## STA 360: Lab 9

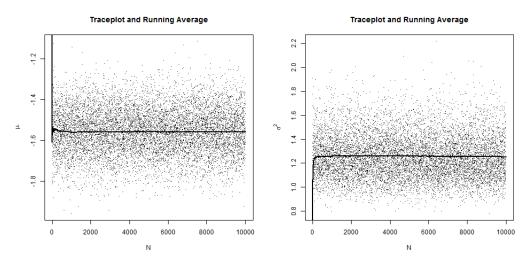
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## April 2, 2015

1. See below for priors used and the resulting posterior distributions for  $\mu$  and  $\sigma^2$ .

4	**
	19th
12	
	Diven data has Lognormal distribution:
	$f_{x}(x,\mu,\sigma) = \frac{1}{x\sqrt{2\pi}} \exp\left\{-\frac{(\ln x - \mu)^{2}}{2\sigma^{2}}\right\}, x>0$
	fy(x) 11 (r) = x = exp3 - 252 ) x>0
	, , , , , , , , , , , , , , , , , , , ,
	Suppose priors of Mand or one:
	^ 1
	p(u) = Normal (u (uo, oo2)
	$p(\sigma^2) = InVGanne(\sigma^2   x, B)$
	P(0°) = InVGamma(0 (x,B)
	20 0
	Therefore,
	0
	1 1 1 2 1 1 2 2
	$p(\mu   X_{in}) = p(\mu) p(X_{in}   \mu, \sigma^2)$
	$\alpha \exp\left\{-\frac{(\mu-\mu_0)^2}{2\sigma^2}\right\} \prod_{i=1}^{\infty} \exp\left\{-\frac{(\mu\times i-\mu)^2}{2\sigma^2}\right\}$
	2002 ): 11 202
1.0	5 (1 × 11)2
	$= exp \left\{ -\frac{(\mu - \mu_0)^2 - \sum_{i=1}^{n} (\ln x_i - \mu)^2}{2\sigma^2} \right\}$
	200
	$= exp \left\{ -\frac{\sigma^{2}(M-M_{\bullet})^{2} + 5\sigma^{2} \sum_{i=1}^{\infty} (\ln x_{i} - M)^{2}}{25\sigma^{2}\sigma^{2}} \right\}$
1,100	= etv) - = =================================
	2000
1,18	$\propto \exp\left\{-\frac{\sigma^{2}(M^{2}-2MM_{0})+\sigma_{0}^{2}\frac{\pi}{2}(M^{2}-2MM_{0})}{2\sigma_{0}^{2}\sigma_{0}^{2}}\right\}$
	X exp } - (10 ) (10)
327	20,00
93	( 22 2/.22 = 1)
	- aug > - Mo-2440+ to: (nu-242 hxi) }
191	$= erp \left\{ - \frac{\mu^2 \sigma^2 - 2\mu\mu_0 \sigma^2 + \sigma_0^2 (n\mu^2 - 2\mu\Sigma_0 hx_0)}{2\sigma_0^2 \sigma^2} \right\}$
	$0  2^2  2  2^2  2^2  1$
	= eng { - 4202-244.02+42no.2240.25 hx;}
	$= exp \left\{ -\frac{(\sigma^2 + n\sigma_0^2)\mu^2 - 2\mu(\mu_0\sigma^2 + \sigma_0^2 \leq 1.4 \times 1)}{2\sigma^2\sigma^2} \right\}$
	= em> - (5+1100)M - 2M(MOT+ 00 51,221)
	$2\sigma^2\sigma^2$
-	1 2 2. /11 2+- 15 lax.)
	12- 21 (MOO) 200 Z MAI)
	= exp {
	20,202/1221253
	$= exp \left\{ -\frac{\mu^2 - \frac{2\mu \left(\mu_0 \sigma^2 + \frac{1}{2} \sigma_0^2 + \frac{1}{2} m_0^2 + \frac{1}{2} m$
	1/ 1 11-52 1 x - 2 2 1
	× nomml(µ( 1002+1002 ) 00202)
	1000 pl 02+n002 J 02+n002)

