Walking without thinking about it

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Abstract— We demonstrate in this paper our motion genera-
tion sheme for the generation of stable bipedal walking motions
and we expand it to enhance its flexibility and independency.
An algorithm for the control of appropriate orientations of
the feet and the trunk permits the robot to turn in a natural
and safe way. Polygonal constraints on the positions of the
computed feet positions serve to improve its reliability. A logic
for the succession of the support phases and an algorithm for
the automatic control of their orientations bridge the gap to
more autonomy and to more practicability.

Keywords: Walking Humanoid Robot, Linear Model Pre-
dictive Control

I. INTRODUCTION

Walking can be realized only with the help of contact
forces between the feet and the ground, and feet can only
push on the ground. This involves *unilateral contact* [1],
which limits the motions that a human or robot can realize.
In the case of walking on a flat ground, this corresponds
to the fact that the Center of Pressure (CoP) can only lie
within the support polygon [18]. One class of control laws
which can handle such constraints explicitly and efficiently
and still generate stable motions is Model Predictive Control
(MPC) [9], [10], which maintains a model-based prediction
of the future motion of the system on a time horizon which
can be of varying length. Such control schemes involve either
an objective function which is minimized over the prediction
horizon or a constraint on the state of the system at the end
of the horizon, or a combination of both. We can observe
that the control laws for dynamic walking machines proposed
in [2], [5], [6], [8], [7], [11], [12], [14], [13], [15], [16], [17],
[19], [20] all belong to this class of MPC schemes, although
not presented as such in some cases.

A first category considers only constraints on the state
of the system at the end of the horizon, which completely
specify the generated motion, such as in [5], [11]. A frequent
option is to consider cyclicity at the end of the horizon, in [8],
[16], [17]. Another interesting option is to consider that the
prediction horizon always ends in a state where the robot
is standing still so that we are sure to always have a safe
option at hand in case a problem occurs [14]. A downside
of these approaches is that being entirely determined by
the state of the system at the beginning and end of the
horizon, the generated motions may lack some flexibility

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that could help tackle secondary objectives such as absorbing
perturbations efficiently or following precise speed require-
ments. A relaxation of these constraints has been proposed
in [15], but in such a form that the generated motion is still
entirely specified by the objective state given at the end of the
prediction horizon, resulting in the same lack of flexibility.

A second category considers only an objective function
minimized over the prediction horizon, as introduced orig-
inally in [7]. A combination of an objective function with
constraints at the end of the horizon has been proposed
in [12], but this combination is not necessary. It has been
shown indeed in [20] that a wide class of objective functions
gives rise to stable walking motions without constraints at the
end of the horizon, and numerous experiments have shown
the real-world efficiency of this approach [13]. This approach
introduces a lot of flexibility in the generated motions,
which can be used for example to absorb perturbations
efficiently [19], dealing explicitly with the constraints on
the contact forces. It is therefore this approach that we're
going to follow here. As advocated in [9], it is when dealing
directly with constraints that MPC schemes really shine.
This introduces here inequality constraints in the underlying
optimization problem, what can degrade computation time.
But what takes time in that case is to discover which
inequality constraints hold as equalities at the optimum, and
we can hopefully make a good guess about this, and solve
this optimization problem with appropriate algorithms in
nearly the same time as without inequality constraints [3].

Additional flexibility can also be introduced in the foot
step placement [2], what can be used altogether to track
precisely any given reference speed [6]. This opens the way
to *walking without thinking about it*: only a desired direction
of locomotion needs to be precised and the whole walking
motion is taken care of seamlessly, including reactive mo-
tions of the feet and CoM in case of perturbations or change
of desired direction. This allows introducing for example
continuous visual control of the walk [4], but:

- Rotations of the robot haven't been considered so far
  within these MPC schemes, so we introduce here an
  MPC scheme generating also rotating motions of the
  body and feet when required, automatically and safely.
- The safety of the foot step placements generated au-
  tomatically by the MPC scheme must be ensured.
  Kinematic feasibility of the steps can be enforced easily
  as in [14]. We propose here to go a bit further and ensure
  as well the long-term dynamic feasibility of the steps.
- The time sequence of steps, including start and stop
  motions, is usually decided externally from the MPC
  schemes. We propose here a brief formalization of this

sequence in the form of a Finite State Machine (FSM), which is incorporated inside the numeric construction of the MPC scheme.

