EE4704 Image Processing and Analysis

Semester 1, 2020/21

Tutorial Set G

1. Given A and B in the figure, obtain

   (a) $A \oplus B$

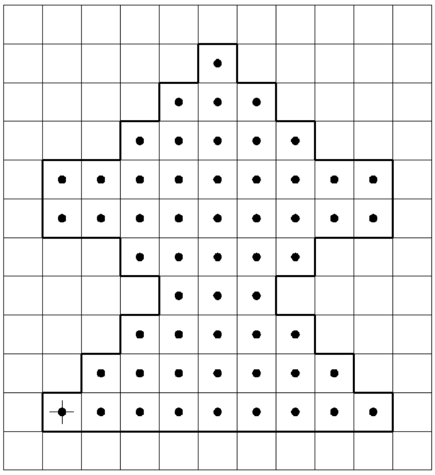
   (b) $A \ominus B$

   (c) $A \circ B$

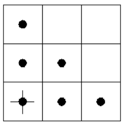
   (d) $A \bullet B$

   (e) $(A \circ B) \bullet B$

   (f) $(A \bullet B) \circ B$

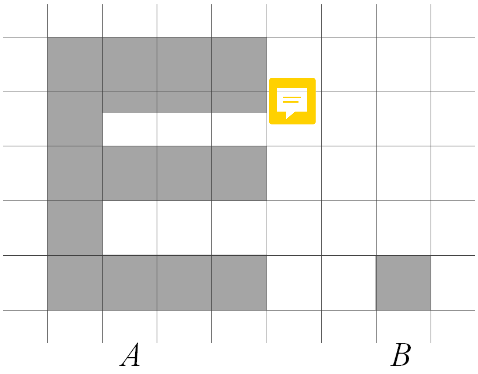


A



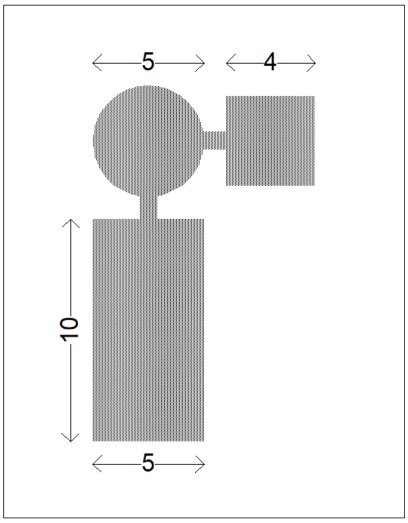
B

2. Given the continuous sets A and B below, obtain $P = A \bullet B$ and $Q = A^c \circ \hat{B}$ to illustrate the duality property of opening and closing.

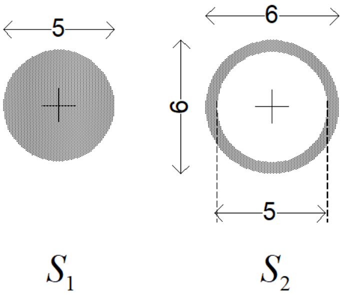


3. Binary image I in the figure consists of a square, a rectangle and a circle connected by narrow strips 1 unit long. Show that the location of the circle (i.e., its centroid) is obtained by this sequence of morphological operations:

$$\begin{aligned} I_1 &= I \circ S_1 \\ I_2 &= (I_1 \ominus S_1) \cap (I_1^c \ominus S_2) \end{aligned}$$



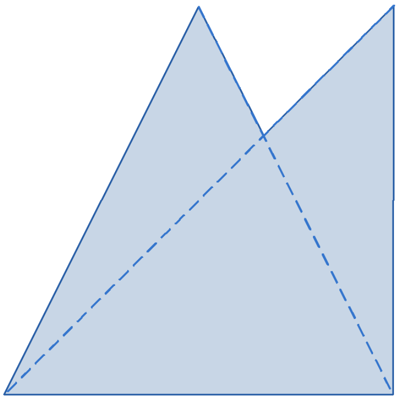
I



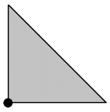
4. Given the sets A and B in the figure, obtain $A \bullet B$ using the following:

(a) $A \bullet B = (A \oplus B) \ominus B$

(b) $(A \bullet B)^c = \cup\{(\hat{B})_x \mid (\hat{B})_x \subseteq A^c\}$



A



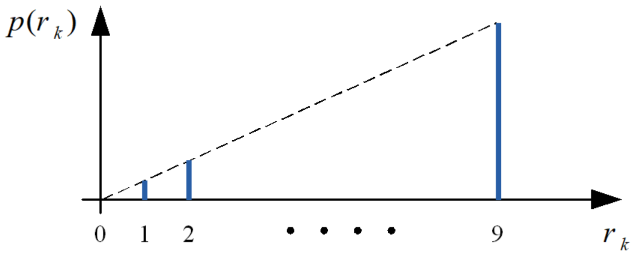
B

5. (a) An event occurs with probability $P(E) = 0.2$. What is the information associated with that event (i) in bits, (ii) in Hartleys?

   (b) Calculate (in bits) the entropy of a 100×100 10-level image in which the pixels are distributed equally among all the gray levels.

   (c) Calculate (in bits) the entropy of a 100×100 10-level image in which the pixels are distributed equally among alternate gray levels $0, 2, 4, 6, 8$.

   (d) Calculate (in bits) the entropy of a 100×100 10-level image in which the histogram values increase linearly with gray level (see figure).



6. The distribution of an 8-level 100×100 image is given in the table below.

   (a) Compute the entropy of the image. What is the coding efficiency if a natural 3-bit binary code is used?

   (b) Derive a Huffman code for this image. What is the average code length and the code efficiency?

   (c) With the original 3-bit pixels, how many bits does the image occupy? What savings does the Huffman code give?

Gray level:	0	1	2	3	4	5	6	7
No. of pixels:	4000	800	800	2000	1200	800	300	100

7. (a) Decode the message 0.23355 given the source probabilities:

$$\begin{array}{lll} P(a_1) = 0.2, & P(a_2) = 0.3, & P(a_3) = 0.1, \\ P(a_4) = 0.2, & P(a_5) = 0.1, & P(\text{EOM}) = 0.1 \end{array}$$

(b) I_1 and I_2 are 6×6 6-level images to be transmitted by arithmetic coding (AC). The pixels are transmitted from left to right, top to bottom, and there is no EOM symbol. In AC, the size of the initial range $[0, 1)$ is $R_0 = 1$, and this is narrowed by the encoder after each pixel is coded. What is the size of the range R_6 after the sixth pixel for each image?

0	1	2	3	4	5
0	1	2	3	4	5
0	1	2	3	4	5
0	1	2	3	4	5
0	1	2	3	4	5
0	1	2	3	4	5

I_1

1	1	3	3	5	5
1	1	3	3	5	5
1	1	3	3	5	5
1	1	3	3	5	5
1	1	3	3	5	5
1	1	3	3	5	5

I_2

8. Run length encoding may not provide any compresssion for "busy" images (those in which the pixel values are rapidly varying, and there is very little correlation between adjacent pixels). Consider a 16×16 checkerboard image with alternate black (0) and white (255) squares. Starting with a pure white image, then an image with 4 squares (each 8 pixels by 8 pixels) and so on, at what stage does run-length encoding start to increase the data (i.e. more than 256 bytes)? Calculate the compression ratio that is achievable for each image. (The unit of storage is the byte.)