Answers

Exercise 1.1

1 **a)** Integers: –7, 27, 1534, 0, –12

    b) Natural numbers: 27, 1534

2 **a)** $\dfrac{17}{20}$ is a fraction, so it is rational

    b) 0.46 is a terminating decimal, so it is rational

    c) $\sqrt{\dfrac{2}{25}} = \dfrac{\sqrt{2}}{5}$ and $\sqrt{2}$ is irrational, so $\sqrt{\dfrac{2}{25}}$ is irrational

    d) π is irrational, so 5π is irrational

    e) 3.141 59 is a terminating decimal, so it is rational

    f) $-0.2\dot{3}\dot{4}$ is a recurring decimal, so it is rational

    g) $\sqrt{\dfrac{4}{25}} = \dfrac{2}{5}$ which is a fraction, so it is rational

    h) $\sqrt{225} = 15$ which is a integer, so it is rational

    i) $2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$ and $3\sqrt{3}$ is irrational, so $2\sqrt{3} + \sqrt{3}$ is irrational

3 **a)** $\sqrt{169} = 13$ which is a integer, so it is rational

    b) 0.49 is a terminating decimal, so it is rational

    c) $\sqrt{3}$ is irrational, so $5 + \sqrt{3}$ is irrational

    d) -2.718 is a terminating decimal, so it is rational

    e) π is irrational, so $5\pi + 2$ is irrational

    f) $\dfrac{4\pi}{3\pi} = \dfrac{4}{3}$ which is a fraction, so it is rational

    g) $\sqrt{27} = 3\sqrt{3}$ and $3\sqrt{3}$ is irrational, so $\sqrt{27}$ is irrational

    h) $\sqrt{1\dfrac{7}{9}} = \sqrt{\dfrac{16}{9}} = \dfrac{4}{3}$ which is a fraction, so it is rational

    i) $\sqrt{2}$ is irrational, so $-6\sqrt{2}$ is irrational

4   **a)** π is an example of an irrational number between 3 and 4

   b) $\sqrt{110}$ is an example of an irrational number between 10 and 11

   c) $\sqrt{390}$ is an example of an irrational number between 19 and 20

Exercise 1.2

1   **a)**   1, 2, 4, 8

   b)   1, 3, 5, 15

   c)   1, 3, 9, 27

   d)   1, 2, 3, 6, 9, 18, 27, 54

2   2, 3, 5, 7, 11, 13, 17, 19, 23, 29

3   **a)**   2, 3

   b)   2, 5

   c)   5, 11

   d)   2, 3, 7

4   **a)** $2^4 \times 3$

   b) $2^3 \times 3^2$

   c) $2 \times 3 \times 5 \times 7$

   d) $2 \times 5^2 \times 7$

   e) 3×5^2

   f) $5^2 \times 11$

   g) $2^3 \times 3 \times 5$

   h) $2 \times 3^2 \times 11$

5   **a)** $3^2 \times 5 \times 11$

   b) $2^2 \times 5 \times 13$

   c) $2^2 \times 3^3 \times 5^2$

   d) $2 \times 7^2 \times 11$

   e) $2^2 \times 3 \times 5 \times 7$

f) $\quad 3^2 \times 5^3$

g) $\quad 2^4 \times 7$

h) $\quad 2^3 \times 5 \times 7^2$

6 **a)** **i)** $\quad 5^2$

ii) $\quad 2^2 \times 3^2$

iii) $\quad 2^2 \times 5^2$

iv) $\quad 2^4 \times 3^2$

b) $\quad$ Prime factors are all in pairs

7 **a)** $\quad 2^5 \times 3$

b) $\quad k = 6$

8 **a)** $\quad 2^3 \times 7^2$

b) $\quad k = 7$

Exercise 1.3

1 **a)** **i)** $\quad 18 = 2 \times 3^2, 24 = 2^3 \times 3$

ii) $\quad$ HCF = 6

iii) $\quad$ LCM = 72

b) **i)** $\quad 64 = 2^6, 100 = 2^2 \times 5^2$

ii) $\quad$ HCF = 4

iii) $\quad$ LCM = 1600

c) **i)** $\quad 50 = 2 \times 5^2, 350 = 2 \times 5^2 \times 7$

ii) $\quad$ HCF = 50

iii) $\quad$ LCM = 350

d) **i)** $\quad 72 = 2^3 \times 3^2, 126 = 2 \times 3^2 \times 7$

ii) $\quad$ HCF = 18

iii) $\quad$ LCM = 504

2 **a)** $\quad$ HCF = 9, LCM = 189

b) HCF = 10, LCM = 100

c) HCF = 12, LCM = 336

d) HCF = 2, LCM = 1600

e) HCF = 7, LCM = 294

3 **a)** **i)** $260 = 2^2 \times 5 \times 13$, $300 = 2^2 \times 3 \times 5^2$

ii) HCF = 20

iii) LCM = 3900

b) **i)** $340 = 2^2 \times 5 \times 17$, $425 = 5^2 \times 17$

ii) HCF = 85

iii) LCM = 1700

c) **i)** $756 = 2^2 \times 3^3 \times 7$, $2100 = 2^2 \times 3 \times 5^2 \times 7$

ii) HCF = 84

iii) LCM = 18 900

d) **i)** $1980 = 2^2 \times 3^2 \times 5 \times 11$, $2376 = 2^3 \times 3^3 \times 11$

ii) HCF = 396

iii) LCM = 11 880

4 **a)** HCF = 252, LCM = 49 896

b) HCF = 504, LCM = 42 336

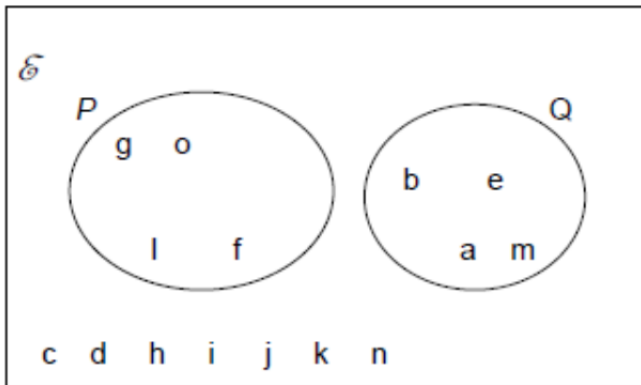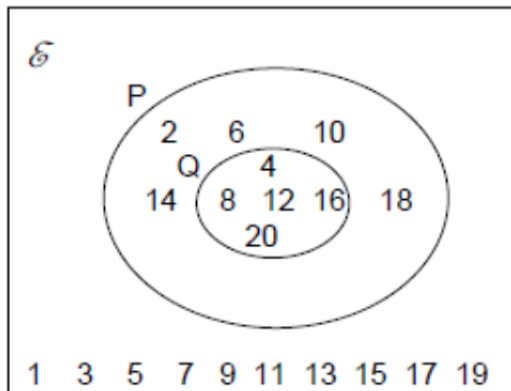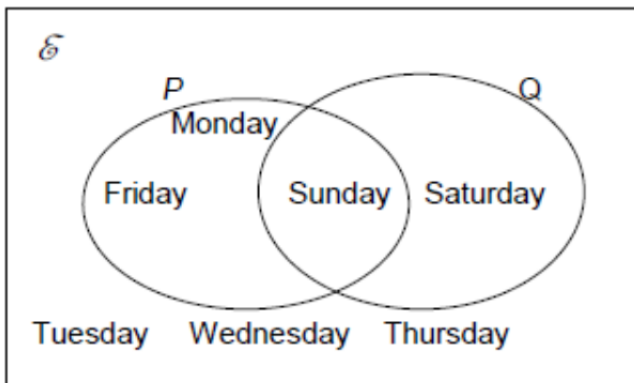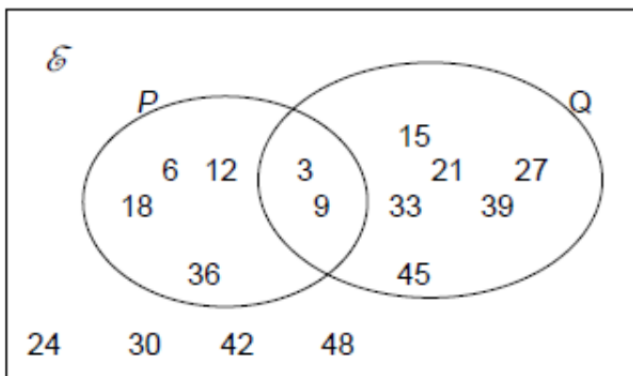**5** **a)** 15

b) 900

6 12 mm by 12 mm

7 10:02:56 p.m.

8 10:15 a.m.

Exercise 2.1

1 11, 12, 13, 14, 15, 16, 17, 18

2 2, 3, 5, 7, 11

3 1, 2, 3, 4, 6, 12

4 8, 16, 24, 32, 40, 48

5 a, e, i, o, u
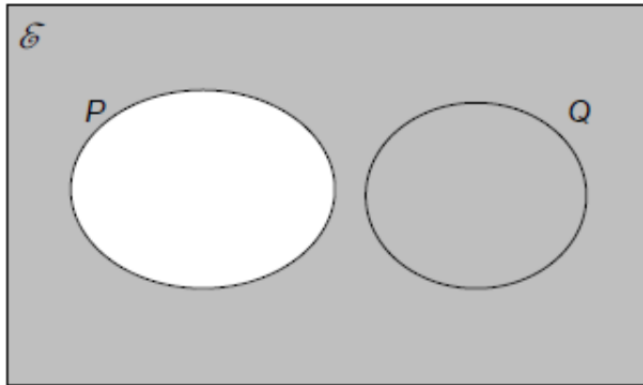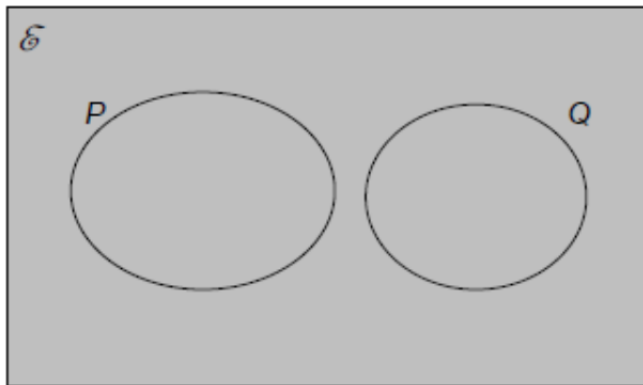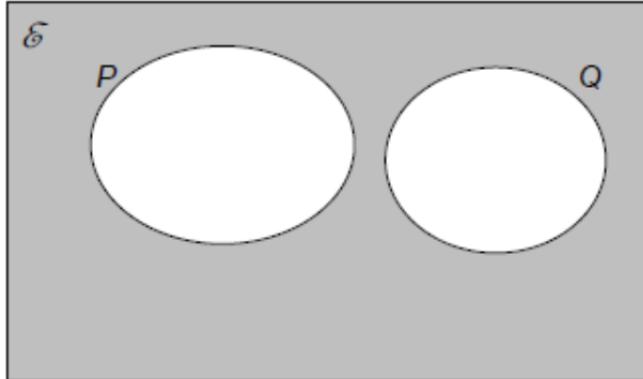
Exercise 2.2

1

2

3

4

5

6

MATHEMATICS **2** Set Language and Notation
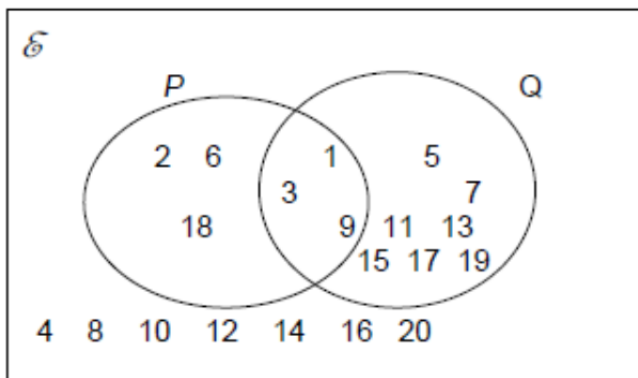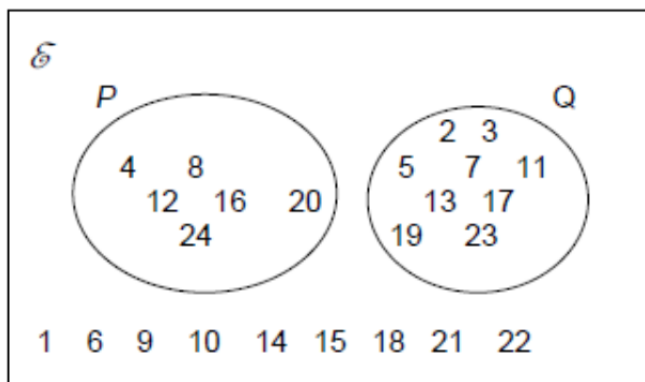
Exercise 2.3

1 **a)** {1, 3, 9}

b) {1, 2, 3, 5, 6, 7, 9, 11, 13, 15, 17, 18, 19}

2 **a)** Ø

b) {2, 3, 4, 5, 7, 8, 11, 12, 13, 16, 17, 19, 20, 23, 24}

3 **a)** Ø

b) {a, b, e, f, g, l, m, o}

4 **a)** {4, 8, 12, 16, 20}

b) {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}

5 **a)** {Sunday}

b) {Sunday, Monday, Friday, Saturday}

6 **a)** {3, 9}

b) {3, 6, 9, 12, 15, 18, 21, 27, 33, 36, 39, 45}

Exercise 2.4

1 **a)** 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

b) 4, 8, 10, 12, 14, 16, 20

2 **a)** 1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24

b) 1, 6, 9, 10, 14, 15, 18, 21, 22

3 **a)** c, d, f, g, h, i, j, k, l, n, o

b) c, d, h, i, j, k, n

4 **a)** 1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15, 17, 18, 19

b) 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

5 **a)** Monday, Tuesday, Wednesday, Thursday, Friday

b) Tuesday, Wednesday, Thursday

6 **a)** 6, 12, 18, 24, 30, 36, 42, 48
b) 24, 30, 42, 48

Exercise 2.5

1   **a)**

b)

c)

2   **a)**
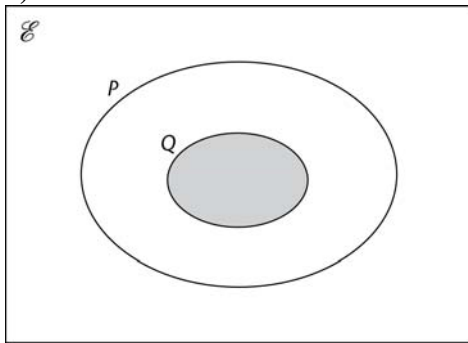
b)
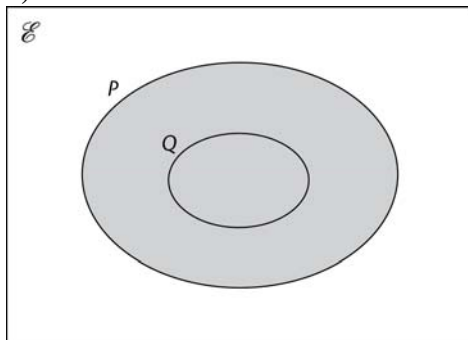
c)
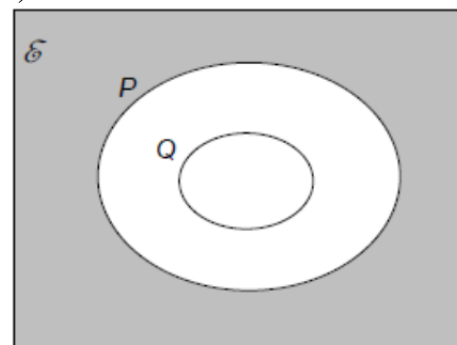
3   **a)**
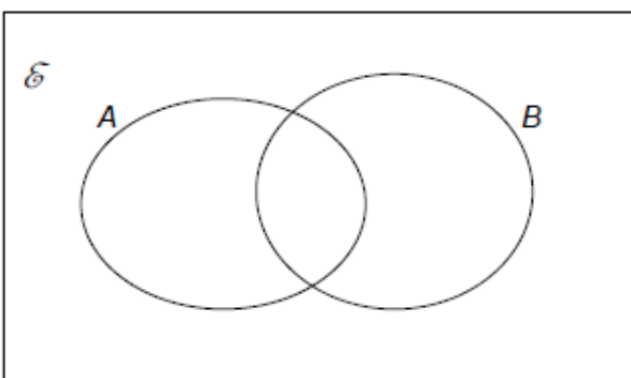
b)   {squares}

4   **a)**   **i) and ii)**
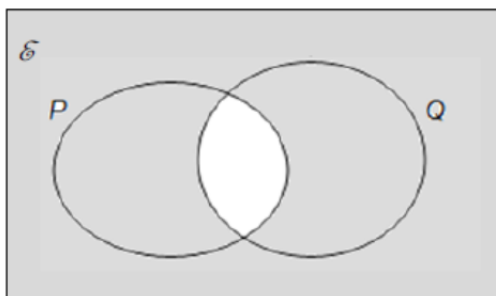
   b)   They are the same

5   **a)**   **i) and ii)**
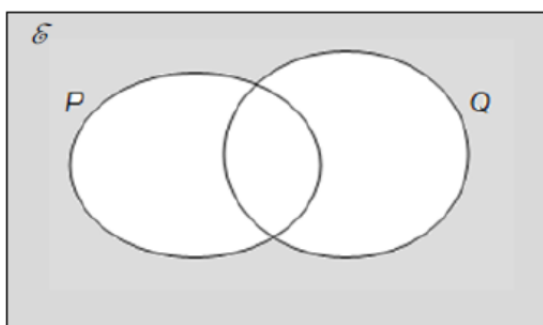
   b)   They are the same

Exercise 2.6

1   Ø, {p}, {q}, {r}, {p, q}, {p, r}, {q, r}, {p, q, r}

2   64

3   **a)**
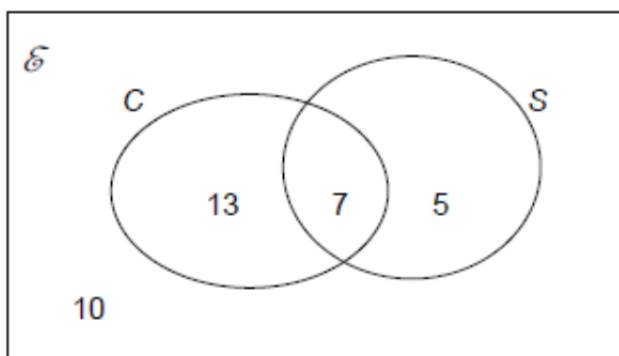
   b)   13
   c)   10
4   **a)**   17

b)
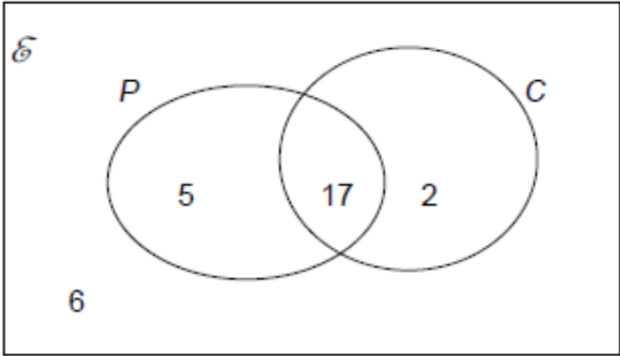
5 **a)**
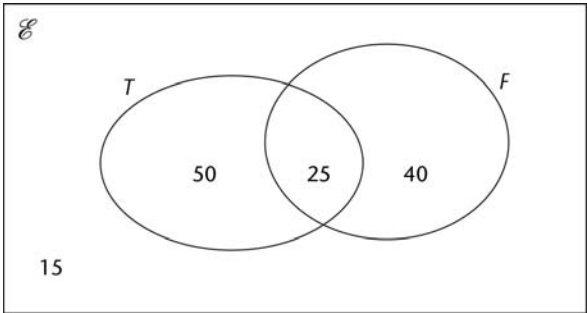
    b)   25

6   12

7   39

8
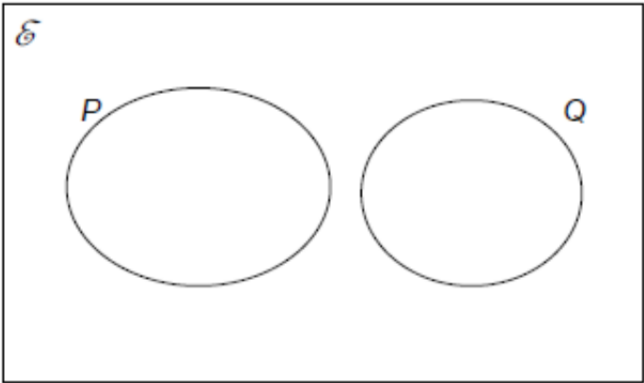
9
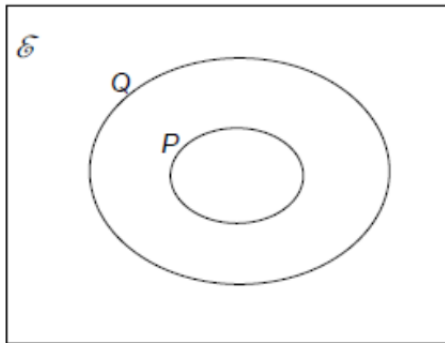
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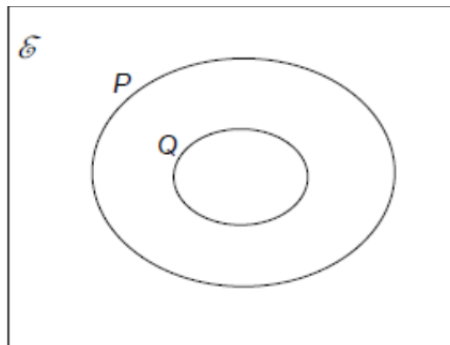
11   −2, −1, 0, 1, 2, 3

Exercise 2.7

1   **a)**
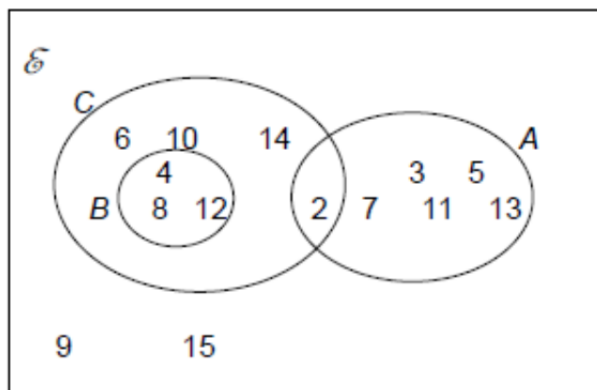
   b)   **i)**   7

          ii)   5

2   **i) – iv)**
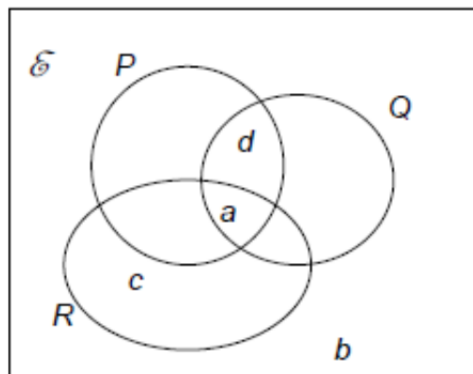
3   **a)** $B \cap C \cap A'$

      b) $(P \cap R) \cup (Q \cap R)$ or $(P \cup Q) \cap R$

4   **a)** $T \cap F$

      b)   E

5   **a)**   9

      b)   1

6   or
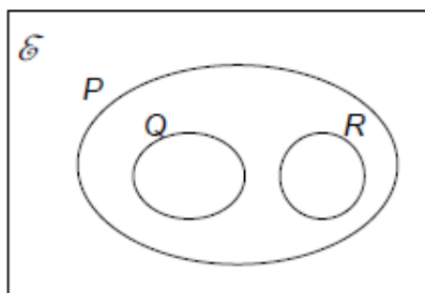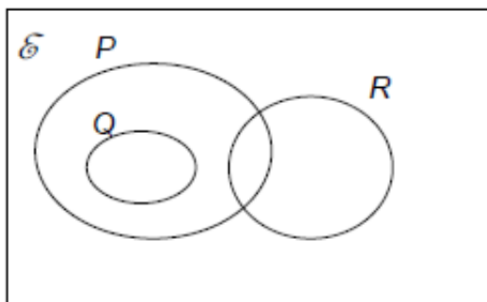
7   **a)** $P \cap Q = \varnothing$

      b) $R \subset Q$

      c) $R \cup Q = Q$

      d) $n(R \cap Q) = n(R)$

      e) $n(P) + n(R) = n(P \cup R)$

Exercise 3.1

1   **a)**   49

b)   144

c)   25

d)   100

e)   81

f)   64

g)   121

h)   9

i)   36

j)   16

2   **a)**   7

b)   11

c)   9

d)   6

e)   5

f)   13

g)   12

h)   15

i)   10

j)   14

3   **a)**   169

b)   121

c)   196

d)   36

e)   81

4   **a)**   9

b)   12

c)   4

d)   10

e)   8

5   **a)**   23

b)   16

c)   18

d)   29

e)   28

6   **a)**   400

b)   625

c)   169

d)   576

e)   1089

7   **a)**   11

b)   13

c)   33

d)   5

e)   41

f)   27

g)   0

h)   63

8   **a)**   360

     b)   525

     c)   185

     d)   1325

Exercise 3.2

1   **a)**   64

     b)   125

     c)   27

     d)   1000

     e)   512

2   **a)**   1

     b)   4

     c)   10

3   **a)**   343

     b)   729

c)   8000

d)   15 625

e)   3.375

f)   19.683

g)   157.464

4   **a)**   7

     b)   9

     c)   11

     d)   100

     e)   6

     f)   12

     g)   8

5   **a)**   3.83

     b)   6.13

     c)   8.09

     d)   10.32

     e)   19.60

6   3.56 cm

Exercise 4.1

1

	Start temperature/°C	Move/°C	End temperature/°C
a)	4	Up 3	7
b)	−2	Down 4	−6
c)	10	Down 14	−4
d)	−5	Down 3	−8
e)	−10	Up 8	−2
f)	10	Down 19	−9
g)	−4	Up 6	2
h)	3	Up 7	10
i)	−3	Down 6	−9
j)	−10	Up 2	−8

