EE4704 Image Processing and Analysis

Tutorial Set F – Solution

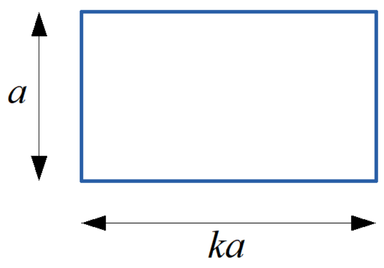
Question 1

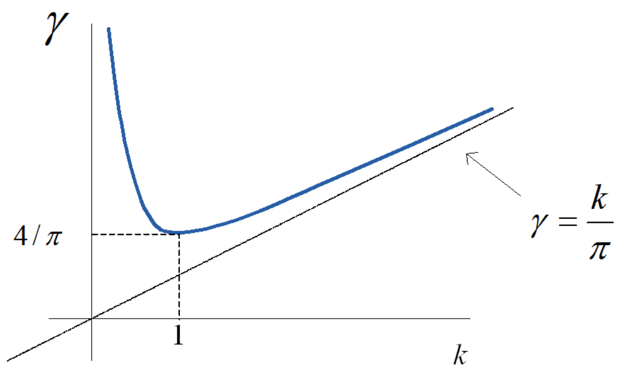
Part (a)

The compactness of the rectangle is

$$\begin{aligned}
\gamma &= \frac{P^2}{4\pi A} \\
&= \frac{(2ka+2a)^2}{4\pi \times ka^2} \\
&= \frac{4(k+1)^2}{4\pi k} \\
&= \frac{(k+1)^2}{\pi k} \qquad (1)
\end{aligned}$$

$d\gamma/dk = 0 \Rightarrow k = 1$, i.e. the minimum value of γ is $4/\pi$ when $k = 1$, i.e., a square.





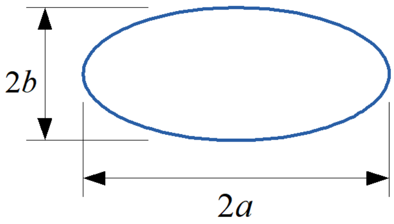
Part (b)

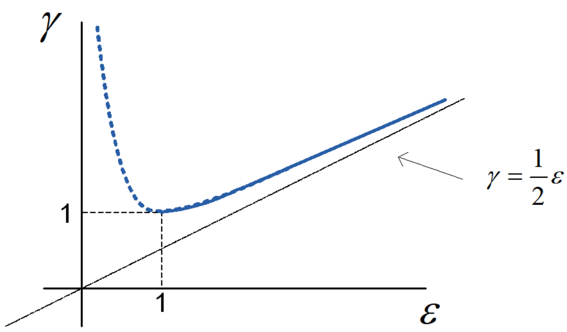
$$\epsilon = \frac{2a}{2b} = \frac{a}{b} \tag{2}$$

$$\tag{3}$$

$$\begin{aligned}\gamma &= \frac{\pi^2 \times 2(a^2+b^2)}{4\pi \times \pi ab} \\ &= \frac{a^2+b^2}{2ab} \\ &= \frac{1+\epsilon^2}{2\epsilon}, \qquad \text{where } \epsilon = \frac{a}{b} \geq 1 \end{aligned} \tag{4}$$

$d\gamma/d\epsilon = 0 \Rightarrow \epsilon = 1$, i.e. the minimum value of γ is 1, when $\epsilon = 1$, i.e., a circle.

