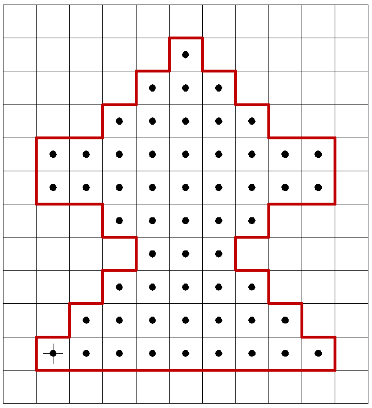
EE4704 Image Processing and Analysis

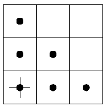
Tutorial Set G – Solutions

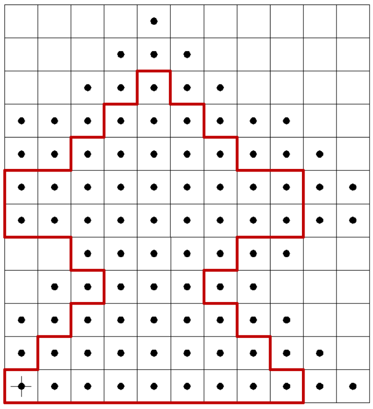
Question 1

A

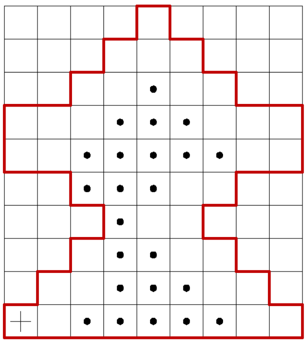


B

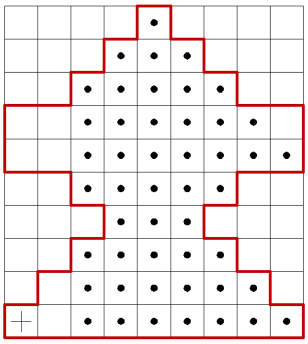




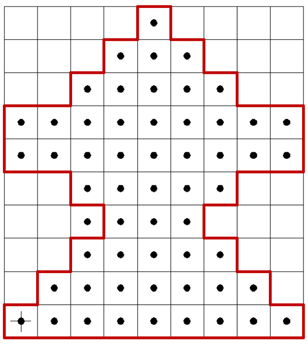
$A \oplus B$



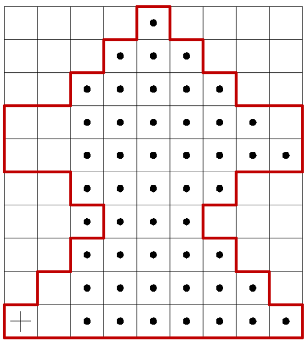
$A \ominus B$