2   **a)**   Floor 4

    b)   Floor 2

3   **a)**   18 °C

    b)   20 °C

    c)   14 °C

    d)   48 °C

    e)   18 °C

    f)   25 °C

g) 18 °C

h) 50 °C

4 147 °C

5 −$166

6 19 855 m

7 64.6°C

8 66.2°C

9 9.9 m

Exercise 5.1

1 a) Improper fraction

b) Mixed number

c) Proper fraction

d) Mixed number

e) Proper fraction

2 a) $1\frac{3}{8}$

b) $2\frac{1}{5}$

c) $2\frac{1}{4}$

d) $3\frac{1}{2}$

e) $2\frac{1}{7}$

f) $3\frac{1}{3}$

g) $2\frac{3}{8}$

h) $5\frac{3}{4}$

i) $3\frac{3}{10}$

j) $4\frac{1}{9}$

3 a) $\dfrac{9}{8}$

b) $\dfrac{21}{8}$

c) $\dfrac{15}{4}$

d) $\dfrac{11}{2}$

e) $\dfrac{29}{9}$

f) $\dfrac{12}{5}$

g) $\dfrac{11}{3}$

h) $\dfrac{21}{10}$

i) $\dfrac{19}{8}$

j) $\dfrac{34}{7}$

Exercise 5.2

1 a) 48

b) 64

c) 25

d) 144

e) 165

2 468

3 15 630

4 $126

5 $\dfrac{7}{10}$ of \$120 = \$84 and $\dfrac{7}{8}$ of \$104 = \$91;

so $\dfrac{7}{8}$ of \$104 is more

6 $\dfrac{3}{8}$ of \$192 = \$72 and $\dfrac{2}{5}$ of \$180 = \$72;

so they are both the same

e) $\dfrac{2}{3}$

f) $\dfrac{5}{6}$

g) $\dfrac{1}{2}$

h) $\dfrac{3}{5}$

Exercise 5.3

1 a) $\dfrac{1}{4} = \dfrac{2}{8} = \dfrac{3}{12} = \dfrac{5}{20}$

b) $\dfrac{1}{5} = \dfrac{2}{10} = \dfrac{4}{20} = \dfrac{7}{35}$

c) $\dfrac{2}{5} = \dfrac{4}{10} = \dfrac{10}{25} = \dfrac{12}{30}$

d) $\dfrac{2}{9} = \dfrac{4}{18} = \dfrac{8}{36} = \dfrac{6}{27}$

e) $\dfrac{1}{7} = \dfrac{2}{14} = \dfrac{5}{35}$

f) $\dfrac{4}{9} = \dfrac{16}{36} = \dfrac{32}{72}$

2 a) $\dfrac{4}{5}$

b) $\dfrac{1}{6}$

c) $\dfrac{5}{7}$

d) $\dfrac{3}{4}$

i) $\dfrac{2}{3}$

j) $\dfrac{1}{2}$

k) $\dfrac{1}{6}$

l) $\dfrac{1}{2}$

m) $\dfrac{4}{5}$

n) $\dfrac{8}{9}$

o) $\dfrac{5}{7}$

p) $\dfrac{2}{3}$

3 $\dfrac{3}{8}$

4 $\dfrac{8}{15}$

5 a) $\dfrac{9}{25}$

b) $\dfrac{21}{50}$

c) $\dfrac{2}{5}$

Exercise 5.4

1 $\dfrac{7}{10}$

2 $\dfrac{29}{100}$

3 $\dfrac{17}{20}$

4 $\dfrac{7}{100}$

5 $\dfrac{39}{125}$

6 $\dfrac{7}{125}$

7 $\dfrac{1}{125}$

8 $\dfrac{1}{50}$

9 $\dfrac{109}{200}$

10 $\dfrac{269}{2000}$

Exercise 5.5

1 a) 0.375

b) 0.3125

c) 0.275

d) 0.316

e) 0.0375

2 a) $0.\dot{6}$

b) $0.8\dot{3}$

c) $0.08\dot{3}$

d) $0.2\dot{6}$

e) $0.\dot{4}\dot{8}$

3 a) $\dfrac{2}{9}$

b) $\dfrac{3}{9}$

c) $\dfrac{5}{9}$

4 a) $0.0\dot{7}\dot{4}$

b) $0.\dot{1}8\dot{5}$

c) $0.\dot{3}7\dot{0}$

d) $0.1\dot{8}$

e) $0.5\dot{4}$

Exercise 5.6

1 a) $\dfrac{7}{20}$

  b) $\dfrac{13}{20}$

  c) $\dfrac{2}{25}$

  d) $1\dfrac{1}{5}$

2 a) 0.16

  b) 0.27

  c) 0.83

  d) 0.07

  e) 0.31

  f) 0.04

  g) 0.17

  h) 0.02

  i) 1.5

  j) 2.5

  k) 0.09

  l) 0.125

3 a) 0.01

  b) 0.17

  c) 0.04

  d) 1.6

  e) 0.125

  f) 0.625

  g) 0.15

  h) 0.425

  i) 0.3125

4 a) 1%

  b) 17%

  c) 4%

  d) 160%

  e) 12.5%

  f) 62.5%

  g) 15%

  h) 42.5%

  i) 31.25%

5 a) 16.7%

  b) 83.3%

  c) 8.3%

  d) 41.7%

  e) 4.3%

6 Any three fractions equivalent to $\dfrac{2}{5}$

7 Any three fractions equivalent to $\dfrac{1}{8}$

8 a) 1.6

  b) $1\frac{3}{5}$

9 58%

10 22%

11 Red: 22% = 0.22; Silver: $\dfrac{3}{20}$ = 0.15; Black: $\dfrac{6}{25}$

  = 0.24. So black is the most popular.

12 Class A: $\dfrac{3}{7}$ = 0.428…; Class B: 45% = 0.45. So

  class B has the higher proportion of boys.

Exercise 6.1

1 a) −5°C, −2°C, 0°C, 3°C, 7°C

   b) −2°C, −1°C, 1°C, 2°C, 5°C

   c) −9°C, −7°C, −3°C, 4°C, 7°C

   d) −8°C, −2°C, 4°C, 7°C, 9°C

   e) −7°C, −4°C, −2°C, 3°C, 5°C

2 a) 31 cm, 1600 mm, 2.42 m, 284 cm, 9 m

   b) 105 mm, 3.2 m, 423 cm, 6100 mm, 804 cm

3 a) 874 g, 1.7 kg, 4000 g, 9.4 kg, 52 000 g

   b) 0.174 kg, 2104 g, 2.79 kg, 3.4 kg, 4123 g

4 a) 51 cl, 80 cl, 1600 ml, 2.4 litres, 9 litres

   b) 51.5 ml, 1 litre, 1500 ml, 180 cl, 3.1 litres

Exercise 6.2

1 a) >

   b) <

   c) <

   d) >

   e) >

   f) >

   g) >

   h) <

Exercise 6.3

1 a) 197, 321, 358, 411, 426, 462

   b) 4621, 6317, 9981, 39 171, 59 042, 89 125

   c) 12, 75, 124, 415, 631, 1792

   d) 1611, 4257, 5218, 6641, 7034, 9425

   e) 1 010 701, 1 020 504, 1 030 504, 1 040 501, 1 050 403, 1 060 504

2 a) 0.102, 0.123, 0.201, 0.231, 0.456

   b) 0.003, 0.01, 0.056, 0.066, 0.1

   c) 0.004 04, 0.0044, 0.0404, 0.044, 0.404

   d) 0.112, 0.149, 0.2, 0.51, 0.71

   e) 0.009 16, 0.090 11, 0.091, 0.0946, 0.913

3 a) 3.001, 3.102, 3.12, 3.201, 3.21

   b) 0.12, 1.21, 2.12, 12.1, 121

   c) 7.015, 7.023, 7.105, 7.41, 7.69

   d) 5.001, 5.0102, 5.02, 5.0201, 5.321

   e) 0.0012, 0.01, 0.0121, 1.201, 12.02

   f) 8.01, 8.04, 8.079, 8.1, 8.79

Exercise 6.4

1 a) $<$

   b) $<$

   c) $>$

   d) $>$

e) <

f) >

2 a) $\dfrac{11}{20}, \dfrac{3}{5}, \dfrac{7}{10}, \dfrac{3}{4}$

b) $\dfrac{7}{12}, \dfrac{3}{4}, \dfrac{5}{6}, \dfrac{7}{8}$

c) $\dfrac{3}{10}, \dfrac{2}{5}, \dfrac{1}{2}, \dfrac{2}{3}, \dfrac{13}{15}$

d) $\dfrac{7}{16}, \dfrac{1}{2}, \dfrac{5}{8}, \dfrac{3}{4}, \dfrac{13}{16}$

e) $\dfrac{3}{8}, \dfrac{2}{5}, \dfrac{17}{40}, \dfrac{9}{20}, \dfrac{1}{2}$

f) $\dfrac{17}{32}, \dfrac{11}{16}, \dfrac{3}{4}, \dfrac{7}{8}$

Exercise 6.5

1 $\dfrac{7}{10}, \dfrac{4}{5}, 0.83, \dfrac{17}{20}, 88\%$

2 $35\%, \dfrac{3}{8}, \dfrac{2}{5}, \dfrac{5}{12}, 0.45$

3 $30\%, \dfrac{3}{5}, \dfrac{2}{3}, 0.7, \dfrac{3}{4}$

4 City:$35\% = 0.35$; United: $\dfrac{3}{8} = 0.375$; Rovers: 0.4

5 Football: $\dfrac{2}{7} = 0.286...$; Gymnastics: $28\% = 0.28$; Rugby: 0.27

MATHEMATICS **7** Standard Form

Exercise 7.1

1 a) 7×10^3

   b) 8.4×10^4

   c) 5.63×10^2

   d) 6.5×10^6

   e) 7.23×10^5

   f) 2.7×10^1

   g) 5.34×10^4

   h) 6.93×10^2

   i) 4.39×10^3

   j) 4.123×10^8

   k) 8×10^6

   l) 3.92×10^7

2 a) 3×10^{-3}

   b) 5.6×10^{-2}

   c) 8×10^{-4}

   d) 6.3×10^{-6}

   e) 8.2×10^{-5}

   f) 6.0×10^{-3}

   g) 3.8×10^{-7}

   h) 7.8×10^{-1}

   i) 3.69×10^{-3}

   j) 6.58×10^{-4}

   k) 5.6×10^{-10}

   l) 7.23×10^{-6}

2 a) 50 000

   b) 370 000

   c) 0.0007

   d) 6 900 000

   e) 0.0061

   f) 47 300

   g) 27 900 000

   h) 0.000 0483

   i) 0.0103

   j) 989 000 000

   k) 0.000 002 61

   l) 370

   m) 3690

   n) 0.000 607

   o) 0.000 000 548

   p) 1 980 000 000

4 7.2×10^9

MATHEMATICS   **7** Standard Form

Exercise 7.2

1 a) 8×10^7

b) 1.2×10^{11}

c) 5.6×10^6

d) 3×10^3

e) 5.2×10^7

f) 4×10^5

g) 7.2×10^5

h) 5×10^5

i) 5.6×10^{-6}

j) 2.5×10^{-9}

k) 6.4×10^4

l) 6.997×10^6

m) 5.83×10^5

n) 4.56×10^9

o) 7.158×10^6

p) 6.88×10^{-4}

2 a) 2.356×10^{13}

b) 1.5×10^5

c) 1.45×10^{11}

d) 1.073×10^{-6}

e) 3.25×10^4

f) 2.03×10^9

g) 2.704×10^{13}

h) 9.61×10^{-8}

i) 3.692×10^6

j) 8.019×10^5

k) 5.202×10^{-3}

l) 6.7484×10^{-3}

m) -4.2368×10^{-3}

n) 7.3164×10^8

Exercise 8.1

1   4

2   2

3   4

4   6

5   1

6   –4

7   –4

8   –7

9   6

10   –4

11   –20

12   30

13   –4

14   –8

15   9

16   –12

17   –2

18   –21

19   36

20   –120

Exercise 8.2

1   14

2   18

3   22

4   27

5   80

6   3

7   55

8   54

9   100

10   10

11   10

12   7

13   1

14   9

15   0

16   2.5

17   7

18   12

19   64

20   –6

21 −8

22 24

23 −21

24 4

25 −4

26 28

27 −12

28 **a)** Hassan has worked out addition before division.

   b) 10

29 **a)** She has worked out multiplication before power.

   b) 12

30 **a)** **i)** $(3 + 6) \times 5 - 1 = 44$

   ii) $3 + 6 \times 5 - 1 = 32$

   iii) $3 + 6 \times (5 - 1) = 27$

   b) **i)** $6 + (4^2 - 16) \div 2 = 6$

   ii) $(6 + 4^2 - 16) \div 2 = 3$

   iii) $(6 + 4)^2 - 16 \div 2 = 92$

   c) **i)** $12 - 8 \div (4 + 4) = 11$

   ii) $(12 - 8) \div 4 + 4 = 5$

   iii) $12 - 8 \div 4 + 4 = 14$

d) **i)** $(18 + 12) \div 6 - 3 = 2$

   ii) $18 + 12 \div 6 - 3 = 17$

   iii) $18 + 12 \div (6 - 3) = 22$

   iv) $(18 + 12) \div (6 - 3) = 10$

Exercise 8.3

1 1794

2 13 363

3 8596

4 22 560

5 11 534

6 28 290

7 20 828

8 27 430

9 27 392

10 46 800

11 1176

12 6300

Exercise 8.4

1 a) 14.553

   b) 1455.3

   c) 14.553

   d) 14.553

e) 145 530

2 a) 1054

   b) 10.54

   c) 1.054

   d) 10.54

   e) 105.4

3 a) 1.5

   b) 0.48

   c) 0.08

   d) 0.014

   e) 0.03

   f) 0.0063

4 a) 63

   b) 35.99

   c) 26.16

   d) 1.872

   e) 3.822

   f) 9.968

5 a) 0.56

   b) 0.05

   c) 0.36

   d) 0.58

Exercise 8.5

1 47

2 38

3 54

4 33

5 17

6 16

7 19

8 32

9 34

10 32

11 69

12 12 (with 8 spare seats)

Exercise 8.6

1 a) 532.5

   b) 0.5325

   c) 53.25

   d) 532.5

   e) 53 250

2 a) 1.928

   b) 0.1928

   c) 192.8

d) 0.019 28

e) 192.8

3 a) 40

   b) 4

   c) 8

   d) 30

   e) 500

   f) 5

4 a) 31

   b) 780

   c) 3.7

   d) 0.425

   e) 365

   f) 0.75

5 a) 2.6

   b) 20

   c) 20

   d) 130

Exercise 8.7

1 a) $\dfrac{6}{7}$

   b) $\dfrac{1}{2}$

c) $\dfrac{11}{12}$

d) $\dfrac{19}{20}$

e) $\dfrac{23}{40}$

f) $\dfrac{23}{20}$ or $1\dfrac{3}{20}$

2 a) $\dfrac{1}{7}$

   b) $\dfrac{1}{2}$

   c) $\dfrac{5}{12}$

   d) $\dfrac{1}{4}$

   e) $\dfrac{7}{24}$

   f) $\dfrac{13}{36}$

3 a) $1\dfrac{7}{10}$

   b) $1\dfrac{7}{10}$

   c) $3\dfrac{3}{10}$

   d) $6\dfrac{4}{5}$

   e) $1\dfrac{7}{10}$

4 a) $4\dfrac{2}{5}$

b) $1\dfrac{1}{6}$

c) $\dfrac{1}{2}$

d) $\dfrac{3}{4}$

e) $\dfrac{9}{10}$

5 a) $5\dfrac{7}{10}$

b) $3\dfrac{1}{8}$

c) $3\dfrac{1}{12}$

d) $7\dfrac{3}{8}$

e) $4\dfrac{7}{12}$

6 a) $7\dfrac{11}{18}$

b) $\dfrac{1}{26}$

c) $5\dfrac{13}{20}$

d) $1\dfrac{11}{14}$

e) $\dfrac{5}{6}$

Exercise 8.8

1 2

2 $3\dfrac{1}{2}$

3 3

4 9

5 2

6 10

7 $2\dfrac{2}{3}$

8 $\dfrac{8}{9}$

9 $2\dfrac{2}{5}$

10 $\dfrac{3}{5}$

11 $\dfrac{1}{6}$

12 $\dfrac{2}{5}$

13 $\dfrac{2}{9}$

14 $\dfrac{2}{9}$

15 $\dfrac{1}{2}$

16 $\dfrac{1}{3}$

17 $\dfrac{5}{12}$

18 $\dfrac{3}{22}$

19 $\dfrac{5}{12}$

20 $\dfrac{1}{4}$

21 $9\dfrac{3}{4}$

22 $5\dfrac{1}{2}$

23   7

24   8

25 $5\dfrac{1}{3}$

Exercise 8.9

1   27

2 $\dfrac{3}{2} = 1\dfrac{1}{2}$

3 $\dfrac{2}{9}$

4 $\dfrac{4}{9}$

5 $\dfrac{2}{9}$

6 $\dfrac{1}{5}$

7   2

8 $\dfrac{1}{12}$

9   7

10 $2\dfrac{2}{3}$

11 $\dfrac{2}{3}$

12   2

13 $\dfrac{4}{7}$

14 $1\dfrac{3}{4}$

15 $1\dfrac{3}{5}$

16 $\dfrac{2}{15}$

17 $\dfrac{9}{14}$

18 $15\dfrac{3}{4}$

19 $3\dfrac{22}{27}$

20 $6\dfrac{3}{5}$

Exercise 9.1

1 a) 2.4 m to 3.5 m

   b) 5 m to 7 m

2 8.5m to 10 m

3 12 m to 15 m

Exercise 9.2

1 a) 4.6

   b) 5.5

   c) 4.6

   d) 8.4

   e) 0.5

   f) 0.1

   g) 82.2

   h) 3.0

   i) 6.2

   j) 0.5

   k) 5.2

   l) 48.0

   m) 0.9

   n) 7.7

   o) 10.0

2 a) 5.48

   b) 12.08

   c) 0.21

   d) 0.57

   e) 9.02

   f) 78.04

   g) 7.01

   h) 0.07

   i) 1.52

   j) 2.13

   k) 9.42

   l) 0.84

   m) 0.28

   n) 0.85

   o) 7.09

   p) 18.63

   q) 7.11

   r) 8.08

   s) 4.66

   t) 3.73

3 a) 9.34

   b) 17.16

c) 2.94

d) 53.96

4 a) 0.1

  b) 0.4

  c) 0.4

  d) 0.4

5 7.4

6 a) 0.333

  b) 0.286

  c) 0.273

  d) 0.308

  e) 4.667

7 7.3

8 850

9 17.7 cm^2

10 40.32 cm^2

11 112.0 cm^3

12 6670 km

Exercise 9.3

1 a) 4

  b) 6

  c) 8

d) 4

e) 6

f) 60

g) 50

h) 30

i) 300

j) 5000

2 a) 4000

  b) 30

  c) 900

  d) 60

  e) 0.002

  f) 6

  g) 5

  h) 0.003

  i) 0.003

  j) 0.05

3 a) 18

  b) 180

  c) 5700

  d) 98 000

  e) 50

f) 0.17

g) 0.039

h) 0.0061

i) 0.031

j) 0.99

4 a) 8.26

b) 69.8

c) 16 300

d) 208

e) 12 500

f) 7.10

g) 50.9

h) 0.416

i) 0.0386

j) 3.14

5 1600

6 The actual number was 35 085, which is 35 000

to 2 significant figures.

Exercise 9.4

1 $200 \times 0.9 = \$180$

2 $6 \times 9 = 54$ cm^2

3 $20 \times 30 = 600c = \$6$

4 Volume $= 4 \times 2 \times 2 = 16$ cm^3

5 $60 \div 10 = 6$ cm

6 $3000 \div 8 = 375$ miles (or 380 miles or 400

miles)

7 $20 \times 20 = \$400$

8 $200 \times 0.7 = \$140$

9 $3 \times 3 \times 3 = 27$ m^2

10   **a)** $6 \times 2 = 12$

b) $20 \div 4 = 5$

c) $40 \times 20 = 800$

d) $10 \div 4 = 2.5$

e) $2 \times 0.8 = 1.6$

f) $3 \times 8 = 24$

g) $100 \times 3 = 300$

h) $5 \times 0.03 = 0.15$

11   **a)** $4 \times 10 \times 20 = 800$

b) $0.9 \times 5 = 4.5$

c) $60 \times 10 = 600$

d) $200 \times 60 \times 3 = 36\,000$

e) $10 \times 0.07 = 0.7$

f) $0.2 \times 0.8 = 0.16$

g) $50 \times 80 = 4000$

h) $7^2 = 49$

i) $40 \times 20 \times 30 = 24\,000$

j) $900 \times 40 = 36\,000$

k) $3 \times 10 = 30$

l) $50 \times (20 + 40) = 3000$

12 **a)** $4 \times 80 = 320$; estimate is smaller because the rounded values of both numbers are smaller than the actual values.

b) $200 \div 10 = 20$; estimate is bigger because the rounded value of the number being divided is bigger than the actual value being divided and the rounded value of the number it is divided by is smaller than the actual value.

c) $20 \times 900 = 18\,000$; estimate is bigger because the rounded values of both numbers are bigger than the actual values.

13 **a)** $5 \times 7 = 35$ cm^2

b) Greater because the rounded values of both lengths are greater than the actual lengths.

14 **a)** $20 \div 6 \approx 3.3$ cm (or 3 cm)

b) Greater because the rounded value of the area is greater than the actual area and the rounded value of the length is less than the actual length.

Exercise 9.5

1 $30 \div 5 = 6$

2 $300 \div 90 \approx 3$

3 $40 \div 9 \approx 4$

4 $50 \div 7 \approx 7$

5 $(2 \times 3) \div 3 = 2$

6 $\dfrac{10 \times 20}{60} \approx 3$

7 $\sqrt{5 \times 5} = 5$

8 $\dfrac{\sqrt{8 \times 2}}{2} \approx 2$

9 $(0.4 \times 90) \div 8 = 4.5$

10 $1 \div 5 = 0.2$

11 $30 \times \sqrt{100} = 300$

12 $\dfrac{60 \times 10}{10 \times 5} = 12$

13 $40 \div 0.09 \approx 444$

14 $\sqrt{81} = 9$

15 $\dfrac{1000}{10} = 100$

16 $\dfrac{2}{20} = 0.1$

17 $\dfrac{30}{40} = 0.75$

18 $\dfrac{300 \times 100}{700} \approx 43$

19 $\dfrac{600 \times 1}{4 \times 2} = 75$

20 $\dfrac{7}{0.05 \times 40} = 3.5$

21 $\sqrt{40 \times 80} \approx 60$

Exercise 9.6

1 a) $3\frac{1}{2}$ minutes

  b) 3 weeks

  c) 3 kg

  d) 2 m

2 a) 5.7 m

  b) 2.8 mm or 3 mm

  c) 6650 km or 6600 km

  d) 67.6 kg or 68 kg

  e) 0.097 cm^2

MATHEMATICS **10** Limits of Accuracy

Exercise 10.1

1 a) i) 26.5 cm, 27.5 cm

ii) 29.5 cm, 30.5 cm

iii) 127.5 cm, 128.5 cm

b) i) 5 cm, 15 cm

ii) 25 cm, 35 cm

iii) 145 cm, 155 cm

c) i) 5.55 cm, 5.65 cm

ii) 0.75 cm, 0.85 cm

iii) 11.95 cm, 12.05 cm

d) i) 1.225 m, 1.235 m

ii) 0.445 m, 0.455 m

iii) 9.075 m, 9.085 m

e) i) 10.615 s, 10.625 s

ii) 9.805 s, 9.815 s

iii) 48.095 s, 48.105 s

2 a) 56.5 kg and 57.5 kg

b) 4.65 m and 4.75 m

c) 467.5 ml and 468.5 ml

d) 34.905 s and 34.915 s

e) 0.6335 kg and 0.6345 kg

3 Each pole is between 99.5 cm and 100.5 cm.

He can be sure of reaching a height of

$10 \times 99.5 = 995$ cm or 9.95 m.

4 a) Greatest perimeter = 36 cm

b) Smallest area = 51.75 cm^2

Exercise 10.2

1 a) 61.2 s

b) 24.51 s

c) 12.4 m

d) 1.747 kg

e) 185 mm

2 a) 61 s

b) 24.49 s

c) 12.38 m

d) 1.745 kg

e) 183 mm

3 a) 704 g

b) 6.7 cm

c) 4.4 s

d) 0.16 s

e) 9000 m

4 a) 702 g

 b) 6.5 cm

 c) 4.2 s

 d) 0.14 s

 e) 8800 m

5 50.80 cm

6 a) 29.20 s

 b) 1.06 s

7 Upper bound = 26.5 cm;

 lower bound = 23.5 cm

8 a) Upper bound = 13.8; lower bound = 13.6

 b) Upper bound = 3.6; lower bound = 3.4

9 No; 595.5 mm is greater than 59.5 cm.

Exercise 10.3

1 75 mm

2 a) Upper bound = 20.507 175 m^2;

 Lower bound = 20.415 675 m^2

 b) Upper bound = 40.1625 m^2;

 Lower bound = 38.8825 m^2

3 a) Upper bound = 1141.7575 cm;

 Lower bound = 1131.2875 cm

 b) Upper bound = 12.676 725 m;

 Lower bound = 12.579 325 m

c) Upper bound = 146.625 km;

 Lower bound = 138.425 km

 d) Upper bound = 11.707 275 m;

 Lower bound = 11.630 375 m

4 a) Upper bound = 5.61 cm/s;

 Lower bound = 5.08 cm/s

 b) Upper bound = 10.7 m/s;

 Lower bound = 10.5 m/s

5 a) 11.7 cm

 b) 10.6 cm

6 Upper bound = 844 people/square km;

 Lower bound = 830 people/square km

7 Upper bound = 536.25 cm^2;

 Lower bound = 490.25 cm^2

8 a) Upper bound = 87.95 cm^3;

 Lower bound = 81.37 cm^3

 b) Upper bound = 1702 g;

 Lower bound = 1566 g

9 Upper bound = 84.5; Lower bound = 20.2

10 Don Quarrie's time is between 9.85 s and 9.95 s.

Calvin Smith's time is between 9.925 s and 9.935 s.

So Quarrie could have run faster, e.g. 9.85 s is faster than 9.925 s.

However if Quarrie's time was 9.925 s (or less) then Smith would definitely be the faster since this is his slowest time.

11 Min Volume: $79.5 \times 74.5 \times 89.5$

$= 530\ 086.125$ cm^3 $= 530.08..$ litres

Max Volume: $80.5 \times 75.5 \times 90.5$

$= 550\ 036.375$ cm^3 $= 550.03..$ litres

Min volume < 550 litres $<$ Max volume; so yes.

MATHEMATICS **11** Ratio, Proportion, Rate

Exercise 11.1

1 a) 2:1

b) 1:3

c) 5:1

d) 2:3

e) 1:5

f) 5:4

g) 3:1

h) 1:12

i) 1:9

j) 1:9

2 a) 1:3

b) 1:20

c) 3:10

d) 10:3

e) 4:1

3 a) 1:20

b) 3:20

c) 16:3

d) 3:20

4 a) 1:3:5

b) 3:6:4

c) 1:4:5

5 a) 1:5:10

b) 1:4:6

c) 12:15:20

6 5:8:10

Exercise 11.2

1 a) 1:3

b) 1:5

c) 1:2.5

d) 1:1.75

e) 1:7.5

f) 1:125

g) 1:0.2

h) 1:500 000

2 a) 1:4

b) 1:2.4

c) 1:50

d) 1:250 000

e) 1:0.4

3 1:250 000

4 1:6

MATHEMATICS   **11** Ratio, Proportion, Rate

Exercise 11.3

1 a) 24 babies

  b) 9 helpers

2 a) 6 litres

  b) 4 litres

3 a) 30 ml

  b) 10 teaspoons

4 a) 100 ml

  b) 36 ml

5 a) 150 g

  b) 48 g

6 $120

Exercise 11.4

1 a) 15 litres

  b) 25 litres

2 a) 50 ml

  b) 250 ml

3 Amit $320; Bree $800; Chris $480

4 400 g copper; 200 g zinc

5 20 boys

6 800 m

7 0.5 litres

8 Blue 312; Orange 156; Green 104

9 a) 400 g

  b) 80 g

Exercise 11.5

1 a) 25

  b) 70

  c) 32

2 a) 80

  b) 54

  c) 57.6

3 a) 35

  b) 37.5

  c) 31.25

4 a) 50

  b) 52.5

  c) 62.5

Exercise 11.6

1 a) $\dfrac{8}{3}$

  b) 440 m

2 a) $\dfrac{12}{27}$ or $\dfrac{4}{9}$

  b) 96 km

3 a) $\dfrac{175}{50}$ or $\dfrac{7}{2}$

  b) $315

4 a) $\dfrac{5}{7}$

  b) 20 rungs

5 a) $\dfrac{15}{27}$ or $\dfrac{5}{9}$

  b) 195 g

6 a) $\dfrac{0.9}{2.5}$ or $\dfrac{9}{25}$

  b) 178.2 kg

Exercise 11.7

1 a) $\dfrac{48}{32}$ or $\dfrac{3}{2}$

  b) 12 minutes

2 a) $\dfrac{12}{8}$ or $\dfrac{3}{2}$

  b) 4 weeks

3 a) $\dfrac{3}{4}$

  b) 24 hours

4 a) $\dfrac{40}{50}$ or $\dfrac{4}{5}$

  b) 55 minutes

5 a) $\dfrac{20}{12}$ or $\dfrac{5}{3}$

  b) 9 days

6 a) $\dfrac{2}{3}$

  b) 9 hours

7 a) $\dfrac{3}{4}$

  b) 16 kilometres per hour

8 a) $\dfrac{14}{18}$ or $\dfrac{7}{9}$

  b) 27 weeks

9 a) $\dfrac{8}{6}$ or $\dfrac{4}{3}$

  b) 13.5 hours

10 a) $\dfrac{15}{9}$ or $\dfrac{5}{3}$

  b) 12 days

Exercise 11.8

1 20 minutes

2 500 kg/minute

3 1164 litres

4 $239.40

5 a) $8.64

  b) $138.24

6 50 km/hour

7 4.5 m/s

8 12 km/hour

9 18 km/hour

10   11.4 seconds

MATHEMATICS **12** Percentages

Exercise 12.1

1 a) $\dfrac{3}{4}$

  b) $\dfrac{3}{25}$

  c) $\dfrac{3}{50}$

  d) $\dfrac{4}{5}$

2 a) 0.27

  b) 0.86

  c) 0.03

  d) 0.08

  e) 1.5

  f) 1.4

  g) 2.5

  h) 1.74

  i) 0.027

  j) 0.036

  k) 0.128

  l) 0.173

Exercise 12.2

1 a) $20

  b) $0.60

  c) $0.72

  d) $6

  e) $18

  f) $1

  g) $60

  h) $5.40

  i) $2.58

  j) $2

  k) $15

  l) $0.34

2 $90

3 15% of $20 = $3; 20% of $15 = $3. Both

   amounts are the same.

