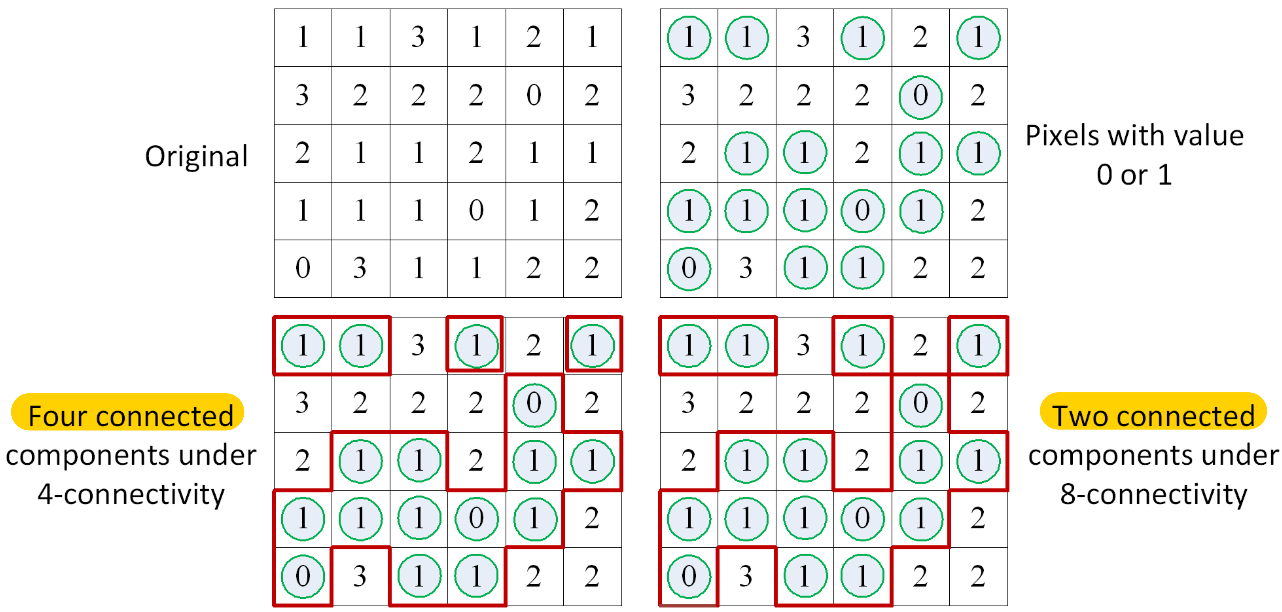
EE4704 Image Processing and Analysis

Tutorial Set A – Solution

Question 1



Question 2

100	150	200	200	200
100	20	200	200	200
100	20	200	200	200
100	20	200	200	200
100	150	200	200	200

I_1

100	150	200	200	200
100	150	30	200	200
100	150	30	200	200
100	150	30	200	200
100	150	200	200	200

I_2

100	150	200	200	200
100	85	115	200	200
100	85	115	200	200
100	85	115	200	200
100	150	200	200	200

$A = 0.5(I_1 + I_2)$

0	0	0	0	0
0	130	170	0	0
0	130	170	0	0
0	130	170	0	0
0	0	0	0	0

$B = |I_1 - I_2|$

100	150	200	200	200
100	98	98	200	200
100	98	98	200	200
100	98	98	200	200
100	150	200	200	200

$C = 0.4I_1 + 0.6I_2$

Question 3

Part (a)

The image is given by

$$\begin{aligned} f(x,y) &= i(x,y) \times r(x,y) && (1) \\ &= 512 \exp(-(x/10)^2 - (y/10)^2) && (2) \end{aligned}$$

