BULK CLASSIFICATION OF TRADING ACTIVITY

David Easley[1]
dae3@cornell.edu

Marcos M. López de Prado[2]
marcos.lopezdeprado@tudor.com

Maureen O'Hara[3]
mo19@cornell.edu

March, 2012

ABSTRACT

The classification of the aggressor's side of a trade is a critical concern in Market Microstructure Theory. Among other uses, it is a key input necessary to identify information asymmetries and the presence of toxic order flow. Although some Exchanges have recently started to report the "aggressor" flag, historical databases usually lack this piece of data. Thus the researcher and/or practitioner still needs to infer the aggressor side from existing information, typically level 1 Tick Data. This poses the additional problem of having to parse hundreds of millions of records per instrument and year. In this paper we propose a new *Bulk Volume Classification* methodology that does not require working with Tick Data. Instead, it uses Time or Volume Bars, which for a small fraction of the records needed by the Tick rule delivers a classification with greater accuracy. The implication is that working with Tick Data for inferring the aggressor classification is not only inefficient and costly, but also does not offer greater accuracy compared to Time or Volume Bars.

Keywords: Aggressor side, Bulk Volume Classification, flow toxicity, volume imbalance, market microstructure.

JEL codes: C02, D52, D53, G14.

We thank Robert Almgren, Robert Engle, Craig Holden, David Leinweber, Pam Moulton, Mark Ready, Riccardo Rebonato, John Wu and the CIFT group at the Lawrence Berkeley National Laboratory for providing useful comments. We are grateful to Sergey Kosyakov and Steven Jones for their research assistance.

* The authors have applied for a patent on 'Bulk Volume Classification' and have a financial interest in it.

[1] Scarborough Professor and Donald C. Opatrny Chair Department of Economics, Cornell University.
[2] Head of High Frequency Futures Trading at Tudor Investment Corp., and Postdoctoral Fellow of RCC at Harvard University.
[3] Purcell Professor of Finance, Johnson Graduate School of Management, Cornell University.

INTRODUCTION

Every trade involves at least one buyer and one seller, and knowing which side initiated the trade is valuable information to market participants. Trade direction (i.e. buy or sell) can be used to identify asymmetric information, and a sustained imbalance of trade can signal the presence of toxic order flow and its potential consequent effects on market liquidity. Determining who initiated the trade, however, has never been straightforward, and in high frequency settings this task is even more challenging. Trading now takes place largely in electronic markets where designated liquidity providers are not present, and practices such as order splitting, hidden orders, and fragmentation of trading make drawing inferences from trade data problematic. Even when order data is available, however, the sheer volume of trading results in massive data files which are expensive to buy and store, let alone to manipulate. For practitioners and researchers alike, what is needed is a reliable, inexpensive mechanism to assign trade direction in high frequency markets.

In this paper, we propose a new approach for classifying trading activity. Our classification technique is inspired by the principle that bulk counting and measuring may be more accurate than item counting when measurement error is likely.[4] The *"Bulk Volume Classification"* (BVC) algorithm aggregates trades over short time intervals (denoted time bars) or volume intervals (denoted volume bars) and then uses the standardized price change between the beginning and end of the interval to approximate the percentage of buy and sell volume. Unlike traditional trade classification algorithms that assign trades to be either buys *or* sells, our approach

[4] For example, suppose that you have an unbiased instrument for measuring distances, which makes random errors whose standard deviation is σ. If we are allowed only two measurements on two items of different lengths, A and B, there is a more accurate procedure of measuring these items than measuring independently. Indeed, if you apply your first measurement on A+B and your second measurement on A-B, the measuring error will be half that if you measure A and B independently! See Mosteller [1965] for a formal proof.

apportions trades into buy volume and sell volume. As we demonstrate in this paper, this approach provides a parsimonious, accurate, and highly tractable way to assign trade direction.

Our research can be viewed as providing a new tool for the high frequency world that now characterizes trading. As we discuss in Easley et al. [2011a; 2011b; 2012], high frequency markets are not simply the "old" markets speeded up. Massive numbers of trades occurring in extremely short time intervals challenge conventional time-based approaches of analyzing data. Indeed, one intriguing result of our research is that classifying trading using volume-bars is more accurate than using time-bars, especially in the case of commodities.

We also show that using either time-bars or volume bars is more accurate than standard tick rule classification schemes based on individual trade data. Using futures data, we find that a tick rule algorithm works reasonably well in the e-mini S&P 500 Futures, correctly classifying 86.43% of the data, but it does significantly less well in oil futures (78.95%) and in gold futures (67.18%). We attribute this degradation of performance to the smaller trading volume, lower liquidity (i.e. thinner books) and greater dynamicity (number of quote changes per fill) in these contracts. In contrast, we find that the BVC approach can (depending on the time or volume bar specification) correctly classify 91% of e-mini S&P 500 Futures volume, 91.59% of oil futures volume, and 87.36% of gold futures trading activity. Moreover, the time-series evidence shows that while all three classification approaches (tick rule, time bars, volume bars) work less well on extremely active days, in the e-mini contract BVC time-bars beat the tick rule on 82.11% of days while volume-bars does so on 99.19% of days.

Another advantage of our approach is its parsimonious use of data. One might have conjectured that an advantage of the high frequency world is more extensive databases providing information such as trade direction. Yet, while such databases are now becoming available in some markets, they have serious drawbacks. The massive number of trades in markets makes such data expensive both to buy and to manipulate.[5] Multiple exceptions and nuances make dealing with this data a research project in its own right, as will become apparent later in this paper. Fragmentation of trading also requires such data from every market in which a security trades, something unlikely to occur for asset classes like equities. Using time-bars, our approach allows the researcher to work with a tiny fraction of the original tick dataset. Moreover, time bar data are available at low cost from vendors such as Bloomberg. We believe this new approach will be useful for a wide variety of applications in high frequency markets.

This paper is organized as follows. Section 1 discusses existing trade classification algorithms, their historical accuracy in different market settings, and the complications posed by high frequency markets for trade classification. Section 2 introduces our BVC algorithm and the role played by time bars and volume bars. Section 3 investigates the accuracy of tick rule versus BVC in the context of the e-mini S&P500 futures, the WTI crude oil futures, and Gold futures. Section 4 addresses the stability of these classification approaches by looking at their time-series behavior. Section 5 discusses the accuracy of trade classification if we apply the empirical distribution of price changes. Section 6 summarizes the main conclusions. The Appendix provides an algorithm in Python Language.

---

[5] Databases that include information such as buyer and seller indicators involve the processing of hundreds of millions of records per year and instrument. This is not a trivial problem, and is one of the reasons behind the S.E.C.'s plea for computational help from the U.S. Department of Energy's National Laboratories (see Bethel, Leinweber, Ruebel and Wu [2011]).

1. TRADE CLASSIFICATION ALGORITHMS

The problem of how to classify trades into buys and sells is long-standing. Data providing the time-stamped trade price and, often, quantity have typically been available for exchange-traded products such as futures. The advent of the national market system (and the consequent development of the consolidated tape and consolidated quote systems) allowed similar time-stamped quote data for equity markets. Based on such data, a variety of approaches have been developed to classify trades into buys and sells. In this section, we provide a brief taxonomy of these approaches, their historical accuracy, and the circumstances under which they falter.

1.1. TICK RULE

The Tick rule classifies a trade as a buy if the trade price is greater than the previous trade price, and as a sell if the trade price is lower than the previous trade price. If the trade price is equal to the previous price, the trade is assigned to the same side as the previous trade. This method is the least data demanding among all tick-based algorithms, as it only requires price data (referred to in the market as Level 1 data). Another advantage of this approach is that it leaves no trades unclassified.

By focusing on price movements, the tick rule captures the natural intuition that buyers pay a higher price and sellers get a lower price. There are, however, a variety of reasons why such a simple approach can fail to provide accurate classifications. One such reason is movements within the order book. A trade at a lower price may be a sale, but it can also be a buy if time has passed and the spread has moved down. In a high frequency setting, a related problem can arise as price movements up or down in rapid succession tend to deplete the book and cause orders to replenish. This can cause buys to be mis-classified as sells, and conversely. Trades at no price movement are particularly challenging. The convention signs the trade in the direction of the previous trade but practices such as hidden orders introduce errors. For example, a trade at the midpoint (say classified as a buy) followed by a hidden sell order at the midpoint will incorrectly classify the latter trade as a buy.

In the case of futures and options, calendar trades pose a challenge of their own. These are exchange-guaranteed spreads traded in own book. However, every trade in that book impacts two other books simultaneously. These implied trades are generated by the exchange at an arbitrary level (only the difference of "spread" matters to the exchange and traders involved). Although they may be for a large size that can sweep the market several levels, they are typically not reported by most vendors, thus adding uncertainty regarding the meaning of the next trade's sign.