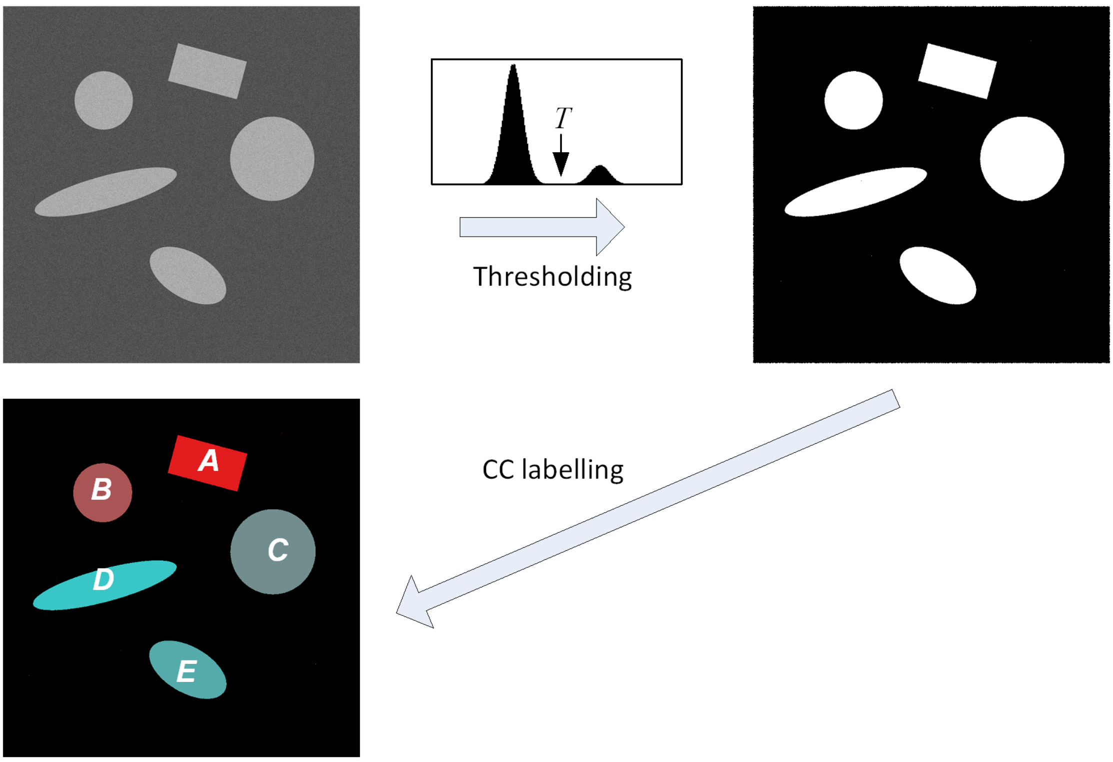




Question 2

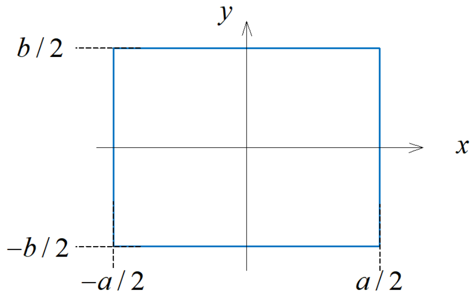
Procedure

1. Find a suitable threshold, T, by determining the valley between the two modes, or use the inter-means algorithm.
2. Threshold image at gray level T:
$$g(x,y) = \begin{cases} 1 & \text{if} \quad f(x,y) > T \\ 0 & \text{if} \quad f(x,y) \leq T. \end{cases} \tag{5}$$
3. Apply component labelling algorithm to $g(x,y)$ with $V = 1$.
4. This will result in 5 connected components labelled A to E. (As an additional step, small regions due to noise may be discarded by imposing a minimum-area requirement, or by using morphology.)
5. For each R_i, compute m_{00}, m_{01} and m_{10} to obtain the centroid $(\bar{x}, \bar{y})$:
$$\bar{x} = \frac{m_{10}}{m_{00}} \qquad \bar{y} = \frac{m_{01}}{m_{00}}$$
For example, for Region A:
$$m_{pq} = \sum\sum_{(x,y)\in A} x^p y^q f(x,y) \tag{6}$$



Question 3

Part (a)



Since ϕ_1 is invariant to translation, we can for convenience centre the rectangle at the origin.

$$
\begin{aligned}
m_{00} &= ab && (7)\\
\bar{x} &= \bar{y} = 0 && (8)\\
m_{10} &= m_{01} = 0 && (9)
\end{aligned}
$$

$$
\begin{aligned}
\mu_{00} &= m_{00} && (10)\\
\mu_{20} &= m_{20} - \bar{x}m_{10} = m_{20} && (11)\\
\mu_{02} &= m_{02} - \bar{y}m_{01} = m_{02} && (12)
\end{aligned}
$$

$$
\begin{aligned}
m_{20} &= \iint x^2 f(x,y)\,dx\,dy && (13)\\
&= \int_{-a/2}^{a/2} x^2\,dx \int_{-b/2}^{b/2} dy && (14)\\
&= [x^3/3]_{-a/2}^{a/2}\,[y]_{-b/2}^{b/2} && (15)\\
&= \frac{1}{12}a^3 b && (16)\\
\eta_{20} &= \frac{\mu_{20}}{\mu_{00}^2} && (17)\\
&= \frac{1}{12}\frac{a}{b} && (18)
\end{aligned}
$$