The magnitude of $f(x,y)$ spans the range 0 to 512; this range is quantised to 256 levels

$$0, 1, 2, \ldots, 255$$

The scheme for uniform quantisation is shown in Figure 3a. The quantisation interval is

$$\Delta I = \frac{512}{256} = 2 \qquad (3)$$

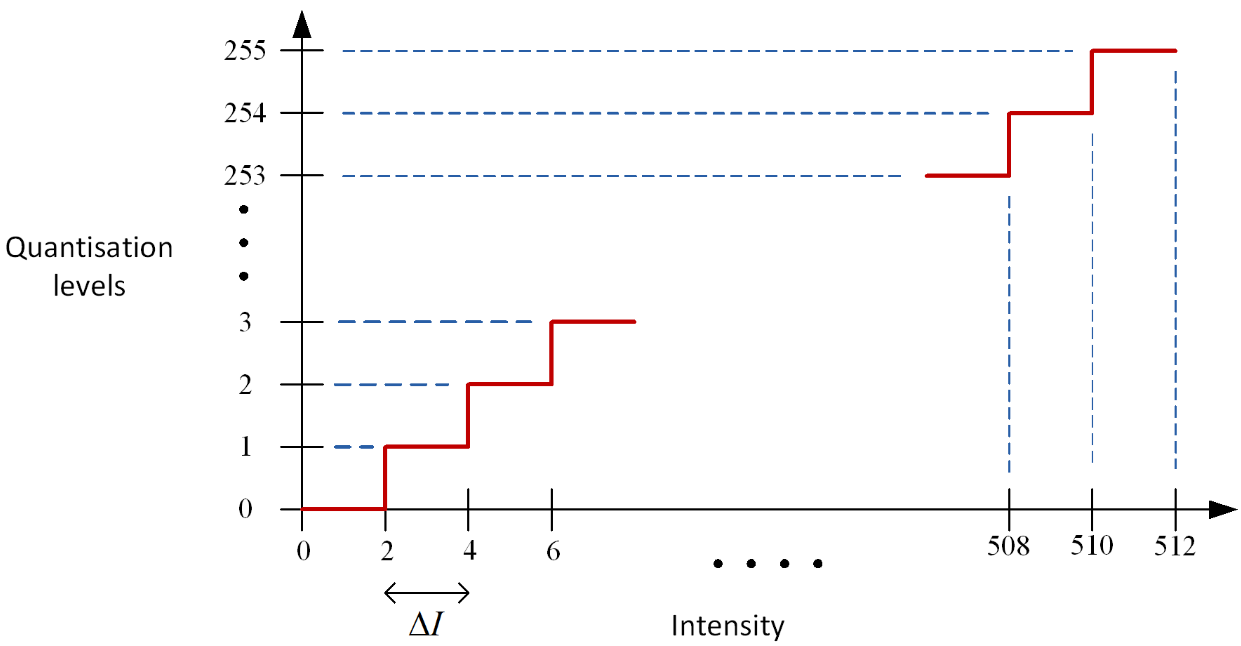


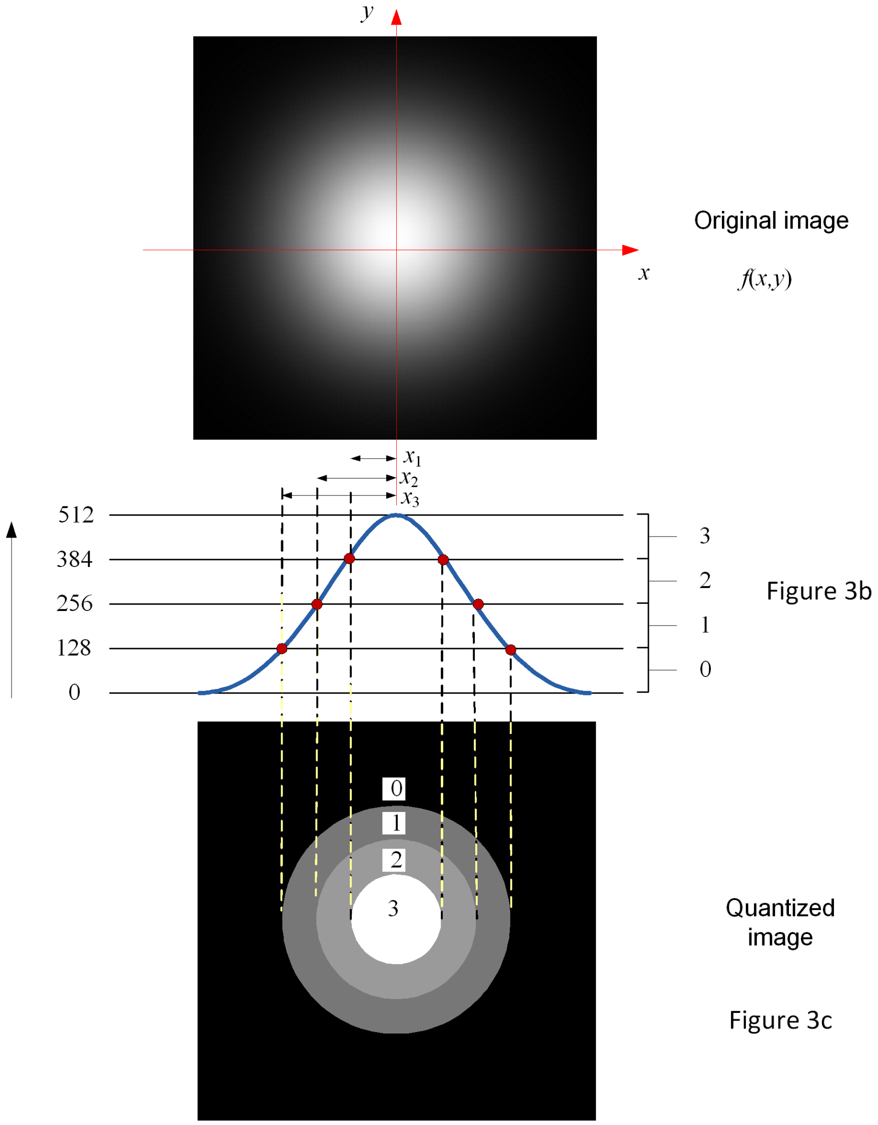
Figure 3a

Part (b)

The use of two bits produces 4 gray levels, 0, 1, 2 and 3, spanning the intensity range 0 to 512. A cross section of $f(x, y)$ through $(0, 0)$ is shown in Figure 3b. The equation describing the profile can be obtained from Eq.(2):

$$f(x, 0) = 512 \exp(-(x/10)^2) \tag{4}$$

A sketch of the quantised image is shown in Figure 3c. Note the false contours.