4 $1

5 $400

6 160

7 108

8 78

9 $1150

10 $295.32

11   **a)**   299.25

   b) 20.25

   c) $230.40

   d) 351.5 m

   e) 113.16

   f) $170.28

   g) 20.28

   h) 2.76

   i) $2.56

   j) $2.10

12 $157.50

13   46 seats (round 45.05 up to nearest integer)

Exercise 12.3

1 a) 12%

   b) 8%

   c) 40%

   d) 25%

   e) 40%

   f) 15%

2 a) 16%

b) 24%

c) 25%

d) 30%

e) 73%

f) 8%

g) 90%

h) 20%

i) 5%

j) 130%

3 56%

4 24%

5 15%

6 20%

7 32%

8 55%

9 a) 12.5%

   b) 26.7%

   c) 47.5%

   d) 85%

   e) 79.2%

   f) 66.4%

10   42.5%

11   14.7%

12   93.3%

13   59.4%

14   93.3%

15   42%

Exercise 12.4

1 20%

2 12%

3 52%

4 27%

5 15%

Exercise 12.5

1 a) $480

  b) $580

  c) $424

  d) $720

2 a) $140

  b) $170

  c) $194

  d) $80

3 $30 000

4 $12 480

5 $10.40

6 $1290

7 1.68 m

8 $173.90

9 $327.60

10 $2655

Exercise 12.6

1 a) 1.13

  b) 1.2

  c) 1.68

  d) 1.08

  e) 1.02

  f) 1.175

  g) 2.5

2 a) 0.86

  b) 0.8

  c) 0.55

  d) 0.93

  e) 0.97

  f) 0.77

  g) 0.835

3 $4.78

4 $29.40

5 $32.40

Exercise 12.7

1 75 tonnes

2 40

3 $50

4 56 500

5 $12 500

6 $14 200

7 $8800

8 $27 000

9 $24 000

10 $480

Exercise 13.1

1 0.368 to 3 s.f.

2 –4.03 to 3 s.f.

3 –22.572

4 0.191 to 3 s.f.

5 5

6 3.84

7 4.5

8 9.2

9 4.48

10   1.20 to 3 s.f.

11   1.23 to 3 s.f.

12   34.123

13   1.3

14   3.45 to 3 s.f

15   2.68 to 3 s.f

16   2.65 to 3 s.f

17   1.78 to 3 s.f

18   1.85 to 3 s.f

19   17.4 to 3 s.f

20   3.77 to 3 s.f

21   0.0673 to 3 s.f.

22   1.44 to 3 s.f

Exercise 13.2

1 a) 2.35×10^{13}

b) 4.69×10^{5}

c) 1.87×10^{11}

d) 2.55×10^{-6}

e) 2.61×10^{5}

f) 2.76×10^{8}

g) 3.17×10^{11}

h) 7.67×10^{-7}

2 6.4×10^{10} km^2

3 a) 6.32×10^{7}

b) 259 people/km^2 or 260 people/km^2

4 a) 1.5×10^{4} seconds

b) 4.35×10^{9} km

5 a) 4.299×10^{9}

b) 99 people/km^2 or 100 people/km^2

6 a) 1.5×10^{11}

b) 495 seconds (8 minutes 15 seconds)

MATHEMATICS **13** Use of an Electronic Calculator

Exercise 13.3

1 a) $40 \times 20 = 800$; so could be correct.

b) Wrong. Dividing by a number greater than 1 reduces the value.

c) Wrong. $8000 \times 10 = 80\,000$.

d) Wrong. $12 \times 9 = 108$.

e) $5 \times 20 = 100$; so could be correct.

2 a) Negative ÷ negative = positive.

b) $12.4 \times 1 = 12.4$, so the answer should be less than 12.4

c) $30 \times 4 = 120$, so $30 \times 40 = 1200$

d) $8 = \sqrt{64}$ and $9 = \sqrt{81}$, so the answer should lie between 8 and 9.

e) The square of a number between 0 and 1 is smaller than the number.

f) $16 \div 8 = 2$.

g) Dividing 125 by a number between 0 and 1 has an answer that is greater than 125.

h) Positive × negative = negative.

i) The last digit should be $7 + 2 = 9$.

j) Dividing 76 by a number between 0 and 1 has an answer that is greater than 76.

Exercise 13.4

1 5.55 litres

2 0.57 m to 2 d.p.

3 a) $62.50

b) $8.37

c) $11.41

d) $27.94

4 $99.60

Exercise 13.5

1 a) 1.75 hours

b) 2.2 hours

c) 0.65 hours

2 a) 5.25 minutes

b) 3.45 minutes

c) 0.7 minutes

3 a) 3 hours 30 minutes

b) 1 hour 18 minutes

c) 4 hours 51 minutes

4 a) 3 minutes 45 seconds

b) 2 minutes 24 seconds

c) 1 minutes 54 seconds

5 2 hours 40 minutes

Exercise 14.1

1 a) 11.45 a.m.

b) 5.53 a.m.

c) 1.40 a.m.

d) 9.20 a.m.

e) 12.10 a.m.

f) 10.41 a.m.

g) 2.32 a.m.

h) 4.30 a.m.

i) 11.20 a.m.

j) 12.48 a.m.

2 a) 1.45 p.m.

b) 3.53 p.m.

c) 9.40 p.m.

d) 10.59 p.m.

e) 12.10 p.m.

f) 2.40 p.m.

g) 5.23 p.m.

h) 7.40 p.m.

i) 8.19 p.m.

j) 12.03 p.m.

3 a) 01 50

b) 14 40

c) 11 49

d) 18 30

e) 00 02

f) 03 20

g) 14 08

h) 00 49

i) 09 35

j) 23 02

4 a) 3.45 a.m.

b) 2.56 p.m.

c) 11.40 p.m.

d) 11.59 a.m.

e) 12.55 p.m.

f) 4.35 a.m.

g) 3.16 p.m.

h) 9.40 p.m.

i) 1.59 a.m.

j) 2.52 p.m.

Exercise 14.2

1 15 30

2 6 hours and 45 minutes

3 10.35 a.m.

4 2 hours and 35 minutes

5 1 hours and 45 minutes

6 2 hours and 33 minutes

7 a) i) 1 hours and 25 minutes

ii) 2 hours

iii) 2 hours and 30 minutes

iv) 3 hours and 40 minutes

b) i) 12 10

ii) 13 55

8 7.05 p.m.

9 7.55 a.m.

10 09 50

Exercise 14.3

1 a) 5 hours 24 minutes

b) 3 hours 16 minutes

c) 2 hours 50 minutes

d) 12 hours 46 minutes

2 a) 4.25 hours

b) 1.83 hours

c) 0.75 hours

d) 10.65 hours

e) 3.63 hours

3 a) 4 minutes 42 seconds

b) 5 minutes 15 seconds

c) 25 minutes 18 seconds

d) 24 seconds

4 18 36 or 6.36 p.m.

5 42.64 km/h

6 19 24 or 7.24 p.m.

7 16 05 or 4.05 p.m.

8 7 hours

Exercise 15.1

Other comparisons are possible but the same conclusion should be reached.

1 25.8 cents/litre, 43.5 cents/litre; 5 litres is best value.

2 $1.33/litre, $1.35/litre, $1.14/litre, $1.30/litre; 1 litre is best value.

3 3.17 cents/ml, 2.89 cents/ml; 200 ml is best value.

4 0.33 cents/ml, 0.16 cents/ml, 0.15 cents/ml; 12 × 330 ml is best value

5 0.12 cents/gram, 0.1175 cents/gram; 1.2 kg is best value

6 0.757 cents/gram, 0.671 cents/gram; 325 g is best value

7 0.762 cents/gram, 0.796 cents/gram; 680 g is best value

8 A: $18.75, B: $19; restaurant A is best value

Exercise 15.2

1 a) 24 340.80 PKR

   b) $34.51

2 a) 1164.80 AED

   b) €204.33

3 a) 114 862.50 JPY

   b) 161.03 AUD

4 a) 22 941 MUR

   b) £68.65

5 a) Internet

   b) i) 705 PKR

   ii) $6.95

6 a) 1.54 Singapore dollars

   b) €0.65

   c) €155.77 (accept €155 to €156)

MATHEMATICS **16** Personal and Small Business Finance

Exercise 16.1

1 $918

2 $838

3

Trouser sizes	Size	Quantity	Colour	Price	Total
10, 12, 14, 16, 18	**14**	1	Red	$14.99	$14.99
20, 22, 24, 26	**20**	2	Blue	$16.99	$33.98
Postage & packing					**$1.95**
Total					**$50.92**

4

Item	Cost	Total
2.4 kg potatoes	$1.60 per kg	$3.84
6 oranges	26c each	$1.56
1.4 kg onions	$2.20 per kg	$3.08
8 bananas	31c each	$2.48
	Total	**$10.96**

Exercise 16.2

1 a) $915

   b) $10 140

2 $28

3 $1475

4 22.7%

5 Bob's Budget bargains by 0.65%

6 60%

7 7.83%

Exercise 16.3

1 $285.84

2 a) $12.58

   b) $1548.10

   c) $730.81

   d) $2924.26

3 $6336.80

4 $1754.79

5 a) $3646.52

   b) $9621.41

   c) $4231.80

6 $16 488

7 3525

8 $35 500

9 $38 774

10 a) 180 419 dubs

   b) 365 760 dubs

11   **a)** $4051.69

    b) $5033.40

12   SI: $680; CI: $665.50. Difference = $14.50

13   5 years: $1469.33; 4 years: $1411.58. So 5 years at 8% is better.