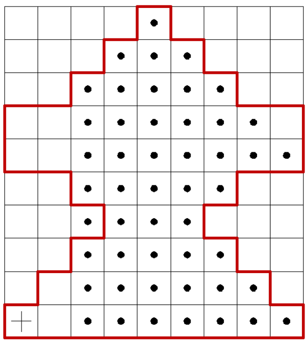
$A \circ B$



$A \bullet B$

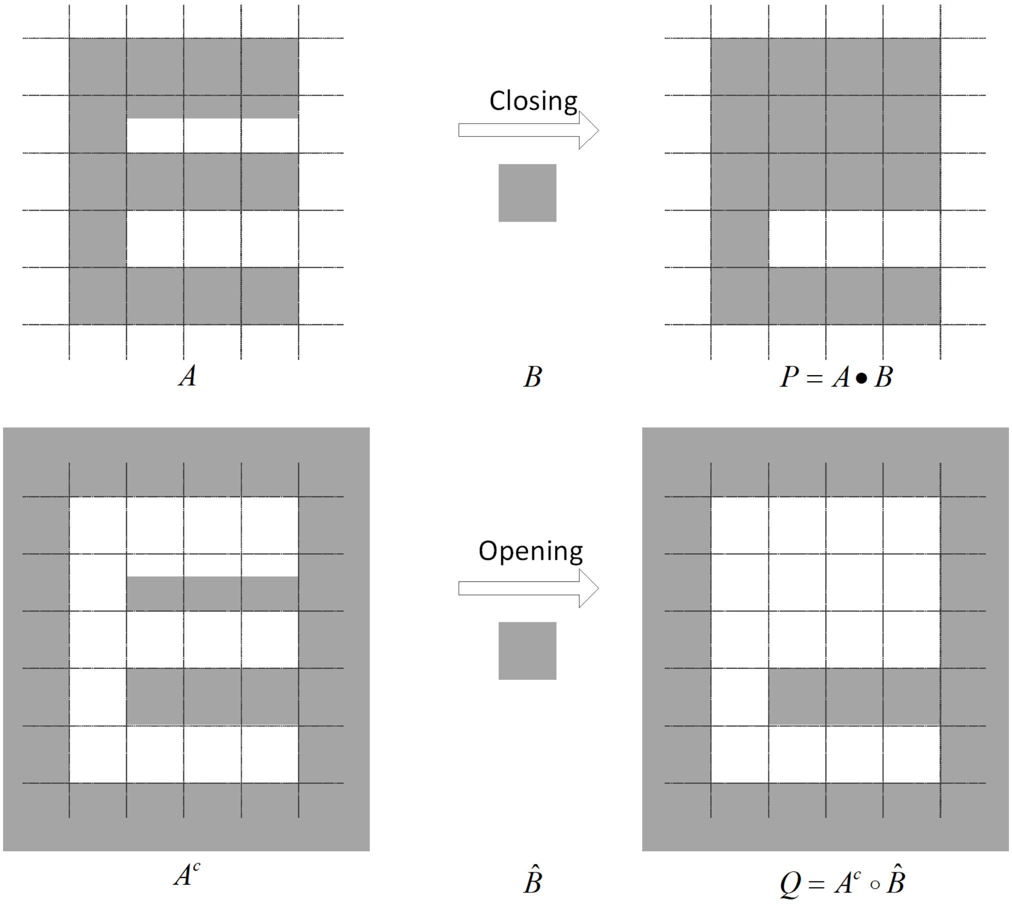


$(A \circ B) \bullet B$



$(A \bullet B) \circ B$

Question 2



From the results, we see that

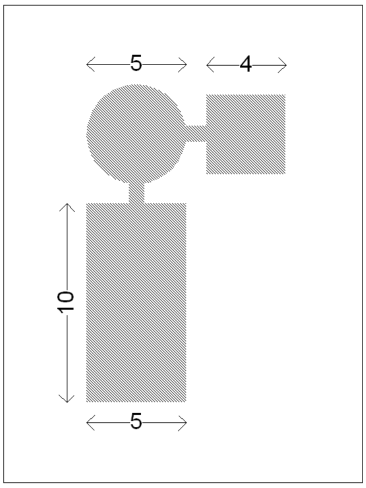
$$P^c = Q$$

i.e.

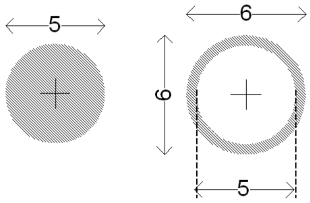
$$(A \bullet B)^c = A^c \circ \hat{B}$$

which is the dual property of opening and closing.

