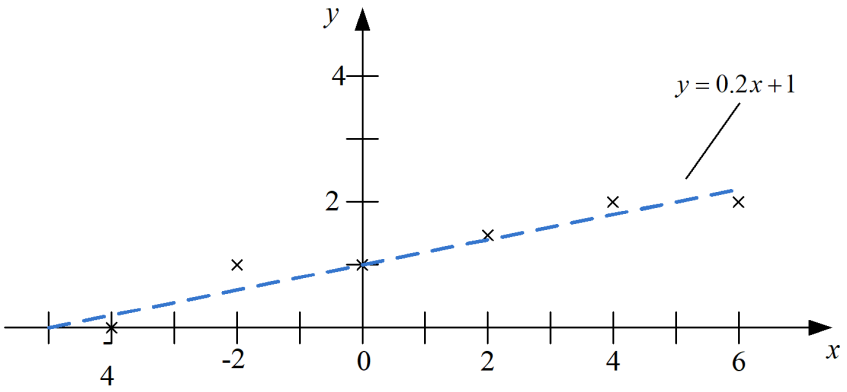
EE4704 Image Processing and Analysis

Tutorial Set E – Solution

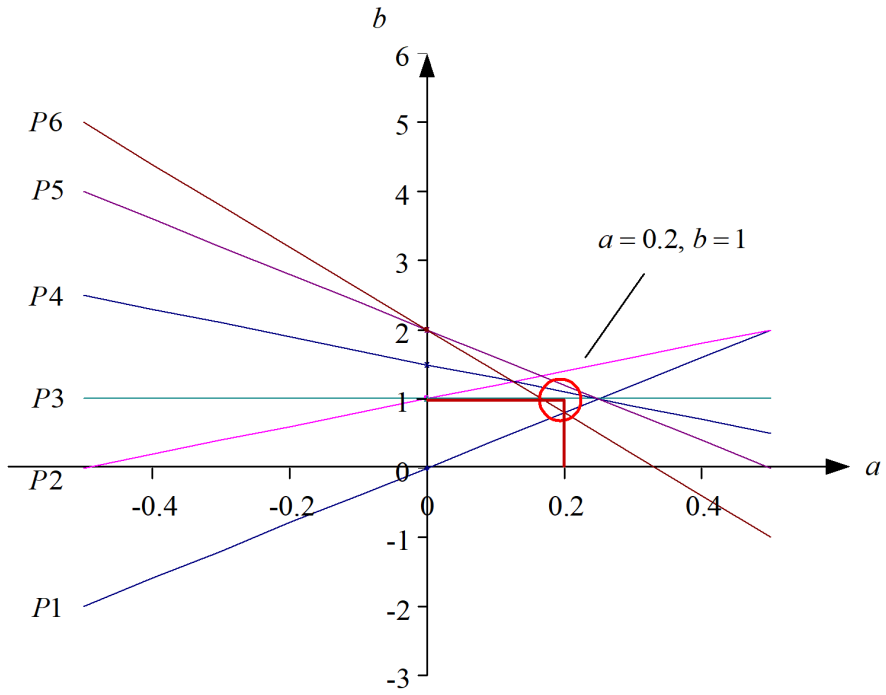
Question 1

Part (a)