As with most trade classification studies, research on tick rule accuracy has focused on implementation in equities markets (there is, to our knowledge, no comparable study of tick rule accuracy in futures markets). Odders-White [2000] reports 78.6% accuracy when applying this algorithm on the NYSE's 1991 TORQ database. Ellis, Michaely and O'Hara [2000] report 80% accuracy on Nasdaq data from the late 1990's. Aitken and Frino [1996] estimate 75% accuracy on Australian stocks in the 1990's. Whether these tick rule classification accuracies persist in high frequency settings is not clear, but recent evidence from Chakrabarty, Moulton and Shkilko [2012] (discussed later in the paper) suggests that accuracies have degraded substantially

1.2. QUOTE RULE

The Quote Rule classifies a trade that occurs above the best-bid-or-offer (the BBO) midpoint as a Buy, a trade that occurs below the BBO midpoint as a Sell, and leaves as unclassified trades that occur at the midpoint. This method is more data demanding than the Tick rule as it requires Level 2 data (tick trades and BBO quotes). In principle, the availability of more data should make trade classification more accurate, but in reality quote rule algorithms are problematic. In equity markets, for example, the large number of trades occurring at the mid-quote results in large classification errors using this approach. Equally challenging is the difficulty of aligning trades to the correct prevailing quotes. In particular, quotes change more frequently than trades execute, meaning that a trade may appear to be at the current ask when in fact it took place at what was then the prevailing bid.[6] This problem is likely to be particularly acute for less liquid securities with thinner order books.

Empirical investigations find substantial errors using the quote rule for trade classification. Odders-White [2000] reports that this algorithm left 15.9% of the trades unclassified, and it misclassified an additional 9.1% when applied to NYSE's TORQ database. Ellis, Michaely and O'Hara [2000] report 78% accuracy on Nasdaq data, counting unclassified trades as misclassified.

1.3. LEE-READY ALGORITHM

The most widely-used classification method for stocks, the Lee and Ready [1991] algorithm combines features from both the tick and quote rules. This classification algorithm was designed to sign trades on the NYSE where different reporting systems collected quote information and trade information from the specialists. The Lee-Ready algorithm applies the quote rule on all transactions away from the midpoint, and a tick rule for those transactions at the midpoint. Like the tick rule, the Lee-Ready

[6] In electronic markets, quote changes will often occur when orders are cancelled, and not just when orders execute. Hasbrouck and Saar [2009] find that 98% of orders on the NYSE are cancelled, underscoring the difficulties of this approach.

algorithm leaves no trades unclassified, however it requires Level 2 data for implementation.[7]

Odders-White [2000] reports a 85% accuracy when applying this algorithm on NYSE's 1991 TORQ database. Ellis, Michaely and O'Hara [2000] report 83% accuracy on 1996 Nasdaq data. Chakrabarty, Moulton, and Shkilko [2012], using 2005 data from INET (an ECN now part of Nasdaq), find that Lee-Ready classified correctly only 68-69% of trades. This dramatic fall-off in classification accuracy is likely to be even greater today given that high frequency trading was nascent in 2005.

The original Lee-Ready algorithm specified a 5-second delay in comparing trades to quotes, highlighting even in pre-high frequency days the difficulty of aligning times for trade and quote reports. This problem is substantially more difficult now. In equities, for example, there are now 13 exchanges (and a variety of trade reporting facilities (TRFs)) sending data to the consolidated tape. The tape marks trade and quote times when it receives the data, but this will be different from the actual trade or quote time depending on the latency of the venue's reporting technology. Moreover, the aggregate tape is then a blend of all of these reports, delivered at various latencies.[8] Most professional traders now purchase direct feeds of data from the individual venues rather than rely on the much slower, and arguably less accurate, tape data.[9]

In summary, extant trade classification algorithms use time-stamped individual trade and quote reports to infer trade direction. While arguably accurate "enough" in the past, the speed, volume and fragmentation of current high frequency

---

[7] Since all futures trades must occur at the bid or ask (and spreads are typically only one tick), Lee-Ready reduces to the Tick Rule for these products and needs only Level 1 data.

[8] Holden and Jacobsen (2011) discuss these tape timing issues in more detail.

[9] The problems with the consolidated tape then translate into problems with TAQ data as it is derived from the tape. In addition to the issues raised above, O'Hara, Yao, Ye [2011] show that the tape also does not report odd lot transactions, which in turn means that the sequence of trades is incorrect as well as incomplete.

markets pose serious challenges to any trade-based classification approach. Even apart from accuracy concerns, however, current practices such as algorithmic trading result in massive numbers of trades, quotes and messages that can overwhelm even the most sophisticated data handling systems.[10] In the next section, we propose a new approach to classifying trading activity.

2. BULK VOLUME CLASSIFICATION (BVC)

In our analysis, we aggregate trades over short time or volume intervals (denoted time bars and volume bars respectively), assign each interval the last price included in it, and then use the standardized price change between the two consecutive intervals to determine the percentage of buy and sell volume.[11] For example, a bar τ is assigned the price change $P_\tau - P_{\tau-1}$, where P_τ is the last price included in bar τ, and $P_{\tau-1}$ the last price included in bar $\tau - 1$.[12] Aggregation mitigates the effects of order splitting, and using the standardized price change allows volume classification in probabilistic terms (which we call *Bulk Volume Classification*, or BVC for short). Let

$$
\begin{aligned}
V_\tau^B &= V_\tau \cdot Z\left(\frac{P_\tau - P_{\tau-1}}{\sigma_{\Delta P}}\right) \\
V_\tau^S &= V_\tau \cdot \left[1 - Z\left(\frac{P_\tau - P_{\tau-1}}{\sigma_{\Delta P}}\right)\right] = V - V_\tau^B
\end{aligned}
\qquad (1)
$$

where V_τ is the volume traded during (time or volume) bar τ, and which we wish to classify in terms of buy and sell volume V_τ^B *and* V_τ^S respectively. Z is the CDF of the standard normal distribution, $P_\tau - P_{\tau-1}$ is the price change between two consecutive

[10] For this reason, exchanges are not able to ensure that events are broadcasted in the same sequence as they took place. This phenomenon is particularly detrimental to the accuracy of Tick-based classification rules.
[11] See also Easley, et al. [2011b] where we apply this technique in estimating VPIN measures.
[12] The first bar starts with the second transaction in our sample, so that the algorithm has a P_0 for initialization.

bars and $\sigma_{\Delta P}$ is an estimate of the volume-weighted standard derivation of price changes between bars. Our procedure splits the volume in a bar equally between buy and sell volume if there is no price change from the beginning to the end of the bar. Alternatively, if the price increases, the volume is weighted more toward buys than sells and the weighting depends on how large the price change is relative to the distribution of price changes.

A key difference between BVC and the Tick-based algorithms is that the latter signs volume as either buy *or* sell, whilst the former assigns a fraction of the volume as buys *and* the remainder as sells. In other words, the Tick-based algorithms provide a discrete classification, while the BVC algorithm is continuous. Because most trading now arises from algorithms which slice and dice orders into tiny pieces, it seems sensible that trading intentions are now better captured by looking at the aggregation of trades rather than the individual units.

It is also useful to note that because the BVC approach uses the normal approximation it works well with large numbers of trades, a feature now describing most high frequency markets. In low transactional settings[13], however, BVC will not be appropriate as it is not designed to sign individual trades. In such markets, standard tick-based algorithms are the more appropriate approach.

3. CLASSIFICATION ACCURACY

How well does this classification algorithm work? To establish this, we need to compare our estimated buy and sell categorizations with actual buy-sell data. Standard equity databases do not provide such information, but it is available (for a price) for futures contracts trading on GLOBEX. Apart from availability, futures data

[13] As we will see later, accuracy increases with greater data granularity per bin, because measuring errors offset each other. If bins are extremely small because only a few trades occur per bin, accuracy will decline.

have several advantages relative to equity data. One is that futures are not fragmented, so selection biases in the data are nonexistent. Another advantage is that all trades must occur either at the best bid or the best offer. Consequently, quote data provides little additional information for purposes of trade classification.[14] For this reason, we will limit our analysis to comparing the accuracy of two Level 1 algorithms: Tick-rule and BVC.

We focus our investigation on three contracts: the e-mini S&P 500 future, the WTI Crude Oil future, and the Gold futures. The e-mini S&P 500 Futures trades on the Chicago Mercantile Exchange (CME) and is the most actively traded index futures contract, with an average daily volume of 2.2 million contracts. The WTI Crude Oil Futures trades on the New York Mercantile Exchange (NYMEX) and is the most actively traded commodities contract. Gold Futures trade on the Commodities Exchange (COMEX) and, while active, their trading volume is approximately one-fifth that of the e-mini. Trade classification algorithms can be sensitive to market structure, book dynamics and liquidity among other factors, so we use these three distinct contracts to look for general trends in trade classification accuracy.