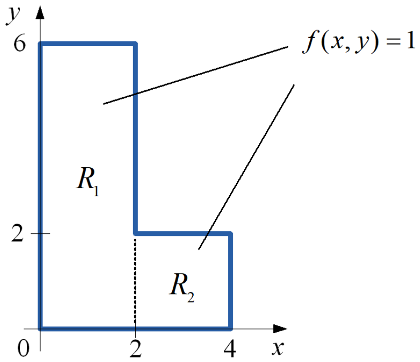
Similarly,

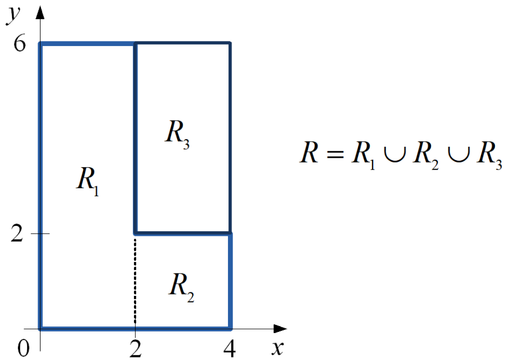
$$
\begin{aligned}
m_{02} &= \frac{1}{12}ab^3 && (19)\\
\eta_{02} &= \frac{1}{12}\frac{a}{b} && (20)
\end{aligned}
$$

Hence

$$\phi_1 = \eta_{20} + \eta_{02} = \frac{1}{12}\left(\frac{1}{b} + \frac{b}{a}\right) \qquad (21)$$

Part (b)





$$f(x, y) = \begin{cases} 1 & (x, y) \in R_1, R_2 \\ 0 & \text{elsewhere} \end{cases}$$

$$m_{pq} = \int\int_{R_1 \cup R_2} x^p y^q dx dy = \int\int_{R_1} x^p y^q dx dy + \int\int_{R_2} x^p y^q dx dy \qquad (22)$$

or

$$m_{pq} = \int\int_{R_1 \cup R_2} x^p y^q dx dy = \int\int_{R} x^p y^q dx dy - \int\int_{R_3} x^p y^q dx dy \qquad (23)$$

We use Eq. (22) and take the origin to be the lower left corner.

$$\begin{aligned}
m_{00} &= \text{area of object} = 16 \\
m_{10} &= \iint x\,dx\,dy = \int_0^6 \left\{ \int_0^2 x\,dx \right\} dy + \int_0^2 \left\{ \int_2^4 x\,dx \right\} dy \\
&= \int_0^6 \left[x^2/2 \right]_0^2 dy + \int_0^2 \left[x^2/2 \right]_2^4 dy \\
&= \int_0^6 2\,dy + \int_0^2 6\,dy \\
&= 12 + 12 \\
&= 24 \\
m_{01} &= \iint y\,dx\,dy = \int_0^6 y \left\{ \int_0^2 dx \right\} dy + \int_0^2 y \left\{ \int_2^4 dx \right\} dy \\
&= \int_0^6 2y\,dy + \int_0^2 2y\,dy \\
&= \left[y^2 \right]_0^6 + \left[y^2 \right]_0^2 \\
&= 36 + 4 \\
&= 40
\end{aligned}$$

$$\begin{aligned}
\bar{x} &= m_{10}/m_{00} = 1.5 \\
\bar{y} &= m_{01}/m_{00} = 2.5
\end{aligned}$$

$$\begin{aligned}
\mu_{11} &= \iint (x-1.5)(y-2.5)\,dx\,dy \\
&= \int_0^6 \left\{\int_0^2 (x-1.5)(y-2.5)\,dx\right\}\,dy + \int_0^2 \left\{\int_2^4 (x-1.5)(y-2.5)\,dx\right\}\,dy \\
&= -3-9 \\
&= -12
\end{aligned}$$

$$\begin{aligned}
\eta_{11} &= \mu_{11}/\mu_{00}^2 \\
&= -12/16^2 \\
&= -0.0469
\end{aligned}$$

Alternatively, use

$$\mu_{11} = m_{11} - \bar{y} m_{10}$$

We have

$$\begin{aligned}
m_{11} &= \iint xy\,dx\,dy = \int_0^6 y\left\{\int_0^2 x\,dx\right\}\,dy + \int_0^2 y\left\{\int_2^4 x\,dx\right\}\,dy \\
&= \int_0^6 \left[x^2/2\right]_0^2 y\,dy + \int_0^2 \left[x^2/2\right]_2^4 y\,dy \\
&= \int_0^6 2y\,dy + \int_0^2 6y\,dy \\
&= \left[y^2\right]_0^6 + \left[3y^2\right]_0^2 \\
&= 36+12 \\
&= 48
\end{aligned}$$