$(x_i, y_i) \rightarrow b = -x_i a + y_i$

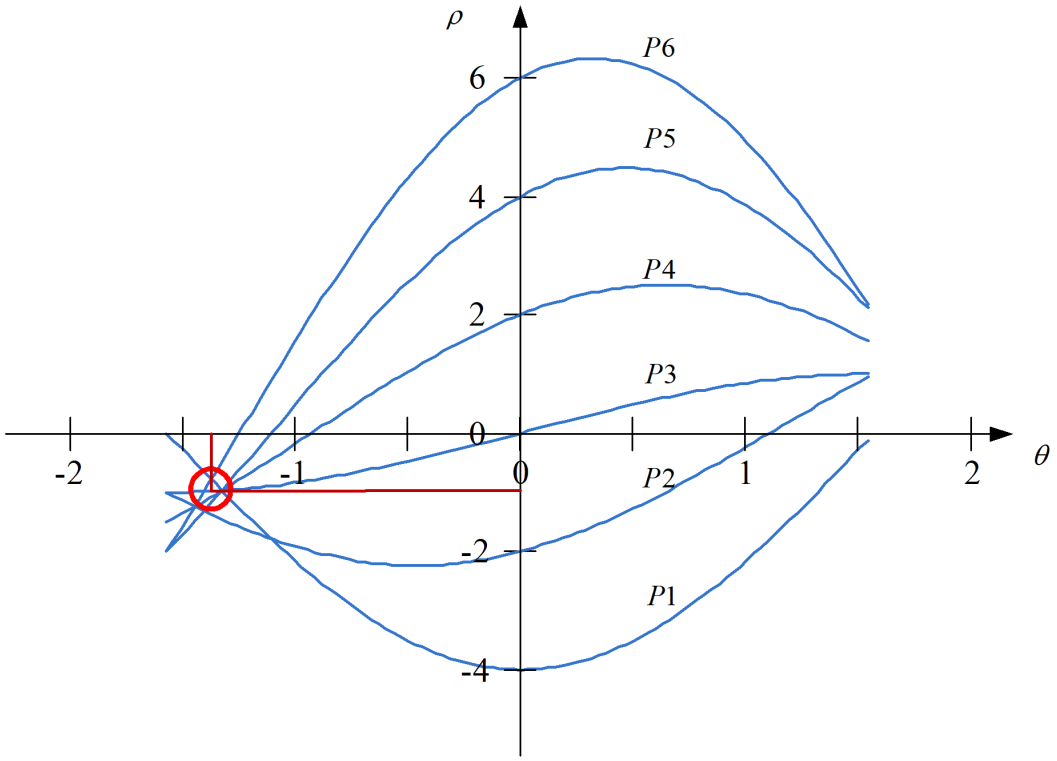
$P1: \ (-4,0) \rightarrow b = 4a$
$P2: \ (-2,1) \rightarrow b = 2a + 1$
$P3: \ (0,1) \rightarrow b = 1$
$P4: \ (2,1.5) \rightarrow b = -2a + 1.5$
$P5: \ (4,2) \rightarrow b = -4a + 2$
$P6: \ (6,2) \rightarrow b = -6a + 2$



Part (b)

$$(x_i, y_i) \quad \rightarrow \quad \rho = \sqrt{x_i^2 + y_i^2}\,\cos(\theta - \tan^{-1}(y_i/x_i))$$

$$\begin{array}{lcll}
P1: & (-4,0) & \rightarrow & \rho = 4\cos(\theta - \tan^{-1}(\frac{0}{-4})) = 4\cos(\theta - 3.14) \\
P2: & (-2,1) & \rightarrow & \rho = \sqrt{5}\cos(\theta - \tan^{-1}(\frac{1}{-2})) = \sqrt{5}\cos(\theta - 2.68) \\
P3: & (0,1) & \rightarrow & \rho = 1\cos(\theta - \tan^{-1}(\frac{1}{0})) = 1\cos(\theta - 1.57) \\
P4: & (2,1.5) & \rightarrow & \rho = \sqrt{6.25}\cos(\theta - \tan^{-1}(\frac{1.5}{2})) = \sqrt{6.25}\cos(\theta - 0.64) \\
P5: & (4,2) & \rightarrow & \rho = \sqrt{20}\cos(\theta - \tan^{-1}(\frac{2}{4})) = \sqrt{20}\cos(\theta - 0.46) \\
P6: & (6,2) & \rightarrow & \rho = \sqrt{40}\cos(\theta - \tan^{-1}(\frac{2}{6})) = \sqrt{40}\cos(\theta - 0.32)
\end{array}$$



Detected line parameters:

$$\rho = -1, \quad \theta = -1.4 \text{ rad}$$

Line equation:

$$\begin{aligned}
x\cos(-1.4) + y\sin(-1.4) &= -1 \\
y &= 0.17x + 1.0
\end{aligned}$$

Question 2

$$(x_i, y_i) \quad \rightarrow \quad b = -x_i a + y_i$$

The accumulator array has limits

$$-1 \leq a \leq +1, \quad -2 \leq b \leq +14$$

with $\Delta a = 0.2$ and $\Delta b = 1$.