3.1. TRANSACTIONS DATA

Our empirical study begins with an analysis of tick data for the CME e-mini S&P500 Futures contract from November 7th 2010 to November 6th 2011. *DataMine Market Depth* is the CME Group database that provides (at a cost) all messages needed to recreate the book (multiple-depth) and trade data for any CME GLOBEX trade

[14] We do not entirely discard the applicability of Level 2 algorithms to other products, especially those with a sparse book, but in the case of full and deep books like the E-mini S&P500, Level 1 algorithms seem to make the most adequate use of the data.

product, time-stamped to the millisecond, following the FIX/FAST protocol[15]. This level 3 data was purchased directly from the CME, and was delivered as 357 zip files containing 2272 flat files. This represents about 21.6GB of compressed data, and about 220GB uncompressed.[16]

There are a variety of challenges in working with this data. The data comes in a highly irregular format in which a single line can contain an arbitrary number of messages. Among these messages, we find anywhere between 1 and 19 trades per line. Most messages relate to requests to modify or cancel quotes. Another problem is that a trade cannot be identified by any particular FIX tag, but only by a combination of them (for example, when tag 269=2 after another tag 279=0, then tag 270 contains the price, tag 271 the traded size, tag 5797 the aggressor side, tag 52 the UTC transmission time and tag 107 the instrument). A further complication is that the files mix messages from all e-mini S&P500 Futures contracts trading at that time, not only the front contract, requiring care in separating trades from the different expirations. Finally, some reported trades are fictional, and the only way to tell the difference with the real trades is by checking the trade time (they are time stamped during periods when the Exchange was actually closed).[17]

It should be noted that Exchanges do not always report trades in the sequence they occurred, particular when their networks are overloaded with dense traffic. Book updates are incremental. Losing or misplacing a message within the sequence of events means that the researcher will not reconstruct the book correctly, which will

[15] This protocol receives frequent updates and modifications. In the context of this paper, we will always refer to version 2.19, dated 12/09/11.

[16] We mention these numbers to signal the difficulty of working with this data using standard commercial packages.

[17] Fictional trades can arise as part of the algorithm testing process. Another oddity in the data is that 27,419 trades (or 0.0213% of the total) reported at 4.30pm EST on weekdays and 6pm on Sundays are matched in the opening auction, and therefore have no aggressor flag. These were deleted from our study.

impact the accuracy of the Tick-based algorithms but it will not affect BVC's accuracy. Tag 34 ("Integer Message Sequence Number") helps to identify whether a message was broadcasted out of sequence.

In short, a rather complex data handler needs to be programmed in order to extract the fields we need: Time, Price[18], Volume, Aggressor and Instrument.[19] Once the data is parsed, we are left with 128,579,415 trades (Level 1 data) stored in a SQLite3 database, which occupies 3.87GB of memory. A similar database with Level 2 data would consist of 20.7GB of memory containing 629,897,077 records (trades and BBO updates). There is an average of about 3.9 BBO updates per trade.

Most of these trades are small, averaging 4.50 contracts per reported fill. Figure 1 plots the frequency of trades per trade size. About 51.56% of the trades are for one contract. Because the CME applies a FIFO (First In, First Out) matching algorithm for e-mini S&P500 futures, reducing the size of the order does not place it lower in the queue[20]. The frequency line quickly decays as a function of the trade size, with the exception of round trade sizes (5, 10, 20, 25, 50, 100, 200, etc.).

That round trade sizes are much more common than their neighbors may be attributed to so-called 'mouse' or 'GUI' traders, i.e. human traders that send orders by clicking buttons on a GUI (Graphical User Interface). As an interesting aside, this footprint of 'GUI traders' could be used by machines to learn the patterns of their human competitors, and eventually anticipate them to the advantage of the 'silicon traders'. For example, size 10 is 2.9 times more frequent than size 9. Size 50 is 10.86

[18] Prices are recorded in cents of the index level, to avoid decimals.

[19] Unfortunately every exchange makes slightly different implementations of the FIX protocol, which means that a CME data handler may not work on EUREX FIX-formatted messages.

[20] That is not the case for all CME products. The CME reports the matching algorithm through FIX tag 1142. For instance, CME matches Eurodollar short futures following an Allocation algorithm. This is an enhanced pro-rata algorithm that incorporates a priority (TOP order) to the first incoming order that betters the market. CME follows a Pro-Rata algorithm to match orders on FX Futures Spreads. The CME applies 10 different matching algorithms, depending on the product.

times more likely than size 49. Size 100 is 16.78 times more frequent than size 99. Size 200 is 27.18 times more likely than size 199. Size 250 is 32.5 times more frequent than size 249, and size 500 is 57.06 times more frequent than size 499. Such patterns are not typical of 'silicon traders' who usually randomize trades to disguise their footprint in markets.

[FIGURE 1 HERE]

3.2. TIME BARS AND VOLUME BARS

Dealing with (time or volume) bars is much easier, faster, and cheaper. To begin with, most data vendors offer time bars pre-processed, which saves the time and expense associated with FIX-formatted tick data. Bloomberg users, for example, can download up to 6 months of time bars for any security at no extra cost. In addition, One-Tick and TickWrite users can form sub-minute time bars as well as volume and tick bars.

Compressing the data into either time bars or volume bars dramatically reduces the amount of data needed for analysis. For example, our 128,579,415 original trades result in only 443,653 time bars of size 1-minute. This is a much more manageable series, constituting just 0.35% of the original data size. Finer time bars also allow significant compression, with 1-second time bars reducing the data points to 8,732,580, or 6.89% of the original data. In our analysis, we evaluate trade accuracy using time bars from 1 second to 300 seconds. Volume bars are defined over specific quantities and in our study we look at volume bars ranging from 1,000 to 10,000 contracts. Using as few as 1000 contracts, however, we can compress the data to 0.45% of its original size. Figure 2 shows how the various time and volume bar settings affects the compression of data for the e-mini S&P 500 future.

[FIGURE 2 HERE]

3.3. DOES IT WORK? TESTING THE ACCURACY OF BULK VOLUME AND THE TICK RULE

We have shown that the BVC approach dramatically reduces the amount of data needed to classify buy and sell trading activity, but we have not yet shown how accurate it is. In this section, we investigate this issue by computing the accuracy of the BVC approach as well as the accuracy of the Tick Rule algorithm. Our BVC approach yields a fraction of trade in each bar that is classified as buyer or seller initiated. As it does not classify individual trades we cannot compute accuracy on a trade-by-trade basis. Instead, for both the Tick Rule and BVC approach, we compare the computed fraction of buy and sell trades in each bar with the actual fraction of buy and sell trades in the bar.

To do this comparison we compute the buy and sell volume implied by the Tick Rule and compare the accuracy of this estimate with the accuracy of BVC's estimates. If (V_τ^B, V_τ^S) is the actual buy and sell volume in bar τ and $\left(\widehat{V_\tau^B}, \widehat{V_\tau^S}\right)$ is the estimated buy and sell volume in bar τ , then the accuracy ratio (*Ar*) is defined to be

$$Ar = \frac{\sum_\tau\left[min\left(\widehat{V_\tau^B}, V_\tau^B\right) + min\left(\widehat{V_\tau^S}, V_\tau^S\right)\right]}{\sum_\tau V_\tau} \qquad (2)$$

Table 1 provides accuracy ratios for the Tick Rule and BVC on the e-mini S&P 500 Futures contract.

[TABLE 1 HERE]

We first note that using the tick-rule on e-mini S&P 500 Futures delivers an accuracy ratio of 86.43%. This is remarkably close to the results reported in the aforementioned studies on stocks, and it suggests that the tick-rule provides a reasonably accurate, though heavily data intensive way, to classify buy and sell trades in e-mini futures.[21]

The BVC results vary depending upon the time bar or volume bar selected, but the table clearly shows that the BVC algorithm beats the tick rule for time bars of 60 seconds or longer and for volume bars of 3000 shares or more. Moreover, using the BVC approach we can correctly classify approximately 87.6% of volume using time bars, and 90.72% of volume using volume bars. Thus, the BVC algorithm is more accurate than the standard tick-based approach.[22]

This greater accuracy comes with a more parsimonious use of data. Using volume bars, we can achieve a 90% accuracy rate using only 0.6% of the data needed for the tick-rule. Focusing on a 60 second time bar, we can achieve 86.61% accuracy using 0.35% of the data. In other words, less data granularity is at least as informative as full granularity for the purpose of trade classification.

What may be surprising is that volume bars are more accurate than time bars. This is consistent with arguments in Easley, López de Prado, and O'Hara [2011a; 2011b] that in high frequency markets a volume clock is a more appropriate metric than a time clock. Volume bars fill more quickly when markets are active, and more slowly when it is inactive. Volume bars always contain the same amount of volume, so the aggregation properties of the BVC approach will generally work better than it

[21] Note that this is accuracy of the tick rule's estimate about the volume of buy and sell trades, and not its accuracy on a trade-by-trade basis. Calculating this latter trade-by-trade accuracy, the tick rule correctly classifies 86.14% of trades.

[22] It is worth noting that if all trades generated price changes of the same absolute size, varying between negative for sells and positive for buys, then the tick rule and BVC would yield identical estimates for the fraction of buy versus sell volume. In reality, some price changes are zero, some are small and some are large, each associated with different trade sizes, and so the two procedures yield differing estimates. We thank Craig Holden for prompting us to think about this point.

will with the varying amounts of trade found in time bars.[23] This suggests that volume bars may be a better choice for classifying trading activity.

A related issue is the optimal time or volume bar size to use. Accuracy increases with bar size, but we are also giving up some of the variability in trading patterns that is apparent with shorter bars (although this is less of a problem with volume bars). Depending on the research question posed, some researchers will favor a particular bar type and size over others, taking into account factors such as data compression and accuracy.

