos 2\pi(ax + by)$. Show that the Fourier transform of $f(x, y)$ is given by

$$F(u, v) = \frac{1}{2}\delta(u - a, v - b) + \frac{1}{2}\delta(u + a, v + b)$$

   Obtain and sketch the Fourier spectrum of

   (a) $f_1(x, y) = \cos(20\pi x)$

   (b) $f_2(x, y) = \sin(40\pi y)$

   (c) $f_3(x, y) = \sin(30x + 40y)$

   (d) $f_4(x, y) = \sin(30x + 40y + 30)$

4. Consider the N-point sequence $f(x)$ and its DFT $F(u)$. Show that multiplying $f(x)$ by $(-1)^x$, i.e.,

$$f'(x) = f(x)\,(-1)^x, \qquad x = 0, 1, \ldots, N-1$$

prior to taking the transform shifts the origin of the transform, $u = 0$, to the point $u = N/2$. (*Hint:* use the translation property of the DFT.)

5. You are given an 8-point sequence

$$f(x) = 1, 1, 1, 1, 0, 0, 0, 0$$

   (a) Obtain $F(u)$, the DFT of $f(x)$.

   (b) Sketch the magnitude and phase components for $-8 \leq u \leq 15$.

   (c) Find the $G(u)$, the DFT of $g(x) = (-1)^x f(x)$. Sketch $|G(u)|$.

6. Compute the DFTs of the following functions:

   (a) $f_1(x) = 1, 1, 1, 1$

   (b) $f_2(x) = 1, 0, 0, 0$

   (c) $f_3(x) = 0, 1, 0, 0$

   (d) $f_4(x, y) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

   (e) $f_5(x, y) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

   (f) $f_6(x, y) = f_5(x-1, y-2)$

This may be useful:

$$\sum_{x=0}^{N-1} \exp(-j2\pi ux/N) = \begin{cases} N & u = 0 \\ 0 & u \neq 0 \end{cases}$$

where u, x are integers and N is an even integer.

7. Consider the 200×200 image $f(x, y)$ shown in the figure (the origin is at the top left corner). The following operations are applied to $f(x, y)$:

Step 1 – Compute the DFT
Step 2 – Take the complex conjugate of the transform
Step 3 – Compute the inverse DFT

(a) Obtain an expression for the output image $g(x, y)$ in terms of $f(x, y)$. Sketch $g(x, y)$ for $0 \leq x, y \leq 199$.

(b) The coordinates of point A in $f(x, y)$ are $(20, 90)$. What are its coordinates in $g(x, y)$?