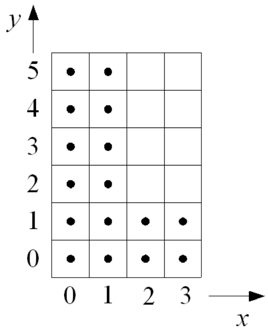
$$\begin{aligned}
\mu_{11} &= 48 - 2.5\times 24 \\
&= -12
\end{aligned}$$

$$\begin{aligned}
\eta_{11} &= \mu_{11}/\mu_{00}^2 \\
&= -12/16^2 \\
&= -0.0469
\end{aligned}$$

$$
\begin{aligned}
m_{20} &= \iint x^2\,dx\,dy = \int_0^6 \left\{\int_0^2 x^2\,dx\right\}\,dy + \int_0^2 \left\{\int_2^4 x^2\,dx\right\}\,dy \\
&= 16 + \frac{112}{3} \\
&= 53.3333 \\
m_{02} &= \iint y^2\,dx\,dy = \int_0^6 y^2 \left\{\int_0^2\,dx\right\}\,dy + \int_0^2 y^2 \left\{\int_2^4\,dx\right\}\,dy \\
&= \frac{432}{3} + \frac{16}{3} \\
&= 149.3333
\end{aligned}
$$

$$
\begin{aligned}
\mu_{20} &= m_{20} - \bar{x}m_{10} \\
&= 53.3333 - 1.5 \times 24 = 17.3333 \\
\mu_{02} &= m_{02} - \bar{y}m_{01} \\
&= 149.3333 - 2.5 \times 40 = 49.3333 \\
\eta_{20} &= \mu_{20}/\mu_{00}^2 = 0.0677 \\
\eta_{02} &= \mu_{02}/\mu_{00}^2 = 0.1927 \\
\phi_1 &= \eta_{20} + \eta_{02} \\
&= 0.26
\end{aligned}
$$

Part (c)



Choose origin at bottom left pixel.

$$
\begin{aligned}
m_{00} &= 16 \\
m_{10} &= \sum\sum x^1 y^0 f(x,y) = \sum\sum x f(x,y) = \sum_{x=0}^{3} x \left\{ \sum_{y=0}^{5} f(x,y) \right\} \\
&= 0 \times \sum_{y=0}^{5} f(x,y) + 1 \times \sum_{y=0}^{5} f(x,y) + 2 \times \sum_{y=0}^{5} f(x,y) + 3 \times \sum_{y=0}^{5} f(x,y) \\
&= (0 \times 6) + (1 \times 6) + (2 \times 2) + (3 \times 2) \\
&= 16 \\
m_{01} &= \sum\sum x^0 y^1 f(x,y) = \sum\sum y f(x,y) = \sum_{y=0}^{5} y \left\{ \sum_{x=0}^{3} f(x,y) \right\} \\
&= (0 \times 4) + (1 \times 4) + (2 \times 2) + (3 \times 2) + (4 \times 2) + (5 \times 2) \\
&= 32 \\
\bar{x} &= m_{10}/m_{00} = 1 \\
\bar{y} &= m_{01}/m_{00} = 2
\end{aligned}
$$

$$
\begin{aligned}
\mu_{11} &= \sum\sum (x-1)(y-2) f(x,y) = \sum_{y=0}^{5} (y-2) \left\{ \sum_{x=0}^{3} (x-1) f(x,y) \right\} \\
&= (-2)(-1+0+1+2) + (-1)(-1+0+1+2) + (0)(-1+0) \\
&\quad +(1)(-1+0) + (2)(-1+0) + (3)(-1+0) \\
&= -12 \\
\eta_{11} &= \mu_{11}/\mu_{00}^2 = -0.0469
\end{aligned}
$$

$$
\begin{aligned}
m_{20} &= \sum\sum x^2 f(x,y) = \sum_x x^2 \left\{\sum_y f(x,y)\right\} \\
&= (0^2 \times 6) + (1^2 \times 6) + (2^2 \times 2) + (3^2 \times 2) \\
&= 32 \\
m_{02} &= \sum\sum y^2 f(x,y) = \sum_y y^2 \left\{\sum_x f(x,y)\right\} \\
&= (0^2 \times 4) + (1^2 \times 4) + (2^2 \times 2) + (3^2 \times 2) + (4^2 \times 2) + (5^2 \times 2) \\
&= 112
\end{aligned}
$$