14 $34 696

15   5 years

Exercise 17.1

1 a) 9

  b) 6

  c) 3

  d) 11

  e) 10

  f) 12

  g) 10

  h) 19

  i) 2

  j) 17

  k) 28

  l) 14

  m) 1

  n) 8

  o) 8

  p) 40

  q) 40

  r) 18

  s) 2

  t) 25

  u) 3

v) 10

w) 20

x) 12

y) 100

z) 8

2 a) 5

  b) −1

  c) 15

  d) 5

  e) 11

  f) 4

  g) 9

  h) 90

  i) 15

  j) 27

3 a) 19

  b) 39

  c) 4

  d) 6 ½

  e) 14.5

4 a) 10

b) 10

c) 232

5 a) 4

   b) 6

   c) 8.5

   d) 8

Exercise 17.2

1 a) $V = -21$

   b) $P = 55$

   c) $T = 2$

   d) $M = 10$

   e) $R = 24$

   f) $L = 2\dfrac{1}{6}$ or $\dfrac{13}{6}$

   g) $D = \dfrac{8}{25}$

   h) $A = 34$

   i) $P = 38$

   j) $B = 4$

2 $M = 0.56$

3 a) $S = 720$

   b) $S = 30.625$

4 $A = 111.5$

5 $E = 22.1$

6 $f = 42$

Exercise 17.3

1 $C = xy$

2 $A = mn$

3 $h = nt$

4 $F = 32 + 1.8C$

5 $A = ½\, bh$

6 $I = \dfrac{V}{R}$

7 $C = np$

8 $w = nq$

9 $N = \dfrac{L}{t}$

10 $Q = ½R + 1$

Exercise 17.4

1 $C = 180$

2 $A = 42$

3 $h = 340$

4 $F = 104$

5 $A = 15$

6 $I = 5.44$

7 $C = 3500$

8 $w = 30\ 000$

9 $N = 30$

10 $Q = 19$

Exercise 17.5

1 $2x + 3 = 23$, $x = 10$; their ages are 10 and 13

2 $3a + 15 = 180$, $a = 55$; the angles are 55°, 55° and 70°

3 $3x = 45$, $x = 15$; Chan spends \$15 and Ali spends \$30

4 a) $330 + 3x = 540$, $x = 70$

b) The angles are 150°, 150°, 70°, 70° and 100°

5 a) $2x + 15 = 53$, $x = 19$

b) 19 boys and 34 girls

6 $2x - 10 = 140$, $x = 75$; 75 men and 65 women

7 $x = 2(2x - 30)$, $x = 20$; angles are 20° and 10°

8 $3(x + 4) = 27$, $x = 5$

9 $2n - 5 = 3(n - 2)$, $n = 1$

10 $4(c + 12) = 28c$, $c = 2$; children pay \$2, adults \$14

Exercise 17.6

1 a) $b = a + c$

b) $x = \dfrac{3a - y}{w}$

c) $t = \dfrac{v - u}{a}$

d) $T = AH$

e) $T = \dfrac{P - C}{3}$

f) $u = 2P - v$

g) $r = \dfrac{C}{2\pi}$

h) $q = \dfrac{A}{p} - r$ or $q = \dfrac{A - pr}{p}$

i) $q = p - 2r$

j) $r = \dfrac{B - s}{5}$

k) $t = 2u - s$

l) $q = \dfrac{ms}{pr}$

m) $G = \dfrac{2F + L}{2}$ or $G = F + \dfrac{L}{2}$

n) $n = \dfrac{Ft - m}{4}$

o) $S = 2aT$

p) $y = \dfrac{tx - A}{2t}$

2 a) $l = \dfrac{P}{2} - w$

   b) 14 m

3 a) $n = \dfrac{C - A}{32}$

   b) 56

4 a) $w = \dfrac{T - 40}{45}$

   b) 2.4 kg

5 a) $r = \dfrac{S}{2\pi h}$

   b) 2.4 cm

6 a) $h = \dfrac{3V}{\pi r^2}$

   b) 27 cm

6 a) $h = \dfrac{3V}{\pi r^2}$

   b) 27.0 cm

7 a) $n = \dfrac{C - 40}{5}$ or $n = \dfrac{C}{5} - 8$

   b) 39

8 a) $h = \dfrac{S - 2\pi r^2}{2\pi r}$ or $h = \dfrac{S}{2\pi r} - r$

   b) 7.3 cm

Exercise 17.7

1 $t = \dfrac{s}{a + 2b}$

2 $b = \dfrac{s}{a - c}$

3 $t = \dfrac{bP}{b - a}$

4 $u = \sqrt{v^2 - 2as}$

5 $r = \sqrt{\dfrac{A}{4\pi}}$

6 $y = \dfrac{7 - 3a}{3 - b}$ or $y = \dfrac{3a - 7}{b - 3}$

7 $a = \dfrac{cd}{b - c}$

8 $a = \dfrac{b + 2}{2b + 2}$ or $a = \dfrac{b + 2}{2(b + 1)}$

9 $r = \pm\sqrt{\dfrac{s + 1}{2}}$

10 $x = \dfrac{bs + s}{2a + b}$ or $x = \dfrac{s(b + 1)}{2a + b}$

11 $d = \dfrac{bc - ab}{a + c}$ or $d = \dfrac{b(c - a)}{a + c}$

12 $t = \dfrac{ab}{1 - bs}$

13 $c = \sqrt[3]{\dfrac{V - 5ab^2}{3}}$

14 $P = \dfrac{100A}{100 + RT}$

15 $v = \dfrac{su}{u - s}$

16 $a = \dfrac{1}{s - b}$

17 $c = \dfrac{1 - ab}{a}$

18 $d = \dfrac{b + c - a}{a - b}$

19 $b = \dfrac{100a}{m + 100}$

20 $p = \dfrac{a}{1 - a}$

21 $x = \dfrac{b - a}{2a - b}$

22 $L = \dfrac{T^2 g}{4\pi^2}$

23 $x = \sqrt{\dfrac{y + 4}{3}}$

24 $r = \sqrt{\dfrac{3V}{\pi h}}$

25 **a)** $d = 5.39$

    b) 3.68 m

    c) $x = \sqrt{d^2 - y^2 - z^2}$

    d) 0.71 m

MATHEMATICS 18 Algebraic Manipulation

Exercise 18.1

1 −1

2 4

3 1

4 9

5 1

6 3

7 4

8 6

9 1

10   −4

10   60

11   −28

12   −3

13   7

14   56

15   10.5

16   −21

17   4

18   2

19   28

20   −12

Exercise 18.2

1 14

2 6

3 16.5

4 −11

5 −2

6 −12

7 10

8 16

9 −18

Exercise 18.3

1 $5x$

2 $3y + 2z$

3 $2x + 2y$

4 $3a + 2b$

5 $5x$

6 $7p$

7 $6b$

8 $3p$

9 $4s$

10 $a^2 + b^2$

11 $3a + 3b$

12 $m + 2n$

13 $2x + y$

14 $x^2 - 2x - 15$

15 $12pq$

16 $8b - 4a$

17 $8a$

18 $6a + 9b$

19 $3x$

20 $3c + 4d$

Exercise 18.4

1 $4ab$

2 $2ac + 6ab$

3 0

4 $6ab - 4ac$

5 $2b^2 - a^2$

6 $2x^2 - 4xy + y^2$

7 $2b^2 - a^2$

8 $9a^2 + 2ab - 6b^2$

9 $ab + bc$

10 $pq - 3p^2$

11 Cannot be simplified

12 $2ab + bc$

13 $-2a^3$

14 Cannot be simplified

15 $3a^3 + 7a^2$

16 $3ab^2 + 3a^2b$

17 $2x^2$

18 $13a^3 - 6a^2$

19 abc

20 $11ab$

Exercise 18.5

1 $2a + 2b$

2 $8x + 4$

3 $2p + 6$

4 $9x - 3$

5 $4x - 6$

6 $21y + 7z$

7 $12 - 32a$

8 $20a + 30b$

9 $15e - 40f$

10 $2 - 2x$

11 $5p - 5q$

12 $a^2 + 2a$

13 $y^2 - y$

14 $3y - 2y^2$

15 $2x - x^2$

16 $-2y - y^2$

17 $3c^2 + 12c$

18 $-10x^2 + 6x$

19 $6i + 8j - 10k$

20 $20m - 12n + 8p$

Exercise 18.6

1 $12a + 3 = 3(4a + 1)$

2 $9a + 18 = 9(a + 2)$

3 $5y - 30 = 5(y - 6)$

4 $6b - 4 = 2(3b - 2)$

5 $4x + 16 = 4(x + 4)$

6 $y^2 + 2y = y(y + 2)$

7 $2b + 6b^2 = 2b(1 + 3b)$

8 $8a^2 + 20a = 4a(2a + 5)$

9 $2(x + 3)$

10 $4(x - 5)$

11 $3(3 - 4x)$

12 $x(3x + 5)$

13 $5(a^2 + 2b)$

14 $12(2 + 3a^2)$

15 $10x(x - 10)$

16 $8(3x + 4y)$

17 $5a(3b - 4c)$

18 $6f(5f - 3g)$

19 $7a(6b + 5a)$

20 $5ab(a + 2b)$

21 $a(3b - 2c + 3d)$

22 $x^2 y(2y - 3x)$

23 $5(x^2 - 3x + 3)$

24 $2(6x - 3y + 4z)$

25 $3b(3a+2b)$

26 $2ac(2a-c)$

27 $4xy(3x+2-y)$

28 $3a^2b(1-3ab)$

29 $5abc(abc-2)$

30 $a^2b(2-3b^2+7a^2)$

31 $a(4bc-3c^2+2ab)$

Exercise 18.7

1 $(a+b)(2x+3)$

2 $(a-2b)(5x-3)$

3 $(p+q)(p+r)$

4 $(a+b)(x-y)$

5 $(a+b)(a-c)$

6 $(a-b)(a-c)$

7 $(b+3)(4a+3)$

8 $(2a+3)(x-2y)$

9 $(ax-b)(5x-2)$

10 $(a^2+2b)(3a-2b)$

11 $(5a+3b)(2x-y)$

12 $(4c-3d)(2a-3b)$

13 $(4a-3b)(2a+5c)$

14 $(a+1)(a+b)$

15 $(a+2b)(c+3d)$

16 $(2a-3b)(5c-d)$

17 $(5a+3d)(2b-c)$

18 $(2x-3y)(3x-z)$

19 $(5x^2-3y)(2-5y)$

20 $(x-3)(2x-1)$

Exercise 18.8

1 x^2+5x+6

2 $a^2+7a+12$

3 a^2+3a+2

4 $x^2+3x-10$

5 $x^2+4x-21$

6 $x^2-11x+30$

7 $x^2+3x-18$

8 $x^2-9x+20$

9 x^2+4x+3

10 a^2+6a+9

11 $a^2 + 3a + 2$

12 $x^2 - x - 2$

13 $p^2 + 2p - 8$

14 $a^2 + 15a + 56$

15 $x^2 - 2x - 24$

16 $x^2 - 12x + 27$

17 $x^2 + 9x - 10$

18 $x^2 + 6x + 9$

19 $a^2 - 10a + 25$

20 $b^2 + 2b + 1$

21 $x^2 - 4x + 4$

22 $a^2 + 4a + 4$

23 $x^2 - 20x + 100$

24 $x^2 + 16x + 64$

25 $b^2 - 14b + 49$

26 $x^2 + 8x + 16$

27 $x^2 - 4$

28 $x^2 - 36$

29 $x^2 - 16$

30 $x^2 - 1$

31 $x^2 - 100$

32 $x^2 - 49$

Exercise 18.9

1 $x^2 - x - 30$

2 $10x^2 - 22x + 4$

3 $6x^2 - 19x + 10$

4 $10x^2 - 3x - 18$

5 $2x^2 + 3xy + y^2$

6 $3x^2 - 17xy + 20y^2$

7 $2x^2 - 7xy + 6y^2$

8 $42x^2 + 20xy - 32y^2$

9 $4g^2 - 20gh + 21h^2$

10 $6j^2 - 37jm + 56m^2$

11 $10m^2 + 23mn - 42n^2$

12 $6r^2 - rn - 15n^2$

13 $4q^2 - 4pq - 63p^2$

14 $6r^2 - 37rs + 56s^2$

15 $4s^2 - 20st + 21t^2$

Exercise 18.10

1 $(x+5)(x-5)$

2 $(x+2)(x-2)$

3 $(2a+b)(2a-b)$

4 $(3+4y)(3-4y)$

5 $(5x+7y)(5x-7y)$

6 $(3x+8)(3x-8)$

7 $(1+7t)(1-7t)$

8 $(10x+1)(10x-1)$

9 $(xy+4a)(xy-4a)$

10 $(y+13)(y-13)$

11 $(11x+12y)(11x-12y)$

12 $2(2+x)(2-x)$

13 $3(x+8)(x-8)$

14 $5(3-2x)(3+2x)$

15 $5(xy+2)(xy-2)$

16 $3(x+2)(x-2)$

17 $5(x+3)(x-3)$

18 $7(x+7)(x-7)$

19 $10(x+20)(x-20)$

20 $8(x+5)(x-5)$

Exercise 18.11

1 $(x+3)(x+2)$

2 $(x+5)(x+1)$

3 $(x+3)(x+1)$

4 $(x+4)(x+2)$

5 $(x+4)(x+1)$

6 $(x+4)(x+5)$

7 $(x+1)(x+1)$

8 $(x-6)(x-1)$

9 $(x-6)(x-3)$

10 $(x-2)(x-5)$

11 $(x-3)(x-1)$

12 $(a-1)(a-1)$

13 $(y-7)(y-2)$

14 $(x-4)(x-2)$

15 $(a+6)(a+2)$

16 $(a-3)(a-3)$

17 $(b-8)(b-4)$

18 $(x+3)(x+8)$

19 $(x-4)(x-5)$

20 $(x-7)(x-8)$

Exercise 18.12

1 $(x+2)(x-4)$

2 $(x+5)(x-1)$

3 $(x+2)(x-3)$

4 $(x+6)(x-1)$

5 $(x+3)(x-1)$

6 $(x+3)(x-6)$

7 $(x+4)(x-7)$

8 $(x+1)(x-10)$

9 $(x+2)(x-11)$

10 $(x+8)(x-4)$

11 $(y+11)(y-2)$

12 $(x+4)(x-3)$

13 $(x+5)(x-4)$

14 $(a+10)(a-2)$

15 $(a+3)(a-9)$

16 $(a+2)(a-8)$

17 $(b+20)(b-1)$

18 $(x+13)(x-2)$

19 $(x+10)(x-3)$

20 $(x+2)(x-9)$

Exercise 18.13

1 $(x+1)^2$

2 $(x-2)^2$

3 $(x-5)^2$

4 $(a+10)^2$

5 $(3-2y)^2$

6 $(2x+y)^2$

7 $(7a+2)^2$

8 $(6x-5y)^2$

9 $(4a-3b)^2$

10 $(2ab+c)^2$

Exercise 18.14

1 $2(x+2)(x+1)$

2 $(3x+1)(x+2)$

3 $(2x+1)(x+4)$

4 $(2x+3)(x+2)$

5 $3(2x-1)(x-2)$

6 $3(x-2)^2$

7 $(3x-2)(x-3)$

8 $(x-1)(3x-10)$

9 $(3x-5)(x-2)$

10 $(2x-5)(2x-3)$

11 $(2x+1)(2x+3)$

12 $(7x+3)(x+1)$

13 $(5x-3)(x-2)$

14 $(5x-2)(x-4)$

15 $(3x-2)(2x-5)$

16 $(4x-3)(2x-3)$

17 $(3x+5)(x+4)$

18 $(2x+3)(x+2)$

19 $(3x+1)(x+4)$

20 $(5x+3)(x+3)$

21 $2(2x+1)(x+1)$

22 $(3x+5)(x+2)$

23 $(2x+1)(x+2)$

24 $(4x+5)(x+3)$

Exercise 18.15

1 $(3x-5)(x+2)$

2 $(2x-1)(x+3)$

3 $(3x+4)(x-2)$

4 $(3x+1)(x-4)$

5 $(2x-1)(x+5)$

6 $(3x-5)(x+3)$

7 $5(x-5)(x+2)$

8 $(5x-2)(x+3)$

9 $(2x+1)(2x-3)$

10 $(7x-4)(x+2)$

11 $(3x-7)(x+2)$

12 $(3x+4)(x-5)$

13 $(2x-7)(x+3)$

14 $(2x+1)(x-8)$

15 $(2x-7)(3x+2)$

16 $(6x+5)(x-3)$

17 $(2x+5)(x-3)$

18 $(3x+7)(x-2)$

19 $(5x+3)(x-4)$

20 $(3x+4)(x-3)$

21 $(4x+5)(x-2)$

22 $(2x+3)(x-5)$

23 $(4x+1)(x-2)$

24 $(3x+2)(x-6)$

Exercise 18.16

1 $\dfrac{5ab^2}{2}$

2 $3a^2b$

3 $\dfrac{3x^3y}{20}$

4 $\dfrac{3y^4}{2}$

5 $\dfrac{x-4}{2x}$

6 $\dfrac{3x}{x+3}$

7 $\dfrac{x+1}{x-1}$

8 $\dfrac{3x-4}{5(x-1)}$

9 $\dfrac{3}{x-2}$

10 $\dfrac{6}{x+2}$

11 $\dfrac{x-2}{x-1}$

12 $\dfrac{x-4}{x-5}$

13 $\dfrac{x+1}{x+3}$

14 $\dfrac{3(x+2)}{x+4}$

15 $\dfrac{3x+2}{2x-3}$

16 $\dfrac{2x-3}{x-1}$

17 $\dfrac{3x}{2x-1}$

18 $\dfrac{5(x+3)}{x-3}$

19 $x+2$

20 $\dfrac{x+2}{2x-3}$

Exercise 18.17

1 $\dfrac{11x}{10}$

2 $\dfrac{x}{15}$

3 $\dfrac{3x+1}{10}$

4 $\dfrac{17x-12}{30}$

5 x

6 $\dfrac{3x-1}{x(x-1)}$

7 $\dfrac{5x+3}{x(x+1)}$

8 $\dfrac{x^2+6x+1}{(x+1)(x+3)}$

9 $\dfrac{4x^2-x+3}{(x-1)(x+2)}$

10 $\dfrac{-x^2+5x+2}{(x-1)(x+2)}$

11 $\dfrac{7x^2-8x-10}{5x(x+1)}$

12 $\dfrac{3x^2-17x-15}{5x(x+1)}$

13 $\dfrac{23x^2+4x+51}{9(x+3)(x+2)}$

14 $\dfrac{3(x+1)(2x+3)}{(2x+1)(x+2)}$

15 $\dfrac{2(x^2+7x-3)}{(x+3)(x-3)}$

Exercise 19.1

1 a) 6^4

  b) 7^3

  c) 8^5

  d) 4^4

  e) 2^6

  f) 10^2

2 a) $5^2 \times 4^3$

  b) $3^2 \times 5^3$

  c) $2^3 \times 3^2 \times 4^5$

  d) $7^3 \times 8^2 \times 9^3$

3 a) 64

  b) 64

  c) 81

  d) 100 000

Exercise 19.2

1 5^5

2 6^9

3 10^7

4 3^{11}

5 8^5

6 4^5

7 9^9

8 6^8

9 4^8

10 $5^7 \times 6^2$

11 $2^9 \times 3^8$

12 $7^8 \times 8^5$

Exercise 19.3

1 a) 10^3

  b) 3^3

  c) 8^2

  d) 7^2

  e) 6

2 a) 3^2

  b) 2^2

  c) 5^4

  d) 4^3

  e) 2^7

  f) 6^6

MATHEMATICS **19** Indices

Exercise 19.4

1 a) a^5

  b) a^9

  c) a^6

  d) a^9

2 a) a^2

  b) a^4

  c) a^6

  d) a^3

3 a) $6a^5$

  b) $12a^9$

  c) $12a^6$

  d) $15a^9$

4 a) $3a^2$

  b) $2a^4$

  c) $2a^6$

  d) $3a^3$

5 a) $9a^6$

  b) $8a^3$

  c) $25a^8$

  d) $32a^{15}$

6 a) x^4

  b) x^2

  c) x^2

  d) x^{12}

7 a) $6a^5b^3$

  b) $6a^5b^7$

  c) Cannot simplify

8 a) $5ab^2$

  b) $2ac$

  c) $27b^4$

9 a) $\dfrac{4a^2c}{3b^2}$

  b) $\dfrac{8x^2y^2}{5z^2}$

  c) $\dfrac{6a^7b^3}{5c^3}$

Exercise 19.5

1 a) $\dfrac{1}{6}$

  b) $\dfrac{1}{9}$

  c) 1

  d) $\dfrac{1}{25}$

e) $\dfrac{1}{1000}$

2 a) 2

   b) 9

   c) $\dfrac{3}{2} = 1\dfrac{1}{2}$

   d) $\dfrac{16}{9} = 1\dfrac{7}{9}$

   e) $\dfrac{125}{8} = 15\dfrac{5}{8}$

3 a) 3

   b) 5

   c) 36

   d) $\dfrac{1}{8}$

Exercise 19.6

1 a) $n^{\frac{1}{3}}$

   b) $x^{\frac{1}{6}}$

   c) $m^{\frac{5}{3}}$

   d) $p^{\frac{4}{5}}$

2 a) $\dfrac{1}{4}$

   b) 2

   c) 1

   d) $\dfrac{1}{16}$

   e) 8

3 a) 2

   b) $\dfrac{1}{8}$

   c) 16

   d) 64

   e) 8

4 a) $\dfrac{1}{9}$

   b) 3

   c) 1

   d) $\dfrac{1}{81}$

   e) 27

5 a) 3

   b) 81

   c) $\dfrac{1}{27}$

   d) $\dfrac{1}{3}$

   e) 1

6 a) 8

b) $\dfrac{1}{4}$

c) 1

d) $\dfrac{1}{16}$

e) 32

7 a) 4

b) $\dfrac{1}{2}$

c) 1

d) 64

e) $\dfrac{1}{128}$

8 a) 28

b) 6

c) $\dfrac{1}{4}$

d) 5

9 a) 12

b) 64

c) $\dfrac{1}{3}$

d) 27

10 a) 9

b) 16

c) $\dfrac{16}{3} = 5\dfrac{1}{3}$

d) 15

11 a) 125

b) 6

c) 100

d) $\dfrac{343}{3} = 114\dfrac{1}{3}$

12 a) 1000

b) $\dfrac{2}{25}$

c) $\dfrac{455}{4} = 113\dfrac{3}{4}$

d) $-\dfrac{24}{5}$

Exercise 19.7

1 a) $2^{\frac{5}{2}}$

b) $2^{-\frac{7}{4}}$

c) 2^{-6}

d) 2^{-3}

e) $2^{-\frac{2}{3}}$

f) $2^{\frac{1}{2}}$

2 a) 3^{3}

b) 3^{-1}

c) $3^{\frac{3}{2}}$

d) 3^6

e) 3^2

f) 3^{11n}

3 a) 2^5

 b) 2^2

 c) 2^3

 d) 2^{-2}

 e) 2^{3n}

 f) 2^{3n-8}

4 a) 5^4

 b) 5^2

 c) 5^{-1}

 d) $5^{-\frac{5}{2}}$

 e) 5^{4-n}

 f) 5^{7n}

5 a) $2^3 \times 3$

 b) $2^6 \times 3^2$

 c) $2^{\frac{1}{3}} \times 3^{\frac{2}{3}}$

d) $2^2 \times 3^{-2}$ or $\dfrac{2^2}{3^2}$

e) $3^3 \times 2^{-1}$ or $\dfrac{3^3}{2}$

f) $2^{4n} \times 3^{2n}$

6 a) x^5

 b) x^{2n}

 c) x^2

 d) $x^{\frac{5}{2}}$

 e) $x^{\frac{29}{6}}$

 f) $x^{\frac{5}{4}}$

7 a) $x = 4$

 b) $x = \dfrac{1}{3}$

 c) $x = 4$

 d) $x = -\dfrac{1}{2}$

 e) $x = -3$

 f) $x = -\dfrac{1}{2}$

8 a) $10a^{-1}$

 b) $\dfrac{1}{2}a^3$

c) $\dfrac{3}{2}a^{-\frac{1}{2}}$

d) $\dfrac{8}{3}a^{-\frac{7}{2}}$

9 a) $\dfrac{3b}{a}$

   b) $\dfrac{a}{3b^2}$

   c) $\dfrac{9a^2}{b^2}$

   d) $\dfrac{b^2}{5a^3}$

10 a) $2y^4$

   b) $2y^{-7}$

   c) $\dfrac{1}{2}y^{\frac{3}{2}}$

   d) $\dfrac{5}{4}y^{-\frac{5}{2}}$

Exercise 20.1

1 $x = 4$

2 $x = 4$

3 $x = 3$

4 $a = 4$

5 $y = 3$

Exercise 20.2

1 $x = 4$

2 $x = 4$

3 $x = -2$

4 $x = 0$

5 $x = 8$

6 $x = -4$

7 $x = 8\frac{1}{2}$

8 $x = 3$

9 $x = 3.4$

10 $x = 1\frac{3}{4}$

11 $x = 2$

12 $x = 4$

13 $x = -\frac{1}{4}$

14 $x = 10$

15 $x = 1$

16 $x = 3$

17 $x = 2$

18 $x = 8$

19 $x = 4$

20 $x = 5\frac{1}{2}$

Exercise 20.3

1 $x = 4$

2 $x = 3$

3 $x = -3$

4 $x = 8$

5 $x = 1$

6 $x = 1$

7 $x = 2$

8 $x = 4$

9 $x = -13$

10 $x = 2$

11 $x = 1$

12 $x = 4\frac{1}{2}$

13 $x = 4$

14 $x = 1$

15 $x = 2$

16 $x = 7$

17 $x = 5$

18 $x = 4$

19 $x = 9$

20 $x = -7$

Exercise 20.4

1 $x = 4$

2 $x = 6$

3 $x = 5$

4 $x = 2\frac{1}{2}$

5 $x = \frac{6}{7}$

6 $x = 12$

7 $x = 24\frac{1}{2}$

8 $x = 20$

9 $x = 50$

10 $x = 6\frac{1}{2}$

11 $x = 23$

12 $x = 7$

13 $x = 4$

14 $x = 1\frac{1}{4}$

15 $x = 1\frac{1}{2}$

16 $x = \frac{7}{10}$

17 $x = \frac{5}{9}$

18 $x = 12$

19 $x = 9$

20 $x = 24$

Exercise 20.5

1 $x = 7$

2 $x = 4$

3 $x = 25\frac{1}{2}$

4 $x = 4$

5 $x = 3$

6 $x = 1$

7 $x = -1$

8 $x = 1$

9 $x = -\frac{1}{5}$

10 $x = \dfrac{1}{2}$

11 $x = 3$

12 $x = 0$

13 $x = \dfrac{80}{9}$

14 $x = 3\dfrac{1}{2}$

15 $x = -5\dfrac{1}{2}$

Exercise 20.6

1 e.g. $x = -3, -4$

2 a) $x = -4, -3, -2, -1$

 b) $x = 2, 3, 4, 5$

 c) $x = 2, 3, 4$

 d) $x = -4, -3, -2, -1$

3 $x \le 7$

4 $x < 4$

5 $x \le 1$

6 $x > 1.5$
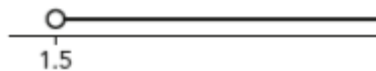
7 $x \le 4$
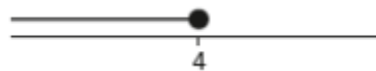
8 $x \ge 5$

9 $x > 0.5$

10 $x > 2.5$

11 $x \ge 4$

12 $a > 1$

13 $x < 4$

14 $x < -2$

15 $x > 7$

16 $x \le 1$

17 $x < 2$

18 $x \ge 2$

19 $x < 4$

20 $x > -3$

21 $x < -8.5$

22 $x > -6$

23 $x > -3$
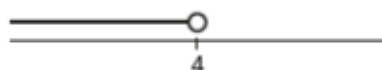
24 $x \geq -10$

25 $x < -18$

26 $x \leq 4$

27 $x < 17$

28 $2 < x < 5$

29 $-1 < x \leq 2.4$

30   **a)** $x = 2, 3$

   b) $x = -2, -1, 0, 1, 2, 3$

Exercise 20.7

1 $x = 4, y = 1$

2 $x = 2, y = 3$

3 $x = 2, y = 2$

4 $x = 4, y = 1$

5 $x = 2, y = 3$

6 $x = 5, y = 1$

7 $x = 4, y = 1$

8 $x = 2, y = 2$

9 $x = 2, y = 1$

10 $x = 3, y = 2$

11 $x = 5, y = 7$

12 $x = 4, y = 1$

13 $x = 1, y = 5$

14 $x = 2, y = 1$

15 $x = 2, y = -1$

16 $x = 2\frac{1}{2}, y = 1\frac{1}{2}$

17 $x = \frac{1}{2}, y = 2\frac{1}{2}$

18 $x = 2\frac{1}{2}, y = 1$

19 $x = 3, y = -1$

20 $x = -3, y = -2$

21 $x = -1\frac{1}{2}, y = 1\frac{1}{2}$

Exercise 20.8

1 $x = 2, y = 3$

2 $x = 1, y = 1$

3 $x = 4, y = 1$

4 $x = 2, y = 3$

5 $x = 1, y = 2$

6 $x = 5, y = 6$

7 $x = 5, y = 2$

8 $x = 4, y = 1$

9 $x = -1, y = 2$

10 $x = -2, y = -3$

11 $x = -2, y = 3$

12 $x = \frac{1}{2}, y = 2\frac{1}{2}$

13 $x = 2\frac{9}{26}, y = 3\frac{5}{26}$

14 $x = 5, y = -2$

15 $x = 2.6, y = 3.8$

Exercise 20.9

1 $x = 2$ or $x = 3$

2 $x = 1$ or $x = 5$

3 $x = 1$ or $x = 3$

4 $x = \pm 10$

5 $x = -2$ or $x = -4$

6 $x = -1$ or $x = -4$

7 $x = -4$ or $x = -5$

8 $x = \pm 5$

9 $x = -1$ (repeated)

10 $x = 1$ or $x = 6$

11 $x = 3$ or $x = 6$

12 $x = 0$ or $x = 8$

13 $x = -3$ or $x = -4$

14 $x = 0$ or $x = -3$

15 $x = 0$ or $x = -6$

16 $x = 4$ or $x = 6$

17 $x = 2$ or $x = 4$

18 $x = \pm 13$

19 $x = \pm 15$

20 $x = -3$ or $x = 1$

21 $x = -5$ or $x = 1$

22 $x = 0$ or $x = 10$

23 $x = -3$ or $x = 4$

24 $x = -6$ or $x = 1$

25 $x = 0$ or $x = 1$

26 $x = -3$ or $x = 5$

27 $x = -3$ or $x = 6$

28 $x = -1$ or $x = 10$

29 $x = 2$ or $x = 15$

30 $x = -8$ or $x = 4$

Exercise 20.10

1 $x = -1.5$ or $x = 4$

2 $x = -\frac{2}{3}$ or $x = 4$

3 $x = -1.5$ or $x = -1$

4 $x = -1$ or $x = 2.5$

5 $x = -1$ or $x = \frac{1}{3}$

6 $x = -\dfrac{1}{2}$ or $x = -5$

7 $x = 1.5$ or $x = 5$

8 $x = -\dfrac{4}{3}$ or $x = \dfrac{1}{2}$

9 $x = -6$ or $x = 5$

10 $x = 1$ or $x = 3$

11 $x = -2$ or $x = 3$

12 $x = -\dfrac{4}{3}$ or $x = 6$

13 $x = \pm 2$

14 $x = \pm 3$

15 $x = \pm 5$

Exercise 20.11

1 $x = 0.76$ or $x = 5.24$

2 $x = -0.53$ or $x = -9.47$

3 $x = 6.70$ or $x = 0.30$

4 $x = -6.61$ or $x = 0.61$

5 $x = 0.64$ or $x = 9.36$

6 $x = 0.44$ or $x = 4.56$

7 $x = 0.19$ or $x = 1.31$

8 $x = -0.79$ or $x = 2.12$

9 $x = -0.26$ or $x = -5.74$

10 $x = -1.22$ or $x = 0.55$

11 $x = -0.48$ or $x = 1.68$

12 $x = -0.35$ or $x = 2.85$

13 $x = -0.36$ or $x = 0.56$

14 $x = -0.10$ or $x = -0.65$

15 **a)** $(x+6)^2 - 24$

   b) -24

   c) $x = -6 \pm 2\sqrt{6}$

16 **a)** $2(x+2)^2 - 3$

   b) $5(x+\dfrac{1}{2})^2 - 3\dfrac{1}{4}$

   c) $2(x+\dfrac{9}{2})^2 - 17\dfrac{1}{2}$

Exercise 20.12

1 $x = -0.84$ or $x = -7.16$

2 $x = -0.82$ or $x = 1.82$

3 $x = -1.85$ or $x = 0.18$

4 $x = 0.54$ or $x = 1.86$

5 $x = -2.32$ or $x = 0.52$

6 $x = -0.19$ or $x = 5.19$

7 $x = -0.74$ or $x = -2.26$

8 $x = -0.63$ or $x = -6.37$

9 $x = -0.85$ or $x = 2.35$

10 $x = -1.22$ or $x = 0.55$

11 $x = 0.76$ or $x = 1.84$

12 $x = -0.44$ or $x = -1.36$

13 Width $= 2.40$ m and length $= 10.40$ m

14 **a)** $20x - 2x^2$

    b) 7.24 m by 5.53 m or 2.76 m by 14.47 m

    c) 50 m^2

15 **a)** $4x^2 + 74x + 330$

    b) 0.90 m

10 $x = \dfrac{3}{4}$ or $x = 1$

11 $x = -\dfrac{3}{2}$ or $x = \dfrac{5}{3}$

12 $x = \dfrac{1}{2}$ or $x = 4$

13 $x = -11$ or $x = 2$

14 $x = -\dfrac{1}{2}$ or $x = 5$

15 $x = 1.70$ or $x = 5.31$

16 $x = -1.30$ or $x = 2.30$

Exercise 20.13

1 $x = 5$

2 $x = 3$

3 $x = 0$

4 $x = 4$

5 $x = -3$

6 $x = 5$

7 $x = 1$

8 $x = -\dfrac{2}{3}$

9 $x = -1$ or $x = \dfrac{3}{4}$

Exercise 21.1

1 a) $x > 2$

b) $y < -2$

c) $y < 2x$

d) $y \geq x + 4$

e) $3x + 4y > 12$

f) $y \leq 2x - 4$

2
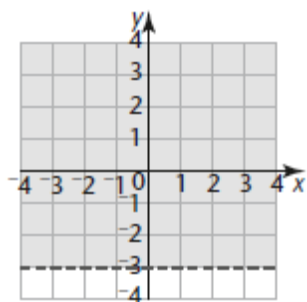
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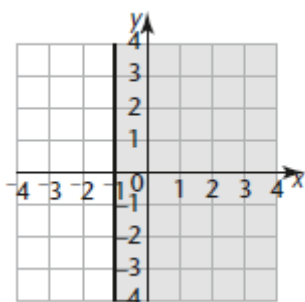
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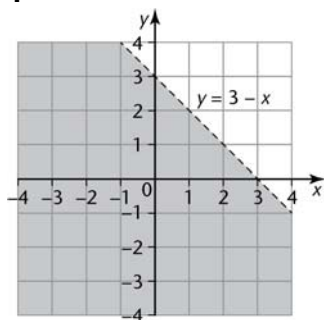
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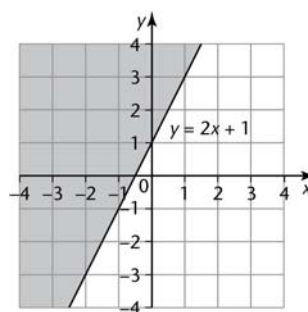
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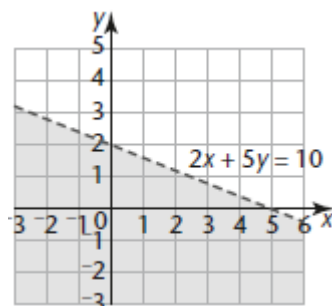
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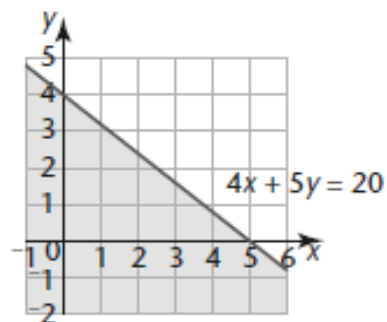
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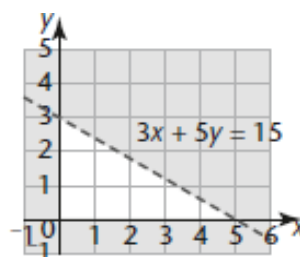
Exercise 21.2

In this exercise, the required region is labelled R

1
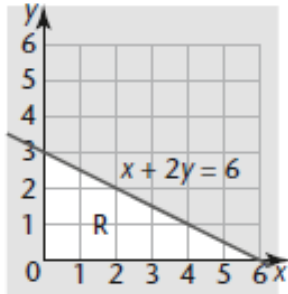
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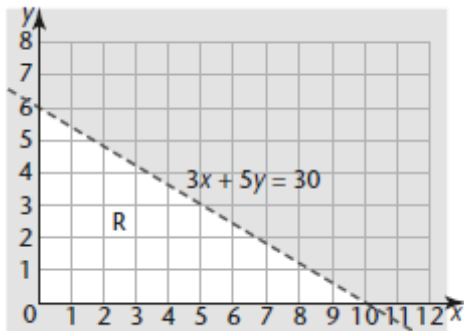
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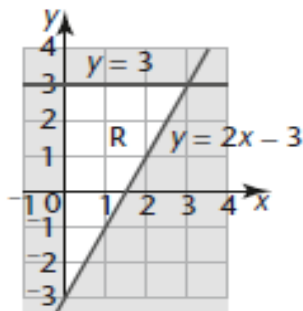
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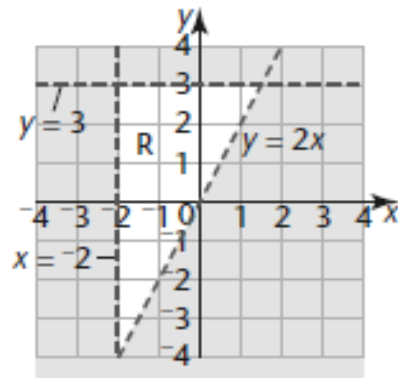
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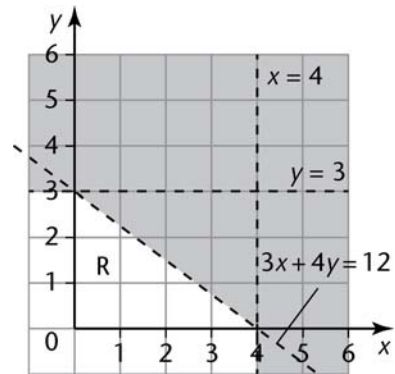
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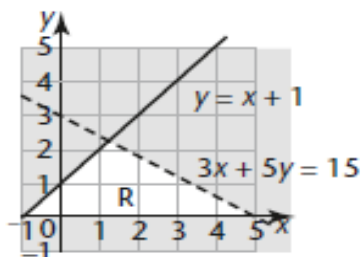
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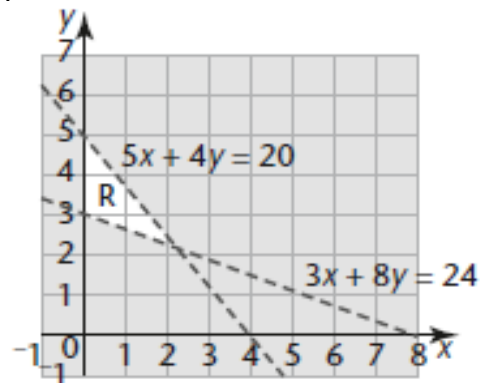
Exercise 22.1

1 a) Add 4; 26

  b) Add 8; 51

  c) Add 5; 34

2 a) Multiply by 3; 729

  b) Divide by 5; 5

  c) Add 1 more each time; 22

3 a) 22, 27, 37

  b) 22, 13, 7

  c) 64, 256

4 a) 77 is in the sequence because the sequence

  is the multiples of 7 and 77 = 11 × 7

  b) 77 is not in the sequence because all the

  numbers in the sequence are even and 77 is

  odd.

5 a) 49

  b) To get from one number to the next you add

  3, 5, 7, 9, … .