3.4. ROBUSTNESS IN OTHER MARKETS: APPLICATION TO WTI CRUDE OIL FUTURES AND GOLD FUTURES

Because order matching protocols differ across market settings, one might expect that the accuracy of classification algorithms will differ as well. In this sub-section, we address this issue by implementing our analysis on two additional futures contracts, the WTI Oil future (trading on NYMEX) and the Gold future (trading on COMEX). Apart from the differences introduced by differing market rules, these contracts also differ with respect to trading volume levels and order book activity, factors that might also be expected to affect accuracy levels.

WTI Crude Oil Futures are the most liquid of all crude contracts, and the futures product with largest volume among all physical commodities. We acquired Level 3 data from NYMEX, from November 28th 2010 to November 27th 2011. A data handler is also needed in this case, as the same data pre-processing problems are present in WTI Crude Oil Futures as we found in the e-mini S&P500 Futures.[24] Once

[23] A variety of authors have noted that volume or transaction based analyses have “better” statistical properties than time based analyses of securities transactions. See for example, Mandelbrot [1967], Clark [1973] or Ané and Geman [2000] among many.

[24] The data also follows the FIX/FAST protocol, and was delivered as 364 zip files containing 4949 flat files. This represents about 50.7GB of compressed data, and about 510GB uncompressed.

the data is parsed, we are left with 78,630,179 signed trades, stored in a SQLite3 database of Level 1 data which occupies 2.29GB of memory. A similar database with level 2 data would consume 47.5GB of memory containing 1,486,832,916 records (trades and BBO updates), or about 17.91 BBO changes per trade.

Gold Futures are traded at the Commodity Exchange (COMEX). Level 3 data was acquired from COMEX, from November 28th 2010 to December 20th 2011.[25] Once the data is parsed, we are left with 27,960,542 signed trades, stored in a SQLite3 database of Level 1 data which occupies 833MB of memory. A similar database with Level 2 data would consist of 36.4GB of memory containing 1,142,557,861 records (trades and BBO updates), or about 39.86 average BBO changes per trade.

Trading in the WTI Oil and Gold futures contracts share much in common with the e-mini S&P 500 Futures contract, but there are some important differences. For example, book dynamics of the WTI contract are quite different from those of the e-mini. Frequent modifications and cancellations of orders make the WTI book much more unstable than the e-mini's. In our particular sample, there is an average of 17.91 BBO updates for each WTI trade, which is strikingly greater than 3.8 BBO updates for the e-mini. In Gold futures, the number is even higher, with an average of 39.86 BBO updates per trade. These book dynamics create problems for tick rule classification, and so we would expect BVC on bars to be even more useful for such contracts. These dynamics also dramatically increase the data needed for tick-by-tick processing, underscoring another advantage of using time or volume bars in these settings.[26]

[25] It also follows the FIX/FAST protocol, and was delivered as 388 zip files containing 3044 flat files. This represents about 28.8GB of compressed data, and about 290GB uncompressed.

[26] In particular, one year of WTI data required 510GB for 78,630,179 transactions, while one year of e-mini S&P500 Futures data needed only 220GB for 128,579,415 transactions.

Trade sizes are also different with these contracts. The average trade size for the WTI Oil is only 1.91 contracts per reported fill. The WTI Crude's contract value is typically about 50% more expensive than e-mini S&P500's, as well as 49% more volatile, so this smaller trade size may reflect the greater costs of transacting in the WTI contract. The average trade size for Gold contract is smaller still, only 1.64 contracts per reported fill. Although the Gold contract is about as volatile as the e-mini S&P500 Futures, its contract value is typically about 50% more expensive than crude's, and almost three times as expensive as e-mini's.

Figure 3 plots the frequency of trades per trade size for WTI crude (panel A) and for Gold Futures (panel B). Both contracts show a similar profile as the e-mini contract, although trades are even more concentrated towards single units. In the WTI contract, 83.02% of the trades were of size 1, while the corresponding number is 79.18% in the gold contract. As we found in the e-mini S&P500 Futures, the frequency line quickly decays as a function of the trade size, with the exception of round trade sizes.[27]

[FIGURE 3 HERE]

We compared the accuracy of tick rule versus BVC for the WTI Oil and Gold Futures using time bars (Table 2) and volume bars (Table 3). As we conjectured, the tick rule does appreciably worse in these contracts, correctly classifying 78.95% of Gold contracts and only 67.18% of WTI contracts. By contrast, BVC can correctly classify more than 91% of gold contracts (using volume bars; 82.9% using time bars) and 87% of WTI contracts (volume bars, 80.3% using time bars). As was the case with the e-mini Futures contract, volume bars outperform time bars, and both outperform the tick rule.

[27] Anyone coming to this market with sizes over one unit is telegraphing her/his urgency for acquiring a position, and drawing the attention of predatory algorithms.

[TABLE 2 AND TABLE 3 HERE]

The Tables also show that this greater accuracy is available using a wide range of time and volume bars. For gold Futures, BVC provides greater accuracy for time bars of 30 seconds or higher and for volume bars of 100 contracts or higher. For WTI Futures, the cut-offs are even lower, with time bars of 2 or more seconds and volume bars of 50 or more contracts providing greater accuracy. The Tables also show the significant compression of data available with BVC. Volume bars use just 1.62% of the data to outperform the tick rule in Gold futures, and 3.82 % of the data to do so in WTI futures. We interpret these results as confirming that BVC provides an accurate and data efficient method for classifying trading activity in markets.

4. THE STABILITY OF CLASSIFICATION RESULTS

Market activity in futures contracts can differ significantly over time, reflecting both differences in trader sentiment and technical factors such as the roll-over to a new front contract. Trading activity may also be particularly high in times of market stress. Our sample period includes August, 2011, a notable period when concerns about the European sovereign debt crisis caused dramatic movements in markets worldwide. A natural question to ask is how well do classification algorithms work in these different market conditions?

We calculated the daily accuracy of the tick-rule, BVC-volume bars, and BVC-time bar for the e-mini, WTI oil futures, and gold futures over our sample period. As the accuracy of the BVC method depends on time and volume bar specifications, we selected those levels that maximized the BVC performance on average as given in Tables 1-3. Thus, for the e-mini Futures contract we chose 2-minute time bars and 8000 contract volume bars; for the WTI oil contract we chose 5-

minute time bars and 5000 contract volume bars, and for the Gold contract we chose 5-minute time bars and 4500 contract volume bars. Figure 4 Panels (a), (b), and (c) plots these accuracy levels along with the daily volume level in each contract.

[FIGURE 4 HERE]

The data reveal a clear rank order: In general, BVC-volume bars provide the highest classification accuracy across all three contracts and across virtually all market conditions, followed by BVC-time bars, and then by tick-rule. For the e-mini, BVC- Time bars beats the Tick-rule on 82.11% of days, and BVC-Volume bars beat the Tick-rule on 99.19% of days. For WTI oil futures, BVC-Time bars beats the Tick-rule on 99.61% of days, and BVC-Volume bars beat the Tick-rule on 100% of days. Similarly, for Gold futures, BVC- Time bars beats the Tick-rule on 87.27% of days, and BVC- Volume bars beats the Tick-rule on 100% of days.

All three algorithms tend to perform worse on highly active days. These activity patterns differ across the contracts, with the e-Mini S&P 500 and Gold Futures being particularly affected by the sovereign debt crisis in August. On August 9th, the e-Mini S&P500 futures hit its highest volume level in the history of the contract. This extraordinary volume resulted in a degradation of BVC relative accuracy: BVC-Time bars did noticeably worse than Tick-rule (71.68% vs. 85.89%), while BVC-Volume bars correctly classified 84.33% of the volume, about the same as Tick-rule. Overall, however, the relative ranking among algorithm accuracy appears reasonably robust to market activity, and this is particularly true for the WTI oil future and gold future contracts.

An interesting feature revealed by Figure 4 is a spike down in classification accuracy for all three methods, around 11/23/10, 01/26/11, 03/25/11, 05/25/11, 07/25/11 and 11/23/11. These are not arbitrary dates; they coincide with the sessions around which volume shifts from one front contract to the next (a.k.a. rolling dates). Trade on these dates is dominated by large calendar trades, typically executed through exchange guaranteed spreads, impacting two books in opposite directions. In order to avoid this phenomenon, we recommend excluding such dates for the purpose of trade classification.

5. USING THE EMPIRICAL DISTRIBUTION OF PRICE CHANGES

The bulk volume classification procedure described in Eq.(1) treats price changes $P_\tau - P_{\tau-1}$ as if they are distributed i.i.d. with

$$P_\tau - P_{\tau-1} \sim N(0, \sigma_{\Delta P}) \tag{3}$$

This normality assumption is a simplification that obviates the need to compute the empirical distribution of price changes. Alternatively, we could compute the true CDF of price changes, F, and then define an alternative bulk volume classification procedure, BVC2, whereby

$$\begin{gathered} V_\tau^B = V_\tau \cdot F(P_\tau - P_{\tau-1}) \\ V_\tau^S = V_\tau \cdot [1 - F(P_\tau - P_{\tau-1})] = V - V_\tau^B \end{gathered} \tag{4}$$

There are a several features of Eq.(4) that lead us to suspect that it will not deliver a particularly accurate classification. First, we are no longer assuming that $F(0) = \frac{1}{2}$. In fact, $F(0) < \frac{1}{2}$. This may be the result of a bullish trend over a particular subsample that is not canceled by a bearish trend over another subsample with our data set. Using the actual distribution of price change would allow that phenomenon to condition our volume classification over the entire sample. Similarly, price gaps originated at particular events, like Nonfarm payroll releases, will introduce significant skewness and kurtosis into the distribution that will condition the entire sample. Because Eq.(4) incorporates more sample information than Eq.(1), the impact of these subsample effects will be more prevalent in BVC2 than in BVC. Finally, $P_\tau - P_{\tau-1}$ is not continuous due to fixed tick sizes, making F a highly discretized distribution. The accuracy results, reported in Table 4, confirm this intuition. The BVC2 biases classification algorithm is much less effective in classifying trading volume than the BVC algorithm. Thus it does not appear to be productive to use the actual distribution of price changes.