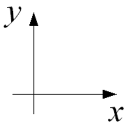
$$
\begin{aligned}
\mu_{20} &= m_{20} - \bar{x} m_{10} \\
&= 32 - 1 \times 16 = 16 \\
\mu_{02} &= m_{02} - \bar{y} m_{01} \\
&= 112 - 2 \times 32 = 48
\end{aligned}
$$

$$
\begin{aligned}
\eta_{20} &= \mu_{20}/\mu_{00}^2 \\
&= 16/16^2 = 0.0625 \\
\eta_{02} &= \mu_{02}/\mu_{00}^2 \\
&= 48/16^2 = 0.1875 \\
\phi_1 &= \eta_{20} + \eta_{02} \\
&= 0.25
\end{aligned}
$$

Question 4

Part (a)

$$
\begin{array}{cccccccc}
 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\
\dots & 0 & 0 & 1 & 1 & 2 & 2 & \dots \\
\dots & 0 & 0 & 1 & 1 & 2 & 2 & \dots \\
\dots & 2 & 2 & 0 & 0 & 1 & 1 & \dots \\
\dots & 2 & 2 & 0 & 0 & 1 & 1 & \dots \\
\dots & 1 & 1 & 2 & 2 & 0 & 0 & \dots \\
\dots & 1 & 1 & 2 & 2 & 0 & 0 & \dots \\
 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots &
\end{array}
$$



Texture P

$$\mathbf{A}_{0,1} = \begin{bmatrix} 6 & 0 & 6 \\ 6 & 6 & 0 \\ 0 & 6 & 6 \end{bmatrix} \qquad \mathbf{A}_{0,2} = \begin{bmatrix} 0 & 0 & 12 \\ 12 & 0 & 0 \\ 0 & 12 & 0 \end{bmatrix}$$

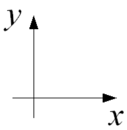
$$\mathbf{C}_{0,1} = \begin{bmatrix} \frac{1}{6} & 0 & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & 0 \\ 0 & \frac{1}{6} & \frac{1}{6} \end{bmatrix} \qquad \mathbf{C}_{0,2} = \begin{bmatrix} 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix}$$

$$
\begin{aligned}
D &= \sum_{i=1}^{3}\sum_{j=1}^{3}(i-j)^2 c_{ij} \\
&= (1-1)^2 c_{11} + (1-2)^2 c_{12} + (1-3)^2 c_{13} \\
&+ (2-1)^2 c_{21} + (2-2)^2 c_{22} + (2-3)^2 c_{23} \\
&+ (3-1)^2 c_{31} + (3-2)^2 c_{32} + (3-3)^2 c_{33}
\end{aligned}
$$

For $\mathbf{C}_{0,1}$: $\quad D = 2^2 \times \frac{1}{6} + 1^2 \times \frac{1}{6} + 1^2 \times \frac{1}{6} = 1.0$

For $\mathbf{C}_{0,2}$: $\quad D = 2^2 \times \frac{1}{3} + 1^2 \times \frac{1}{3} + 1^2 \times \frac{1}{3} = 2.0$

$$
\begin{array}{cccccccc}
 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\
\ldots & 0 & 1 & 2 & 0 & 1 & 2 & \ldots \\
\ldots & 1 & 2 & 0 & 1 & 2 & 0 & \ldots \\
\ldots & 2 & 0 & 1 & 2 & 0 & 1 & \ldots \\
\ldots & 0 & 1 & 2 & 0 & 1 & 2 & \ldots \\
\ldots & 1 & 2 & 0 & 1 & 2 & 0 & \ldots \\
\ldots & 2 & 0 & 1 & 2 & 0 & 1 & \ldots \\
 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots &
\end{array}
$$



Texture Q

$\nwarrow$

$$
\mathbf{A}_{-1,1} = \begin{bmatrix} 0 & 0 & 12 \\ 12 & 0 & 0 \\ 0 & 12 & 0 \end{bmatrix}
$$

