EE4704 Image Processing and Analysis

Semester 1, 2020/21

Tutorial Set D

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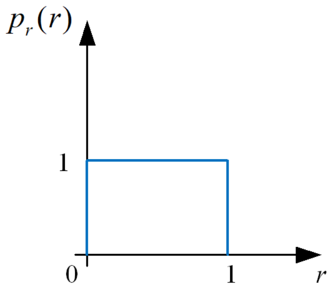
1. Consider a continuous image function $f(x, y)$ with PDF $p(r)$, $0 \le r \le 1$. The exponential function
$$T_\gamma(r) = r^\gamma, \quad \gamma > 0$$
may be used as a transformation function for image enhancement. Sketch
$$s = T_\gamma(r)$$
for $0 < \gamma < 1$, $\gamma = 1$, and $\gamma > 1$. Describe, in general, the effect on overall brightness of applying $T_\gamma(r)$ to an image. Obtain the discrete version of T_γ that can be applied to an image with gray levels $0, 1, 2, ..., 255$.

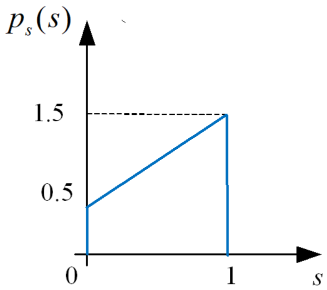
2. A contrast enhancement method is described by the equation below, which relates the output gray levels, s_k, to the input gray levels, r_k:
$$s_k = C(r_k - \mu) + \mu, \qquad k = 0, 1, \ldots, L - 1$$
In this equation, L is the number of gray levels, μ is the mean value of the imput image, and C is a parameter that can be adjusted to obtain the desire amount of contrast. Explain how this algorithm works. Choose an appropriate value of C for the image with the following gray-level distribution:

Gray level:	0	1	2	3	4	5	6	7	8	9	10
Number of pixels:	0	0	200	300	500	1000	1300	1300	1800	0	0

C is chosen such that contrast is maximised without clipping at gray levels 0 or 10.

3. Obtain the transformation function $T(r)$ that can be applied to the histogram $p_r(r)$ to obtain $p_s(s)$.





4. Consider the normalised histogram of a 21-level image:

$$p_r(r_k) = \begin{cases} 0.15 & 0.2 \le r_k \le 0.35 \\ 0.1 & 0.4 \le r_k \le 0.55 \\ 0 & \text{elsewhere} \end{cases}$$

where $r_k = 0, 0.05, 0.1, 0.15, \ldots, 1$. Obtain the transformation function that equalises the image and plot the resulting histogram.

5. Consider a Gaussian lowpass filter

$$H(u, v) = e^{-\omega^2/2\sigma^2}$$

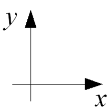
(where $\omega^2 = u^2 + v^2$) that is used to filter an $N \times N$ image with discrete Fourier transform $F(u, v)$. What is the effect of repeatedly filtering this image with $H(u, v)$?

6. In the image below, $f(x, y)$, the object pixels have gray level 1, background pixels gray level 0.

   (a) Using the Sobel operator, obtain the gradient vector for all the pixels in the image. Calculate the gradient magnitude to 1 decimal place.

   (b) Applying the Laplacian operator to $f(x, y)$ to give $g_L(x, y)$. For each row in g_L, do a hoizontal scan and indicate the zero crossing point along the scan line. Repeat for each column (vertical scan line). The zero crossing can be estimated to sub-pixel resolution.

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0
0	0	1	1	1	0	0	0	0	0	0
0	0	1	1	1	1	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0
0	0	1	1	1	1	1	1	0	0	0
0	0	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0



7. Given the continuous image function

$$f(x, y) = \exp(-ax^2 - by^2)$$

Obtain an expression for the gradient vector at (x, y).