The center coordinates of the cells are

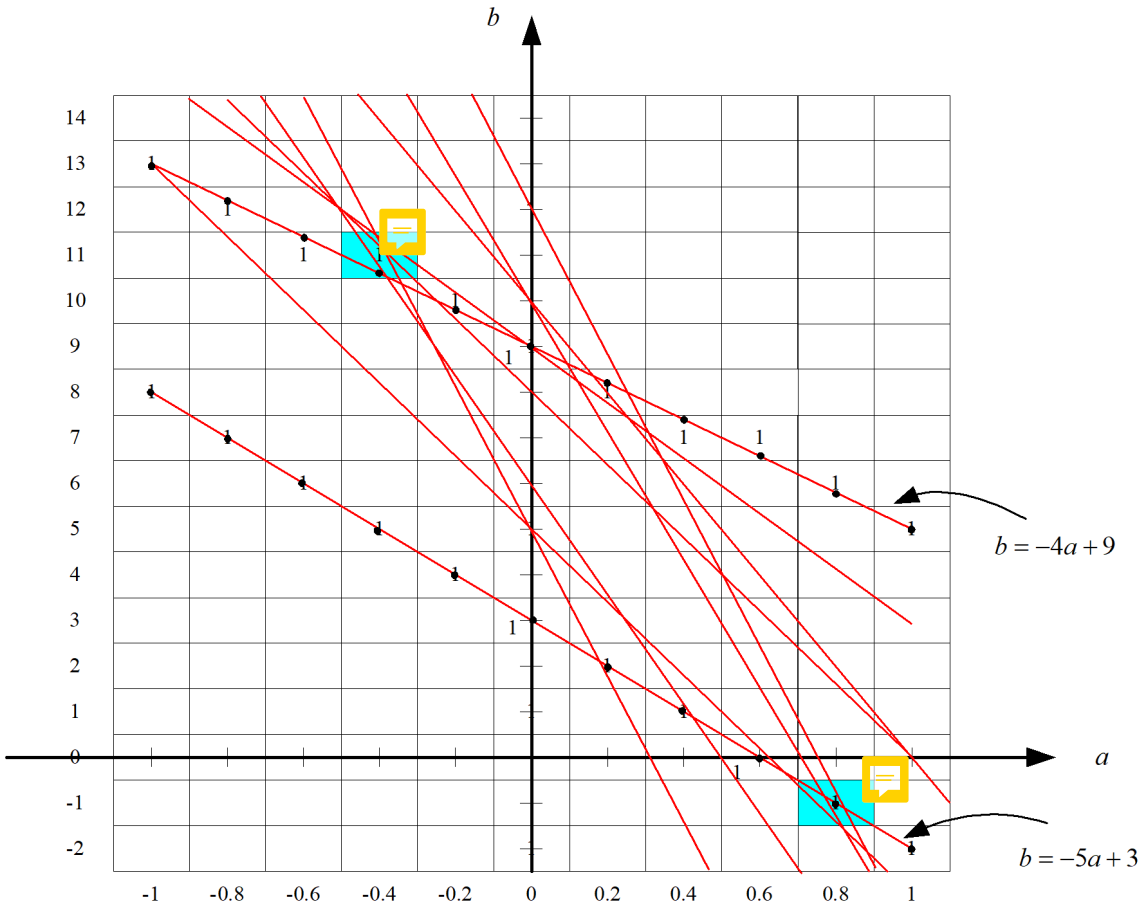
$$\begin{aligned} a_i &= -1, -0.8, -0.6, \ldots, 0, \ldots, 0.8, 1 && (\Delta a = 0.2) \\ b_j &= -2, -1, 0, \ldots, 13, 14 && (\Delta b = 1) \end{aligned}$$