$$\begin{aligned} f(x_1, 0) &= 384 \Rightarrow x_1 = 5.36 & (5) \\ f(x_2, 0) &= 256 \Rightarrow x_2 = 8.33 & (6) \\ f(x_3, 0) &= 128 \Rightarrow x_3 = 11.77 & (7) \end{aligned}$$

Hence the diameters of the circles are 10.7, 16.7 and 23.5.

Question 4

I

0	0	0	0	0	0	0	0	0	0	0
0	0	9	9	9	0	0	0	0	0	0
0	0	9	9	9	9	0	0	0	0	0
0	0	9	9	9	9	9	0	0	0	0
0	0	9	9	9	9	9	9	0	0	0
0	0	0	9	9	9	9	9	9	0	0
0	0	0	0	9	9	9	9	9	0	0
0	0	0	0	0	0	0	0	0	0	0

I_1

0	0	0	0	0	0	0	0	0	0	0
0	3	6	9	6	3	0	0	0	0	0
0	3	6	9	9	6	3	0	0	0	0
0	3	6	9	9	9	6	3	0	0	0
0	3	6	9	9	9	9	6	3	0	0
0	0	3	6	9	9	9	9	6	3	0
0	0	0	3	6	9	9	9	6	3	0
0	0	0	0	0	0	0	0	0	0	0

I_2

0	0	3	3	3	0	0	0	0	0	0
0	0	6	6	6	3	0	0	0	0	0
0	0	9	9	9	6	3	0	0	0	0
0	0	9	9	9	9	6	3	0	0	0
0	0	6	9	9	9	9	6	3	0	0
0	0	3	6	9	9	9	9	6	0	0
0	0	0	3	6	6	6	6	6	0	0
0	0	0	0	3	3	3	3	3	0	0

- M_1 smooths (blurs) in a horizontal direction.
- M_2 smooths (blurs) in a vertical direction.