Exercise 22.2

1 a) Linear

  b) 11, 13

2 a) Not linear

  b) 30, 28

3 a) Linear

  b) 66, 63

4 a) Linear

  b) 17, 21

5 a) Not linear

  b) 36, 49

6 a) Not linear

  b) 45, 66

7 a) Linear

  b) 30, 37

Exercise 22.3

1 2, 3, 4, 5

2 2, 4, 6, 8

3 1, 3, 5, 7

4 6, 7, 8, 9

5 3, 6, 9, 12

6 4, 7, 10, 13

7 2, 7, 12, 17

8 10, 20, 30, 40

9 0, 7, 14, 21

10   1, 0, −1, −2

11 1, 2, 3, 4

12 4, 5, 6, 7

13 4, 8, 12, 16

14 0, 1, 2, 3

15 3, 5, 7, 9

16 2, 5, 8, 11

17 11, 17, 23, 29

18 −1, 1, 3, 5

19 4, 3, 2, 1

20 8, 6, 4, 2

21 1, 4, 9, 16

22 3, 6, 11, 18

23 −4, −1, 4, 11

24 3, 12, 27, 48

25 1, 8, 27, 64

Exercise 22.4

1 n

2 $2n + 2$

3 $4n$

4 $2n - 2$

5 $4n + 3$

6 $6n - 5$

7 $10n + 1$

8 $3n + 2$

9 $100n + 1$

10 $n - 1$

11 $3n - 1$

12 $2n + 5$

13 $5n - 1$

14 $5n + 10$

15 $4n - 5$

16 $2n + 3$

17 $n + 100$

18 $5 - n$

19 $10 - 3n$

20 $27 - 2n$

Exercise 22.5

1 a) $n^2 + 2$

b) $n^2 - 5$

c) $n^2 + n$

d) $2n^2$

e) $n^2 + 3n - 1$

f) $n^2 - 2n + 4$

2 a) $n^3 + 10$

  b) $n^3 - 9$

  c) $2n^3$

  d) $n^3 + n^2$

  e) $n^3 - n$

3 a) $2^n - 1$

  b) $2^n + 12$

  c) $2^n + n$

  d) $2^n - 2n$

4 a) $T_1 = A \times 1^3 + B \times 1^2 = A + B = 1$

  $T_2 = A \times 2^3 + B \times 2^2 = 8A + 4B = 12$

  b) $A = 2, B = -1$

  c) $T_3 = 45, T_4 = 112$

5 $\dfrac{2n^2}{2n+1}$

6

Diagram (n)	1	2	3	4	n
Number of small squares	1	4	9	16	n^2
Numbers of dots	4	9	16	25	$(n+1)^2$
Number of lines	4	12	24	40	$2n^2 + 2n$

Exercise 23.1

1 a) $t \propto \dfrac{1}{s}$

  b) $p \propto w$

  c) $d \propto t$

  d) $b \propto \dfrac{1}{s}$

  e) $t \propto d$

  f) $c \propto m$

2 a) $y \propto x$

  b) $y \propto x$

  c) $y \propto \dfrac{1}{x}$

  d) $y \propto x$

  e) $y \propto x$

  f) $y \propto \dfrac{1}{x}$

  g) $y \propto x$

  h) $y \propto x$

  i) $y \propto \dfrac{1}{x}$

  j) $y \propto x$

Exercise 23.2

1 a) $y = \dfrac{1}{3}x$

  b) $y = 7x$

  c) $y = \dfrac{80}{x}$

  d) $y = 5x$

  e) $y = \dfrac{2}{3}x$

  f) $y = \dfrac{15}{x}$

  g) $y = \dfrac{2}{3}x$

  h) $y = \dfrac{5}{4}x$

  i) $y = \dfrac{50}{x}$

  j) $y = \dfrac{2}{5}x$

2 a) $I = \dfrac{V}{6}$

  b) 10 amperes

3 a) $w = \dfrac{330}{f}$

  b) 0.5 m

4 a) i) $A = \dfrac{2}{5}B$

ii) $A = \dfrac{1000}{B}$

b) i) $B = 62.5$

ii) $B = 40$

Exercise 23.3

1 a) $y = \dfrac{1}{12}x^2$

b) $y = 12$

c) $x = 24$

2 a) $y = \dfrac{64}{x^2}$

b) $y = 1$

c) $x = \pm 1.6$

3 a) $y = \dfrac{8}{3}\sqrt{x}$

b) $y = 16$

c) $x = 2.25$

4 a) $y = \dfrac{40}{x^3}$

b) $y = 0.32$

c) $x = 0.5$

5 a) $y = \dfrac{5}{2}x^3$

b) $y = 160$

c) $x = 6$

6 $y = 4$

7 $y = 0.625$

8 $y = 4.8$

9 $y = 4$

10 $y = 0.36$

11 a) $v = 4\sqrt{l}$

b) 40 m/s

12 a) 35 days

b) $n = \dfrac{105}{t}$

13 4 newtons

14 a) $4p$

b) 96 m

c) 1500%

15 a) $y \propto x^2$

b) $y \propto \dfrac{1}{x^2}$

c) $y \propto \dfrac{1}{x}$

d) $y \propto \dfrac{1}{x^2}$

e) $y \propto x^2$

f) $y \propto \dfrac{1}{x}$

g) $y \propto x$

h) $y \propto \dfrac{1}{x^2}$

Exercise 24.1

1 a)
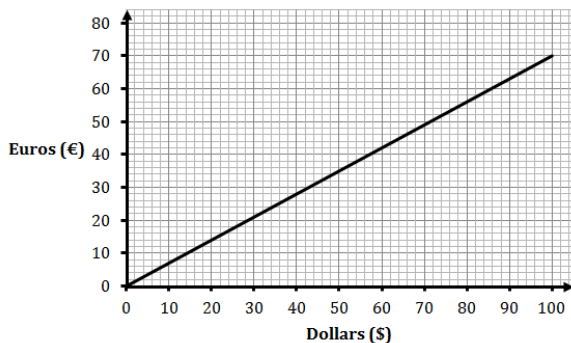
b) i) €24 to €25

   ii) $85 to $86

2 a) i) NZ$60 to NZ$65

   b) i) £31 to £32

   ii) £80

   c) £190 to £194

3 a) i) 24 to 25 miles

   ii) 40 to 41 miles

   b) i) 16 to 17 km

   ii) 89 to 90 km

   c) 240 km

4 a) i) 31 to 33 °F

   ii) 183 to 187 °F

   b) i) 37 to 39 °C

   ii) 76 to 78 °C

Exercise 24.2

1 a) 09 45

   b) 6 minutes

   c) 1.9 km

   d) On the way to the supermarket.

   e) 4 km/h

2 a) 10 10

   b) 4.8 km

   c) 10 minutes

   d) 16 minutes

3 a) 1 hour 30 minutes   **ii)**

   b) 45 minutes

   c) 95 km

   d) 12 48

4 a)

   b) 21 minutes

   c) 8.04 am

MATHEMATICS   **24** Graphs in Practical Situations
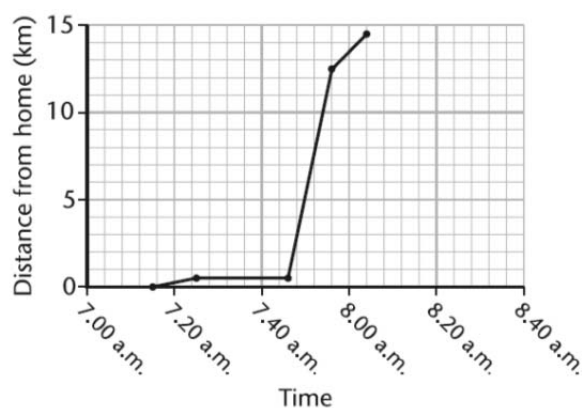
Exercise 24.3

1 08:15 to 09:00: 13.3 km/h;
09:00 to 09:15: 0 km/h;
09:15 to 09:45: 16 km/h;
09:45 to 10:00: 0 km/h;
10:00 to 10:30: 10 km/h

2 0.25 m/s; 2.5 m/s

3 a) 4 m/s

   b) 1 m/s

Exercise 24.4

1 0.75 m/s²; 0.4 m/s²

2 a) 0.6 m/s²

   b) 0.25 m/s²

3 a) 1.9 to 2 m/s²

   b) −2 m/s²

4 a) $t = 0$ to $t = 30$: acceleration is $\frac{1}{3}$ m/s²;

   $t = 30$ to $t = 45$: acceleration is $\frac{2}{3}$ m/s²;

   $t = 45$ to $t = 65$: acceleration is 0 m/s²;
   $t = 65$ to $t = 90$: deceleration is 0.8 m/s².

   b) i)   6.7 m/s to 1 d.p.

   ii)   12 m/s

5 a)
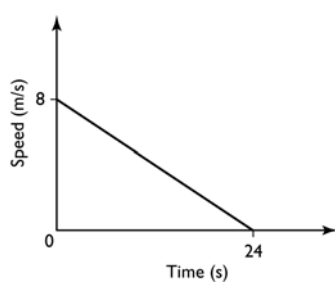
   b) $\frac{1}{3}$ m/s²

c) 5 m/s

6 a) 18 m/s

   b) 20 seconds

Exercise 24.5

1 a) 180 m

   b) 350 m

   c) 250 m

   d) 200 m

2 a) 0.25 m/s²

   b) i)   600 m

   ii) 1800 m

3 a) 690 m

   b) 30 seconds

4 $u = 8$

5 a) i)   10.5 m/s

   ii) 78.75 m

   b)
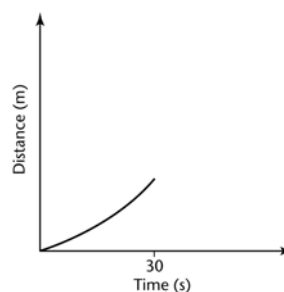
Exercise 25.1

1

x	−4	−3	−2	−1	0	1	2	−1.5
x^2	16	9	4	1	0	1	4	2.25
$+3x$	−12	−9	−6	−3	0	3	6	−4.5
−7	−7	−7	−7	−7	−7	−7	−7	−7
$y = x^2 + 3x - 7$	−3	−7	−9	−9	−7	−3	3	−9.25

2

x	−3	−2	−1	0	1	2	3
x^2	9	4	1	0	1	4	9
$2x^2$	18	8	2	0	2	8	18
−8	−8	−8	−8	−8	−8	−8	−8
$y = 2x^2 - 8$	10	0	−6	−8	−6	0	10

3

x	−6	−5	−4	−3	−2	−1	0	1	2	−2.5
$-x^2$	−36	−25	−16	−9	−4	−1	0	−1	−4	−6.25
$-5x$	30	25	20	15	10	5	0	−5	−10	12.5
+6	6	6	6	6	6	6	6	6	6	+6
$y = -x^2 - 5x + 6$	0	6	10	12	12	10	6	0	−8	12.25

4 a)

x	−1	0	1	2	3	4	5	1.5
x^2	1	0	1	4	9	16	25	2.25
$-3x$	3	0	−3	−6	−9	−12	−15	−4.5
$y = x^2 - 3x$	4	0	−2	−2	0	4	10	−2.25

b)
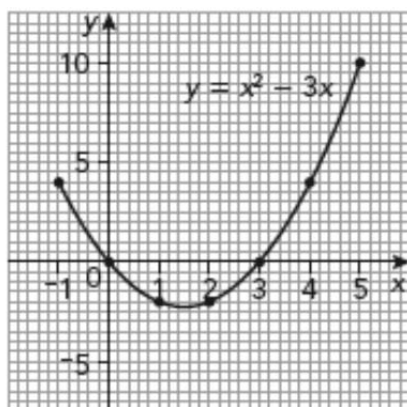
5

x	−1	0	1	2	3	4	5	6
x^2	1	0	1	4	9	16	25	36
$-6x$	6	0	−6	−12	−18	−24	−30	−36
$+5$	5	5	5	5	5	5	5	5
$y = x^2 - 6x + 5$	12	5	0	−3	−4	−3	0	5
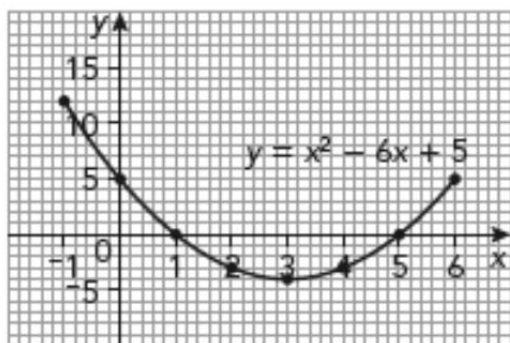
6

x	−1	0	1	2	3	4	5
$-x^2$	−1	0	−1	−4	−9	−16	−25
$+4x$	−4	0	4	8	12	16	20
−3	−3	−3	−3	−3	−3	−3	−3
$y = -x^2 + 4x - 3$	−8	−3	0	1	0	−3	−8
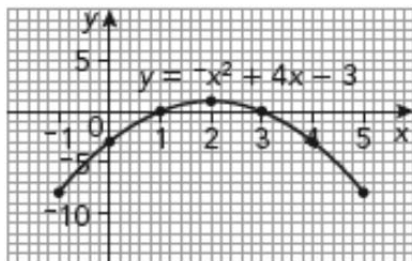
7 a)

x	−1	0	1	2	3	4	5	6	2.5
x^2	1	0	1	4	9	16	25	36	6.25
$-5x$	5	0	−5	−10	−15	−20	−25	−30	−12.5
$+2$	2	2	2	2	2	2	2	2	2
$y = x^2 - 5x + 2$	8	2	−2	−4	−4	−2	2	8	−4.25
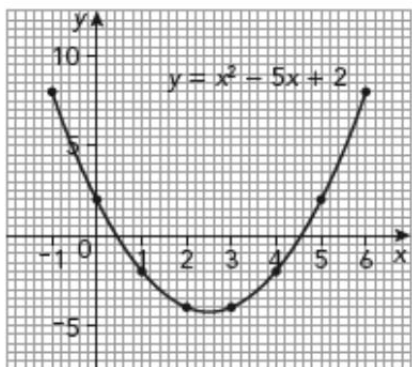
b) $x = 0.4$ (or 0.5) or $x = 4.5$ (or 4.6)

8 a)

x	−2	−1	0	1	2	3	4	1.5
x^2	4	1	0	1	4	9	16	2.25
$2x^2$	8	2	0	2	8	18	32	4.5
$-5x$	10	5	0	−5	−10	−15	−20	−7.5
$+1$	1	1	1	1	1	1	1	1
$y = 2x^2 - 5x + 2$	19	8	1	−2	−1	4	13	−2
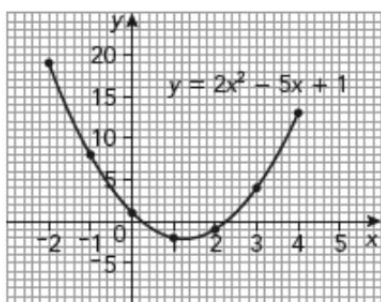
b) $x = 0.2$ or $x = 2.3$

9 a)

x	−1	0	1	2	3	4	5	6	7
x^2	1	0	1	4	9	16	25	36	49
$2x^2$	2	0	2	8	18	32	50	72	98
$-12x$	12	0	−12	−24	−36	−48	−60	−72	−84
$y = 2x^2 - 12x$	14	0	−10	−16	−18	−16	−10	0	14
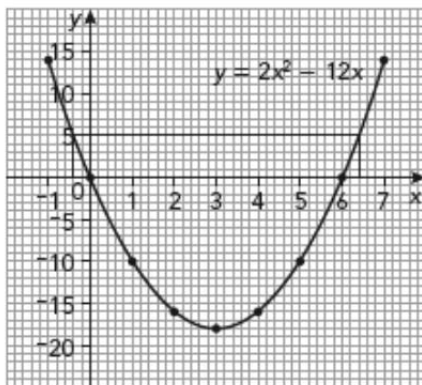
b) $x = -0.4$ or $x = 6.4$

10 **a)**

t	0	1	2	3	4	5
t^2	0	1	4	9	16	25
$d = 5t^2$	0	5	20	45	80	125
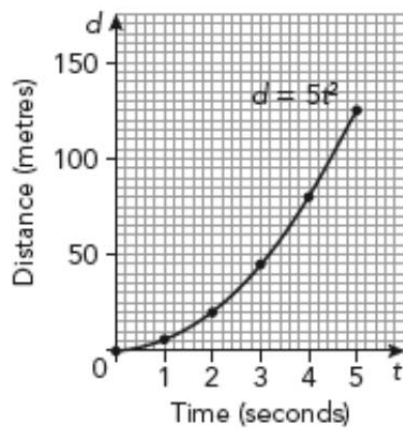
b) 3.6 seconds

Exercise 25.2

Answers are given correct to 1 decimal place.

1 a)
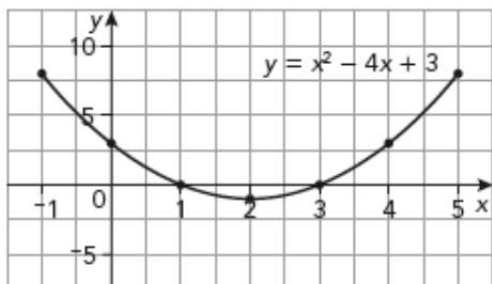
b) $x = 1$ or $x = 3$

2 a)
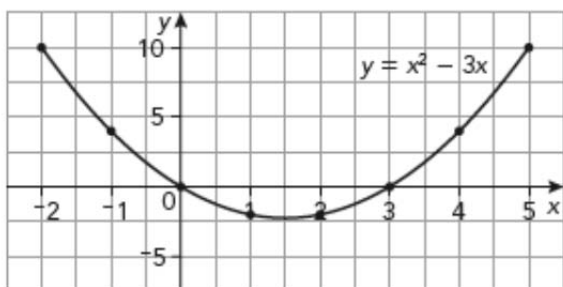
b) $x = 0$ or $x = 3$

3 a)
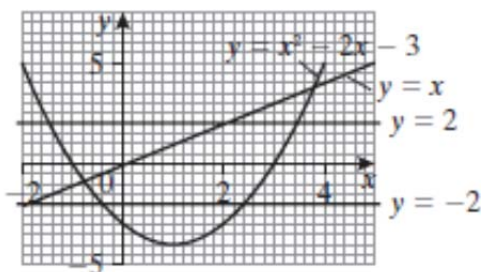
b) i) $x = -1$ or $x = 3$

    ii) $x = -0.4$ or $x = 2.4$

iii) $x = -0.8$ or $x = 3.8$

iv) $x = -1.4$ or $x = 3.4$

4 a)
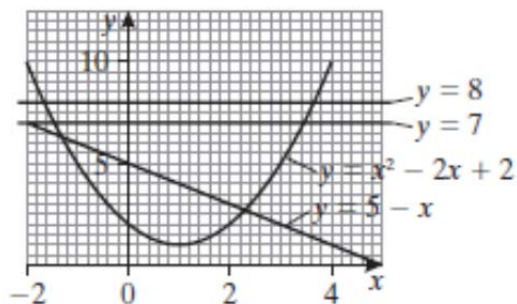
b) i) $x = -1.6$ or $x = 3.6$

ii) $x = -1.3$ or $x = 2.3$

iii) $x = -1.4$ or $x = 3.4$

5 a)
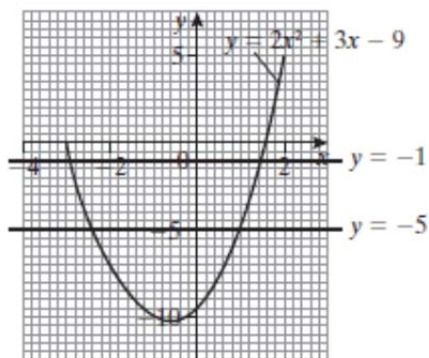
b) i) $x = -2.9$ or $x = 1.4$

ii) $x = -2.4$ or $x = 0.9$

6 a)
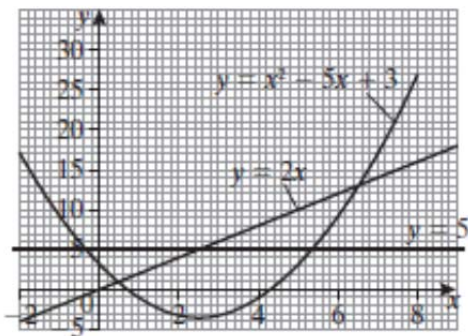
   b) i) $x = 0.7$ or $x = 4.3$

     ii) $x = -0.4$ or $x = 5.4$

     iii) $x = 0.5$ or $x = 6.5$

7 a)
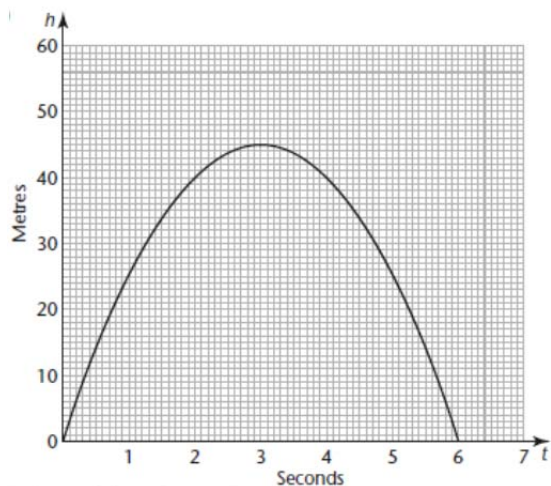
   b) Just – it is at its maximum

   c) 6 seconds

8 $y = -4$

9 $2x^2 - 9x + 4 = 0$

10 $3x^2 + 2x - 6 = 0$

Exercise 25.3

Answers are given correct to 1 decimal place.

1 a)

x	−3	−2	−1	0	1	2	3
x^3	−27	−8	−1	0	1	8	27
+5	5	5	5	5	5	5	5
$y = x^3 + 5$	−22	−3	4	5	6	13	32

b)
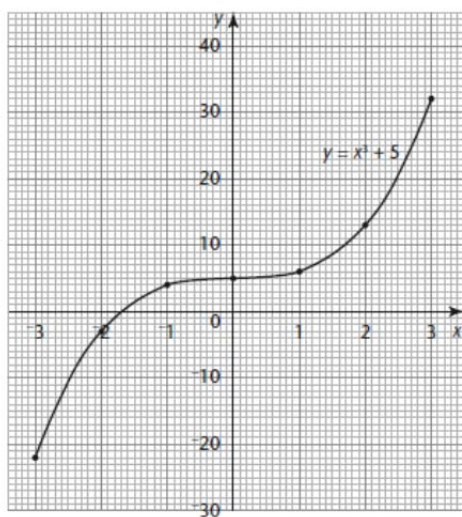
c) $x = -1.7$

2 a)

x	−3	−2	−1	0	1	2	3
x^3	−27	−8	−1	0	1	8	27
$y = -x^3$	27	8	1	0	−1	−8	−27

b)
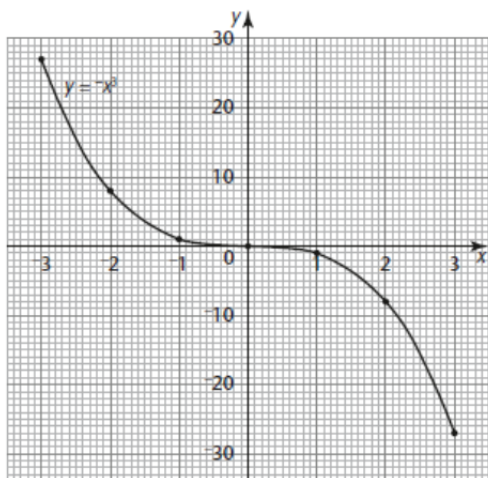
c) $x = -1.8$

3 a)

x	−10	−5	−2	−1	−0.5	−0.1	0.1	0.5	1	2	5	10
$y = \dfrac{1}{x}$	−0.1	−0.2	−0.5	−1	−2	−10	10	2	1	0.5	0.2	0.1

b)
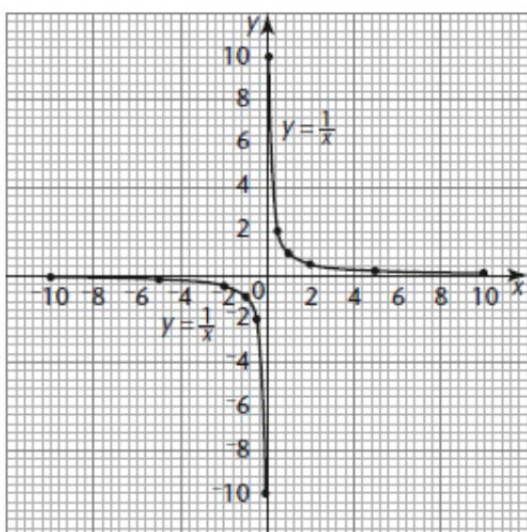
c) *Due to small scale allow a wide tolerance.*

i) $x = 3.3$ (*calculated answer*: $x = 3\frac{1}{3}$)

ii) $x = -0.2$

4 a)

x	−3	−2	−1	0	1	2	3	4
x^3	−27	−8	−1	0	1	8	27	64
$-12x$	36	24	12	0	−12	−24	−36	−48
$+2$	2	2	2	2	2	2	2	2
$y = x^3 - 12x + 2$	11	18	13	2	−9	−14	−7	18

b)
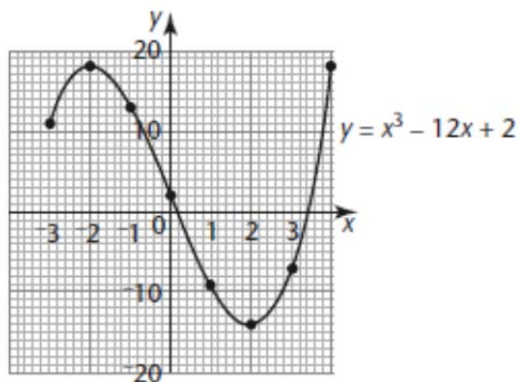
c) The only two values in the range of the graph are $x = 0.2$ and $x = 3.2$

5 a)

x	−3	−2	−1	0	1	2	3	4
x^3	−27	−8	−1	0	1	8	27	64
$-x^2$	−9	−4	−1	0	−1	−4	−9	−16
$-6x$	18	12	6	0	−6	−12	−18	−24
$y = x^3 - x^2 - 6x$	−18	0	4	0	−6	−8	0	24

b)
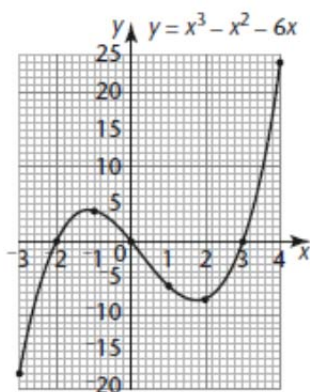
c) The solution is $x = -2$, $x = 0$ or $x = 3$

6 a)

x	−8	−4	−2	−1	1	2	4	8
$y = \dfrac{8}{x}$	−1	−2	−4	−8	8	4	2	1

b)
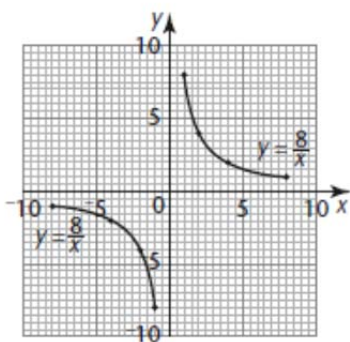
7 a)

x	−12	−8	−6	−4	−3	−2	−1	1	2	3	4	6	8	12
x^2	144	64	36	16	9	4	1	1	4	9	16	36	64	144
$y = \dfrac{12}{x^2}$	0.08	0.19	0.33	0.75	1.33	3	12	12	3	1.33	0.75	0.33	0.19	0.08

b)
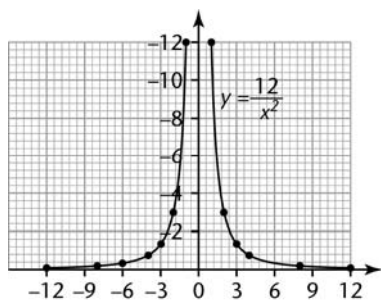
8 a)

x	−5	−4	−2.5	−2	−1	1	2	2.5	4	5
$y = \dfrac{5}{x}$	−1	−1.25	−2	−2.5	−5	5	2.5	2	1.25	1

b) and c)
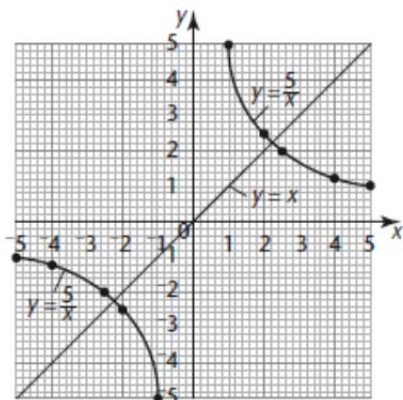
d) $x = -2.2$ or $x = 2.2$

9 a)
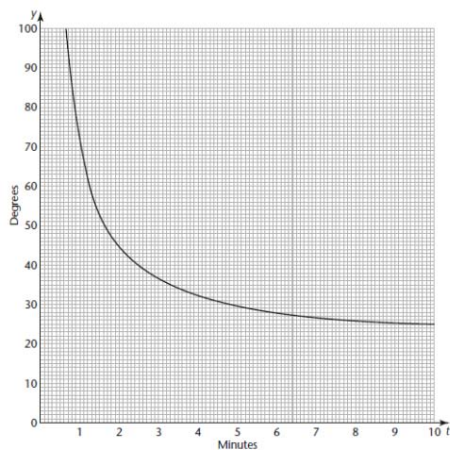
b) 2.5 minutes

c) About 20 °C

10 $y = -x^2 + 4x - 3$

Exercise 25.4

Answers are given correct to 1 decimal place.