$\nearrow$

$$
\mathbf{A}_{1,1} = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix}
$$

$$
\mathbf{C}_{-1,1} = \begin{bmatrix} 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix}
\qquad
\mathbf{C}_{1,1} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}
$$

$$
\begin{aligned}
D &= \sum_{i=1}^{3}\sum_{j=1}^{3}(i-j)^2 c_{ij} \\
&= (1-1)^2 c_{11} + (1-2)^2 c_{12} + (1-3)^2 c_{13} \\
&+ (2-1)^2 c_{21} + (2-2)^2 c_{22} + (2-3)^2 c_{23} \\
&+ (3-1)^2 c_{31} + (3-2)^2 c_{32} + (3-3)^2 c_{33}
\end{aligned}
$$

For $\mathbf{C}_{-1,1}$: $\quad D = 2^2 \times \frac{1}{3} + 1^2 \times \frac{1}{3} + 1^2 \times \frac{1}{3} = 2.0$

For $\mathbf{C}_{1,1}$: $\quad D = 0$

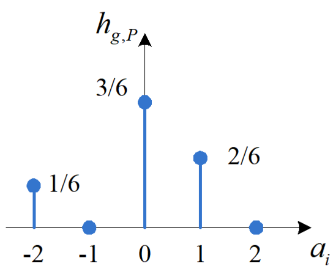
Part (b)

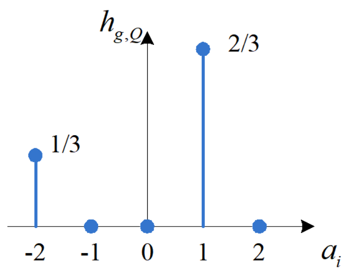
Texture P

$$\underset{f}{\begin{array}{cccccccc} & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 0 & 0 & 1 & 1 & 2 & 2 & \cdots \\ \cdots & 0 & 0 & 1 & 1 & 2 & 2 & \cdots \\ \cdots & 2 & 2 & 0 & 0 & 1 & 1 & \cdots \\ \cdots & 2 & 2 & 0 & 0 & 1 & 1 & \cdots \\ \cdots & 1 & 1 & 2 & 2 & 0 & 0 & \cdots \\ \cdots & 1 & 1 & 2 & 2 & 0 & 0 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{array}} \Rightarrow \underset{g}{\begin{array}{cccccccc} & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 0 & 1 & 0 & 1 & 0 & -2 & \cdots \\ \cdots & 0 & 1 & 0 & 1 & 0 & -2 & \cdots \\ \cdots & 0 & -2 & 0 & 1 & 0 & 1 & \cdots \\ \cdots & 0 & -2 & 0 & 1 & 0 & 1 & \cdots \\ \cdots & 0 & 1 & 0 & -2 & 0 & 1 & \cdots \\ \cdots & 0 & 1 & 0 & -2 & 0 & 1 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{array}}$$

Texture Q

$$\underset{f}{\begin{array}{cccccccc} & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 0 & 1 & 2 & 0 & 1 & 2 & \cdots \\ \cdots & 1 & 2 & 0 & 1 & 2 & 0 & \cdots \\ \cdots & 2 & 0 & 1 & 2 & 0 & 1 & \cdots \\ \cdots & 0 & 1 & 2 & 0 & 1 & 2 & \cdots \\ \cdots & 1 & 2 & 0 & 1 & 2 & 0 & \cdots \\ \cdots & 2 & 0 & 1 & 2 & 0 & 1 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{array}} \Rightarrow \underset{g}{\begin{array}{cccccccc} & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 1 & 1 & -2 & 1 & 1 & -2 & \cdots \\ \cdots & 1 & -2 & 1 & 1 & -2 & 1 & \cdots \\ \cdots & -2 & 1 & 1 & -2 & 1 & 1 & \cdots \\ \cdots & 1 & 1 & -2 & 1 & 1 & -2 & \cdots \\ \cdots & 1 & -2 & 1 & 1 & -2 & 1 & \cdots \\ \cdots & -2 & 1 & 1 & -2 & 1 & 1 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{array}}$$





The nth moment of a about its mean, m is

$$\mu_n(a) = \sum_{i=1}^{K} (a_i - m)^n p(a_i) \quad \text{where} \quad m = \sum_{i=1}^{K} a_i p(a_i)$$

For the resulting histograms $h_{g,P}$ and $h_{g,Q}$, we compute m and μ_2 (the variance).

	m	μ_2
Texture P: $h_{g,P}$	0	1
Texture Q: $h_{g,Q}$	0	2

Hence, the textures can be differentiated by using the μ_2 values.