We're going therefore to begin with describing in Sections II and III the MPC scheme originally introduced in [6], and present in the following Sections these three additions which allow reaching finally this goal of *walking without thinking about it*.

II. KEY ELEMENTS OF A DYNAMIC WALKING MOTION

A. Motion of the Center of Mass

Let's consider a frame $\mathcal{C}$ attached to the position of the Center of Mass (CoM) of the robot and to the orientation of its trunk. The position and orientation of this frame will be noted

$$c = \begin{pmatrix} c^x & c^y & c^z & c^\varphi & c^\psi & c^\theta \end{pmatrix}, \tag{1}$$

with Cardan angles c^φ, c^ψ and c^θ. The acceleration $\ddot{c}$ of this frame has to be continuous for being realized properly with usual actuators. We'll consider here that it is in fact piecewise linear on time intervals of constant length τ, so with a piecewise constant jerk $\dddot{c}$ on these intervals. The trajectory of this frame over longer time intervals of length $n\tau$ can be obtained simply by integrating in time this piecewise constant jerk together with the initial speed and acceleration. For any coordinate $\alpha \in \{x, y, z, \varphi, \psi, \theta\}$, this leads to simple linear relationships (details can be found in [6])

$$C_{i+1}^\alpha = S_p \hat{c}_i^\alpha + U_p \dddot{C}_i^\alpha, \tag{2}$$
$$\dot{C}_{i+1}^\alpha = S_v \hat{c}_i^\alpha + U_v \dddot{C}_i^\alpha, \tag{3}$$
$$\ddot{C}_{i+1}^\alpha = S_a \hat{c}_i^\alpha + U_a \dddot{C}_i^\alpha, \tag{4}$$

where

$$C_{i+1}^\alpha = \begin{bmatrix} c^\alpha(t_{i+1}) \\ \vdots \\ c^\alpha(t_{i+n}) \end{bmatrix}, \dots \dddot{C}_{i+1}^\alpha = \begin{bmatrix} \dddot{c}^\alpha(t_{i+1}) \\ \vdots \\ \dddot{c}^\alpha(t_{i+n}) \end{bmatrix} \tag{5}$$

and

$$\hat{c}_i^\alpha = \begin{bmatrix} c^\alpha(t_i) \\ \dot{c}^\alpha(t_i) \\ \ddot{c}^\alpha(t_i) \end{bmatrix}. \tag{6}$$

B. Motion of the Center of Pressure

The position z of the Center of Pressure (CoP) on the ground can be approximated by considering only the inertial effects due to the translation of the CoM, neglecting the other effects due to the rotations of the different parts of the robot. This proves to be a very effective approximation, which leads to the simple relationships

$$z_i^x = c_i^x - (c_i^z - z_i^z)\ddot{c}_i^x/g, \tag{7}$$
$$z_i^y = c_i^y - (c_i^z - z_i^z)\ddot{c}_i^y/g, \tag{8}$$

where the difference $c_i^z - z_i^z$ along the z coordinate corresponds to the height of the CoM above the ground, and g is the norm of the gravity force. We'll consider here only the simple case where the height of the CoM above the ground

is constant. In that case, we can obtain a relationship similar to (2)-(4):

$$Z_{i+1}^x = S_z \hat{c}_i^x + U_z \dddot{C}_i^x, \tag{9}$$
$$Z_{i+1}^y = S_z \hat{c}_i^y + U_z \dddot{C}_i^y, \tag{10}$$

with

$$S_z = S_p - (c_i^z - z_i^z)S_a/g, \tag{11}$$
$$U_z = U_p - (c_i^z - z_i^z)U_a/g. \tag{12}$$