I_3

0	-1	-2	-3	-2	-1	0	0	0	0	0
0	-2	6	4	5	-3	-1	0	0	0	0
0	-3	4	1	2	4	-3	-1	0	0	0
0	-3	4	1	1	2	4	-3	-1	0	0
0	-2	5	2	1	1	2	4	-3	-1	0
0	-1	-3	4	2	1	1	2	5	-2	0
0	0	-1	-3	5	4	4	4	6	-2	0
0	0	0	-1	-2	-3	-3	-3	-2	-1	0

I_4

0	1	2	3	2	1	0	0	0	0	0
0	2	4	6	5	3	1	0	0	0	0
0	3	6	9	8	6	3	1	0	0	0
0	3	6	9	9	8	6	3	1	0	0
0	2	5	8	9	9	8	6	3	1	0
0	1	3	6	8	9	9	8	5	2	0
0	0	1	3	5	6	6	6	4	2	0
0	0	0	1	2	3	3	3	2	1	0

- M_3 gives a strong (positive or negative) response at the pixels where there is a relatively large change in gray level.

a	b	c
d	e	f
g	h	i

$\xrightarrow{M_1}$

$\frac{1}{3}(a+b+c)$
$\frac{1}{3}(d+e+f)$
$\frac{1}{3}(g+h+i)$

$\xrightarrow{M_2}$

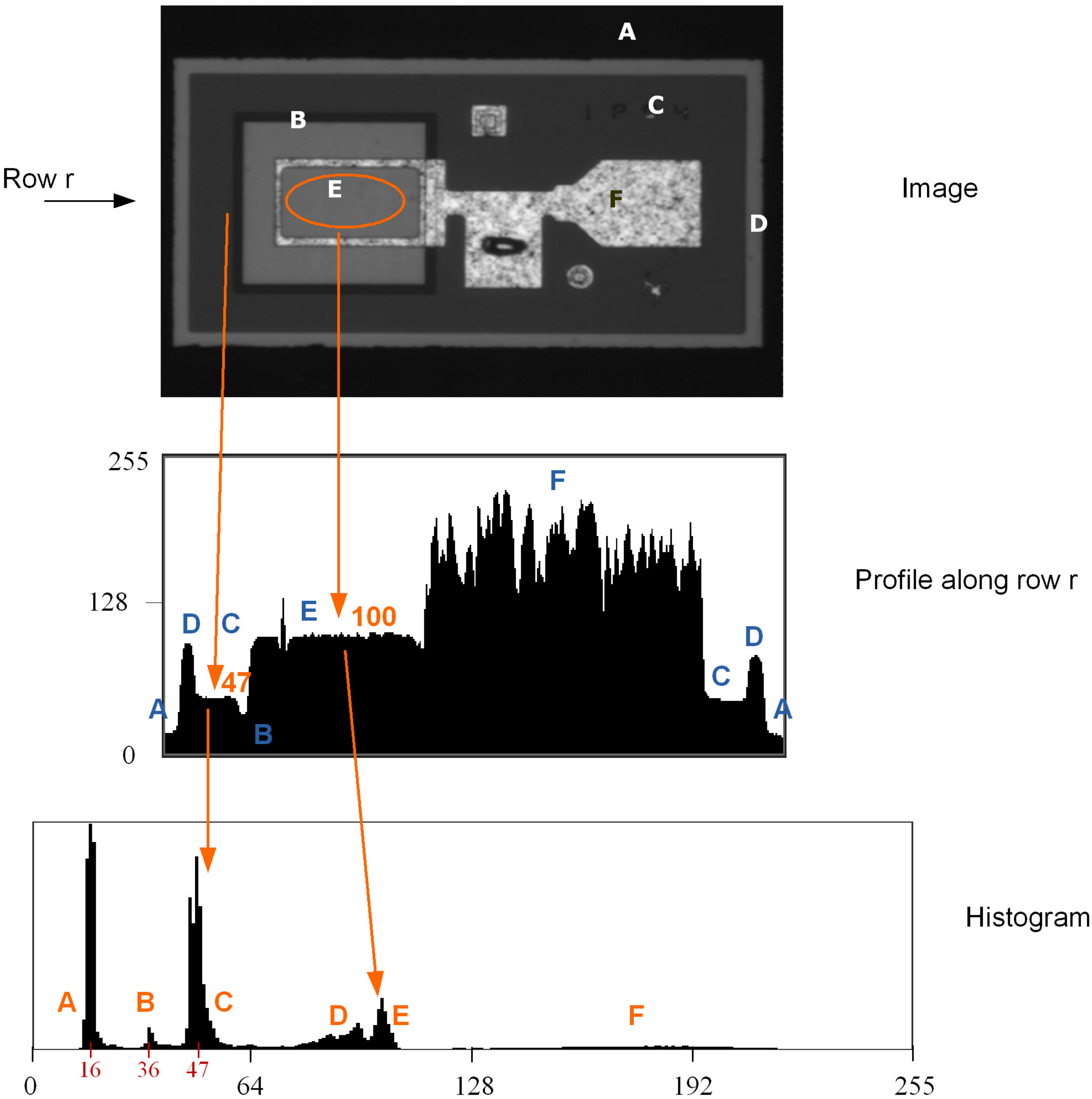
$\frac{1}{9}(a+b+\ldots+i)$

Hence, $M_4 = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