1
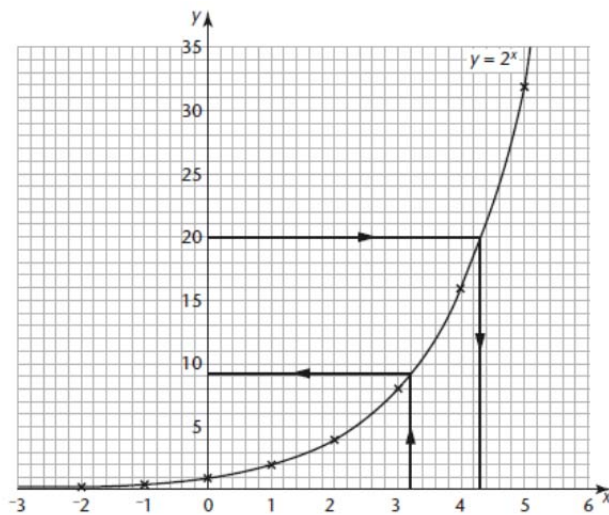
   a) $y = 9.2$

   b) $x = 4.3$

2
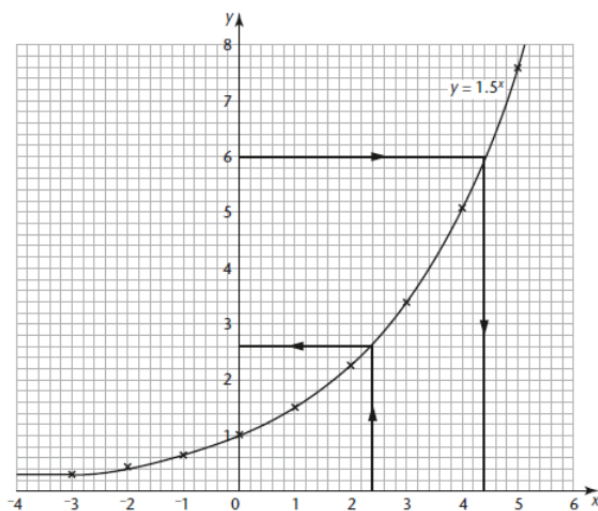
   a) $y = 2.6$

   b) $x = 4.4$

3 a)
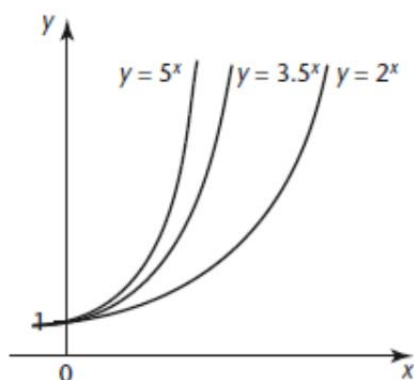
b) i) C

ii) A

iii) B

4 The graph of $y = 3 \times 2^x$ is the same shape as $y = 2^x$ but steeper. It is always above the graph of $y = 2^x$ and

crosses the y axis at $(0, 3)$ instead of $(0, 1)$.

Exercise 25.5

All the answers in this exercise will approximate to the following:

1 Gradient of tangent at $x = 4$ is 4; gradient of tangent at $x = 1$ is −2

2 Gradient of tangent at $x = 4$ is 5; gradient of tangent at $x = 1$ is −1

3 Gradient of tangent at $x = 2$ is 12; gradient of tangent at $x = -1$ is 3

4 Gradient of tangent at $x = 0$ is −12; gradient of tangent at $x = 2$ is 0; gradient of tangent at $x = 3$ is 15

5 a) $x = 1$ or $x = -1$

b) Gradient of tangent at $x = 2$ is −0.25; gradient of tangent at $x = -\dfrac{1}{2}$ is −4

Exercise 26.1

1 a) $f(1) = 7$

  b) $f(-2) = -5$

  c) $f(0) = 3$

2 a) $g(2) = 2$

  b) $g(-3) = 12$

  c) $g(0.5) = 5$

3 a) $h(3) = 3$

  b) $h(-4) = -\dfrac{1}{2}$

  c) $h\left(\dfrac{1}{2}\right) = 13$

4 a) $f(4) = 23$

  b) $f(-3) = 16$

  c) $f\left(\dfrac{1}{2}\right) = 7\dfrac{1}{4}$

5 a) $g(-4) = -50$

  b) $x = -\dfrac{2}{3}$

  c) $5(2 - 3x)$

6 a) $h(2) = 4$

  b) $x = -3$

  c) $\dfrac{6}{x}$

7 a) $f(4) = 5$

  b) $x = 3.5$

  c) $x = 2.5$

8 a) $h(4) = 11$

  b) $4x^2 - 5$

  c) $x = \pm 6$

9 a) $f(4) = -4$

  b) $x = 0$ or $x = 5$

  c) $x = -2$ or $x = 7$

10 a) $g(2) = 1.4$

  b) $x = 1.75$

  c) $x = \dfrac{4}{9}$

Exercise 26.2

1 a) $x = \dfrac{1}{2}$

  b) $f^{-1}(x) = \dfrac{x - 3}{4}$

  c) $f^{-1}(5) = \dfrac{5 - 3}{4} = \dfrac{1}{2}$

2 a) $g^{-1}(x) = \dfrac{6 - x}{2}$

  b) $g^{-1}(4) = \dfrac{6 - 4}{2} = 1$; $g(1) = 6 - 2 \times 1 = 4$

3 a) $h^{-1}(x) = \dfrac{x}{3} + 6$

b) $h^{-1}(9) = 9$

c) $h^{-1}(x) = 6$

4 a) $f^{-1}(x) = \dfrac{5x-1}{2}$

b) $f^{-1}(3) = 7$

c) $f^{-1}\left(\dfrac{4}{5}\right) = 1\dfrac{1}{2}$

5 a) $g^{-1}(x) = 2(x+7)$

b) $g^{-1}(-4) = 6$

c) $g^{-1}(12) = 38$

6 a) $h^{-1}(x) = \dfrac{12}{x} - 1$

b) $h^{-1}(-4) = -4$

c) $h^{-1}(2) = 5$

7 a) $f^{-1}(9) = 4$

b) $f^{-1}(0) = 2\dfrac{1}{2}$

c) $f^{-1}(-18) = -\dfrac{1}{2}$

8 a) $g^{-1}(1) = 2$

b) $g^{-1}(-5) = 4$

c) $g^{-1}(-293) = 100$

9 a) $f^{-1}(x) = \dfrac{3x+1}{6}$

b) $g^{-1}(x) = \dfrac{2-x}{5}$

c) $h^{-1}(x) = \dfrac{6}{x}$

d) $f^{-1}(x) = \dfrac{4(x+7)}{3}$

e) $g^{-1}(x) = \dfrac{x-4}{8}$

f) $h^{-1}(x) = \dfrac{1}{x-2}$

Exercise 27.1

1 a) $\dfrac{4}{7}$

   b) -2.5

   c) 0.2

2 a) 0

   b) $\dfrac{1}{4}$

   c) -1

3 a) -4

   b) 0.75

   c) -3.2

Exercise 27.2

1 a) i) $(1, 6)$

      ii) 4

   b) i) $(4, 4)$

      ii) 6.32

   c) i) $(5, 4.5)$

      ii) 6.71

   d) i) $(5.5, 4.5)$

      ii) 7.07

e) i) (1, 2)

   ii) 6.32

f) i) (−5, −7)

   ii) 8.25

2 (8, 7)

3 a) 12.166

   b) 6.083

   c) Length of line joining midpoints is half *AC*

Exercise 27.3

1

	Gradient	*y*-intercept
a)	3	−2
b)	5	−3
c)	5	2
d)	2	7
e)	−2	7
f)	−3	9

2

	Gradient	*y*-intercept
a)	−2	5
b)	5	1
c)	−2	3.5
d)	$-\dfrac{3}{2}$ or −1.5	4
e)	$-\dfrac{6}{5}$ or −1.2	2
f)	$-\dfrac{2}{5}$ or −0.4	3

3 a) $y = 3x + 2$

   b) $y = 4 - x$

   c) $y = 5x$

   d) $y = 4x - 1$

   e) $y = 5 - 2x$

   f) $y = 3x$

4 a) $y = 4x$

   b) $y = \dfrac{1}{2}x + 1$

   c) $y = -\dfrac{3}{2}x + 1$

   d) $y = -\dfrac{5}{2}x - 5$

5 a) $3x+4y=9$

   b) $y=x+3$

   c) $y=2x-1$

   d) $3x+y=2$

   e) $2x+3y=9$

Exercise 27.4

1 a) $y=2x+2$

   b) $x+2y=4$

   c) $(0, 2)$

2 a) $y=4x+3$ and $4x-y=5$

   b) $2x-3y=5$ and $6y+4x=1$

3 $y=3x+2$

4 $3x+2y=6$

5 $x+3y=16$

6 $3y=2x+9$

7 a) $\dfrac{1}{2}$

   b) $-\dfrac{1}{4}$

   c) $\dfrac{4}{5}$

8 $3y=x+4$

Exercise 28.1

1 a) Acute

b) Right angle

c) Obtuse

d) Acute

e) Reflex

f) Reflex

2 a) Obtuse

b) Acute

c) Reflex

d) Reflex

e) Obtuse

f) Reflex

g) Acute

h) Right angle

3 a) *AC* and *BD*

b) *AB*

Exercise 28.2

1 a) 090°

b) 180°

c) 270°

d) 045°

e) 135°

2 a) 024°

b) 101°

c) 003°

3 *A*: 080°

B: 120°

C: 225°

D: 310°

Exercise 28.3

1 *Check students' diagrams;* 5 diagonals

2 Square, rectangle

3 Rectangle, parallelogram

4 All four angles 90°; opposite sides parallel and

the same length; diagonals same length

5 Square, rhombus

6 Parallelogram, rhombus

7 Opposite angles equal; all sides equal length;

opposite sides parallel; diagonals bisect at 90°

8 Square, rhombus, kite

9 Square, rectangle, isosceles trapezium

10 Square, rectangle, parallelogram, rhombus

Exercise 28.4

1 a) Cuboid

b) Triangular prism

c) Cube

d) Square-based pyramid

2 a) 6 faces, 8 vertices

b) 5 faces, 6 vertices

c) 6 faces, 8 vertices

d) 5 faces, 5 vertices

3 a) Sphere

b) Cylinder

c) Cone

d) Tetrahedron

4 Cuboid, triangluar prism, cube and cylinder

Exercise 28.5

1 Pairs **a)**, **b)**, **d)** and **f)** are congruent

2 Shapes C, F and G

3 35°, 75° and 70°

4 a) No

b) No

Exercise 28.6

1 a) Radius

b) Centre

c) Diameter

d) Chord

e) Tangent

2 11.4 cm

3 25.2 cm

Exercise 28.7

1 a) Cube

b) Square-based pyramid

c) Triangular prism

d) Triangular prism

e) Triangle-based pyramid

f) Cube

2 *The nets are not drawn to size, but lengths are*

marked. Other nets are possible.

a)
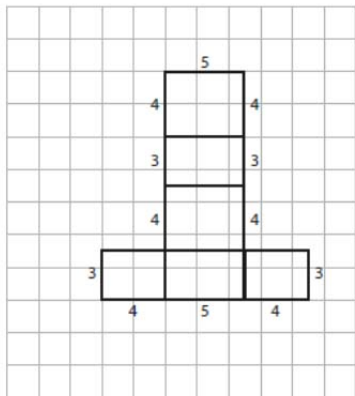
b)
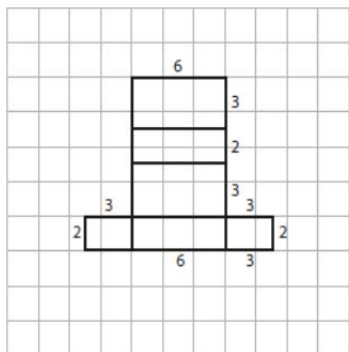
5
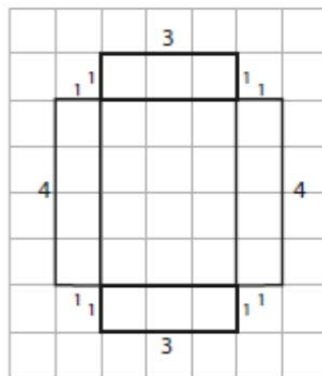
6
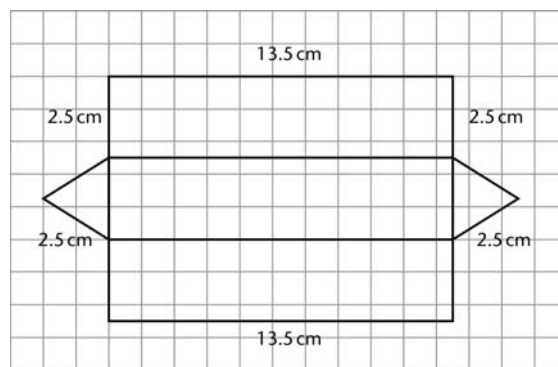
3 a) 12

b) 8

c) 6

4 a) Points K and I

b) Point F

Exercise 29.1

1

	i) Estimated angle	ii) Measured angle
a)	60°–70°	65°
b)	120°–130°	125°
c)	45°–55°	50°
d)	305°–315°	310°

Exercise 29.2

For measurements from students' diagrams, accept ±0.2 cm and ±2° throughout.

1 Check students' diagrams.

2 a) 5.3 cm, 83°, 35°

   b) 2.9 cm, 76.5°, 76.5°

   c) 9.7 cm, 20°, 42°

   d) 2.8 cm, 114°, 33°

3 Check students' diagrams.

4 a) 3.4 cm, 3.7 cm, 117°

   b) 6.4 cm, 3.8 cm, 82°

   c) 6.7 cm, 3.1 cm, 63°

   d) 8.4 cm, 3.9 cm, 29°

Exercise 29.3

For measurements from students' diagrams, accept ±0.2 cm and ±2° throughout.

1 Check students' diagrams.

2 a) 26°, 37°, 117°

   b) 65°, 65°, 50°

   c) 36°, 63°, 81°

4 Check students' diagrams.

5 a) 6.3 cm, 47°, 61°

   b) 6.3 cm, 36°, 93°

6 Check students' diagrams.

   Triangle 1: angle $BCA = 53°$ and angle $ABC = 87°$

   Triangle 2: angle $BCA = 127°$ and angle $ABC = 13°$

7 a) Check students' diagrams.

   b) Angle $D = 113°$

8 a) Check students' diagrams.

   b) $BD = 10$ cm

Exercise 29.4

1 Check students' diagrams.

2 Check students' diagrams.

Exercise 29.5

Check students' diagrams. For measurements from students' diagrams, accept ±0.2 cm and ±2° throughout.

MATHEMATICS   **29** Geometrical Constructions

Exercise 29.6

1 a) 12.4 m

  b) 20.8 m

  c) 27.2 m

  d) 10.4 m

2 a) 42 km

  b) 99 km

  c) 57 km

  d) 28 km

3 Check students' lines. The lines should be the lengths given below.

  a) 5 cm

  b) 5 cm

  c) 12 cm

  d) 7.5 cm

4 a) 12.4 m

  b) Living room: 11.2 m by 6 m

    Dining room: 6.8 m by 4.8 m

    Bedroom 1: 5.6 m by 6 m

    Bedroom 2: 3.4 m by 4.8 m

    Bathroom: 2.4 m by 3.6 m

    Kitchen: 4.6 m by 3.6 m

  c) 13 cm by 7.5 cm

5 a) i)   92 km

  ii)   274 km

  iii)   112 km

  iv)   66 km

  v)   128 km

  vi)   276 km

 b)   4.5 cm

Exercise 30.1

1 Triangle *ABC* is congruenct to triangle *EFD* (SSS)

2 a) Congruent (SAS)

   b) Not congruent

   c) Third angle = 80° therefore congruent (ASA)

   d) Congruent (RHS)

   e) Congruent (SSS)

   f) Not congruent

3 A and (iv) (SAS)

   B and (iii) (ASA)

4 In triangles *ABC* and *ADC*:
   AB = AD (*Given*)
   BC = DC (*Given*)
   AC is common.
   So triangles *ABC* and *ADC* are congruent (SSS).
   Hence Angle *BAC* = Angle *DAC*, and so *AC* bisects Angle *A*, as required.
   Also Angle *BCA* = Angle *DCA*, and so *AC* bisects Angle *C*, as required.

5 Let *D* be the midpoint of *BC*.
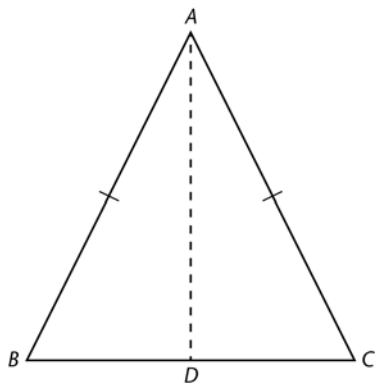
Then in triangles *ABD* and *ACD*:
AB = AC (Equal sides of isoceles triangle)
BD = CD (*D* is midpoint of *BC*)
AD is common.
So triangles *ABD* and *ACD* are congruent (SSS)
Hence Angle *B* = Angle *C*, as required.

6
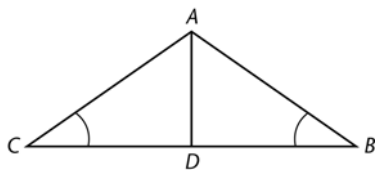
Because triangle *ABC* is isosceles, in triangles *ACD* and *ABD*:

AC = *AB* and Angle *ACD* = Angle *ABD*

CD = *DB* (Given)

Therefore triangle *ACD* is congruent to triangle *ABD* (SAS).

Therefore Angle *CAD* = Angle *BAD* and Angle *ADC* = Angle *ADB*.

But Angles *ADC* and *ADB* lie on a straight line and so add to 180°.

Therefore they are both 90°.

Therefore *AD* bisects angle *BAC* and is perpendicular to *BC*.

7 *ABCD* is a rectangle.
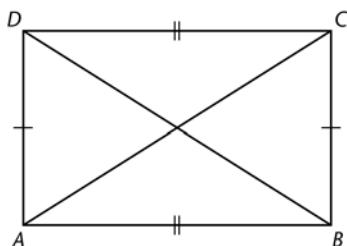
In triangles *ABD* and *ABC*:

AB is common to both, *AD* = *BC*.

Angles *DAB* and *ABC* are both 90°, therefore triangle *ABD* is congruent to triangle *BAC* (SAS).

Therefore *BD* = *AC*.

8 *ABCD* is a rhombus whose diagonals cut at *E*.
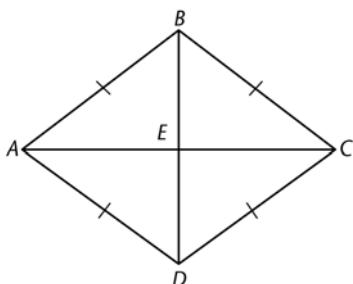
In triangles *BAD* and *BCD*:

BA = *BC* (*ABCD* is a rhombus.)

AD = *CD* (*ABCD* is a rhombus.)

BD is common.

Therefore triangles *BAD* and *BCD* are congruent (SSS).

In triangles *ABE* and *CBE*:

AB = *BC* (*ABCD* is a rhombus.)

BE is common, and because triangles *BAD* and *BCD* are congruent then Angles *ABE* and *EBC* are equal.

Therefore triangles *ABE* and *EBC* are congruent (SAS).

Therefore *AE* = *EC* and Angles *AEB* and *BEC* are equal.

Because they are on a straight line, *AC*, they must be 90°.

Hence, the diagonal *BD* bisects *AC* at right angles.

Similarly you can prove the result for *AC* cutting BD.

Therefore the diagonals bisect each other at right angles.

9 In triangles *ABD* and *CBD*,

$$AB = BC \Big\}\ \text{pairs of equal sides}$$
$$AD = CD \Big\}\ \text{since } ABCD \text{ is a kite.}$$
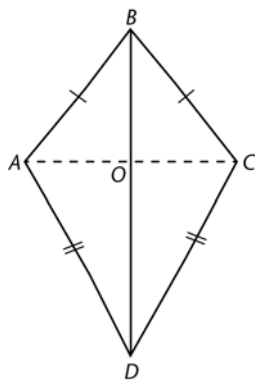
BD is common.

So triangles *ABD* and *CBD* are congruent (SSS).

This means that Angles *ABD* = Angle *CBD* and Angle *ADB* = Angle *CDB*.

i.e. *BD* bisects each of Angles *B* and *D*.

Consider triangle *ABC*.

Let the diagonals intersect at *O*.

$$AB = BC$$
$$\text{Angle } ABO = \text{Angle } CBO \Big\}\ \text{already shown.}$$

BO is common.

So triangles *ABO* and *CBO* are congruent (SAS).

Hence *AO* = *OC* and Angle *AOB* = Angle *COB* = 90°

i.e. the shorter diagonal is bisected at right angles as required.

10   *AX*, *XC*, *AY*, *YB*, *BZ* and *ZC* are all equal.

The angles at *A*, *B* and *C* are equal.
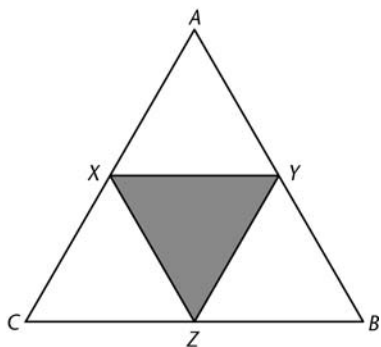
Therefore triangles *CXZ*, *BZY* and *AYX* are congruent (SAS).

Therefore *XZ* = *ZY* = *YX*.

Therefore *XYZ* must be an equilateral triangle.

Exercise 30.2

1 8 cm

2 8 cm

3 Middle mat: 15 cm by 24 cm

Largest mat: 25 cm by 40 cm

4 *PQ* = 4.2 cm; *PR* = 5.88 cm

5 *PQ* = 3.5 cm; *QR* = 6.02 cm

6 3 cm

7 42 m

8 a) Angle *BAC* = Angle *CDE* (alternate angles)

Angle *ACB* = Angle *DCE* (vertically opposite angles)

Angle *ABC* = Angle *CED* (alternate angles)

Corresponding angles are equal so triangle *ABC* is similar to triangle *CDE*.

b) *AC* = 13.5 cm; *BC* = 27 cm

9 Although the lengths of corresponding sides are in constant proportion, the angle between two sets of corresponding sides in the two quadrilaterals is different. Therefore, the two shapes cannot be similar.

10 a) In triangles ADC and BDA,

Angle ADC = Angle BDA = 90°

Angle ABD + Angle ACB = 90°

Angle DAC + Angle ACB = 90°

So Angle ABD = Angle DAC and similarly Angle BAD = Angle ACB

So corresponding angles are equal and therefore traingles ADC and BDA are similar.

b) BD = 1.8 cm

Exercise 30.3

1 a) 4

b) 9

c) 25

d) 16

e) 36

f) 100

2 a) 1000

b) 64

c) 125

d) 8

e) 27

f) 512

3 a) 4

   b) 6

   c) 8

   d) 10

4 a) 72.5 cm^2

   b) 18.1 m^2

5 25.9 cl

6 360 cm^2

7 1 : 50

8 27 : 64 : 125

9 2.48 m

10   **a)** 15

       b) 225

       c) 4.52 m^2

11   15.1 cm

12   0.0226 m^2 or 226 cm^2

13   77.44 cm^2

14   693 cm^2

Exercise 31.1

1 a) 4

  b) 0

  c) 3

2 a)
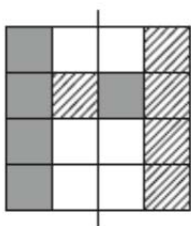
  b)
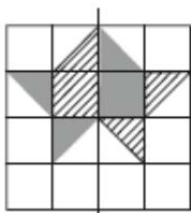
  c)
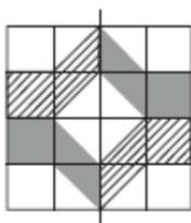
3 a)
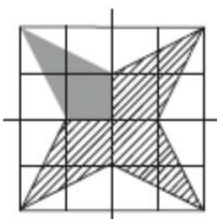
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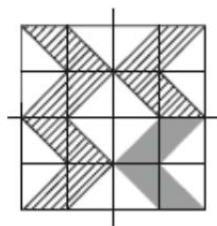
c)
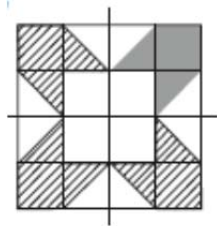
Exercise 31.2

1
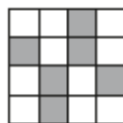
2
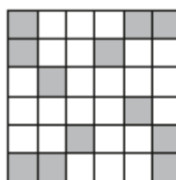
3 Check students' diagrams.

  Pattern with rotational symmetry order 2 and no lines of symmetry.

4

Exercise 31.3

1 a) Two lines of symmetry, rotational symmetry

of order 2

b) Three lines of symmetry, rotational

symmetry of order 3

c) Five lines of symmetry, rotational symmetry

of order 5

2 a) 9

b) An infinite number

3 2 planes of symmetry, 1 axis of symmetry

4 Check students' diagrams.
Sketch of octagon with 2 lines of symmetry and

rotational symmetry of order 2

5 a) Square

b) Kite or isosceles trapezium

6 Sphere

Exercise 31.4

1 Kite
Reason: $AT = BT$ (equal tangents)
and $AO = BO$ (equal radii).

2 Triangles OXQ, OYR and OYS
Reason: $PQ = RS$ (equal chords are equidistant
from centre)
So $PX = XQ = RY = YS$
$OP = OQ = OR = OS$ (equal radii).

3 $M\hat{C}A = 62°$

Reason: $A\hat{M}C = 90°$ (perpendicular from centre
to chord)

So $M\hat{C}A = 62°$ (angle sum of triangle = 180°)

Exercise 32.1

1 a) $a = 19°$ (*The sum of the angles on a straight line is 180°*)

b) $b = 143°$ (*The sum of the angles at a point is 360°*)

c) $c = 48°$ (*The sum of the angles on a straight line is 180°*)
$d = 132°$ (*Vertically opposite angles are equal* or *the sum of the angles on a straight line is 180°*)

d) $e = 45°$ (*The sum of the angles on a straight line is 180°*)

f) $f = 65°$ (*f + 71° = 136°. Vertically opposite angles are equal*)

g) $g = 117°$ (*The sum of the angles on a straight line is 180°*)
$h = 63°$ (*Vertically opposite angles are equal* or *the sum of the angles on a straight line is 180°*)

h) $k = 80°$ (*The sum of the angles at a point is 360°*)

2 $113° + 123° + 134° = 370°$. Karim has meaured incorrectly as the angles at a point total 360°.

Exercise 32.2

1 $x = 74°$ (*Alternate angles are equal*)
$y = 137°$ (*Corresponding angles are equal*)
$z = 67°$ (*Allied angles add up to 180°*)

2 $a = 110°, b = 70°, c = 70°$
$d = 105°, e = 75°, f = 63°, g = 63°$
$p = 82°, q = 67°, r = 31°$

3
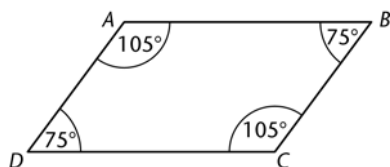
4
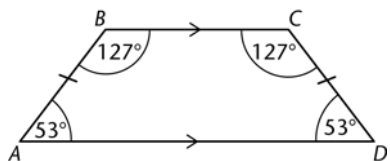
5 $a = 40°$ (*The sum of the angles on a straight line is 180°*)

   $b = 72°$ (*Alternate angles are equal*)

   $c = 68°$ (*The sum of the angles in a triangle is 180°* or *corresponding angles are equal*)

   $d = 81°$ (*Alternate angles are equal*)

   $e = 57°$ (*The sum of the angles in a triangle is 180°*)

   $f = 57°$ (*The sum of the angles on a straight line is 180°* or *corresponding angles are equal*)

6 Angle $ABE = 51°$, Angle $CDE = 94°$ (*The sum of the angles in a triangle is 180°*)

   Corresponding angles BEA and CDE (or ABE and BCD) are equal therefore lines BE and CD are parallel.