[TABLE 4]

6. CONCLUSIONS

It may seem counterintuitive to expect greater accuracy from lower data granularity, but in fact bulk measurements have proven more accurate in a number of settings, such as bulk weighting and counting. Inspired by this principle, we propose a new *Bulk Classification* algorithm to identify the aggressor side of a collection of trades. This departs from previous attempts in the literature, which targeted classifying individual trades. Instead, for a given set of trades bundled in bars, we label a fraction of the volume as buys *and* the remainder as sells.

Bundling ticks in terms of time bars allows the researcher to work with a small fraction of the original tick dataset (only 0.35% for 1-minute bars in the e-mini S&P 500 Futures). Most researchers would prefer working with 0.35% of a dataset composed of hundreds of millions of records, as long as there is no substantial loss of accuracy. What we have found is that, for the purpose of volume classification, BVC of Time and Volume bars is more accurate than the *Tick Rule* of tick data. That more information can be extracted from less data granularity has important theoretical implications for studies of high frequency market microstructure, and of course presents an important practical advantage for researchers not accustomed to dealing with large databases.

We caution against overreaching the implications of these findings. Tick-based classification still has a place in addressing a number of questions. For example, a researcher may be more interested in classifying a particular set of non-contiguous trades, which cannot be bundled in bars, like the activity of a particular market participant (revealed through FIX tag 50, where available). With trading now dominated by computer algorithms, however, trading is rapidly converging to single trade sizes. Order splitting is now the norm, and researchers interested in discerning trading intentions may find using time or volume bars a more natural unit of analysis than individual trades for a wide range of issues. We encourage researchers to investigate whether a principle similar to bulk estimation can be successfully applied to other computational problems in high frequency markets.

One additional issue to consider is that researchers often want to classify trades as buys or sells in order to infer information-based trade. In cases in which trade occurred primarily through a specialist posting bid and ask prices that outsiders hit this classification could provide insight into informed trading. But in electronic limit order book markets this inference is far less obvious. Eisler, Bouchaud and Kockelkoren [2011] make the point that informed traders may choose to place limit orders, aiming at decreasing execution costs. In that case, which side is the aggressor on an individual trade does not necessarily reveal which side is informed although the overall buy and sell volumes should. Thus, in addition to providing a parsimonious approach to trade classification, BVC could potentially be even more informative than aggressor flags for the purpose of determining informational asymmetry.

REFERENCES

Aitken, M. and A. Frino, 1996, *The accuracy of the tick test: Evidence from the Australian Stock Exchange*, Journal of Banking and Finance, 20, 1715-1729

Ané, T., and H. Geman, 2000, *Order Flow, Transaction Clock and Normality of Asset Returns*. Journal of Finance 55:2259–84.

Asquith, P., R. Oman and C. Safaya, 2010, *Short Sales and Trade Classification Algorithms*, Journal of Financial Markets, 13, 157-173.

Bethel, W., D. Leinweber, O. Ruebel and K. Wu, 2011, *Federal Market Information Technology in the Post Flash Crash Era: Roles for Supercomputing*, http://ssrn.com/abstract=1939522

Chakrabarty, B., P. C. Moulton, and A. Shkilko, 2012, *Short Sales, Long Sales, and the Lee-Ready Trade Classification Algorithm Revisited*, Journal of Financial Markets, *forthcoming*.

Clark, P. K., 1973, *A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices*. Econometrica 41:135–55.

Easley, D., M. López de Prado and M. O'Hara, 2011a, *The Microstructure of the Flash Crash*, Journal of Portfolio Management, Winter. http://ssrn.com/abstract=1695041

. Easley, D., M. López de Prado and M. O'Hara2011b, *Flow Toxicity and Liquidity in a High Frequency World*, Review of Financial Studies, Forthcoming. http://ssrn.com/abstract=1695596

Easley, D., M. López de Prado and M. O'Hara, 2012, The Volume Clock: Insights into the High Frequency Paradigm, Working Paper, Eisler, Zoltan, J.-P. Bouchaud and J. Kockelkoren, 2011, *The Impact of order book events: Market orders, limit orders and cancellations*, working paper, August. http://ssrn.com/abstract=1373762

Ellis, K., R. Michaely and M. O'Hara, 2000, *The Accuracy of Trade Classification Rules: Evidence from Nasdaq*, Journal of Financial and Quantitative Analysis, 35(4), December.

Hasbrouck, J. and G. Saar, 2009, *Technology and Liquidity Provision: The Blurring of Traditional Definitions*, Journal of Financial Markets.

Holden, C. and S. Jacobsen, 2011, *The Breakdown of Standard Microstructure Techniques and What to Do About It*, Working paper.

Lee, C. and M. Ready, 1991, *Inferring Trade Direction from Intraday Data*, Journal of Finance, 46, 733-746.

Mandlebrot, B., and M. Taylor, 1967, *On the Distribution of Stock Price Differences*. Operations Research 15(5):1057–62.

Mosteller, F., 1965, *Fifty challenging problems in probability with solutions*, Dover Publications, pp.79-80.

Odders-White, E., 2000, *On the Occurrence and Consequences of Inaccurate Trade Classification*, Journal of Financial Markets, 3, 259-286.

O'Hara, M., 1995, *Market Microstructure Theory*, Blackwell.

APPENDICES

A.1. TICK-RULE IMPLEMENTATION

Here we present a simple implementation of the Tick Rule in Python language. More efficient implementations exist, but we believe the one outlined below is the clearest. *queryCurs* is assumed to contain the output of a SQL query such as

queryCurs.execute('SELECT Price, Volume, VolBuy FROM ' + tablename + ' ORDER BY Instrument, Time')

VolBuy is the field that stores the Volume from traders initiated by an aggressive buyer, as reported by the Exchange. The *tick* list variable will accumulate the amount matched over the entire volume. The rest of the code is self-explanatory.

```
a =queryCurs.fetchone()
flag, price, tick=1, a[0], [0,0]
while True:
    try:
      a=queryCurs.fetchone()
      # tick rule
      if a[0]>price:
        flag=1
      elif a[0]<price:
        flag=2
      if flag==1:
        tick[0]+=a[2]  #correctly classified as buy
      else:
        tick[0]+=a[1]-a[2]  #correctly classified as sell
      tick[1]+=a[1]  #volume to be classified
      # reset price
      price=a[0]
    except:
      break
```

A.2. BULK VOLUME CLASSIFICATION IMPLEMENTATION

An equivalent codification of the BVC algorithm would be as follows. *stDev* is a real variable storing the volume weighted Standard Deviation of price changes across bars. The amount matched over the entire volume is stored in the list variable *bulk*.

```
a =queryCurs.fetchone()
price, bulk=a[0], [0,0]
while True:
    try:
      a=queryCurs.fetchone()
      # bulk classification
      z=float(a[0]-price)/stDev
      z=scipy.stats.norm.cdf(z)
      bulk[0]+=min(a[1]*z,a[2])  #correctly classified as buy
      bulk[0]+=min(a[1]*(1-z),a[1]-a[2])  #correctly classified as sell
      bulk[1]+=a[1]  #volume to be classified
      # reset price
      price=a[0]
    except:
      break
```

TABLES

Table 1 – Classification Accuracy for e-mini S&P futures contracts

The upper panel gives results of BVC accuracy using time-bars, while the lower panel gives the same results using volume-bars. In both cases, bar size of 0 corresponds to classification results applying the tick-rule algorithm.