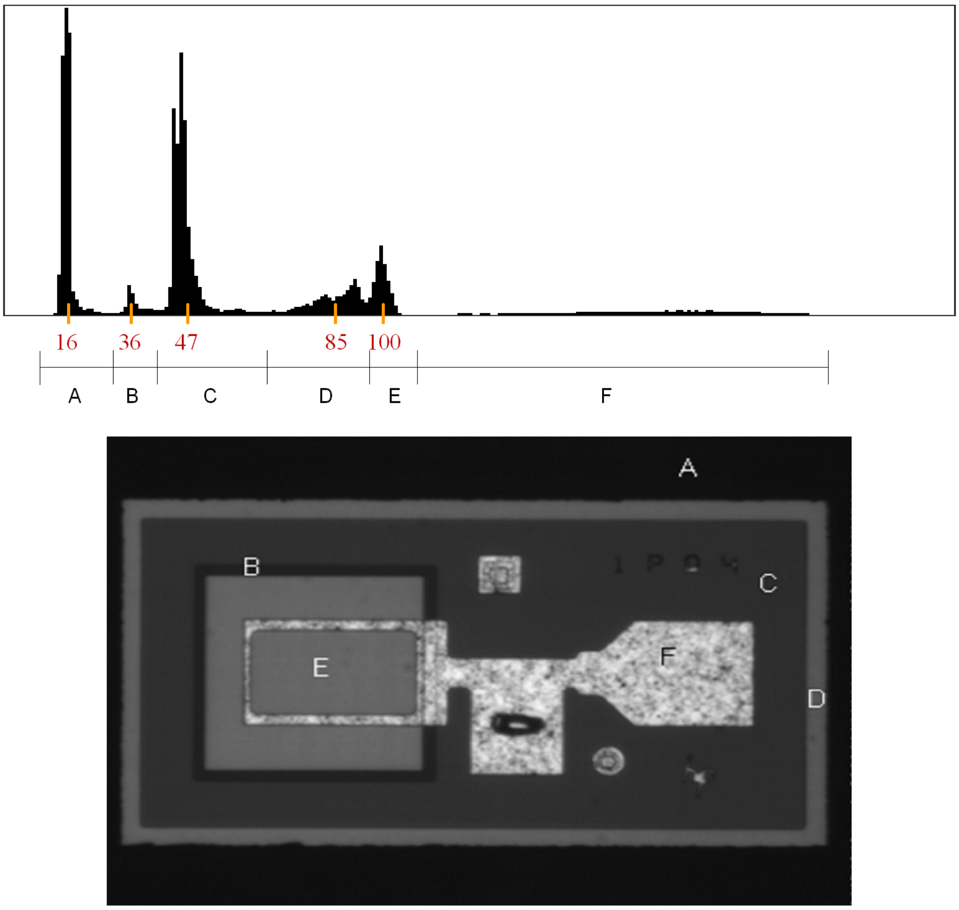
Question 5

Part (a)

Six distinct modes (A to F) can be readily seen in the histogram. By examining the gray levels, it is possible to identify the region in the image corresponding to each mode. For example, from the profile, we see that the small square region has a gray level of about 100, which corresponds to mode E.



We can relate all the histogram modes to the corresponding regions of the image in a similar manner.



Part (b)

The histogram is shifted to the right by 128. Note that clipping occurs at gray level 255.

Question 6

This histogram is

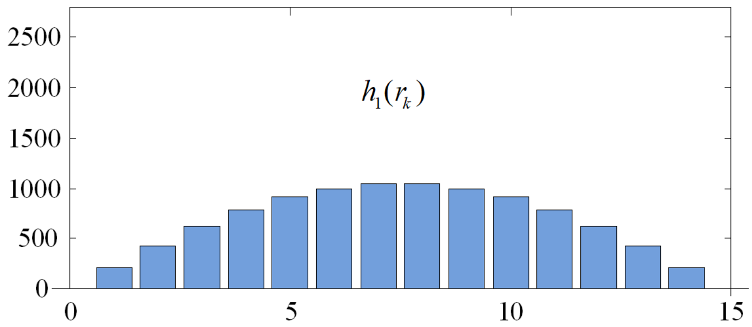
$$h_1(r_k) = A \sin\left(\frac{\pi}{15}k\right) \tag{8}$$

The summation $\sum_{k=0}^{15} h_1(r_k)$ is equal to the number of pixels in the image, 10,000. Hence

$$\sum_{k=0}^{15} h_1(r_k) = A \sum_{k=0}^{15} \sin\left(\frac{\pi}{15}k\right) = 10,000 \tag{9}$$

from which we obtain $A = 1051$.

r_k	$h_1(r_k)$	$p_1(r_k)$ $(\times 10^{-4})$
0	0	0
1	219	219
2	428	428
3	618	618
4	781	781
5	910	910
6	1000	1000
7	1045	1045
8	1045	1045
9	1000	1000
10	910	910
11	781	781
12	618	618
13	428	428
14	219	219
15	0	0



To apply the transformation, we first have to determine the mapping of the individual gray levels. This is done by using the transformation function, which is given by the equations of the two straight-line segments, AB and BC:

$$\begin{array}{llll} AB: & s_k = \frac{2}{5} r_k & & 0 \leq r_k \leq 10 \\ BC: & s_k = \frac{11}{5} r_k - 18 & & 10 \leq r_k \leq 15 \end{array} \tag{10}$$