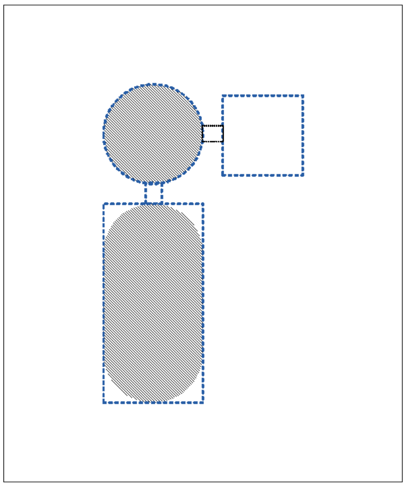
Question 3



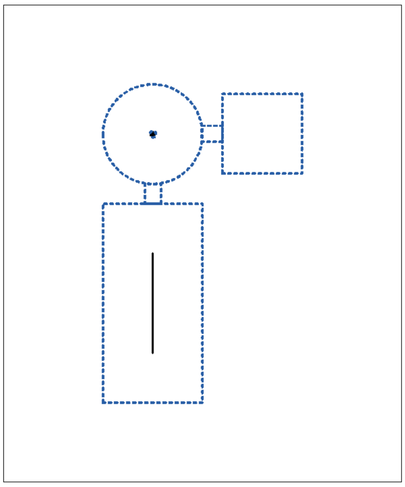
I



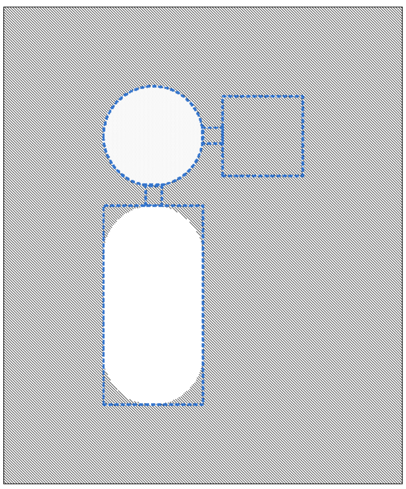
S_1 S_2



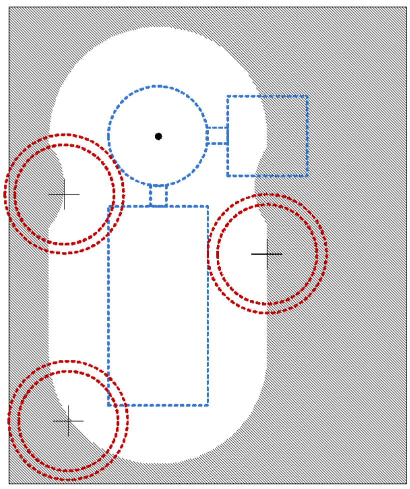
$I_1 = I \circ S_1$



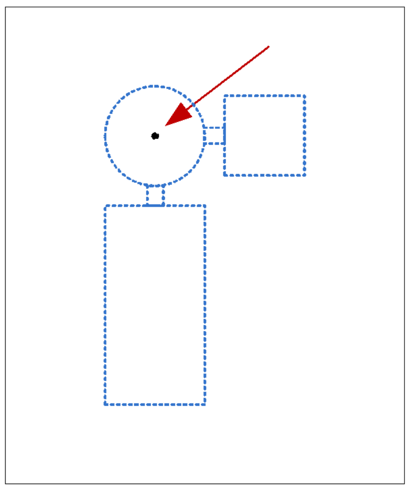
$I_1 \ominus S_1$



I_1^c

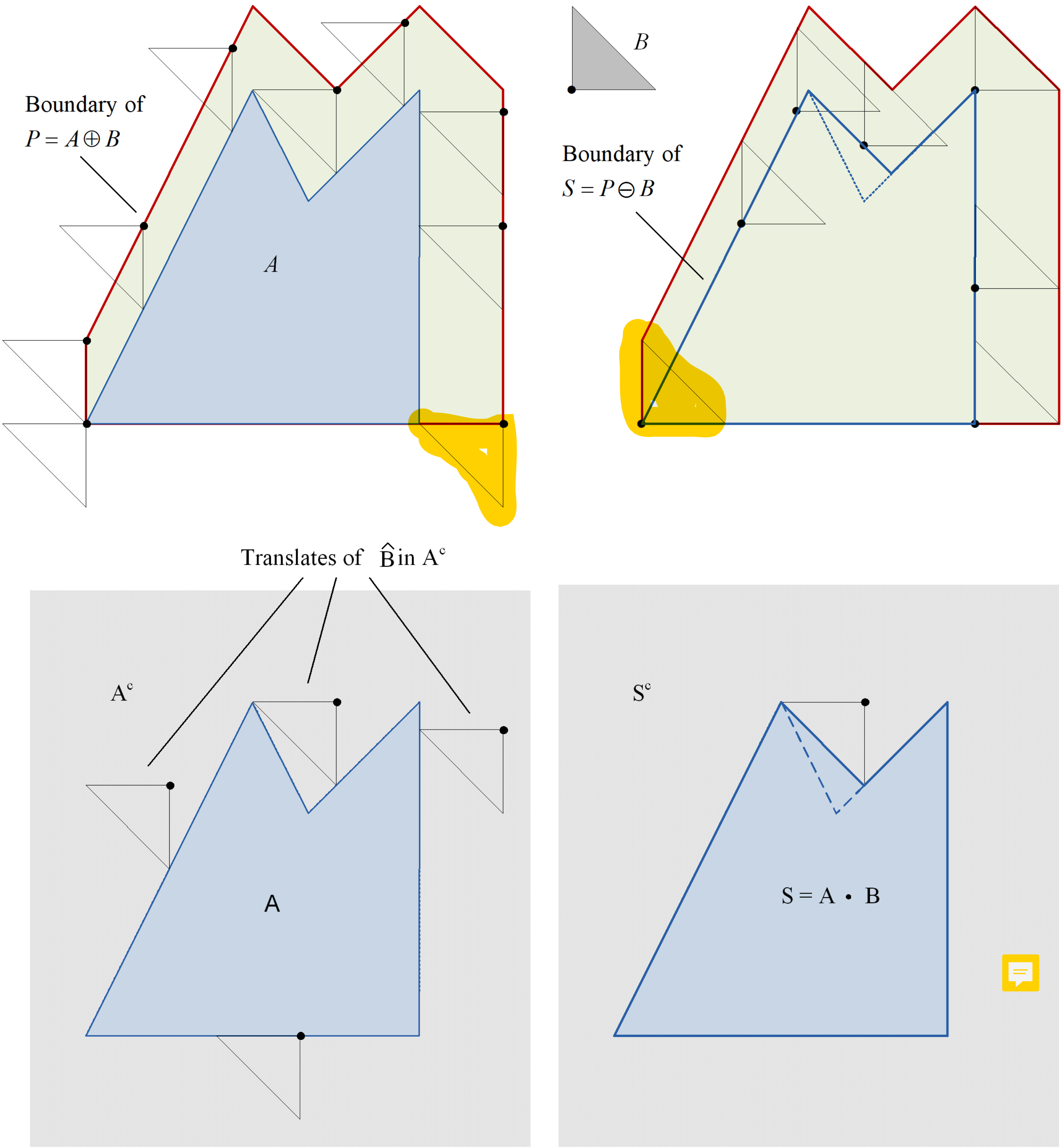


$I_1^c \ominus S_2$



$I_2 = (I_1 \ominus S_1) \cap (I_1^c \ominus S_2)$

Question 4

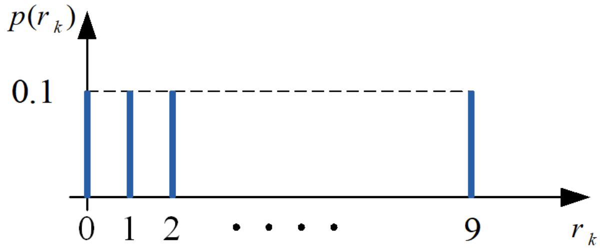

Boundary of
P = A ⊕ B
A
B
Boundary of
S = P ⊖ B
Translates of B̂ in Aᶜ
Aᶜ
A
Sᶜ
S = A • B

Question 5

Part (a)

$$
\begin{aligned}