Exercise 32.3

1 $a = 70°$ (*The sum of the angles in a triangle is 180°*)

2 $b = 21°$ (*The sum of the angles in a triangle is 180°*)

3 $c = 71°$ (*Equal angles in an isosceles triangle*)

   $d = 38°$ (*The sum of the angles in a triangle is 180°*)

4 $e = f = 72°$ (*Equal angles in an isosceles triangle* and *the sum of the angles in a triangle is 180°*)

5 $g = 60°$ (*Angles in an equilateral triangle are all 60°*)

   $h = 120°$ (*The sum of the angles on a straight line is 180°*)

6 $b = 113°$ (*The sum of the angles in a triangle is 180°*)

7 $c = 60°$ (*The sum of the angles in a triangle is 180°*)

8 $a = 50°$ (*The sum of the angles in a triangle is 180°*)

   $b = 130°$ (*The sum of the angles on a straight line is 180°*)

9 $c = 20°$ (*The sum of the angles on a straight line is 180°*)

   $d = 130°$ (*The sum of the angles in a triangle is 180°*)

10 $e = 137°$ (*The sum of the angles on a straight line is 180°*)

   $f = 25°$ (*The sum of the angles in a triangle is 180°*)

   $g = 68°$ (*The sum of the angles on a straight line is 180°*)

11 $h = 128°$ (*The sum of the angles on a straight line is 180°*)

   $i = 64°$ (*The sum of the angles in a triangle is 180°*)

12 $a = 74°$ (*Equal angles in an isosceles triangle* and *the sum of the angles in a triangle is 180°*)

13 $b = 41°$ (*Equal angles in an isosceles triangle* and *the sum of the angles in a triangle is 180°*)

14 $a = 54°$ (*The sum of the angles in a triangle is 180°*)

15 $b = 45°$ (*The sum of the angles in a triangle is 180°*)

16 $c = 36°$ (*The sum of the angles in a triangle is 180°*)

17 $a = 126°$ (*The sum of the angles in a quadrilateral is 360°*)

18 $i = 115°$ (*The sum of the angles in a quadrilateral is 360°*)

19 $j = 123°$ (*The sum of the angles in a quadrilateral is 360°*)
    $k = 57°$ (*The sum of the angles on a straight line is 180°*)

20 $l = 100°$ (*The sum of the angles on a straight line is 180°*)
    $m = 84°$ (*The sum of the angles in a quadrilateral is 360°*)

Exercise 32.4

1 a) 58°

  b) 110°, 121°, 97°, 90°, 122°

2 a) 78°

  b) 126°, 132°, 115°, 145°, 100°, 102°

3 a) 61°

  b) 113°, 137°, 89°, 143°, 119°, 119°

4 Exterior angle = 40°; interior angle = 140°

5 a) 150°

  b) 162°

6 15 sides

7 107°

8 Exterior angle = 24°; interior angle = 156°

9 12 sides

10   130°

Exercise 32.5

1 $a = 140°$ (*Angle at centre = 2 × angle at circumference*)

2 $b = 45°$ (*Angle at circumference = ½ angle at centre*)

  $c = 45°$ (*Equal angles in an isosceles triangle* and *the sum of the angles in a triangle is 180°*)

3 $d = 100°$ (*Equal angles in an isosceles triangle* and *the sum of the angles in a triangle is 180°*)

  $e = 50°$ (*Angle at circumference = ½ angle at centre*)

4 $f = 60°$ (*The sum of the angles on a straight line is 180°*)

  $g = 120°$ (*Angle at centre = 2 × angle at circumference*)

5 $h = 25°$ (*Angle at circumference = ½ angle at centre = 45°; equal angles in an isosceles triangle*)

6 $i = 22°$ (*Angle at centre = 2 × angle at circumference = 136°; equal angles in an isosceles triangle*)

7 $j = 45°$ (*Angle at circumference = ½ angle at centre*)

  $k = 135°$ (*The sum of the angles on a straight line is 180°*)

8 $l = 42°$ (*Angle at circumference = ½ angle at centre*)

9 $m = 30°$ (*Angle in a semicircle is 90°* and *the sum of the angles in a triangle is 180°*)

10 $n = 120°$ (*Angle at circumference = ½ angle at centre*)

Exercise 32.6

1 $a = 50°$ (*Angles in the same segment are equal*)

$b = 100°$ (*Angle at centre = 2 × angle at circumference*)

2 $c = 110°$ (*Angles around a point sum to 360°* and *angle at circumference = ½ angle at centre*)

$d = 110°$ (*Angles in the same segment are equal*)

3 $e = 45°$ (*Angle at circumference = ½ angle at centre*)

$f = 45°$ (*Angles in the same segment are equal*)

4 $g = 45°$ (*Angle at circumference = ½ angle at centre*)

$h = 45°$ (*Angles in the same segment are equal*)

$i = 45°$ (*Equal angles in an isosceles triangle*)

5 $j = 80°$ (*Angle at circumference = ½ angle at centre*)

$k = 100°$ (*Angles in opposite segments are supplementary*)

6 $l = 140°$ (*Angle at centre = 2 × angle at circumference*)

$m = 110°$ (*Angles in opposite segments are supplementary*)

7 Obtuse angle at centre $= 120°$ (*Angles in isosceles triangle*)

Reflex angle at centre $= 240°$ (*Angles about a point*)

$n = 120°$ (*Angle at circumference = ½ angle at centre*)

8 $o = 20°$ (*Angles in the same segment are equal*)

$p = 40°$ (*Angles in the same segment are equal*)

$q = 60°$ (*External angle of triangle*)

9 $r = 104°$ (*Angles in opposite segments are supplementary*)

$s = 85°$ (*Angles in opposite segments are supplementary*)

10 $t = 50°$ (*Angles in the same segment are equal*)

$u = 56°$ (*Sum of the angles in a triangle is 180°* and *angles in the same segment are equal*)

$v = 34°$ (*Angles in opposite segments are supplementary*)

11 $w = 45°$ (*Angles in opposite segments are supplementary*)

$x = 60°$ (*Angles in opposite segments are supplementary*)

12 $y = 40°$ (*Angles in the same segment are equal*)

$z = 80°$ (*Angle at centre $= 2 \times$ angle at circumference*)

$a = 50°$ (*Equal angles in an isosceles triangle* and *the sum of the angles in a triangle is 180°*)

13 $b = 95°$ (*Angles on a straight line* and *angles in opposite segments are supplementary*)

$c = 126°$ (*Angles in opposite segments are supplementary* and *angles on a straight line*)

14 $d = 30°$ (*Angle in a semi-circle, angle sum of triangle and angles in same segment*)

15 $e = 90°$ (*Angle in a semicircle is 90°*)

$f = 40°$ (*Angles in the same segment are equal*)

$g = 32°$ (*Angle sum of triangle is 180°*)

Exercise 32.7

Other reasons may be equally valid.

1 $a = 40°$ (*Equal angles in an isosceles triangle* and *the sum of the angles in a triangle is 180°*)

$b = 40°$ (*Angles in the same segment are equal* or *angle in a semi-circle is 90°*)

$c = 50°$ (*Angle in a semi-circle is 90°*)

2 $d = 60°$ (*Angles in the same segment are equal*)

$e = 20°$ (*Angles on straight line, angles in a kite, angles in a triangle*)

$f = 20°$ (*Angles in the same segment are equal*)

3 $g = 69°$ (*Angle at circumference $= \frac{1}{2}$ angle at centre*)

$h = 34.5°$ (*Angles on straight line, angles in an isoceles triangle*)

4 $i = 90°$ (*Angle in a semi-circle is 90°*)

$j = 80°$ (*Angles in an isoceles triangle, angles on straight line*)

$k = 50°$ (*Angles in an isoceles triangle*)

5 $l = 50°$ (*Angles in the same segment are equal*)

$m = 40°$ (*Angle at centre $= 2 \times$ angle at circumference, angles in an isoceles triangle*)

6 $n = 40°$ (*Base angles in an isoceles triangle are equal*)

7 $o = 55°$ (*Angles in the same segment are equal*)

$p = 27.5°$ (*Angles in opposite segments are supplementary, angles in an isoceles triangle*)

8 $q = 90°$ *(Angle in a semi-circle is 90°)*

$r = 30°$ *(Angle in a semi-circle is 90°, angles sum of triangle)*

$s = 30°$ *(Angles in the same segment are equal)*

$t = 40°$ *(Angles on straight line, angles in a triangle)*

9 $u = 18°$ *(Angle in a semi-circle is 90°, angle sum of triangle, angles in the same segment are equal)*

Exercise 32.8

Other reasons may be equally valid.

1 $a = 50°$ *(Angle between tangent and radius is 90°)*

2 $b = 50°$ *(Angle between tangent and radius is 90°, angle sum of quadrialteral)*

3 $c = 70°$ *(Angle between tangent and radius is 90°)*

$d = 20°$ *(Angles in the same segment are equal)*

4 $e = 15°$ *(Angle between tangent and radius is 90° so 6e = 90°)*

$f = 30°$ *(Angles in an isosceles triangle)*

5 $g = 45°$ *(Angle between tangent and radius is 90°)*

6 $h = 50°$ *(Angle between tangent and radius is 90°, angle sum of triangle)*

$i = 30°$ *(Angle between tangent and radius is 90°, angle sum of triangle)*

7 $j = 40°$ *(Angle between tangent and radius is 90°)*

$k = 50°$ *(Angle in a semi-circle is 90°, angles sum of triangle)*

$l = 40°$ *(Angles in the same segment are equal)*

8 $m = n = 55°$ *(Angles in an isosceles triangle and kite, angle between tangent and radius)*

Exercise 33.1

Check the accuracy of students' drawings.

The diagrams in these answers are not accurate but are given as a guide.

1 The locus is made up of four straight lines and four quarter circles, all 1 cm outside the rectangle.
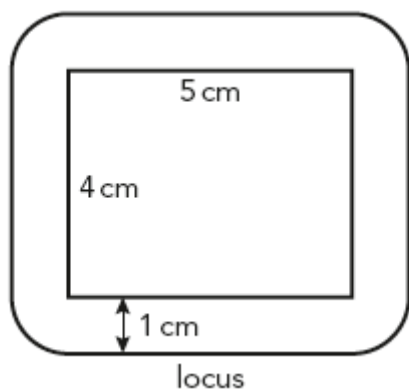
2 The line drawn is the perpendicular bisector of *AB*.

The region shaded is the locus.
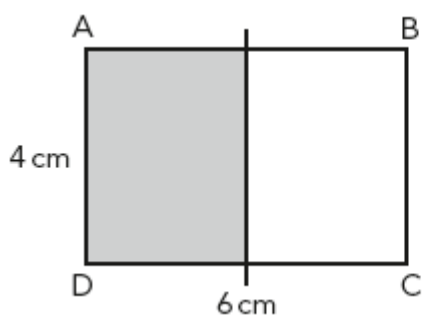
3 The locus is the line parallel to the two given lines.
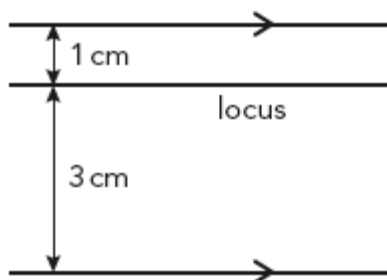
4 The circle is centred on A and has radius 4 cm.

The region shaded is the locus.
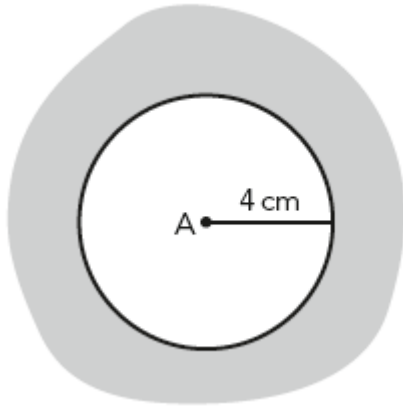
5 Two lines are drawn parallel to the given line and 2 cm away from it.

Two semicircles of radius 2 cm are drawn at the ends.

The region shaded is the locus.
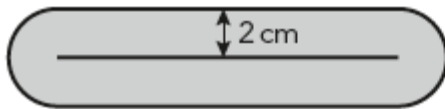
6 The region shaded is the locus.
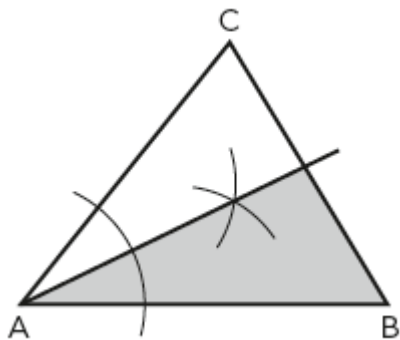
7 The locus passes through *B* and *D*.
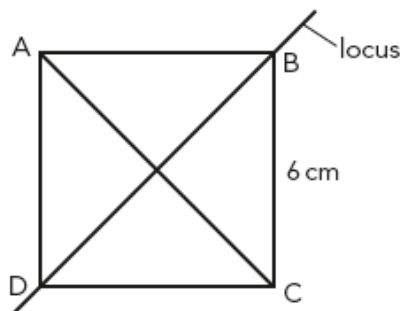
8 A quarter circle (quadrant) is centred on A and has radius 3 cm.
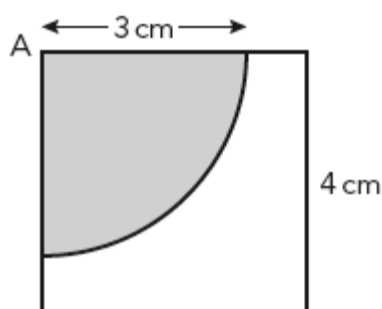
9 The locus is the angle bisector of *CAB*.
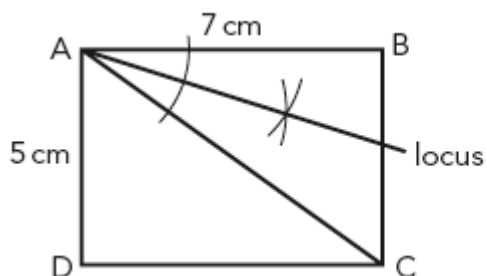
10   The locus is the perpendicular bisector of *AB*.
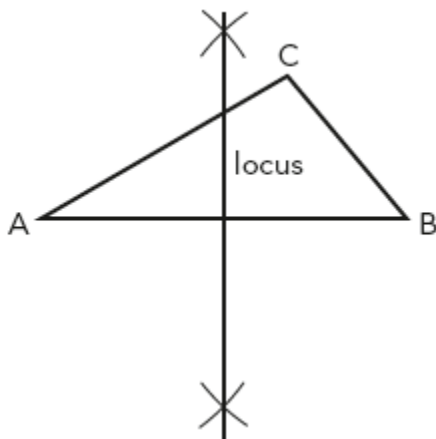
Exercise 33.2

Check the accuracy of students' drawings.

The diagrams in these answers are not accurate but are given as a guide.

*The regions **not required** are shaded in these answers.*

1 Draw a circle centred on *A,* radius 3 cm and a circle centred on *B*, radius 4 cm.

2 Draw a circle centred on *B,* radius 3.5 cm and the perpendicular bisector of *AD* and *BC*.
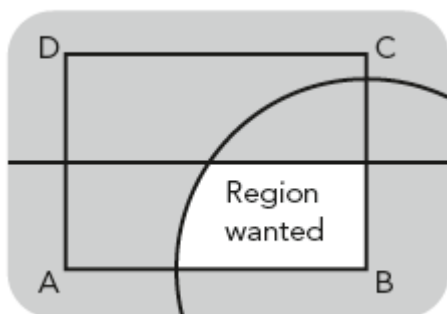
3 The point equidistant from all sides is the intersection of the bisectors of the angles.
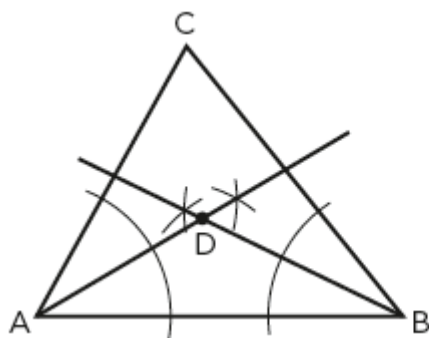
4 Construct the angle bisector of *ABC*.

Draw arcs centred on *A* of radius 3.5 cm.

The points are marked *E* and *F*.
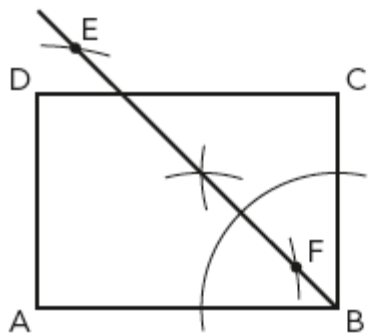
5 Construct the angle bisector of *ABC*.

Draw an arc centred on *C* of radius 3 cm.

The region required is labelled R.
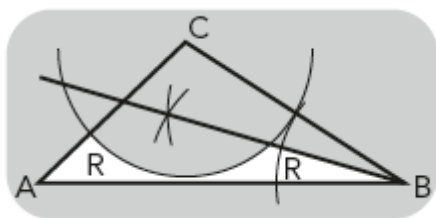
6 Draw two circles centred on *A,* radius 2 cm and radius 3 cm.

The region required is labelled R.
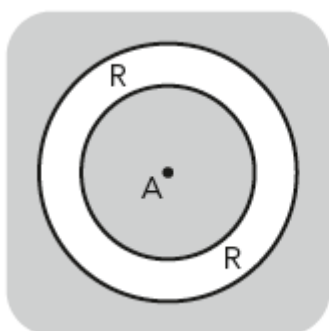
7 Draw a circle centred on *A,* radius 2.5 cm and a circle centred on *B*, radius 3 cm.

The region required is labelled R.
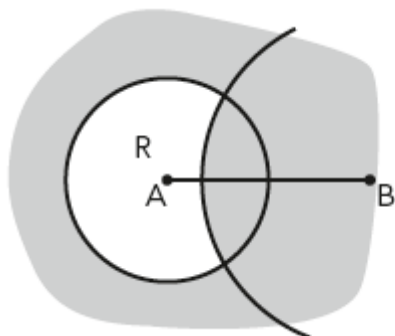
8 Construct the perpendicular bisector of *AB*.

Draw an arc centred on *C* of radius 4 cm.

The point is marked *D*.
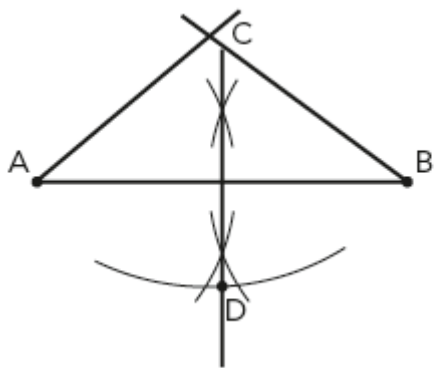
9 Draw four quarter circles centred on each vertex, radius 2 cm.

The region required is labelled R.
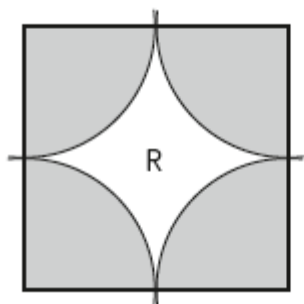
10   Construct the perpendicular bisector of *AB* and the angle bisector of *BAC*.

The region required is labelled R.
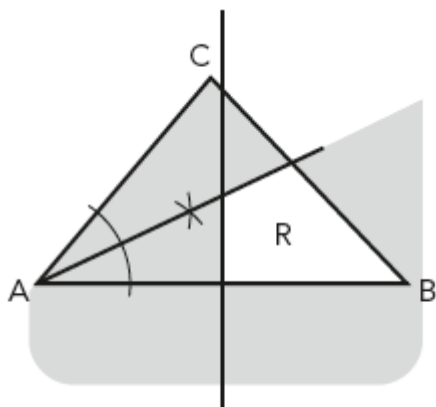
11   Draw two lines 1 cm away from each of the longer sides.

Draw an arc radius 2.5 cm centred on the midpoint of one of the shorter sides.

The region required is labelled R.
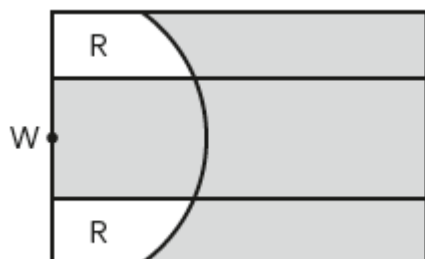
12   Draw an arc radius 3 cm centred on *A*.

Construct the angle bisector of *BCD*.

The region required is labelled R.
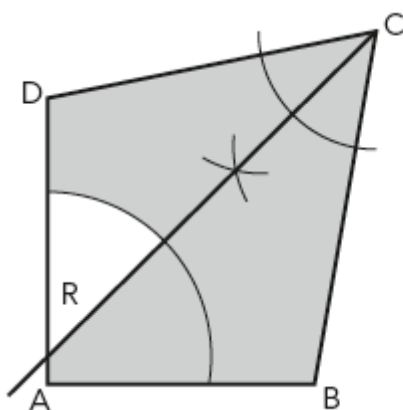
13   Draw a line 1 cm away from *AB*.

Draw an arc radius 4 cm centred on *C*.

The region required is labelled R.
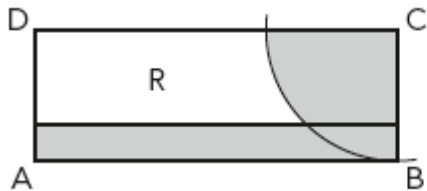
14   Construct the angle bisector of *BAD*.

Draw an arc centred on *A* of radius 3 cm.

The region required is labelled R.
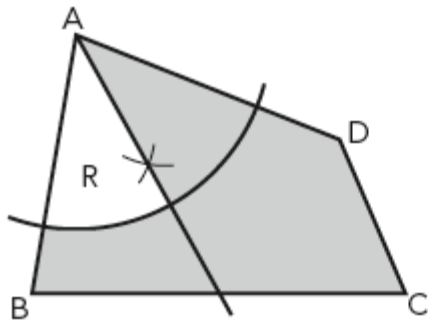
Exercise 34.1

1 a) millimetres or centimetres

 b) metres or centimetres

 c) metres

 d) millimetres or centimetres

2 a) 45.2 mm

 b) 20 mm

 c) 45 mm

 d) 93.5 mm

 e) 2101 mm

 f) 3000 mm

 g) 2239 mm

 h) 9100 mm

3 a) 5200 cm

 b) 500 cm

 c) 232 cm

 d) 1816 cm

 e) 66 cm

 f) 7 cm

 g) 31 cm

 h) 4.6 cm

4 a) 2.1463 m

 b) 5.142 m

 c) 5.7 m

 d) 1.146 m

5 a) 31 cm, 1600 mm, 2.42 m, 284 cm, 9 m

 b) 105 mm, 3.2 m, 423 cm, 6100 mm, 804 cm

6 6.7 km

7 a) 12 g

 b) 7000 g

 c) 1130 g

 d) 2140 g

8 a) 6.6 kg

 b) 8 kg

 c) 6.3 kg

 d) 5.126 kg

9 a) 874 g, 1.7 kg, 4000 g, 9.4 kg, 52 000 g

 b) 0.174 kg, 2104 g, 2.79 kg, 3.4 kg, 4123 g

10 750 g

11 a) 520 ml

 b) 7000 ml

 c) 1520 ml

d) 160 ml

12 **a)** 95.03 litres

b) 2 litres

c) 2.341 litres

d) 46.42 litres

13 **a)** 51 cl, 80 cl, 1600 ml, 2.4 litres, 9 litres

b) 51.5 ml, 1 litre, 1500 ml, 180 cl, 3.1

litres

14 650 ml

Exercise 34.2

1 a) 30 000 cm^2

b) 230 mm^2

c) 95 200 cm^2

d) 1.4 mm^2

2 a) 900 cm^2

b) 81.4 cm^2

c) 720 m^2

d) 9.4 m^2

3 a) 3 200 000 cm^3

b) 0.000 042 m^3

c) 0.005 m^3

d) 6 420 000 cm^3

4 a) 2610 cm^3

b) 9.5 litres

c) 2400 ml

d) 0.91 litres

5 Calculation should be $5 \times 2 \times 0.5$ ($= 5$ m^3) not 5

$\times 2 \times 50$

6 195 mm (or 19.5 cm)

7 1000

8 100

9 147 if the sugar cubes are not crushed; 179 if

the sugar cubes are crushed.

10   50 m

Exercise 35.1

1 a) 10 cm

   b) 38 cm

2 a) Missing lengths: 8 cm and 9 cm; Perimeter

72 cm

   b) Missing lengths: 10 cm, 6 cm and 7 cm;

Perimeter 74 cm

   c) Missing lengths: 120 m and 90 m; Perimeter

660 m

   d) Missing lengths: 3.4 cm, 2 cm, 8.2 cm, 2

cm, 3.4 cm and 3.1 cm; Perimeter 44.2 cm

3 a) 11.4 cm

   b) 12.8 cm

   c) 11.2 cm

4 5 cm

5 30 cm

6 1 cm by 29 cm; 2 cm by 28 cm; 3 cm by 27 cm;
   4 cm by 26 cm; 5 cm by 25 cm; 6 cm by 24 cm;

   7 cm by 23 cm; 8 cm by 22 cm; 9 cm by 21 cm;
   10 cm by 20 cm; 11 cm by 19 cm; 12 cm by 18
   cm;
   13 cm by 17 cm; 14 cm by 16 cm;
   15 cm by 15 cm

Exercise 35.2

1 16.92 cm^2

2 6.76 m^2

3 15.023 cm^2

4 36 m^2

5 a) 2.34 km^2

   b) 6.2 km

6 49 cm^2

7 15 cm

8 a) 444 m^2

   b) 22.2 kg

   c) 9

Exercise 35.3

1 a) 12 cm^2

   b) 20 m^2

   c) 35 cm^2

   d) 31.5 m^2

   e) 30.38 cm^2

   f) 12.6 m^2

   g) 7.5 m^2

   h) 16.38 cm^2

i) 24 m^2

2 a) 40 m^2

  b) 45 cm^2

  c) 49.5 m^2

  d) 17.5 m^2

  e) 13.76 cm^2

  f) 9.86 m^2

  g) 8.16 m^2

  h) 11.25 cm^2

  i) 19.09 m^2

3 a) 24 cm^2

  b) 4.8 cm

Exercise 35.4

1 a) 40 cm^2

  b) 42 cm^2

  c) 30 cm^2

  d) 34.2 cm^2

  e) 37.6 cm^2 to 1 d.p.

  f) 23.8 cm^2

2 a) 46 cm^2

  b) 14 cm^2

  c) 42 cm^2

d) 43.7 cm^2 to 1 d.p.

e) 67.8 cm^2 to 1 d.p.

f) 57.1 cm^2 to 1 d.p.

3 a) $a = 8$

  b) $b = 4$

  c) $c = 8$

4 a) $x = 6$

  b) $y = 4.6$

  c) $z = 4.5$

Exercise 35.5

1 a) 30 cm^2

  b) 16 cm^2

  c) 15 cm^2

  d) 31.5 cm^2 to 1 d.p.

  e) 16.2 cm^2 to 1 d.p.

  f) 19.5 cm^2 to 1 d.p.

2 a) 20 cm^2

  b) 12.5 cm^2

  c) 21 cm^2

  d) 5.2 cm^2 to 1 d.p.

  e) 15.3 cm^2

  f) 12.6 cm^2 to 1 d.p.

3 a) $a = 4$

b) $b = 3$

c) $c = 1.5$

4 a) $x = 2.6$

  b) $y = 3.6$

  c) $z = 2.6$

5 9 cm

6 8 cm

Exercise 35.6

1 a) 69 cm^2

  b) 60 cm^2

  c) 38 cm^2

  d) 96 cm^2

  e) 76 cm^2

  f) 70 cm^2

  g) 33 cm^2

  h) 66 cm^2

  i) 78 cm^2

  j) 125 cm^2

2 a) 28 cm^2

  b) 28.5 cm^2

  c) 32 cm^2

  d) 20 cm^2

  e) 35 cm^2

  f) 21.2 cm^2

Exercise 35.7

1 a) 37.7 cm

  b) 28.3 cm

  c) 62.8 m

  d) 51.2 cm

  e) 47.8 m

  f) 78.5 m

  g) 0.9 cm

  h) 53.4 m

  i) 15.9 m

  j) 20.4 cm

2 a) 31.4 cm

  b) 44.0 cm

  c) 100.5 m

  d) 113.7 m

  e) 33.3 m

  f) 175.9 cm

  g) 20.1 cm

  h) 377.0 m

  i) 11.9 m

j) 458.7 cm

3 57.5 m

4 40 074 km

5 94.2 cm

6 a) 23.9 cm

b) 5.7 cm

c) 15.9 cm

7 95.5 m

8 44.0 m

Exercise 35.8

1 a) 50.3 cm^2 to 1 d.p.

b) 804 m^2 to the nearest m^2

c) 401 m^2 to the nearest m^2

d) 581 m^2 to the nearest m^2

e) 249 cm^2 to the nearest cm^2

2 a) 28.3 cm^2 to 1 d.p.

b) 201 m^2 to the nearest m^2

c) 88.2 cm^2 to 1 d.p.

d) 547 cm^2 to the nearest cm^2

e) 16.6 m^2 to 1 d.p.