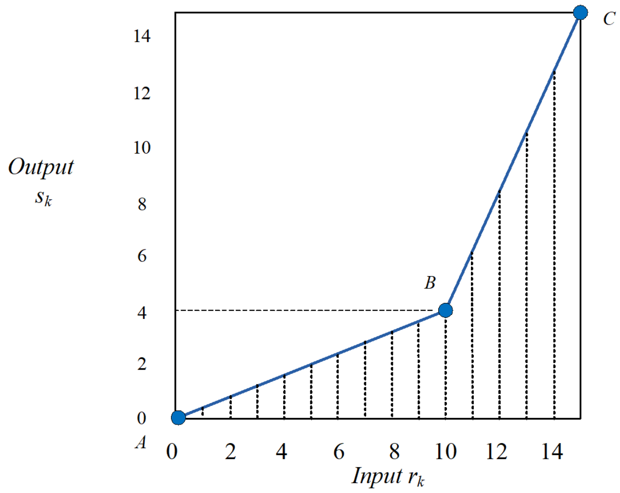
The histogram (h_2) of the transformed image is then readily obtained, bearing in mind that the gray levels r and s take integer values. For example,

$$\begin{array}{llll} r_k = 0 & \rightarrow & s_k = 0 & \\ r_k = 1 & \rightarrow & s_k = 0.4 \rightarrow 0 & \text{(nearest integer)} \end{array}$$

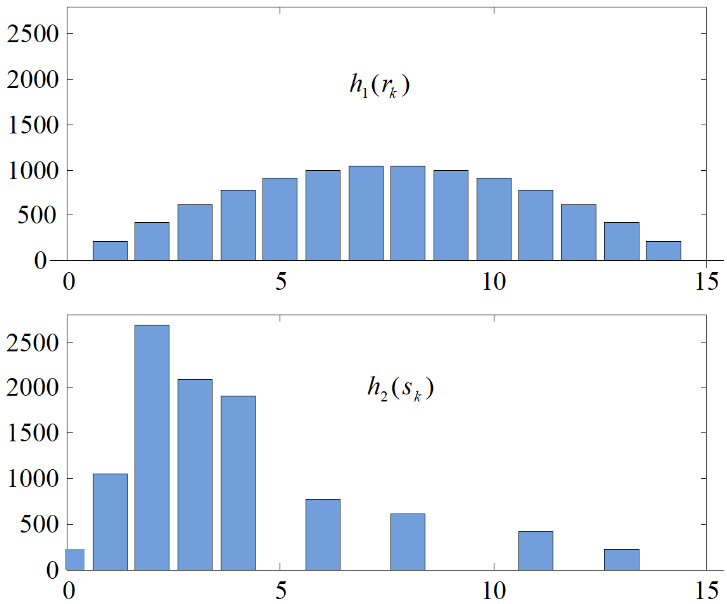
Hence, the number of pixels in output gray level $s_k = 0$ is

$$h_2(0) = h_1(0) + h_1(1) = 0 + 219 = 219$$

We apply the transformation to all the gray levels and tabulate the results.



r_k	$h_1(r_k)$	s_k	$h_2(s_k)$
0	0	0	219
1	219	0	
2	428	1	1046
3	618	1	
4	781	2	2691
5	910	2	
6	1000	2	
7	1045	3	2090
8	1045	3	
9	1000	4	1910
10	910	4	
11	781	6	781
12	618	8	618
13	428	11	428
14	219	13	219
15	0	15	0



The nth moment of r_k about the mean is defined as

$$\mu_n = \sum_{k=0}^{15} (r_k - m)^n p(r_k) \tag{11}$$

where the mean is

$$m = \sum_{k=0}^{15} r_k p(r_k)$$

For example,

$$\mu_2 \;=\; (r_0 - m)^2 p(r_0) + (r_1 - m)^2 p(r_1) + \ldots + (r_{15} - m)^2 p(r_{15}) \tag{12}$$

Applying the equations to the input and output *normalized histograms*,

	Input image	Output image
m	7.50	3.75
μ_2	10.49	7.69
μ_3	0	34.3

- The pixels in the output image are generally concentrated in the lower gray levels; hence the mean is lower and the image appears darker overall.
- The output image has a lower contrast (smaller μ_2) due to the concentration of pixels about gray levels 2 and 3.
- μ_3 is called skewness, which is a measure of the lack of symmetry, with $\mu_3 = 0$ indicating perfect symmetry. Histogram h_2 is asymmetrical, hence its $\mu_3 \neq 0$, unlike histogram h_1.