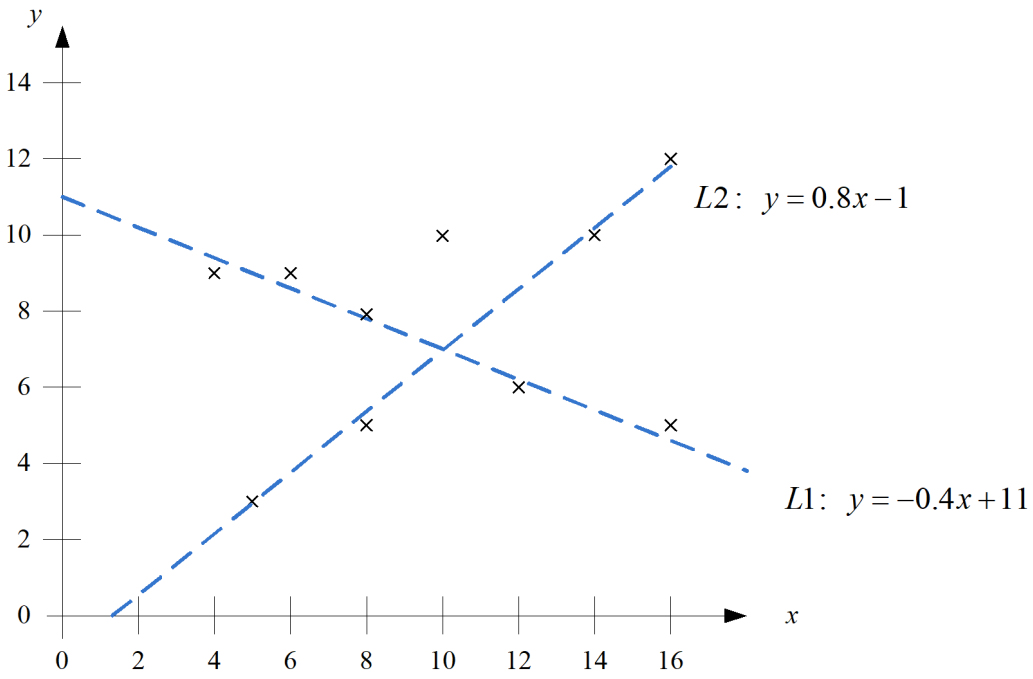
The cell values can be obtained using a table:

	a_i :	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
$(4, 9)$	$b = -4a + 9$	13.0	12.2	11.4	10.6	9.8	9.0	8.2	7.4	6.6	5.8	5.0
$(5, 3)$	$b = -5a + 3$	8	7	6	5	4	3	2	1	0	-1	-2
$(6, 9)$	$b = -6a + 9$	...										
$(8, 5)$	$b = -8a + 5$											
$(8, 8)$	$b = -8a + 8$											
$(10, 10)$	$b = -10a + 10$											
$(12, 6)$	$b = -12a + 6$											
$(14, 10)$	$b = -14a + 10$											
$(16, 5)$	$b = -16a + 5$											
$(16, 12)$	$b = -16a + 12$											

From the completed accumulator array, the largest cell values are 5 obtained at cell $(-0.4, 11)$, and 4 obtained at $(0.8, -1)$. Hence, the corresponding line equations are

$$\begin{aligned} L1: &\quad y = -0.4x + 11 \\ L2: &\quad y = 0.8x - 1 \end{aligned}$$


b
a
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
b = −4a + 9
b = −5a + 3


y
x
14
12
10
8
6
4
2
0
0
2
4
6
8
10
12
14
16
L2 : y = 0.8x − 1
L1 : y = −0.4x + 11

Question 3

Gray level, z_k :	0	1	2	3	4	5	6	7	8	9
No. of pixels, n_k :	400	800	800	1200	400	800	2000	1600	1200	800

Initial estimate

$$T_0 = \text{mean} = \frac{1}{\sum_k n_k} \sum_k n_k z_k = 5.16$$

The histogram is partitioned into two groups:

$$0 \to 5 \qquad \mu_1 = 2.63 \qquad ; \qquad 6 \to 9 \qquad \mu_2 = 7.14$$

$$\begin{aligned} T_1 &= \frac{1}{2}(2,63 + 7,14) \\ &= 4.89 \qquad\qquad (\text{Threshold } = 4) \end{aligned}$$