Data	Time bar size	# Points	Compression	Accuracy
Tick rule	0	128,579,415	0.00%	86.43%
Bulk Volume	1	8,732,580	93.21%	74.05%
Bulk Volume	2	5,978,482	95.35%	76.06%
Bulk Volume	3	4,658,916	96.38%	77.35%
Bulk Volume	5	3,312,436	97.42%	79.18%
Bulk Volume	10	1,994,533	98.45%	81.67%
Bulk Volume	20	1,145,894	99.11%	84.09%
Bulk Volume	30	813,392	99.37%	85.20%
Bulk Volume	60	443,653	99.65%	***86.61%***
Bulk Volume	120	240,790	99.81%	***87.60%***
Bulk Volume	180	168,881	99.87%	***87.56%***
Bulk Volume	300	109,067	99.92%	***87.35%***

Data	Volume bar size	# Points	Compression	Accuracy
Tick rule	0	128,579,415	0.00%	86.43%
Bulk Volume	1000	579,134	99.55%	82.40%
Bulk Volume	2000	289,566	99.77%	86.23%
Bulk Volume	3000	193,043	99.85%	***87.81%***
Bulk Volume	4000	144,782	99.89%	***88.47%***
Bulk Volume	5000	115,825	99.91%	***89.21%***
Bulk Volume	6000	96,521	99.92%	***89.99%***
Bulk Volume	7000	82,730	99.94%	***90.10%***
Bulk Volume	8000	72,389	99.94%	***91.00%***
Bulk Volume	9000	64,346	99.95%	***90.38%***
Bulk Volume	10000	57,911	99.95%	***90.72%***

Table 2 – Classification Accuracy for WTI Crude Oil Futures and Gold Futures:

Tick Rule and Time Bar Results

Panel A gives results of BVC accuracy using time-bars for WTI Crude Futures, and the panel B for Gold Futures. In both cases, bar size of 0 corresponds to classification results applying the tick-rule algorithm.

A. WTI Crude Futures

Data	Time bar size	# Points	Compression	Accuracy
Tick rule	0	78,630,179	0.00%	67.18%
Bulk Volume	1	14,016,205	82.17%	65.99%
Bulk Volume	2	10,971,066	86.05%	***68.28%***
Bulk Volume	3	9,332,011	88.13%	***69.69%***
Bulk Volume	5	7,478,085	90.49%	***71.45%***
Bulk Volume	10	5,359,764	93.18%	***73.72%***
Bulk Volume	20	3,702,204	95.29%	***75.62%***
Bulk Volume	30	2,934,491	96.27%	***76.61%***
Bulk Volume	60	1,921,861	97.56%	***77.97%***
Bulk Volume	120	1,221,265	98.45%	***79.10%***
Bulk Volume	180	924,670	98.82%	***79.74%***
Bulk Volume	300	643,244	99.18%	***80.36%***

B. Gold Futures

Data	Time bar size	# Points	Compression	Accuracy
Tick rule	0	27,960,542	0.00%	78.95%
Bulk Volume	1	6,704,472	76.02%	65.70%
Bulk Volume	2	5,150,890	81.58%	68.61%
Bulk Volume	3	4,269,334	84.73%	70.61%
Bulk Volume	5	3,255,319	88.36%	72.98%
Bulk Volume	10	2,131,823	92.38%	76.16%
Bulk Volume	20	1,328,952	95.25%	78.81%
Bulk Volume	30	995,315	96.44%	***79.92%***
Bulk Volume	60	602,896	97.84%	***81.44%***
Bulk Volume	120	369,632	98.68%	***82.11%***
Bulk Volume	180	280,141	99.00%	***82.53%***
Bulk Volume	300	199,989	99.28%	***82.90%***

Table 3 – Classification Accuracy for WTI Crude Oil Futures and Gold Futures: Tick Rule and Volume Bar Results

Panel A gives results of BVC accuracy using volume-bars for WTI Crude Futures, and panel B for Gold Futures. In both cases, bar size of 0 corresponds to classification results applying the tick-rule algorithm.

A. WTI Crude Futures

Data	Volume bar size	# Points	Compression	Accuracy
Tick rule	0	78,630,179	0.00%	67.18%
Bulk Volume	50	3,007,477	96.18%	***71.48%***
Bulk Volume	100	1,503,724	98.09%	***75.15%***
Bulk Volume	250	601,476	99.24%	***79.56%***
Bulk Volume	500	300,726	99.62%	***82.43%***
Bulk Volume	1000	150,351	99.81%	***84.62%***
Bulk Volume	1500	100,226	99.87%	***85.71%***
Bulk Volume	2000	75,164	99.90%	***86.29%***
Bulk Volume	2500	60,128	99.92%	***86.63%***
Bulk Volume	3000	50,103	99.94%	***86.94%***
Bulk Volume	3500	42,943	99.95%	***87.17%***
Bulk Volume	4000	37,571	99.95%	***87.29%***
Bulk Volume	4500	33,394	99.96%	***87.17%***
Bulk Volume	5000	30,052	99.96%	***87.36%***

A. Gold Futures

Data	Volume bar size	# Points	Compression	Accuracy
Tick rule	0	27,960,542	0.00%	78.95%
Bulk Volume	50	915,833	96.72%	76.65%
Bulk Volume	100	457,907	98.36%	***80.20%***
Bulk Volume	250	183,155	99.34%	***84.17%***
Bulk Volume	500	91,571	99.67%	***86.61%***
Bulk Volume	1000	45,779	99.84%	***88.70%***
Bulk Volume	1500	30,512	99.89%	***89.69%***
Bulk Volume	2000	22,883	99.92%	***90.37%***
Bulk Volume	2500	18,304	99.93%	***90.71%***
Bulk Volume	3000	15,249	99.95%	***91.04%***
Bulk Volume	3500	13,069	99.95%	***91.28%***
Bulk Volume	4000	11,434	99.96%	***91.41%***
Bulk Volume	4500	10,165	99.96%	***91.65%***
Bulk Volume	5000	9,147	99.97%	***91.59%***

Table 4 – BVC2 Classification Accuracy for e-mini S&P futures contracts

This table shows the accuracy rates using Bulk Volume Classification 2 (BVC2) which relies on the empirical distribution of price changes rather than on a normal approximation.

Data	Time bar size	# Points	Compression	Accuracy
Tick rule	0	128,579,415	0.00%	86.43%
BV Empirical	1	8,732,580	93.21%	70.79%
BV Empirical	2	5,978,482	95.35%	71.77%
BV Empirical	3	4,658,916	96.38%	72.19%
BV Empirical	5	3,312,436	97.42%	72.51%
BV Empirical	10	1,994,533	98.45%	72.53%
BV Empirical	20	1,145,894	99.11%	72.07%
BV Empirical	30	813,392	99.37%	71.73%
BV Empirical	60	443,653	99.65%	70.75%
BV Empirical	120	240,790	99.81%	69.77%
BV Empirical	180	168,881	99.87%	69.16%
BV Empirical	300	109,067	99.92%	68.65%

Data	Volume bar size	# Points	Compression	Accuracy
Tick rule	0	128,579,415	0.00%	86.43%
BV Empirical	1000	579,134	99.55%	73.50%
BV Empirical	2000	289,566	99.77%	76.21%
BV Empirical	3000	193,043	99.85%	77.00%
BV Empirical	4000	144,782	99.89%	77.25%
BV Empirical	5000	115,825	99.91%	77.22%
BV Empirical	6000	96,521	99.92%	77.33%
BV Empirical	7000	82,730	99.94%	77.25%
BV Empirical	8000	72,389	99.94%	77.18%
BV Empirical	9000	64,346	99.95%	77.05%
BV Empirical	10000	57,911	99.95%	76.89%

FIGURES

Figure 1 – Percentage of trades as a function of the trade size (in log scale)

[E-mini S&P500 futures]