C. Motion of the feet on the ground

It has been proposed in [6] to check the feasibility of the motion of the robot only at the instants t_i. It can be easily shown that this is sufficient even for a time interval τ as long as 100 ms in the case of standard walking motions. These t_i need however to be synchronized with the contact transitions, when each foot reaches or leaves the ground. Choosing then a time interval τ corresponding exactly to the duration of the double support, it has been concluded that each time we check the feasibility of the motion at the instants t_i, only one foot is firmly on the ground. We need therefore to consider only cases with one foot on the ground.

Attaching a frame $\mathcal{F}$ to the foot on the ground, let's consider the current position and orientation f_i and the position and orientation of the following steps $\bar{F}_{i+1}$. The positions and orientations F_{i+1} at all instants $t_{i+1} \dots t_{i+n}$ can be expressed then in a compact form

$$F_{i+1}^\alpha = V_{i+1} f_i^\alpha + \bar{V}_{i+1} \bar{F}_{i+1}^\alpha \tag{13}$$

for all coordinates $\alpha \in \{x, y, z, \varphi, \psi, \theta\}$ with $V_{i+1}, \bar{V}_{i+1}$ being selection matrices indicating which sampling time t_{i+j} falls within which step (sampling times corresponding to rows and steps to columns) and therefore which foot position is active at what time.

III. FOLLOWING A REFERENCE TRANSLATION SPEED

A. A Model Predictive Control scheme

It has been proposed in [6] to generate walking motions by directly following a reference speed $\dot{c}_{ref}$. Only horizontal translations were considered however, and two options were proposed, either regulating the instantaneous speed $\dot{c}(t)$ or the mean speed over two steps. We consider here a combination of these two options. Secondary objectives were also introduced to help obtain a more satisfying behaviour: centering the position of the feet with respect to the position of the CoP, and minimizing the jerk $\dddot{c}(t)$ in order to slightly smooth the resulting trajectory. Taking these objectives into account over a prediction horizon of length $n\tau$, with motions of the CoM, CoP and feet on the ground as introduced in the previous Section, we will consider the following optimization

problem,

$$\min \frac{\alpha}{2}\left\|\dot{C}_{i+1}^x - \dot{c}_{ref}^x\right\|^2 + \frac{\alpha}{2}\left\|\dot{C}_{i+1}^y - \dot{c}_{ref}^y\right\|^2$$
$$+ \frac{\beta}{2}\left\|EC_{i+1}^x - \dot{c}_{ref}^x\right\|^2 + \frac{\beta}{2}\left\|EC_{i+1}^y - \dot{c}_{ref}^y\right\|^2$$
$$+ \frac{\gamma}{2}\left\|F_{i+1}^x - Z_{i+1}^x\right\|^2 + \frac{\gamma}{2}\left\|F_{i+1}^y - Z_{i+1}^y\right\|^2$$
$$+ \frac{\varepsilon}{2}\left\|\dddot{C}_i^x\right\|^2 + \frac{\varepsilon}{2}\left\|\dddot{C}_i^y\right\|^2 \quad (14)$$

with a double diagonal matrix

$$E = \begin{bmatrix} -I & 0 & I \end{bmatrix}/2\tau_{step} \quad (15)$$

which computes the mean speed of the CoM over two steps out of C_{i+1}^x and C_{i+1}^y, with τ_{step} the duration of a step.