I(E) &= -\log_r P(E) \\
&= -\log_2(0.2) = 2.322 \text{ bits} \\
&= -\log_{10}(0.2) = 0.699 \text{ Hartleys}
\end{aligned}
$$

Part(b)

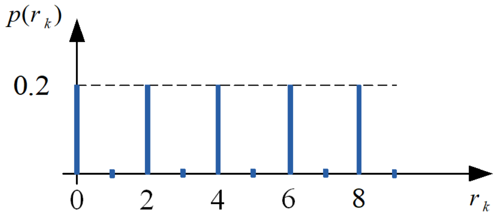


Probability values : $p(r_k) = \frac{1}{L}, \qquad r_k = 0, 1, 2, \ldots, L-1$

The entropy is

$$
\begin{aligned}
H &= -\sum_{k=0}^{L-1} p(r_k) \log p(r_k) \\
&= -\sum_{k=0}^{L-1} \frac{1}{L} \log(\frac{1}{L}) \\
&= \frac{1}{L} \log(L) \sum_{k=0}^{L-1} \\
&= \frac{1}{L} \log(L) \times L \\
&= \log(L) \\
&= \log(10) \\
&= 3.322 \text{ bits}
\end{aligned}
$$

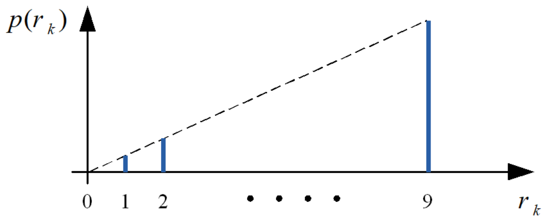
Part (c)