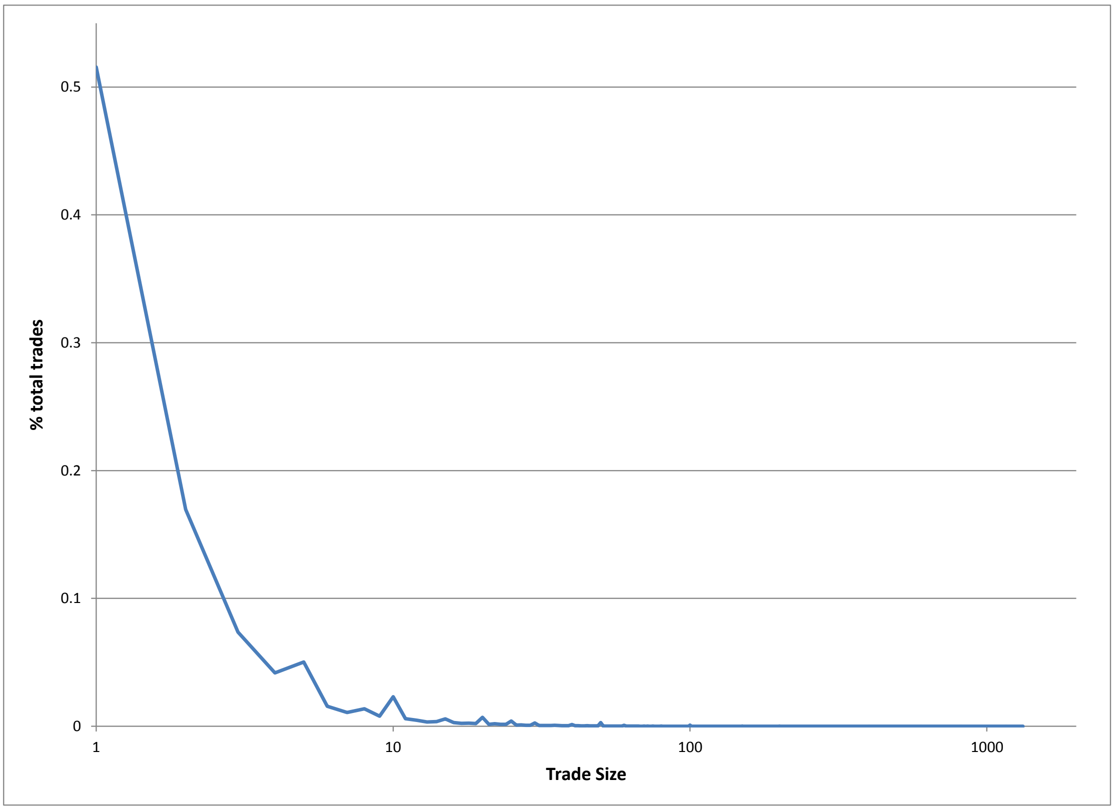


Figure 2 – Compression of data in the E-mini for various time and volume bars

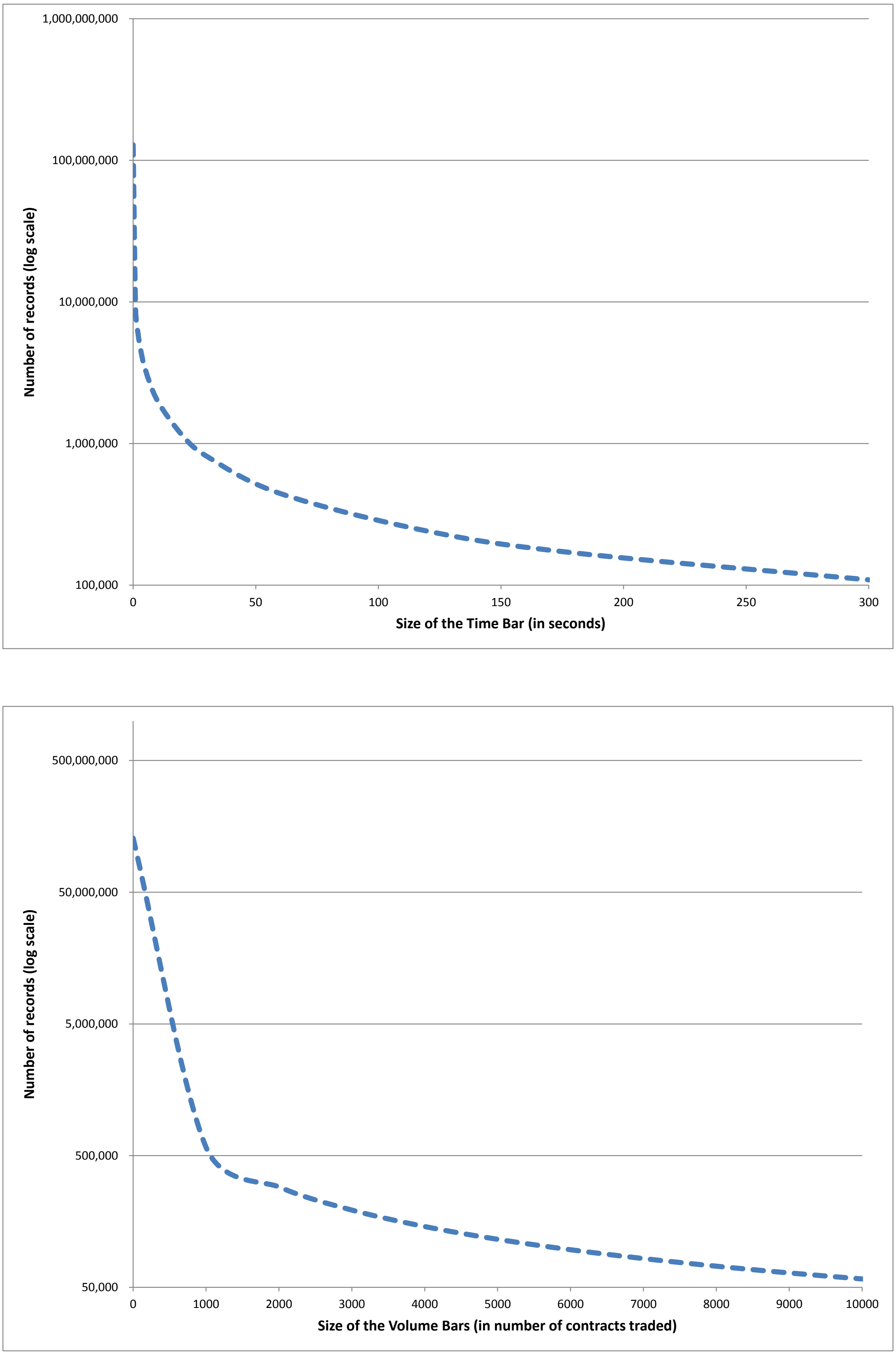
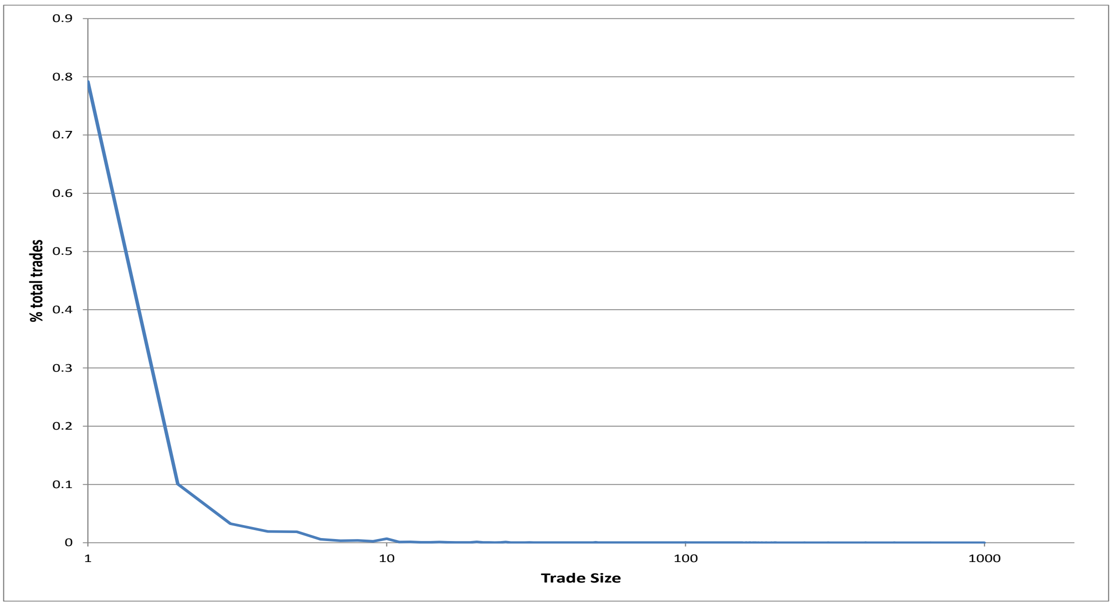


Figure 3 – Percentage of trades as a function of the trade size (in log scale)