Considering more precisely the motions (2)-(4), (9)-(10) and (13), this optimization problem can be expressed as a canonical QP

$$\min_{u_i} \frac{1}{2}u_i^T Q_i u_i + p_i^T u_i \quad (16)$$

over the vector

$$u_i = \begin{bmatrix} \dddot{C}_i^x \\ \bar{F}_{i+1}^x \\ \dddot{C}_i^y \\ \bar{F}_{i+1}^y \end{bmatrix} \quad (17)$$

of motion parameters which gathers the jerk $\dddot{C}_i$ of the CoM and the future steps $\bar{F}_{i+1}$, with a cyclically varying quadratic term because of the cyclically varying matrix $\bar{V}_{i+1}$:

$$Q_i = \begin{bmatrix} Q_i' & 0 \\ 0 & Q_i' \end{bmatrix}, \quad (18)$$

$$Q_i' = \begin{bmatrix} \alpha U_v^T U_v + \beta U_p^T E^T E U_p + \gamma U_z^T U_z + \varepsilon I & -\gamma U_z^T \bar{V}_{i+1} \\ -\gamma \bar{V}_{i+1}^T U_z & \gamma \bar{V}_{i+1}^T \bar{V}_{i+1} \end{bmatrix} \quad (19)$$

and

$$p_i = \begin{bmatrix} \alpha U_v^T (S_v \hat{c}_i^x - \dot{c}_{ref}^x) + \beta U_p^T E^T (ES_p \hat{c}_i^x - \dot{c}_{ref}^x) + \\ \gamma U_z^T (S_z \hat{c}_i^x - V_{i+1} f_i^x) \\ -\gamma \bar{V}_{i+1}^T (S_z \hat{c}_i^x - V_{i+1} f_i^x) \\ \alpha U_v^T (S_v \hat{c}_i^y - \dot{c}_{ref}^y) + \beta U_p^T E^T (ES_p \hat{c}_i^y - \dot{c}_{ref}^y) + \\ \gamma U_z^T (S_z \hat{c}_i^y - V_{i+1} f_i^y) \\ -\gamma \bar{V}_{i+1}^T (S_z \hat{c}_i^y - V_{i+1} f_i^y) \end{bmatrix}. \quad (20)$$

B. Constraints on the Center of Pressure

Since the feet of the robot can only push on the ground, the CoP can lie only within the support polygon, the convex hull of the contact points between the feet and the ground [18]. Any trajectory not satisfying this constraint can't be realized properly, so this needs to be taken into account when computing a walking motion with the MPC scheme (14). Following [6], we'll consider that the foot on the ground has a polygonal shape, potentially depending on the support foot

$s \in \{Left, Right\}$, so that this constraint can be expressed as a set of linear constraints on the position of the CoP:

$$\begin{bmatrix} d_s^x(f^\theta) & d_s^y(f^\theta) \end{bmatrix} \begin{bmatrix} z^x - f^x \\ z^y - f^y \end{bmatrix} \le b(f^\theta) \quad (21)$$

which are linear with respect to the position (f^x, f^y) of the foot on the ground but nonlinear with respect to its orientation f^θ. The column vectors d^x and d^y gather the x and y coordinates of the normal vectors to the edges of the feet and the column vector b corresponds to the positioning of these edges.

Considering this constraint at all instants $t_{i+1} \ldots t_{i+n}$ can be expressed with the help of (13) as

$$D_{i+1} \begin{bmatrix} Z_{i+1}^x - V_{i+1}f_i^x - \bar{V}_{i+1}\bar{F}_{i+1}^x \\ Z_{i+1}^y - V_{i+1}f_i^y - \bar{V}_{i+1}\bar{F}_{i+1}^y \end{bmatrix} \le b_{i+1}, \quad (22)$$

with the simple double block-diagonal matrix

$$D_{i+1} = \begin{bmatrix} d_{s_{i+1}}^x(f_{i+1}^\theta) & 0 & d_{s_{i+1}}^y(f_{i+1}^\theta) & 0 \\ & \ddots & & \ddots \\ 0 & d_{s_{i+n}}^x(f_{i+n}^\theta) & 0 & d_{s_{i+n}}^y(f_{i+n}^\theta) \end{bmatrix} \quad (23)$$

and the vector

$$b_{i+1} = \begin{bmatrix} b(f_{i+1}^\theta) \\ \vdots \\ b(f_{i+n}^\theta) \end{bmatrix}. \quad (24)$$