Next, we have

$$0 \to 4 \qquad \mu_1 = 2.11 \qquad ; \qquad 5 \to 9 \qquad \mu_2 = 6.88$$

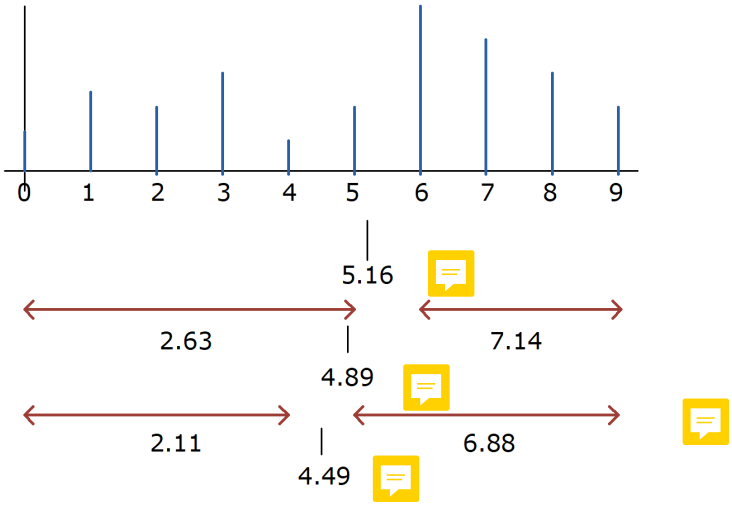
$$\begin{aligned} T_2 &= \frac{1}{2}(2.11 + 6.88) \\ &= 4.49 \qquad\qquad (\text{Threshold } = 4) \end{aligned}$$

The threshold value is unchanged; hence the selected threshold is

$$T = 4$$

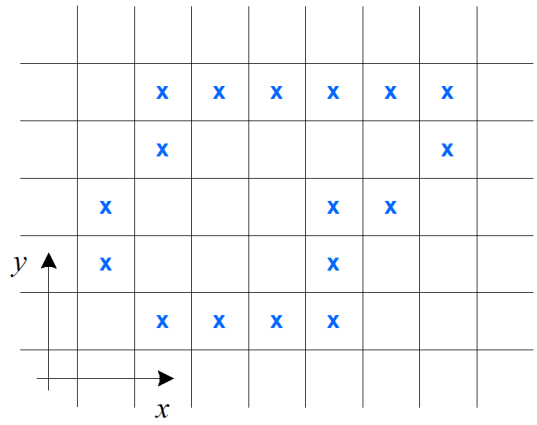
i.e.

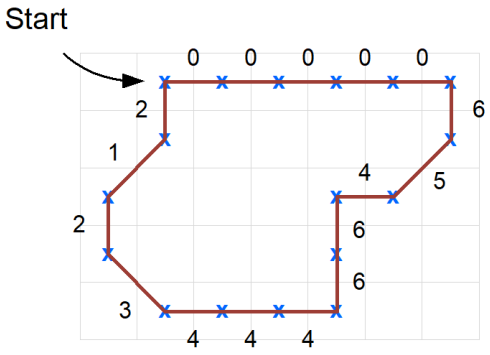
$$g(x,y) = \begin{cases} 1 & z > 4 \\ 0 & z \leq 4 \end{cases}$$



Question 4

Part (a)





Starting from the leftmost pixel of the top row, we have

00000654664443212

This code is normalised for starting point.

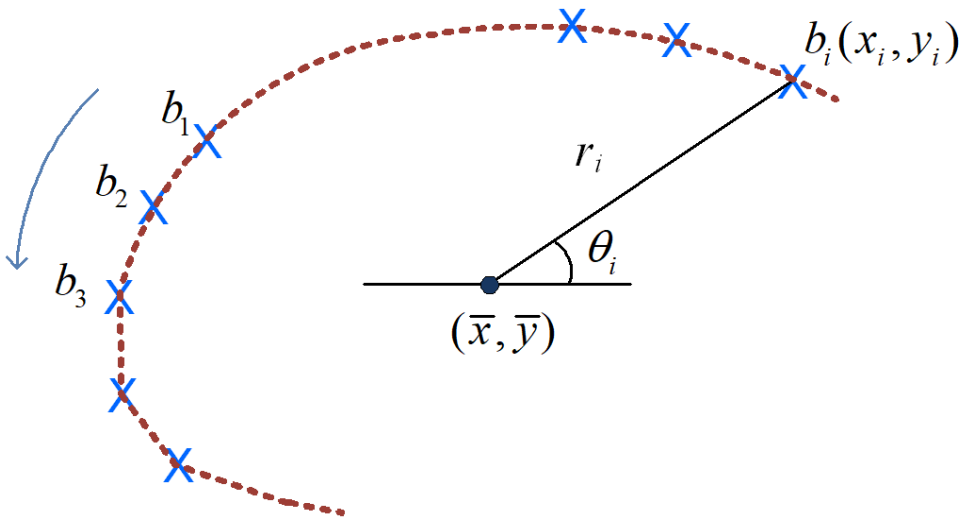
Part (b)