Question 7

Part (a)

The intensity, I, of a pixel depends on the proportion of the object area (a) to the background area (b) in a photoelement (Fig. 7a). We have

$$I = \frac{a \times 80 + b \times 20}{a + b}$$

For a boundary pixel, the ratio $a : b$ is either $3 : 1$ or $1 : 3$.

For pixel P, $a = 3, b = 1 \Rightarrow I = 65$.

For pixel Q, $a = 1, b = 3 \Rightarrow I = 35$.

Hence we obtain the image of Fig. 7b. After thresholding at 50, we obtain the binary image of Fig. 7c.

Part (b)

The result is shown in Fig. 7d. Note that there is no need to compute I for the boundary pixels. For each boundary pixel, we estimate by eye whether the object portion (a) or the background portion (b) is bigger. If $a > b$, the pixel value after thesholding $= 1$, and $= 0$ otherwise. Note that the two triangles do not have the same shape after sampling and thresholding. Sampling at higher resolutions will lead to a more accurate representation of the object.

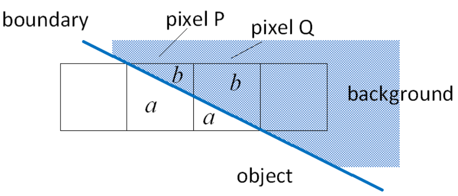


Figure 7a

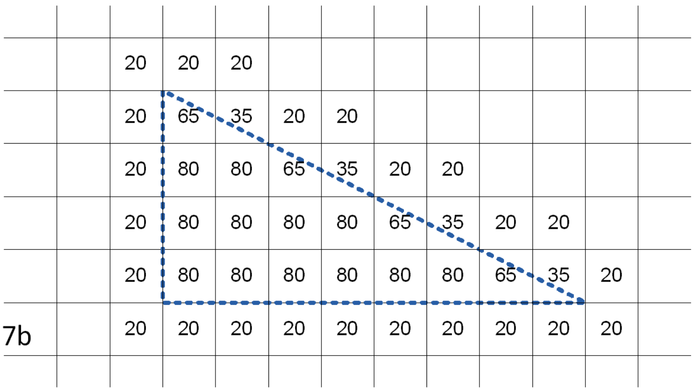


Figure 7b

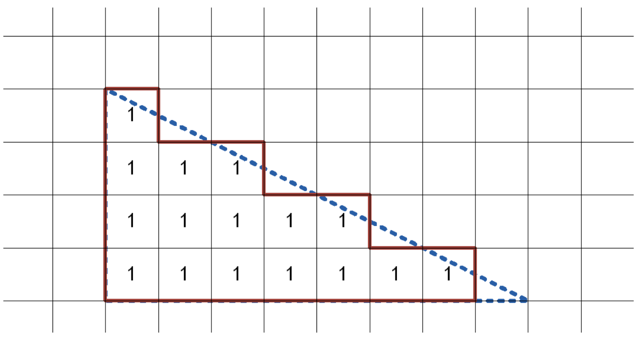


Figure 7c

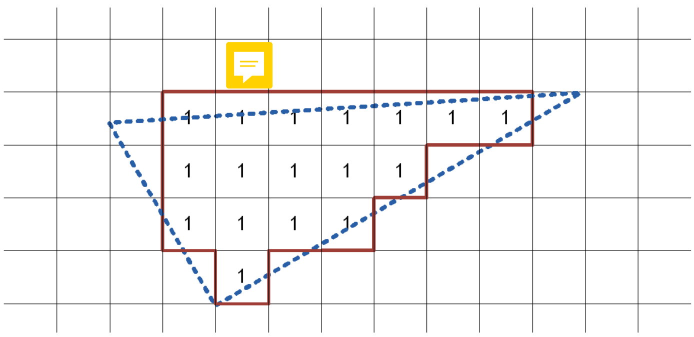


Figure 7d