STA 360: Assignment 1

Michael Lin

January 11, 2015

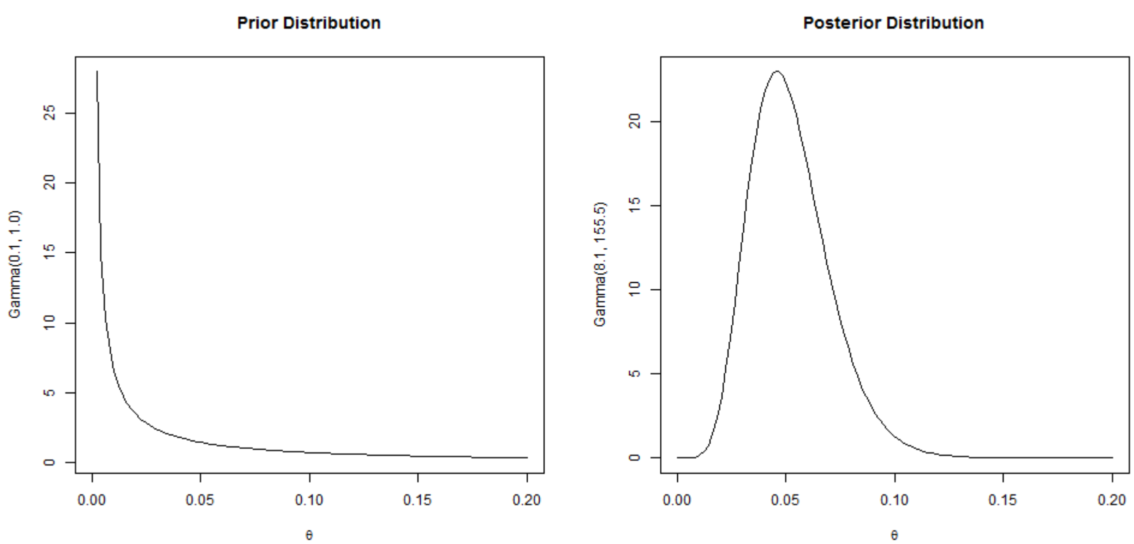
1. The formula for the posterior density, $p(\theta|x_{1:n})$, is derived using Bayes' theorem and plugging in the likelihood and prior:

$$\begin{aligned}
p(\theta|x_{1:n}) &= p(x_{1:n}|\theta)p(\theta) \\
&= \prod_{i=1}^{n} \theta \exp(-\theta x_i)\mathbb{1}(x_i > 0)\frac{b^a}{\Gamma(a)}\theta^{a-1}\exp(-b\theta)\mathbb{1}(\theta > 0) \\
&= \theta^n \exp(-\theta \sum_{i=1}^{n} x_i)\mathbb{1}(x_i > 0)\frac{b^a}{\Gamma(a)}\theta^{a-1}\exp(-b\theta)\mathbb{1}(\theta > 0) \\
&\propto \theta^{a+n-1}\exp\{-\theta(b + \sum_{i=1}^{n} x_i)\}\mathbb{1}(\theta > 0) \\
&\propto \text{Gamma}(\theta|a+n, b+\sum_{i=1}^{n} x_i)
\end{aligned}$$

Thus, the posterior density is

$$\text{Gamma}(\theta|a+n, b+\sum_{i=1}^{n} x_i)$$

2. The prior density is Gamma(0.1,1.0). Using the formula derived from part 1, we have that the posterior density is Gamma(8.1,155.5). Their plots are displayed below.



See below for R code used to plot prior and posterior p.d.f.s.

```
x.data=c(20.9, 69.7, 3.6, 21.8, 21.4, 0.4, 6.7, 10.0);
a=0.1; b=1  #prior
n=length(x.data); y=sum(x.data)  #data

x.prior=seq(0, 0.2, length=100);
hx.prior=dgamma(x.prior, shape=a, rate=b);
x.post=seq(0, 0.2, length=100);
hx.post=dgamma(x.post, shape=a+n, rate=b+y)

png("prior.png");
plot(x.prior, hx.prior, type="l", lty=1, xlab=expression(theta),
     ylab="Gamma(0.1, 1.0)", main="Prior Distribution");
dev.off()

png("post.png");
plot(x.post, hx.post, type="l", lty=1, xlab=expression(theta),
     ylab="Gamma(8.1, 155.5)", main="Posterior Distribution");
dev.off()
```