Denote the 17 boundary pixels by $b_i(x_i, y_i)$, $i = 1, 2, \ldots, 17$. The centroid is estimated by

$$\begin{aligned} \bar{x} &= \frac{1}{17}\sum_{i=1}^{17} x_i \\ &= 4.00 \\ \bar{y} &= \frac{1}{17}\sum_{i=1}^{17} y_i \\ &= 3.235 \end{aligned}$$

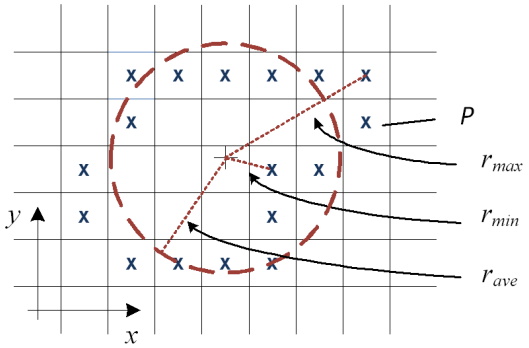
$$\begin{aligned} \text{Radial distance:} \quad r_i &= \sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2} \\ \text{Polar angle:} \quad \theta_i &= \tan^{-1}\frac{y_i - \bar{y}}{x_i - \bar{x}} \end{aligned}$$

where $i = 1, 2, \ldots, 17$.

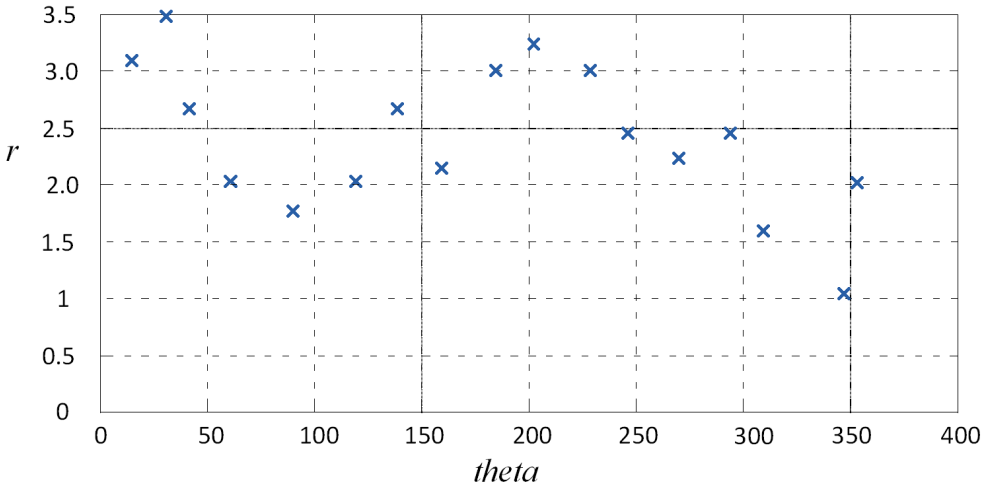


Starting at pixel P

Point	theta	r
(7,4)	14	3.1
(7,5)	30	3.48
(6,5)	41	2.67
(5,5)	60	2.03
(4,5)	90	1.77
(3,5)	120	2.03
(2,5)	139	2.67
(2,4)	159	2.14
(1,3)	184	3.01
(1,2)	202	3.24
(2,1)	228	3
(3,1)	246	2.45
(4,1)	270	2.24
(5,1)	294	2.45
(5,2)	309	1.59
(5,3)	347	1.03
(6,3)	353	2.01



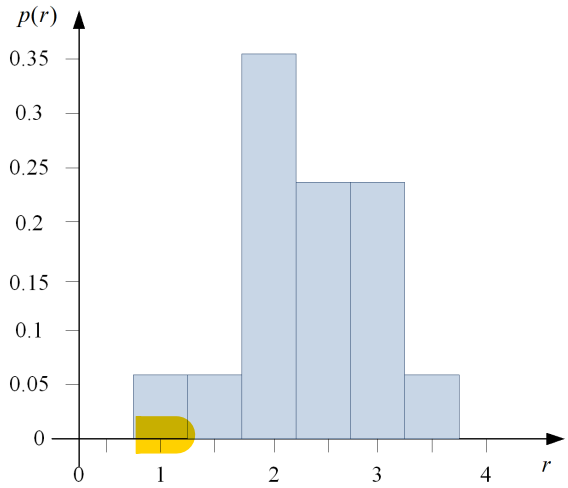
centroid = (4, 3.235)