The entropy is

$$
\begin{aligned}
H &= -\sum_{k=0}^{L-1} p(r_k)\log p(r_k) \\
&= \frac{L}{2}\left\{\frac{2}{L}\log(\frac{L}{2})\right\} \\
&= \log(\frac{L}{2}) \\
&= \log(5) \\
&= 2.322 \text{ bits}
\end{aligned}
$$

Part (d)



Probability values : $p(r_k) = Kr_k, \qquad r_k = 0, 1, 2, \ldots, 9$

$$\sum_k p(r_k) = 1 \Rightarrow K = \frac{1}{45}$$

Hence,

$$p(r_k) = \frac{1}{45} r_k$$

The entropy is

$$
\begin{aligned}
H &= -\sum_{k=0}^{9} p(r_k)\log p(r_k) \\
&= -\sum_{k=0}^{9} \frac{r_k}{45}\log\frac{r_k}{45} \\
&= 2.96 \text{ bits}
\end{aligned}
$$

Question 6

Symbol :	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
Gray level :	0	1	2	3	4	5	6	7
Probability :	0.4	0.08	0.08	0.2	0.12	0.08	0.03	0.01

Part (a)

The entropy is

$$\begin{aligned} H &= -\sum P(a_i)\log_2 P(a_i) \\ &= -0.4\log 0.4 - 3\times 0.08\log 0.08 - 0.12\log 0.12 \\ &\quad -0.2\log 0.2 - 0.03\log 0.03 - 0.01\log 0.01 \\ &= 2.453 \text{ bits} \end{aligned}$$

Coding efficiency using the natural binary code is

$$2.453/3 = 81.8\%$$

Part (b)

Original source			Source reduction											
Symbol	Prob.		1		2		3		4		5		6	
a_0	0.4	1	0.4	1	0.4	1	0.4	1	0.4	1	0.4	1	0.6	0
a_3	0.2	000	0.2	000	0.2	000	0.2	000	0.24	01	0.36	00	0.4	1
a_4	0.12	010	0.12	010	0.12	010	0.16	001	0.2	000	0.24	01		
a_1	0.08	0010	0.08	0010	0.12	011	0.12	010	0.16	001				
a_2	0.08	0011	0.08	0011	0.08	0010	0.12	011						
a_5	0.08	0110	0.08	0110	0.08	0011								
a_6	0.03	01110	0.04	0111										
a_7	0.01	01111												

Gray Level	Prob.	Straight binary code	Huffman code	L
0	0.4	000	1	1
1	0.08	001	0010	4
2	0.08	010	0011	4
3	0.2	011	000	3
4	0.12	100	010	3
5	0.08	101	0110	4
6	0.03	110	01110	5
7	0.01	111	01111	5

Average code length for the Huffman code is

$$\begin{aligned}\bar{L} &= (1 \times 0.4) + (4 \times 0.08) + (4 \times 0.08) + (3 \times 0.2) \\ &\quad +(3 \times 0.12) + (4 \times 0.08) + (5 \times 0.03) + (5 \times 0.01) \\ &= 2.520 \text{ bits}\end{aligned}$$

