STA 360: Practice Question

Michael Lin

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Suppose $X_{1:n}$ have negative binomial distribution with pmf:

$$p(x|\theta) = \binom{x-1}{4}\theta^5(1-\theta)^{x-5}$$

and suppose θ has a Beta(4,6) prior distribution. Find the posterior distribution of θ given $X_{1:n}$.

$$\begin{aligned}
p(\theta|x_{1:n}) &\propto p(x_{1:n}|\theta)\, p(\theta) \\
&= \left[\prod_{i=1}^{n}\binom{x_i-1}{4}\theta^5(1-\theta)^{x_i-5}\right]\cdot\frac{\Gamma(4+6)}{\Gamma(4)\Gamma(6)}\theta^3(1-\theta)^5 \\
&\propto \theta^{5n}(1-\theta)^{\left(\sum x_i\right)-5n}\cdot\theta^3(1-\theta)^5 \\
&= \theta^{(5n+3)}\cdot(1-\theta)^{\left(\sum x_i\right)-5n+5} \\
&\propto \text{Beta}\left(5n+4,\ \left(\sum_{i=1}^{n}x_i\right)-5n+6\right)
\end{aligned}$$

for $\theta \in (0,1)$.