(a) WTI Crude Oil Futures



(b) Gold Futures

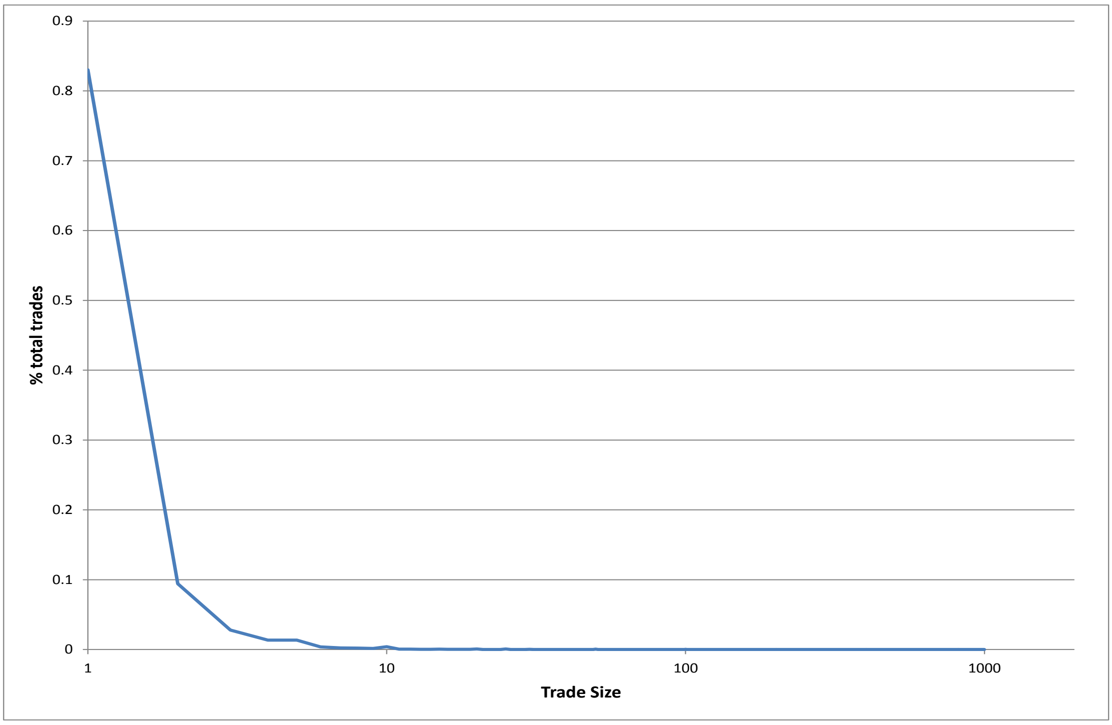
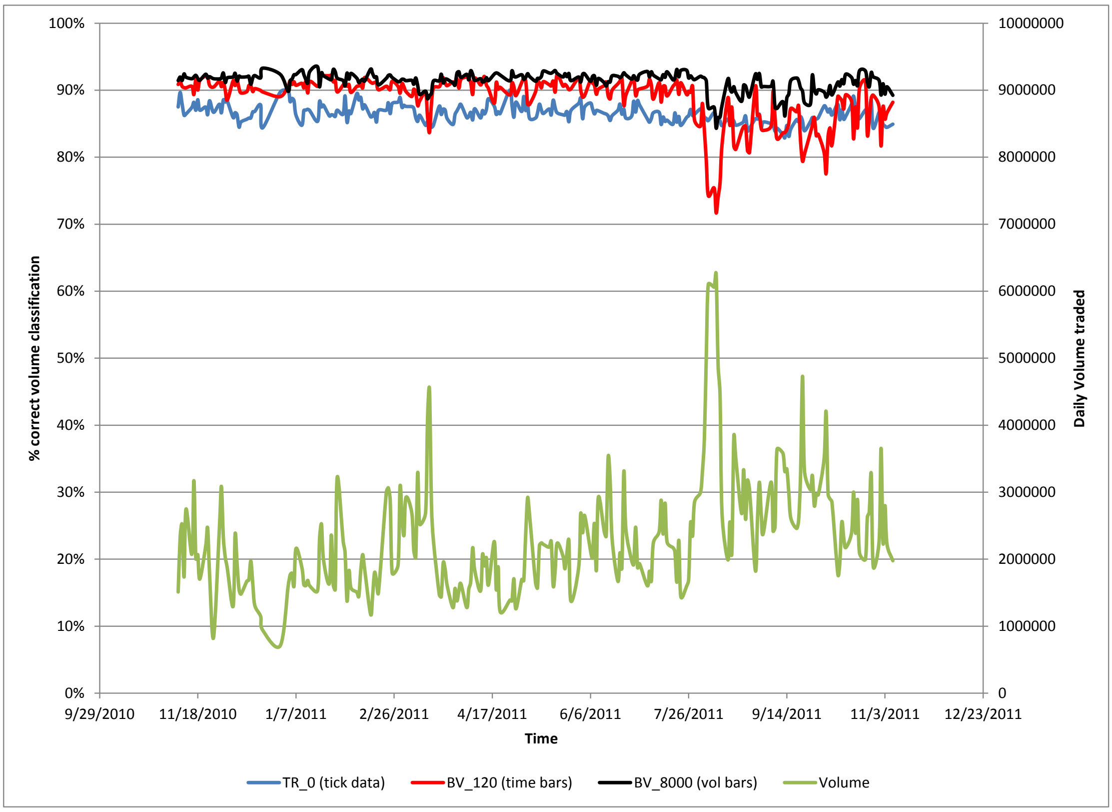
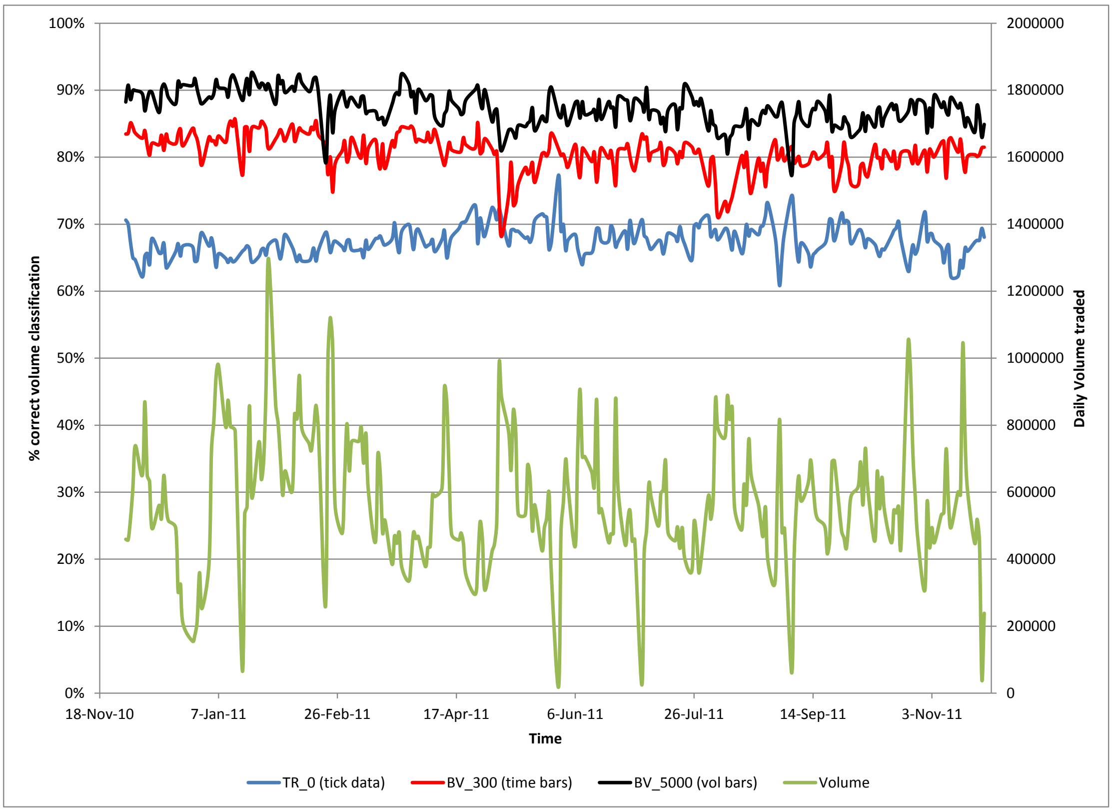


Figure 4 – Classification accuracy over time

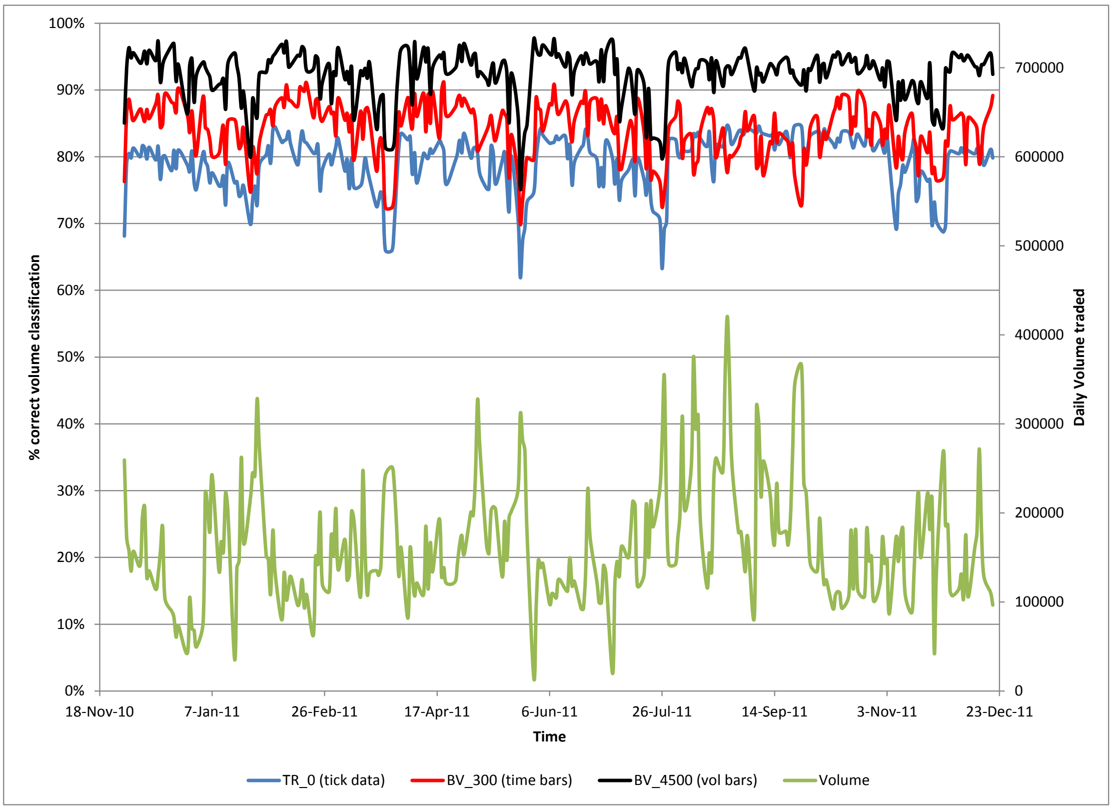
(a)[E-mini S&P500 Futures]



(b) WTI Crude Oil Futures



(c) Gold Futures



DISCLAIMER

The views expressed in this paper are those of the authors and not necessarily reflect those of Tudor Investment Corporation. No investment decision or particular course of action is recommended by this paper.