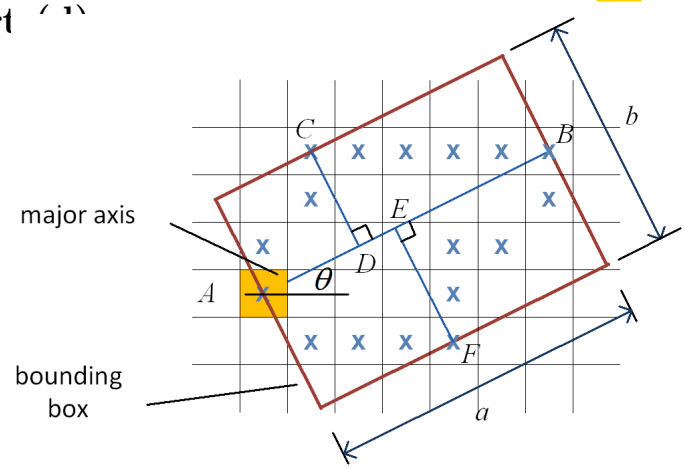
Minimum radial distance, r_{min} $= 1.03$
Maximum radial distance, r_{max} $= 3.48$
Average radial distance, r_{ave} $= 2.41$
Eccentricity $= r_{max}/r_{min}$ $= 3.38$

Part (c)

r	$n(r)$	$p(r)$
0.25 – 0.75	0	0
0.75 – 1.25	1	0.059
1.25 – 1.75	1	0.059
1.75 – 2.25	6	0.353
2.25 – 2.75	4	0.235
2.75 – 3.25	4	0.235
3.25 – 3.75	1	0.059
3.75 – 4.25	0	0



Part ( )



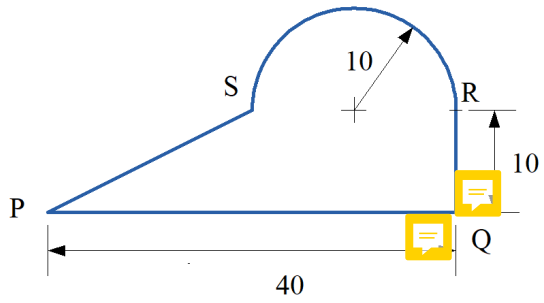
With A as the origin,
equation of AB is $x - 2y = 0$,
C is the point $(1,3)$ and F is the point $(4,-1)$

$$\begin{aligned} \text{Length of major axis} \quad a &= \overline{AB} \\ &= \sqrt{3^2 + 6^2} \\ &= 6.71 \end{aligned}$$

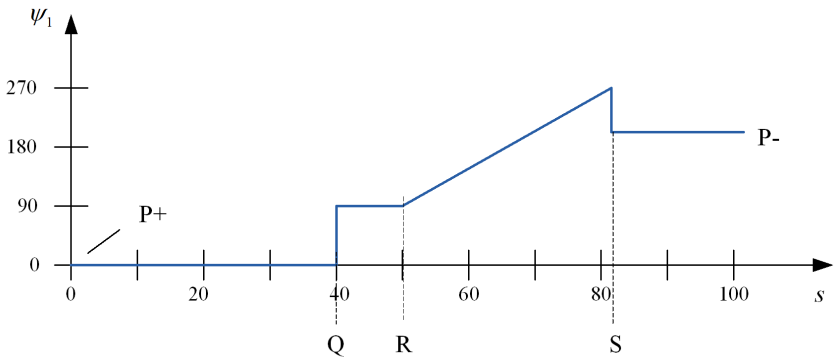
$$\begin{aligned} \text{Orientation,} \quad \theta &= \tan^{-1}(3/6) \\ &= 26.6^\circ \end{aligned}$$

$$\begin{aligned} \text{Length of minor axis} \quad b &= \overline{CD} + \overline{EF} \\ &= \frac{|1-6|}{\sqrt{1^2+2^2}} + \frac{|4+2|}{\sqrt{1^2+2^2}} \\ &= \frac{11}{\sqrt{5}} \\ &= 4.92 \end{aligned}$$