(g<sup>2</sup>)<sup>-1/2</sup>  $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...}) = p(\sigma^{2}) p(X_{1:..} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1...} \mid \mu_{1} \sigma^{2}),$   $P(\sigma^{2} \mid X_{1} \mid \mu_{1} \sigma^{2}$  $(\sigma^{2})^{-x-\frac{n}{2}-1} exp \left\{ -\frac{2}{5} + \frac{5}{5} (\ln x_{i} - \mu)^{2} \right\}$   $= (\sigma^{2})^{-(\alpha+\frac{n}{2})-1} exp \left\{ = \frac{3}{5} + \frac{1}{2} \frac{5}{5} (\ln x_{i} - \mu)^{2} \right\}$   $= \left[ \left( \sigma^{2} \right)^{-(\alpha+\frac{n}{2})-1} \right] + \frac{1}{2} \frac{5}{5} \left[ \left( \ln x_{i} - \mu \right)^{2} \right]$   $= \left[ \left( \sigma^{2} \right)^{-(\alpha+\frac{n}{2})-1} \right] + \frac{1}{2} \frac{5}{5} \left[ \left( \ln x_{i} - \mu \right)^{2} \right]$  2. See below for trace plots. We used  $\alpha, \beta = 0$  for  $\sigma^2$  because it's a Jeffery's prior. We also set  $\mu_0 = 0$  and  $\sigma_0^2 = 0$  to allow large prior variance (i.e. uninformative prior). The Gibbs sampler appears to have converged and explored the space reasonably well.



3. See below for respective 95% confidence interval.

$$CI(mean) = [0.7066048, 0.1072423]$$

$$CI(variance) = [0.1586194, 1.0671797]$$

See below for R code:

```
1
    # load data
    x = read.table("data.txt", header = F)[,1]
    library(MCMCpack)
3
 4
    ## Define prior parameters ##
5
    a = 0; b = 0;
    mu.0 = 0; sig2.0 = 10;
 7
    n = length(x)
9
10
    ## Define prior ##
    mu.temp = rnorm(1, mean = (mu.0*sig2.0 + sig2.0*sum(log(x)))/(sig2.0 + n*sig2.0),
11
                     sd = sqrt((sig2.0*sig2.0)/(sig2.0 + n*sig2.0)))
    sig2.temp = rinvgamma(1, shape = a + n/2,
13
                           scale = b + 1/2*sum((log(x) - mu.temp)^2))
14
15
16
    N = 10000
17
    MU = rep(0, N)
    SIG2 = rep(0, N)
18
19
    ## Gibbs Sampler ##
20
    for(i in 1:N){
21
22
      mu.temp = rnorm(1, mean = (mu.0*sig2.temp + sig2.0*sum(log(x)))/(sig2.temp + n*sig2.0),
23
                  sd = sqrt((sig2.0*sig2.temp)/(sig2.temp + n*sig2.0)))
24
      sig2.temp = rinvgamma(1, shape = a + n/2,
25
                             scale = b + 1/2*sum((log(x) - mu.temp)^2))
26
      MU[i] = mu.temp
27
      SIG2[i] = sig2.temp
28
29
    ## Running Avg ##
30
31
    MU.sum = rep(0, N)
    MU.avg = rep(0, N)
32
    MU.sum[1] = MU[1]
34
    for(i in 2:N){
35
      MU.sum[i] = MU[i] + MU.sum[i-1]
      MU.avg[i] = MU.sum[i]/i
36
37
    }
38
39
    SIG2.sum = rep(0, N)
    SIG2.avg = rep(0, N)
40
    SIG2.sum[1] = MU[1]
41
    for(i in 2:N){
43
      SIG2.sum[i] = SIG2[i] + SIG2.sum[i-1]
44
      SIG2.avg[i] = SIG2.sum[i]/i
45
    }
46
    ## Plot ##
47
    png("MUplot.png")
49
    plot(MU, pch = '.', xlab = "N", ylab = expression(mu),
50
         main = "Traceplot and Running Average")
    lines(MU.avg, lwd = 2)
51
    dev.off()
```

```
53
54
    png("SIGplot.png")
    plot(SIG2, pch = '.', xlab = "N", ylab = expression(sigma^{2}),
56
         main = "Traceplot and Running Average")
57
    lines(SIG2.avg, lwd = 2)
58
    dev.off()
    ## Confidence Interval ##
60
    mean.vec = exp(MU + SIG2/2)
61
    var.vec = (exp(SIG2)-1)*exp(2*MU + SIG2)
62
63
64
   mean.ci = quantile(mean.vec, c(0.025, 0.975))
    var.ci = quantile(var.vec, c(0.025, 0.975))
65
```