Code efficiency is

$$\eta = \frac{2.453}{2.520} = 97.3\%$$

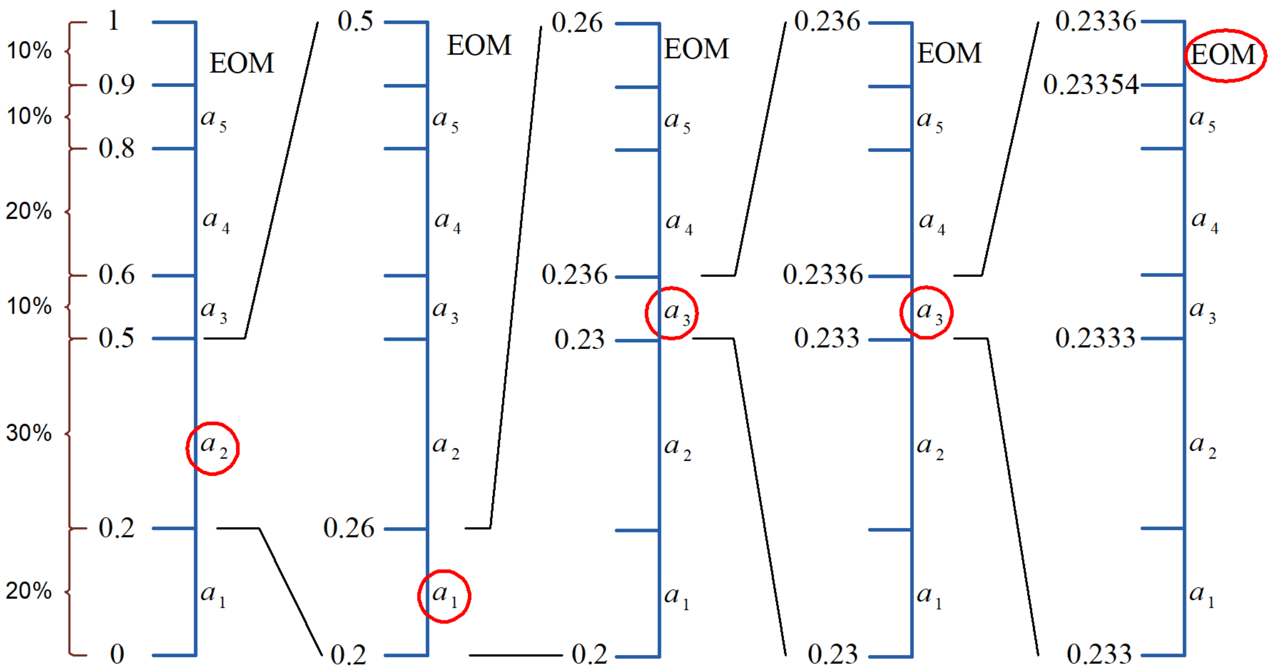
Part (c)

With 3 bits/pixel, the image occupies 3×10^4 bits. Therefore,

$$\text{savings} = (3 - 2.52) \times 10^4 = 4,800 \text{ bits} \quad (16\%)$$

Question 7

Part (a)



Hence, $0.23355 \longrightarrow a_2\ a_1\ a_3\ a_3$ (EOM)

Part (b)

0	1	2	3	4	5
0	1	2	3	4	5
0	1	2	3	4	5
0	1	2	3	4	5
0	1	2	3	4	5
0	1	2	3	4	5

I_1

1	1	3	3	5	5
1	1	3	3	5	5
1	1	3	3	5	5
1	1	3	3	5	5
1	1	3	3	5	5
1	1	3	3	5	5

I_2

The symbol probabilities are:

Image I_1		
Symbol	Gray-level	Prob.
a_0	0	1/6
a_1	1	1/6
a_2	2	1/6
a_3	3	1/6
a_4	4	1/6
a_5	5	1/6

Image I_2		
Symbol	Gray-level	Prob.
a_0	0	0
a_1	1	1/3
a_2	2	0
a_3	3	1/3
a_4	4	0
a_5	5	1/3

Image I_1:
After the first symbol, $R_1 = \left(\frac{1}{6}\right)$.
After the second symbol, $R_2 = \left(\frac{1}{6}\right)^2$.
...
After the sixth symbol, $R_6 = \left(\frac{1}{6}\right)^6 = 2.143 \times 10^{-5}$

Image I_2:
After the first symbol, $R_1 = \left(\frac{1}{3}\right)$.
After the second symbol, $R_2 = \left(\frac{1}{3}\right)^2$.
...
After the sixth symbol, $R_6 = \left(\frac{1}{3}\right)^6 = 1.372 \times 10^{-3}$.

After the 36th pixel, R_{36} for I_1 would be much smaller than R_{36} for I_2. Since more decimal digits are needed for a smaller range, Image I_1 would require more digits for transmission.