Question 5

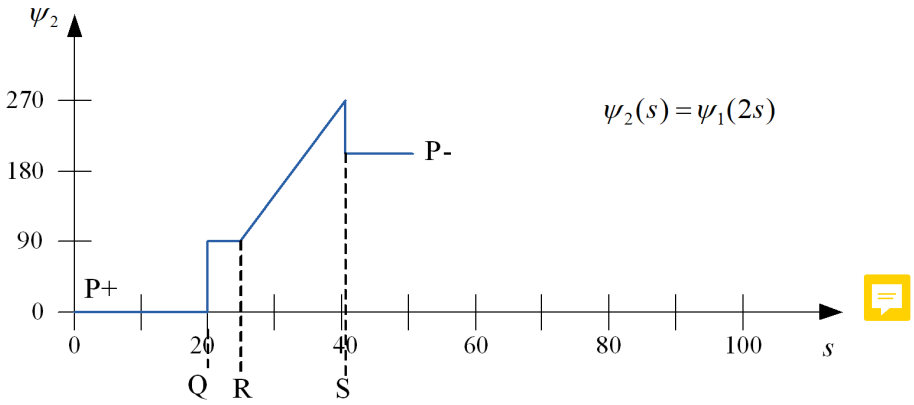


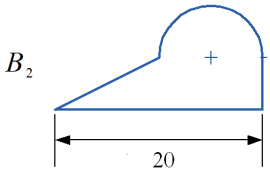
Part (a)

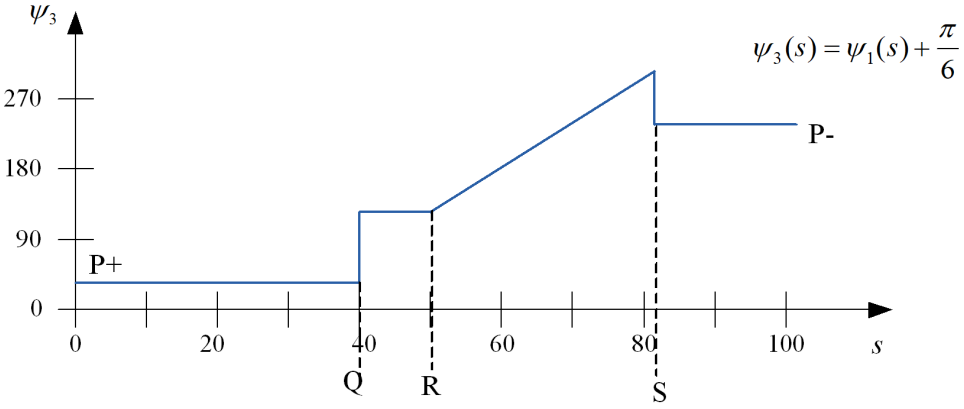


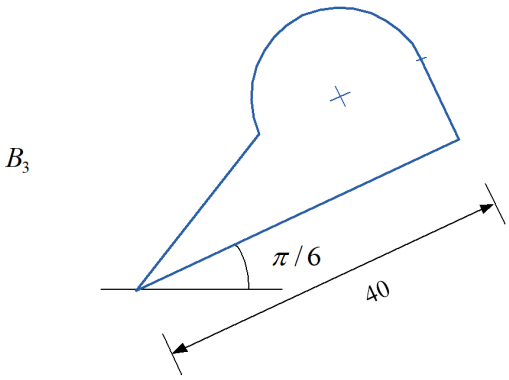
Point	s	ψ
$P+$	0	0°
$Q-$	40	0°
$Q+$	40	90°
R	50	90°
$S-$	81.4	270°
$S+$	81.4	207°
$P-$	103.7	207°

Part (b)


ψ_2
270
180
90
0
P+
P-
$\psi_2(s)=\psi_1(2s)$
0
20
40
60
80
100
s
Q
R
S


B_2
+
20


ψ_3
270
180
90
0
P+
P-
$\psi_3(s)=\psi_1(s)+\frac{\pi}{6}$
0
20
40
60
80
100
s
Q
R
S


B_3
+
$\pi/6$
40