With respect to the vector u_i introduced in (17), this constraint takes the following form

$$D_{i+1} \begin{bmatrix} U_z & -\bar{V}_{i+1} & 0 & 0 \\ 0 & 0 & U_z & -\bar{V}_{i+1} \end{bmatrix} u_i$$
$$\le b_{i+1} + D_{i+1} \begin{bmatrix} V_{i+1}f_i^x - S_z \hat{c}_i^x \\ V_{i+1}f_i^y - S_z \hat{c}_i^y \end{bmatrix}, \quad (25)$$

which can be introduced directly in the QP (16).

IV. FOLLOWING A REFERENCE ROTATION SPEED

Considering also the rotations of the frame c around the vertical axis z in (14) can be done straightforward by employing the same linear relationships (2)-(4) as for the translational motions.

When rotating the robot has to take care of appropriate orientations of the feet that have to follow the rotating trunk in one way or another. Since the most efficient walk is in the sagittal direction the feet should be aligned with the trunk the most of the time. We propose therefore the following optimization problem that minimizes the squared sum of the angular deviations between the feet and the trunk for the whole preview period:

$$\min \frac{\alpha}{2}\left\|\dot{C}_{i+1}^{\theta}-\dot{c}_{ref}^{\theta}\right\|^2 + \frac{\beta}{2}\left\|EC_{i+1}^{\theta}-\dot{c}_{ref}^{\theta}\right\|^2 \tag{26}$$

$$+\frac{\gamma}{2}\left\|\sum_i(f_i^{\theta}-c_i^{\theta})\right\|^2 \tag{27}$$

$$s.t. \begin{bmatrix} E_L & 0 \\ 0 & E_R \end{bmatrix}\begin{bmatrix} \dot{F}_{i+1}^{\prime\theta,L} \\ \dot{F}_{i+1}^{\prime\theta,R} \end{bmatrix} = 0, \tag{28}$$

with $\dot{F}_{i+1}^{\prime\theta} = S_v\hat{f}_i^{\theta} + U_v\ddot{F}_i^{\theta}$ the complete trajectory of the left or the right foot and $E_{R/L}$ the selection matrices that select all elements of $\dot{F}_{i+1}^{\prime\theta}$ that fall into the support time of the corresponding foot.

Naturally, aligning the feet as much as possible with the trunk reduces also the probability of a violation of physical constraints but it might not be enough so that we can add hard constraints to the above QP:

$$\|F_{i+1}^{\prime\theta,R} - F_{i+1}^{\prime\theta,L}\| < \theta_{max}^{rl} \tag{29}$$

$$\|F_{i+1}^{\prime\theta} - C_{i+1}^{\theta}\| < \theta_{max}^{FT} \tag{30}$$

$$\|\dot{F}_{i+1}^{\prime\theta} - \dot{C}_{i+1}^{\theta}\| < \dot{\theta}_{max}^{FT} \tag{31}$$

$$\|\ddot{F}_{i+1}^{\prime\theta} - \ddot{C}_{i+1}^{\theta}\| < \ddot{\theta}_{max}^{FT}. \tag{32}$$

The constraint (30) would limit the intersection angles of the feet to prevent self collisions. Constraints (30)-(32) would limit the angles, the velocity and the acceleration of the feet with respect to the trunk.

Contrary to the translational motions of the feet (section II-C) we use for their orientations the relationships (2)-(4) instead of (13). Thus we can generate smoother trajectories of the feet orientations and their derivatives and to apply the constraints (30)-(32) also for the feet in the air.

As mentioned in section III-B, including the orientations $f^{\theta,s}$ of the feet in contact with the ground as free variables would result in nonlinear CoP constraints (22). In order to keep the linear form of the original QP we choose not to extend it but to predetermine the orientations of the feet and the trunk by solving the above QP prior to (14).