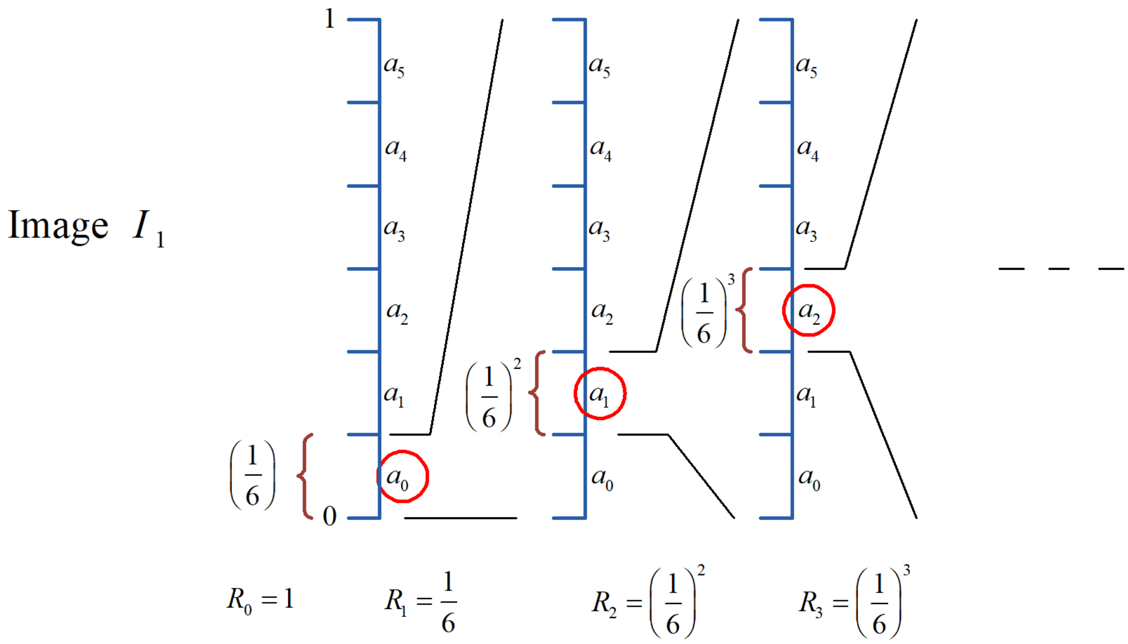


Image I_1
1
0
a_5
a_4
a_3
a_2
a_1
a_0
$\left(\frac{1}{6}\right)$
$\left(\frac{1}{6}\right)^2$
$\left(\frac{1}{6}\right)^3$
$R_0 = 1$
$R_1 = \frac{1}{6}$
$R_2 = \left(\frac{1}{6}\right)^2$
$R_3 = \left(\frac{1}{6}\right)^3$