V. CONSTRAINTS ON THE FOOT PLACEMENT

We need to assure that the footsteps decided by (14) are feasible with respect to joint angles and speed limitations, self-collision and over-stretching avoidance and similar geometric and kinematic limitations. What we need to do, in order to keep the Linear MPC structure of our algorithm, is to derive simple approximations of all these limitations that can be expressed in the form of linear constraints on the vector u_i defined in III-A.

We can derive for example simple linear constraints on the positions of the feet one with respect to the other with minimum and maximum values preventing collision on one side and over-stretching of the legs on the other side:

$$A_{j+1} = \begin{bmatrix} -1\,1 & 0 & -1\,1 & 0 \\ & \ddots & & \ddots \\ 0 & -1\,1 & 0 & -1\,1 \end{bmatrix}\begin{bmatrix} f_i^x \\ \bar{F}_{i+1}^x \\ f_i^y \\ \bar{F}_{i+1}^y \end{bmatrix} \le b_{j+1} \tag{33}$$
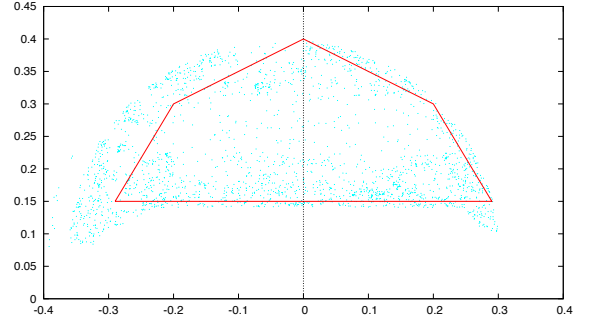

Fig. 1. Polygonal approximation of the feasible reagion of the final position of one foot relative to the support foot.

with

$$A_{j+1} =$$
$$\begin{bmatrix} a_{s,j+1}^x(f_{j+1}^{\theta}) & 0 & a_{s,j+1}^y(f_{j+1}^{\theta}) & 0 \\ & \ddots & & \ddots \\ 0 & a_{s,j+n}^x(f_{j+n}^{\theta}) & 0 & a_{s,j+n}^y(f_{j+n}^{\theta}) \end{bmatrix} \tag{34}$$

To do so, we took in consideration the MPC scheme in [7] already implemented on the HRP-2 robot. Although the regions of feasibility obviously differ, at least slightly, from one MPC to another, what we are looking for is simply a relevant initial heuristic. Later on, if we apply our scheme to HRP-2, we can observe the effects of the initial heuristic, and thus start a hopefully converging sequence of modifications of the polygon, in order to get one more suited to our own Linear MPC structure.

Starting from the standard initial position of HRP-2, we used extensive offline calculations to build a point cloud of positions that can be reached by the center of the left foot with one single step.

The offline computation involved random positions which were used as inputs by the MPC scheme in [7] to generate joint space trajectories. Each trajectory was then checked with an *ad hoc* verification process which declared feasible a trajectory when it does not violate joint limits, avoids self-collisions and respects some other dynamic constraint.

The point cloud on Fig. 1 shows all the positions that lead to feasible trajectories. We define a 5 edges polygon included in this area, as shown on Fig. 1. It is symmetric about the y-axis in order not to take into account specificities of the HRP-2 robot or the MPC scheme in [7] that tend to result in a slightly larger feasible zone when walking backwards.

VI. THE LOGIC OF WALKING

In order to avoid nonlinearities, the MPC scheme introduced in [6] takes into account only simple supports. Because of a sampling synchronized with the contact transitions (see II-C) this is valable during walking but restricting during a stop. To correct this deficiency we propose to stop only with the feet aligned in the lateral plane of the
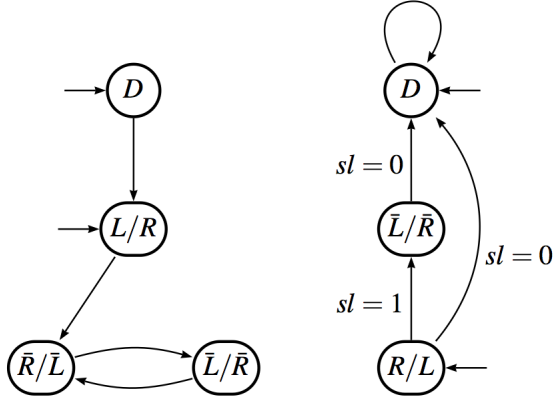
Fig. 2. Evolution of CoP constraints for $\|\dot{X}\| > 0$ (left) and $\|\dot{X}\| = 0$ (right) as a finite state machine.
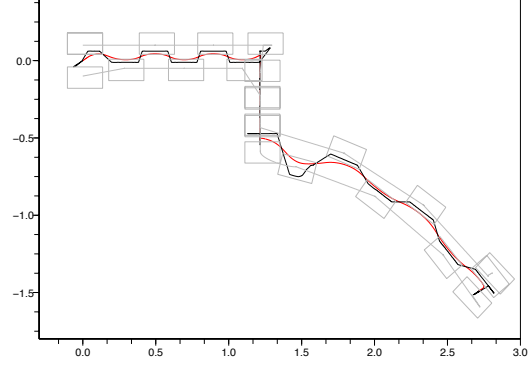

Fig. 3. Motion of the CoM (in red) and CoP (in black) and foot steps generated by our MPC scheme when the system is not submitted to any perturbation. Note an evolution of the CoP from heels to toes very similar to what can be observed in humans.

robot. The double support phase can then be smoothly integrated in the computation of the CoP constraints due to its rectangular form, $s \in \{L, R, D\}$. The assembling of the constraints and the matrices V_{i+1}, $\bar{V}_{i+1}$, E_R, E_L requires for each instant of the whole preview period prior knowledge about the corresponding support phase s_i. This knowledge can be provided by the finite state machine (FSM) in Fig. 2. Every state of this FSM corresponds thus to a vector $\left(d_x^s(f^\theta), d_y^s(f^\theta)\right)$ and a nonzero entry either in $\bar{V}_{i+1}$ or V_{i+1} and E_R or E_L depending on the step number and the previewed sampling time. Given a constant speed reference for the whole preview window the switching process is defined only by the predefined lengths of the simple and double support phases.

A fixed double support phase can require to effectuate one step before the stop in a double support phase, which led to the introduction of the supplementary parameter sl which defines the number of steps to be done before a stop (Fig. 2, right).

VII. SIMULATION RESULTS

Fig. 3 shows a walking gait generated by our MPC scheme when the system is not submitted to any perturbation. The robot starts from rest in a double support phase which is limited to $0.4\,s$ and walks continuously for $20\,s$, making a step regularly every $0.8\,s$. $0.4\,s$ after the switch of the reference velocity to $0.25\,\frac{m}{s}$ the robot starts to walk forward by lifting its right foot first. Five seconds later the reference speed vector is instantly turned by 90 degrees to the right. At the beginning of the tenth second the reference is switched again to $0.25\,\frac{m}{s}$ forward and an additional rotational reference speed of $-\frac{\pi}{20}\,\frac{deg}{s}$ is applied. Finally all references are removed and the robot passes after one additional step into double support with the feet aligned in the same lateral plane and with equal angles. We can see that the QP together with the additional control layers, introduced in the last three sections, manage all requirements including the turning motion and the stop in a double support phase. Especially, when a rotation is desired the algorithm proves to be able

to find appropriate feet angles during walking and for the predetermined double support constraints.