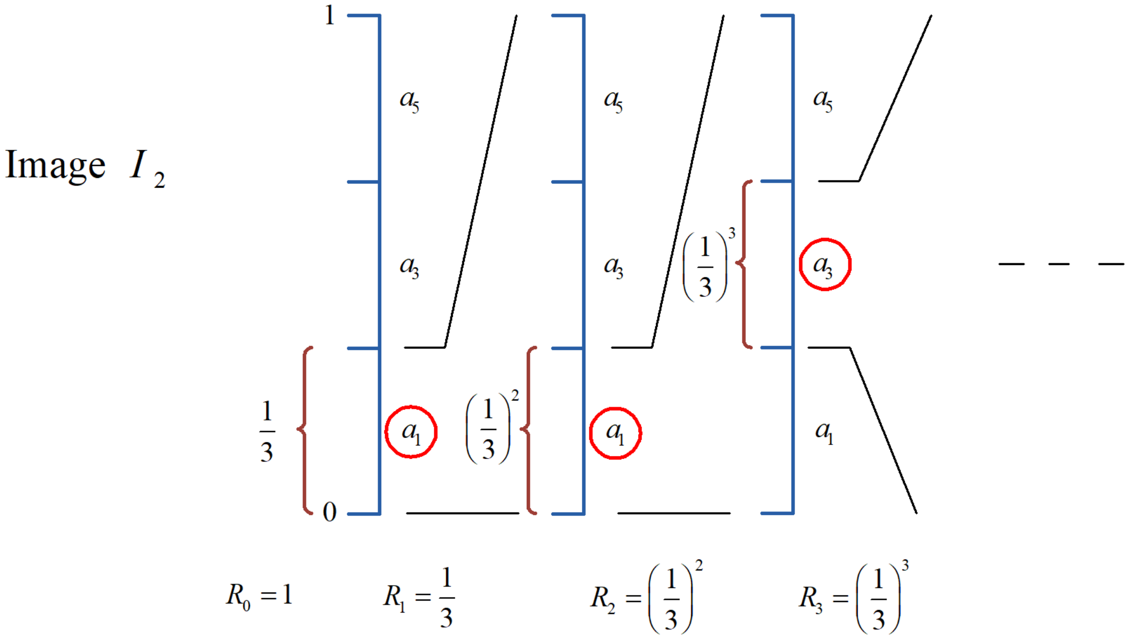


Image I_2
1
0
a_5
a_3
a_1
$\frac{1}{3}$
$\left(\frac{1}{3}\right)^2$
$\left(\frac{1}{3}\right)^3$
$R_0 = 1$
$R_1 = \frac{1}{3}$
$R_2 = \left(\frac{1}{3}\right)^2$
$R_3 = \left(\frac{1}{3}\right)^3$

Question 8

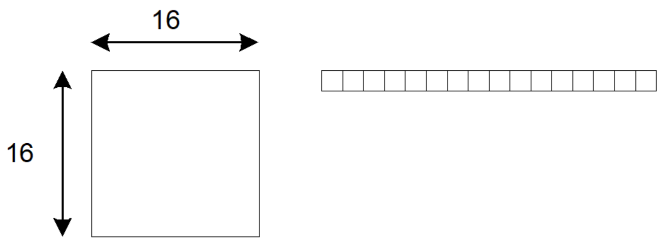
Each pixel is stored as 1 byte. Without run-length coding, the number of bytes required is

$$N_0 = 16^2 = 256$$

Our run-length coding scheme assumes that each row begins with a white pixel, and each run requires 1 byte (to denote the length of the run). For a row starting with a black pixel, an extra byte is needed for the first run (of zero length).

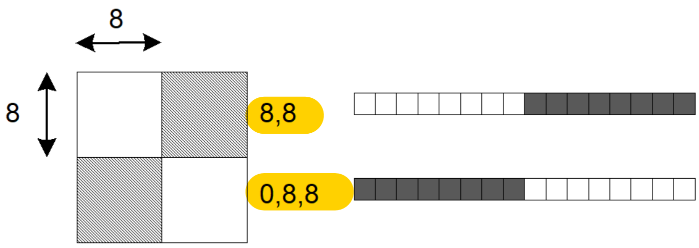
One run per row, 16 rows
Number of bytes required is

$$\begin{aligned} N_1 &= 16 < N_0 \\ C_R &= 16 \end{aligned}$$



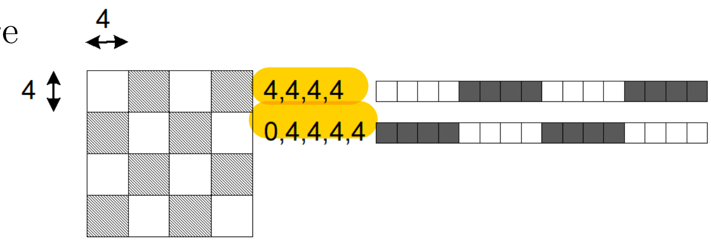
For a row starting with 1, there are two runs
For a row starting with 0, there are three runs
Number of bytes required is

$$\begin{aligned} N_2 &= 8 \times 2 + 8 \times 3 = 40 < N_0 \\ C_R &= 6.4 \end{aligned}$$



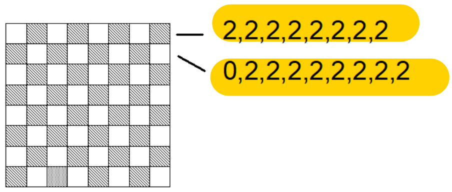
For a row starting with 1, there are four runs
For a row starting with 0, there are five runs
Number of bytes required is

$$\begin{aligned} N_4 &= 8 \times 4 + 8 \times 5 = 72 < N_0 \\ C_R &= 3.6 \end{aligned}$$



For a row starting with 1, there are eight runs
For a row starting with 0, there are nine runs
Number of bytes required is

$$
\begin{aligned}
N_8 &= 8 \times 8 + 8 \times 9 = 136 < N_0 \\
C_R &= 1.9
\end{aligned}
$$



For a row starting with 1, there are sixteen runs
For a row starting with 0, there are seventeen runs
Number of bytes required is

$$
\begin{aligned}
N_{16} &= 8 \times 16 + 8 \times 17 = 264 > N_0 \\
C_R &= 0.97
\end{aligned}
$$