Fig. 4 shows the forward speed of the CoM for three different demanded saggital velocities. We can observe that the QP manages to perfectly realize the desired motion for velocities slower than $0.3\,\frac{m}{s}$. For slightly bigger references oscillations occur but it still manages to maintain the desired mean value. These oscillations occur when step length become such that successive support polygons do not superpose in the saggital plane. Consequently, the Center of Pressure cannot permanently share its lateral plane with the Center of Mass but has to deviate which results in forward acceleration of the CoM. During the next support phase this acceleration has not only to be absorbed but the CoM has to be decelerated so that the desired mean value can be maintained in the best possible way. Constraints on the feet positions together with fixed support lengths limit finally the highest possible speed that the QP is able to realize when a too big velocity reference is applied, as can be seen on the right side of Fig. 4. However, this last part demonstrates one of the most valuable properties of this walking motion generation scheme: safety prevails, in the sense that the generated motion is always kept feasible, even if that means not realizing the desired motion. Here, the goal of the robot is to move forward at a certain speed, but this goal is fulfilled only as much as possible.

To demonstrate the behavior of the feet orientations control layer introduced in section IV we simulated situations where at least one of the angular constraints would be violated for a trunk that changes its velocity instantly. We can see on the left half of Fig. 5 that due to the angular velocity constraint on the feet the robot has to make several steps before being able to effectively minimize the angular deviations between the feet and the trunk. The right support foot on the contrary can not achieve this goal as its angle towards the left foot is limited. None of these violations does affect the rotation of the trunk, contrary to the following decreasing slope where the robot instantly changes its rotation to follow a
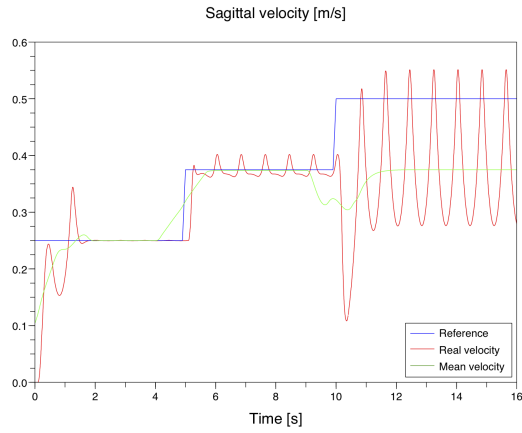
Fig. 4. Forward speed of the CoM (red) for three different reference velocities (blue).
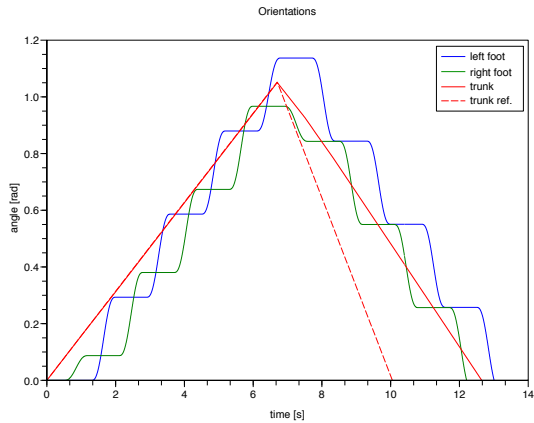

Fig. 5. Orientations of the trunk (red, solid) and the feet (blue and green) for a rotation reference of the trunk which would result in the dashed curve, supposed that the trunk can instantly change its velocity.

higher negative reference. Because it has been previewed that the constraint on the angle between the trunk and at least one previewed foot would be violated, the rotation speed of the trunk has been reduced by (30). This behaviours represents in our view a good trade-off between reliability and performance as the rotation rate of the robots trunk is reduced only when necessary to prevent violations of constraints.

VIII. CONCLUSION

Lacking important features on the side of the reliability and flexibility our LMPC scheme for walking motion generation needed to be completed in order to unfold its full potential. We have therefore defined approximations of admissible zones for the positionning of the feet and we added an inner logic for their succession. An additional layer on top of the original QP contributes a third degree of freedom to the control. The resulting scheme has been successfully implemented and tested for visual servoing tasks in [4].

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