3 7.07 m^2 to 2 d.p.

4 254 cm^2 to the nearest cm^2

5 0.503 m^2 to 3 d.p.

6 1099 m^2 to the nearest m^2

7 124 cm^2 to the nearest cm^2

8 Square: $3.5 \times 3.5 = 12.25$ cm^2; circle: $\pi \times 2^2 = 12.57$ cm^2 to 2 d.p.

So the circle has the larger area.

9 15

10 145 cm^2 to the nearest cm^2

Check: $\pi \times 6.8^2 \approx 3 \times 7^2 = 147$ cm^2 (or 3 × 50 = 150 cm^2)

Exercise 35.9

1 a) 3.56 cm

b) 13.6 cm

c) 27.0 cm

d) 12.4 cm

e) 8.41 cm

f) 5.91 cm

2 a) 9.08 cm^2

b) 25.1 cm^2

c) 139 cm^2

d) 59.5 cm^2

e) 18.1 cm^2

f) 13.9 cm^2

3 a) 25.7 cm

b) 26.3 cm

c) 51.3 cm

4 a) 43°

b) 185°

c) 58°

d) 57°

e) 203°

f) 159°

g) 62°

5 a) 4.91 cm to 3 s.f.

b) 4.84 cm to 3 s.f.

c) 5.57 cm to 3 s.f.

6 a) 6.59 cm to 3 s.f.

b) 1.51 cm to 3 s.f.

c) 1.81 m to 3 s.f.

7 Blue area = 626 mm^2 to the nearest mm^2
Black strip length = 167 mm to the nearest mm

8 57.2957795…°

Exercise 35.10

1 30 cm^3

2 8 cm^3

3 3 m^3

4 4 cm

5 837 cm^3 to the nearest cm^3

6 217 cm^3 to the nearest cm^3

7 402 cm^3 to the nearest cm^3

8 a) 525 cm^3

b) 405 cm^3

c) 67.5 cm^3

9 a) 384 cm^3

b) 168 cm^3

c) 173 cm^3 to the nearest cm^3

10 46.8 cm^3

11 171 m^3 to the nearest m^3

12 110 cm^3 to the nearest cm^3

13 16 cm

14 9.07 cm to 3 s.f.

15 6.44 m to 3 s.f.

16 4.05 litres

Exercise 35.11

1 a) 94 cm^2

b) 144.8 cm^2

2 2400 cm^2

3 1710 cm^2

4 13 600 cm^2

5 240 cm^2

6 a) 496 cm^2

 b) 217 cm^2

 c) 302 cm^2

 d) 430 cm^2

 e) 352 cm^2

7 a) 108 cm^2

 b) 216 cm^2

Exercise 35.12

1 a) 18 cm^3

 b) 54 cm^3

 c) 70 m^3

 d) 50 cm^3

 e) 179 cm^3 to 3 s.f.

 f) 30 cm^3

2 a) 103 cm^3 to 3 s.f.

 b) 314 cm^3 to 3 s.f.

 c) 51.5 cm^3 to 3 s.f.

 d) 154 cm^3 to 3 s.f.

 e) 1010 cm^3 to 3 s.f.

 f) 181 cm^3 to 3 s.f.

3 a) 524 cm^3 to 3 s.f.

 b) 998 cm^3 to 3 s.f.

 c) 33.5 mm^3 to 3 s.f.

 d) 113 cm^3 to 3 s.f.

 e) 435 cm^3 to 3 s.f.

 f) 1990 mm^3 to 3 s.f.

4 12 cm

5 a) 3.64 cm to 3 s.f.

 b) 3.06 cm to 3 s.f.

 c) 6.18 cm to 3 s.f.

6 a) 556 cm^3 to 3 s.f.

 b) 2310 cm^3 to 3 s.f.

 c) 4190 cm^3 to 3 s.f.

7 a) 6.59 cm to 3 s.f.

 b) 12.4 cm to 3 s.f.

8 88

9 12 cm

10 3.17 cm

Exercise 35.13

1 145 cm^2

2 a) 204 cm^2 to 3 s.f.

 b) 58.0 cm^2 to 3 s.f.

 c) 135 cm^2 to 3 s.f.

3 a) 46.1 cm^2 to 3 s.f.

b) 66.8 cm^2 to 3 s.f.

4 a) 314 cm^2 to 3 s.f.

b) 483 cm^2 to 3 s.f.

c) 50.3 mm^2 to 3 s.f.

d) 113 cm^2 to 3 s.f.

e) 278 cm^2 to 3 s.f.

f) 765 mm^2 to 3 s.f.

5 1.95 cm to 3 s.f.

6 173 cm^2 to 3 s.f.

7 375 cm^2 to 3 s.f.

8 255 cm^2 to 3 s.f.

9 7.64 cm to 3 s.f.

10 170 cm^2

11 3.54 cm to 3 s.f.

12 75.2 cm^2 to 3 s.f.

13 130 cm^2 to 3 s.f.

14 6.4×10^{10} km^2

15 484 cm^2 to 3 s.f.

16 124 cm^3 to 3 s.f.

17 **a)** 158 cm^2 to 3 s.f.

b) 6.57 cm to 3 s.f.

c) 121 cm^3 to 3 s.f.

18 **a)** 68.3 cm^2

b) 4.33 cm to 3 s.f.

c) 36.1 cm^3 to 3 s.f.

19 **a)** 3.34 cm to 3 s.f.

b) 72.0 cm^2 to 3 s.f.

20 204 cm^2

Exercise 35.14

1 346 m^3 to 3 s.f.

2 a) 5.74 cm to 3 s.f.

b) 11.8 cm to 3 s.f.

3 a) 3 cm

b) 1230 cm^3 to 3 s.f.

4 *Radius of whole cone = 2r*

Volume of whole cone =
$\frac{1}{3}\pi(2r)^2 \times 2h = \frac{8}{3}\pi r^2 h$

Volume of small cone = $\frac{1}{3}\pi r^2 h$

Volume of frustrum = Volume of whole cone –
volume of small cone

$= \frac{8}{3}\pi r^2 h - \frac{1}{3}\pi r^2 h$

$= \frac{7}{3}\pi r^2 h$ \qquad as required.

5 169 cm^3 to 3 s.f.

6 24.0 cm^2

7 29.6 cm

8 a) The '*top cone*' is similar to the '*whole cone*'

if their radii and heights are in the same ratio.

Ratio of radii is 8 ÷ 10 = 0.8

Ratio of heights is 40 ÷ 50 = 0.8

Hence the '*top cone*' is similar to the '*whole*

cone' and so the base is a frustum.

b) 2.56 litres

9 219 000 m^3 to 3 s.f.

Exercise 36.1

Accept ±2° for all answers involving measurement.

Diagrams are not full size and are intended only as a guide.

1 *A*: 078°; *B*: 112°; *C*: 207°; *D*: 290°

2
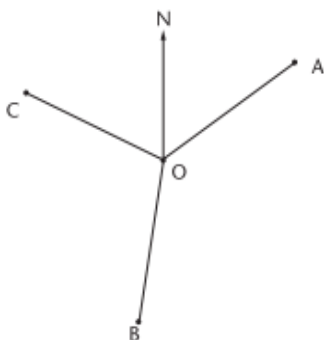
3 304°

4 110°

5 a) 259°

   b) 336°

6
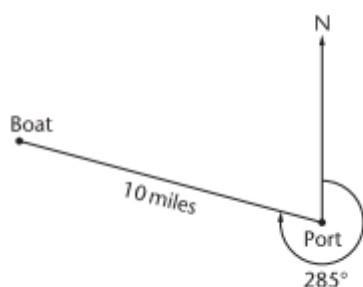
7 a)
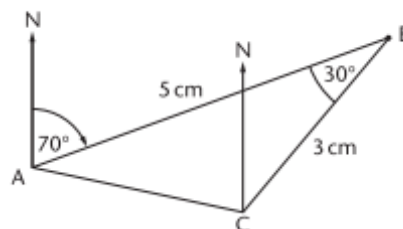
   b) 282°

8 a) 136°

   b) 230°

   c) 050°

Exercise 36.2

1 15 cm²

2 351 cm²

3 168 cm²

4 200 cm²

Exercise 36.3

1 11.18 cm

2 11.31 cm

3 13 cm

4 5.66 cm

5 28.91 cm

6 4 cm

7 8.94 cm

8 5.83 m

9 24 cm

10   6.34 cm

11   9.35 m

12   6.57 cm

Exercise 36.4

1 250.4 m to 1 d.p.

2 28.6 m to 1 d.p.

3 4.9 m to 1 d.p.

4 88.3 cm to 1 d.p.

Exercise 36.5

All answers are correct to 3 s.f.

1 $a = 3.5$ cm

  $b = 6.76$ cm

  $c = 8.18$ cm

  $d = 8.57$ cm

  $e = 2.01$ cm

  $f = 1.33$ cm

  $g = 3.41$ m

  $h = 2.00$ m

2 2.05 m

3 a) 6.88 cm

  b) 68.8 cm^2

4 a) 85.5 km

  b) 235 km

5 Minimum: 5.18 m; maximum: 19.7 m

Exercise 36.6

All answers are correct to 3 s.f.

1 $a = 9.24$ cm

  $b = 13.4$ cm

  $c = 10.5$ cm

  $d = 11.4$ m

  $e = 23.0$ cm

  $f = 10.7$ cm

  $g = 7.71$ m

  $h = 26.7$ m

2 9.53 km

3 a) 1.89 m

  b) 4.73 m^2

4 a)

b) 75.5 km

5 3.48 m

7 *AC* = 231 km; bearing of *C* from *A* is 194°
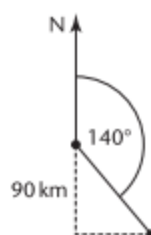
8 13.6 m

Exercise 36.7

All answers are correct to 3 s.f.

1 *a* = 47.2°

 b = 9.46°

 c = 45.6°

 d = 69.2°

 e = 52.8°

 f = 42.7°

 g = 39.6°

 h = 56.7°

2 69.8°

3 36.9°

4 32.3°

5 a)

b) 123°

6 a) 169 m

 b) 24.0°

Exercise 36.8

All answers are correct to 3 s.f.

1 27.9°

2 17.7 m

3 23.1°

4 25.5 m

5 36.4 m

Exercise 36.9

1 a) 0.8660

 b) −0.5

2 17.5° or 162.5° to 1 d.p.

3 45.6° to 1 d.p.

4 a) $\frac{5}{12}$

 b) $-\frac{5}{13}$

5 a) 5

 b) 0.8

 c) −0.6

6 a) 0.8

 b) −0.6

MATHEMATICS **36** Trigonometry

Exercise 36.10

All answers are correct to 3 s.f. unless otherwise

stated.

1 $c = 5.39$ cm

   $A = 46°$

   $a = 5.22$ cm

2 $p = 11.6$ cm

   $R = 26°$

   $r = 5.50$ cm

3 $B = 66.0°$

   $C = 72.0°$

   $c = 7.39$ cm

4 $M = 71.4°$

   $N = 28.6°$

   $n = 6.46$ cm

5 $P = 32.2°$

   $R = 78.4°$

   $r = 7.53$ cm

6 $Y = 35.5°$

   $Z = 48.5°$

   $z = 9.04$ cm

7 $y = 7.10$ cm

   $Z = 45°$

   $z = 7.81$ cm

8 $s = 1.13$ m

   $T = 59°$

   $t = 2.70$ m

9 28.2 cm

10 $B = 94.3°$

11 **a)** $AT = 85.7$ m; $BT = 60.5$ m

   b) 38.9 m

12 **a)** $AB = 25.7$ m; $BC = 42.7$ m

   b) 23.9 m

13 $AC = 43.9$ km; $BC = 25.3$ km

14 **a)** 54° to the nearest degree

   b) 46° to the nearest degree

Exercise 36.11

All answers correct to 3 s.f.

1 14.2 cm

2 3.91 cm

3 48.5°

4 50.7°

5 18.7 cm

6 52.0°

7 39.5°

8 49.3°

9 $A = 45.9°$ (opposite the shortest side)

10 4.79 km

11 9.59 km

12 $x = 11.3$ m; $y = 19.5°$

13 4.85 cm; 6.40 cm

14a) i) 11.7 m

    ii) 10.2 m

    iii) 10.8 m

  b) i) 58.6°

    ii) 67.5°

Exercise 36.12

All answers correct to 3 s.f.

1 a) 8.94 cm^2

  b) 19.7 cm^2

  c) 20.5 cm^2

  d) 34.0 cm^2

  e) 12.1 m^2

2 15 cm^2

3 73.2°

4 44.6 cm

5 35.7 cm^2

6 17.3 cm^2

7 Area of field = 6000 m^2
  (1981.0… m^2 + 4018.6… m^2)

Exercise 36.14

All answers correct to 3 s.f. unless otherwise stated.

1 a) 41.6°

  b) 15 cm

  c) 17 cm

  d) 28.1°

2 a) 11.3 cm

  b) 27.9°

  c) 12.8 cm

  d) 51.3°

3 a) $AC = 102.5$ m to 1 d.p.; $BC = 64.0$ m to 1 d.p.

  b) 120.9 m to 1 d.p.

  c) 328.0° to 1 d.p.

4 a) 45°

  b) 73.4°

  c) 11.3 cm

  d) 5.66 cm

  e) 12.8 cm

  f) 66.2°

5 a) 33.8 cm

   b) 94.3 cm

   c) 21°

   d) 61 300 cm^3

6 a) i) 17 cm

   ii) 13.1 cm

   iii) 69.7°

   b) i) 10.8 cm

   ii) 68.2°

7 a) 21.2 cm

   b) 16.8 cm

   c) 16.8 cm

8 Yes. *CE* = 11.96 m and *AC* = 87.3 m;

   $AC = \sqrt{80^2 + 35^2} = 87.3$ m

9 a) 11.0 cm

   b) 35.5°

10 a) 10.9 m

   b) 68.9°

   c) 11.7 m

Exercise 37.1

1

2 a) Translation of $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$

b) Translation of $\begin{pmatrix} 4 \\ -6 \end{pmatrix}$

c) Translation of $\begin{pmatrix} 1 \\ 8 \end{pmatrix}$

d) Translation of $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$

Exercise 37.2

1 $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$\overrightarrow{CD} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$\overrightarrow{CB} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$
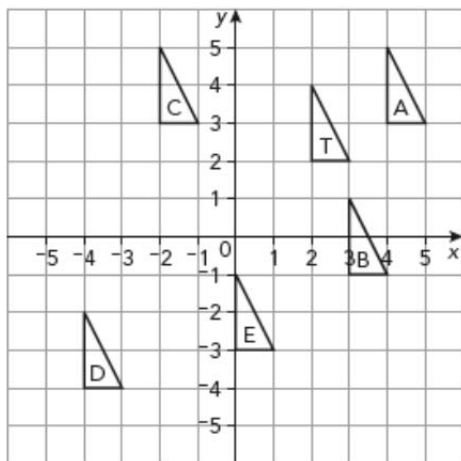
$\overrightarrow{AD} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

$\overrightarrow{CA} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$

2 $\overrightarrow{EF} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$\overrightarrow{GH} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

$\overrightarrow{EH} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$

$\overrightarrow{GF} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$

$\overrightarrow{FH} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

3 a) $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

b) $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$

c) $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

d) $\begin{pmatrix} 1 \\ 7 \end{pmatrix}$

e) $\begin{pmatrix} 8 \\ -6 \end{pmatrix}$

f) $\begin{pmatrix} -6 \\ 4 \end{pmatrix}$

MATHEMATICS **37** Vectors in Two Dimensions

4

	Original point	**Vector**	**New point**
a)	(1, 2)	$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$	(4, 4)
b)	(2, 3)	$\begin{pmatrix} 4 \\ 1 \end{pmatrix}$	(6, 4)
c)	(1, 0)	$\begin{pmatrix} -3 \\ 2 \end{pmatrix}$	(−2, 2)
d)	(4, 2)	$\begin{pmatrix} 0 \\ -3 \end{pmatrix}$	(4, −1)
e)	(−3, 2)	$\begin{pmatrix} -5 \\ -2 \end{pmatrix}$	(−8, 0)
f)	(6, 1)	$\begin{pmatrix} -6 \\ -1 \end{pmatrix}$	(0, 0)

5 $\overrightarrow{AB} = 2\mathbf{a}$

$\overrightarrow{CD} = -\mathbf{a}$

$\overrightarrow{EF} = \dfrac{1}{2}\mathbf{a}$

$\overrightarrow{GH} = \dfrac{3}{2}\mathbf{a}$

$\overrightarrow{PQ} = -\dfrac{1}{2}\mathbf{a}$

$\overrightarrow{RS} = \dfrac{9}{4}\mathbf{a}$

6 $\overrightarrow{AB} = \mathbf{a}$

$\overrightarrow{CD} = -\mathbf{b}$

$\overrightarrow{EF} = 2\mathbf{b}$

$\overrightarrow{GH} = -\dfrac{1}{2}\mathbf{a}$

$\overrightarrow{PQ} = -\dfrac{1}{2}\mathbf{b}$

$\overrightarrow{RS} = 3\mathbf{a}$

7 $\overrightarrow{AB} = 2\mathbf{a}$

$\overrightarrow{CD} = -2\mathbf{a}$

$\overrightarrow{EB} = \mathbf{a}$

$\overrightarrow{GD} = -\mathbf{a}$

$\overrightarrow{HF} = 2\mathbf{a}$

$\overrightarrow{FC} = \mathbf{b}$

8 $\overrightarrow{BC} = \mathbf{b}$

$\overrightarrow{CD} = -\mathbf{a}$

$\overrightarrow{EB} = \dfrac{1}{2}\mathbf{a}$

$\overrightarrow{GD} = -\dfrac{1}{2}\mathbf{a}$

$\overrightarrow{HF} = \mathbf{a}$

$\overrightarrow{FC} = \dfrac{1}{2}\mathbf{b}$

Exercise 37.3

1 a) $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

 b) $\begin{pmatrix} 9 \\ 3 \end{pmatrix}$

 c) $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

 d) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

 e) $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$

2 a) $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$

 b) $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$

 c) $\begin{pmatrix} 0.5 \\ -1.5 \end{pmatrix}$

 d) $\begin{pmatrix} 6 \\ 1 \end{pmatrix}$

 e) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

3 a) $\begin{pmatrix} 3 \\ 12 \end{pmatrix}$

 b) $\begin{pmatrix} 8 \\ 12 \end{pmatrix}$

c) $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$

 d) $\begin{pmatrix} 7 \\ 4 \end{pmatrix}$

 e) $\begin{pmatrix} 7 \\ 18 \end{pmatrix}$

4 a) $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$

 b) $\begin{pmatrix} -6 \\ 3 \end{pmatrix}$

 c) $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

 d) $\begin{pmatrix} -3 \\ -10 \end{pmatrix}$

 e) $\begin{pmatrix} -0.5 \\ 5.5 \end{pmatrix}$

Exercise 37.4

1 $\overrightarrow{OP} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

 $\overrightarrow{OQ} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

 $\overrightarrow{OR} = \begin{pmatrix} -4 \\ -5 \end{pmatrix}$

2 a) $\overrightarrow{OA} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$\overrightarrow{OB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$\overrightarrow{OC} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$

b) i) $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

ii) $\overrightarrow{BC} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

c) $\overrightarrow{AB} = 2 \times \overrightarrow{BC}$. So *ABC* is a straight line and

AB = 2 × *BC* in length.

3 a) $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\overrightarrow{OB} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

$\overrightarrow{OC} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$

b) i) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

ii) $\overrightarrow{CD} = \begin{pmatrix} -4 \\ -6 \end{pmatrix}$

c) $\overrightarrow{CD} = -2 \times \overrightarrow{AB}$. So the line *AB* is parallel to

CD and *CD* = 2 × *AB* in length.

Exercise 37.5

1 a) $\begin{pmatrix} 12 \\ 6 \end{pmatrix}$

b) $\begin{pmatrix} -6 \\ -3 \end{pmatrix}$

c) $\begin{pmatrix} 24 \\ 12 \end{pmatrix}$

d) $\begin{pmatrix} 3 \\ 1.5 \end{pmatrix}$

e) $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$

f) 6.71 correct to 3 s.f.

2 a) $\begin{pmatrix} 3 \\ 9 \end{pmatrix}$

b) $\begin{pmatrix} 4 \\ 7 \end{pmatrix}$

c) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

d) $\begin{pmatrix} 5 \\ 10 \end{pmatrix}$

e) $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

f) 3.16 correct to 3 s.f.

g) 5

3 a) $\begin{pmatrix} -3 \\ -9 \end{pmatrix}$

b) $\begin{pmatrix} 2 \\ -20 \end{pmatrix}$

c) $\begin{pmatrix} 4 \\ -4 \end{pmatrix}$

d) $\begin{pmatrix} -7 \\ 12 \end{pmatrix}$

e) $\begin{pmatrix} 4.5 \\ -1 \end{pmatrix}$

f) 5

g) 3.16 correct to 3 s.f.

4 a) $\begin{pmatrix} 20 \\ 32 \end{pmatrix}$

b) $\begin{pmatrix} -10 \\ -16 \end{pmatrix}$

c) $\begin{pmatrix} 2.5 \\ 4 \end{pmatrix}$

d) $\begin{pmatrix} 45 \\ 72 \end{pmatrix}$

e) $\begin{pmatrix} 2 \\ 3.2 \end{pmatrix}$

f) 9.43 correct to 3 s.f.

5 a) $\begin{pmatrix} 8 \\ 2 \end{pmatrix}$

b) $\begin{pmatrix} 9 \\ 4 \end{pmatrix}$

c) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

d) $\begin{pmatrix} 13 \\ 5 \end{pmatrix}$

e) $\begin{pmatrix} 7 \\ 7 \end{pmatrix}$

f) 9.85 correct to 3 s.f.v

6 a) $\begin{pmatrix} -6 \\ -9 \end{pmatrix}$

b) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

c) $\begin{pmatrix} -7 \\ -4 \end{pmatrix}$

d) $\begin{pmatrix} 14 \\ 30 \end{pmatrix}$

e) $\begin{pmatrix} 3 \\ 8.5 \end{pmatrix}$

f) 7

Exercise 37.6

1 $\overrightarrow{BC} = 2\mathbf{b} - \mathbf{a}$

2 $\overrightarrow{AC} = 2\mathbf{a} - 3\mathbf{b}$

3 a) $\overrightarrow{AB} = -\mathbf{a} - \mathbf{b}$

b) $\overrightarrow{BC} = 3\mathbf{b} - 4\mathbf{a}$

c) $\overrightarrow{AC} = 2\mathbf{b} - 5\mathbf{a}$

4 $\overrightarrow{BC} = \mathbf{b}$

$\overrightarrow{CD} = -\mathbf{a}$

$\overrightarrow{BD} = \mathbf{b} - \mathbf{a}$

$\overrightarrow{AC} = \mathbf{a} + \mathbf{b}$

5 $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

$\overrightarrow{CB} = \frac{1}{3}(\mathbf{b} - \mathbf{a})$

$\overrightarrow{OC} = \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

6 $\overrightarrow{EB} = \frac{1}{2}\mathbf{a} - \mathbf{b}$

7 $\overrightarrow{EB} = \mathbf{b} - \frac{2}{3}\mathbf{a}$

8 a) $\overrightarrow{FA} = \mathbf{b}$

b) $\overrightarrow{BD} = -\mathbf{b} - \mathbf{a}$

c) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

d) $\overrightarrow{AC} = \mathbf{b} - 2\mathbf{a}$

9 a) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

b) $\overrightarrow{AP} = \frac{1}{3}(\mathbf{b} - \mathbf{a})$

c) $\overrightarrow{OP} = a + \frac{1}{3}(\mathbf{b} - \mathbf{a}) = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

10a) $\overrightarrow{AE} = 3\mathbf{a}$

$\overrightarrow{AF} = 3\mathbf{b}$

$\overrightarrow{BC} = \mathbf{b} - \mathbf{a}$

$\overrightarrow{EF} = 3\mathbf{b} - 3\mathbf{a}$

b) $\overrightarrow{EF} = 3 \times \overrightarrow{BC}$ so *EF* and *BC* are parallel and

$EF = 3 \times BC$ in length.

11a) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$; $\overrightarrow{OC} = 4\mathbf{a}$ and $\overrightarrow{OD} = 4\mathbf{b}$ so
$\overrightarrow{CD} = 4\mathbf{b} - 4\mathbf{a} = 4(\mathbf{b} - \mathbf{a})$.

As $\overrightarrow{CD}$ is a multiple of $\overrightarrow{AB}$, *AB* and *CD* are

parallel.

b) $1 : 4$

12a) i) $\overrightarrow{OE} = 2\mathbf{a} + \mathbf{c}$

ii) $\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$

iii) $\overrightarrow{OF} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{c}$

b) *O, F* and *E* are on a straight line.

$OE = 3 \times OF$

Exercise 38.1

1 a) 3×1

b) 2×2

c) 3×2

d) 1×2

2 a) $\begin{pmatrix} 8 & 24 \\ 0 & 4 \\ 12 & 8 \end{pmatrix}$

b) $\begin{pmatrix} 6 & 8 \\ -2 & 14 \end{pmatrix}$

c) $\begin{pmatrix} 15 & -6 & 0 \\ 9 & 3 & -3 \end{pmatrix}$

d) $\begin{pmatrix} 35 & -15 \\ 0 & -5 \end{pmatrix}$

3 a) $\begin{pmatrix} 105 & 70 \\ 140 & 175 \end{pmatrix}$

b) The number of each type of loaf made in 7

days.

Exercise 38.2

1 a) $\begin{pmatrix} 7 & 12 \\ 3 & 5 \end{pmatrix}$

b) $\begin{pmatrix} -3 & 14 \\ -4 & 8 \end{pmatrix}$

c) $\begin{pmatrix} 7 & 15 \\ -4 & 5 \end{pmatrix}$

d) $\begin{pmatrix} -9 & -1 \\ 10 & -1 \end{pmatrix}$

2 a) $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

b) $\begin{pmatrix} 1 & -1 \\ -6 & 2 \end{pmatrix}$

c) $\begin{pmatrix} -2 & 3 \\ 2 & -7 \end{pmatrix}$

d) $\begin{pmatrix} -3 & 0 \\ 6 & -4 \end{pmatrix}$

3 a) $\begin{pmatrix} 9 & 2 & -1 \\ -1 & 5 & 13 \end{pmatrix}$

b) $\begin{pmatrix} -3 & -2 & -1 \\ 9 & -9 & -3 \end{pmatrix}$

c) $\begin{pmatrix} 9 & 0 & -3 \\ 12 & -6 & 15 \end{pmatrix}$

d) $\begin{pmatrix} 12 & 4 & 0 \\ -10 & 14 & 16 \end{pmatrix}$

e) $\begin{pmatrix} 0 & -2 & -2 \\ 13 & -11 & 2 \end{pmatrix}$

f) $\begin{pmatrix} -3 & 2 & -3 \\ -17 & 13 & -7 \end{pmatrix}$

g) $\begin{pmatrix} 18 & 4 & -2 \\ -2 & 10 & 26 \end{pmatrix}$

h) $\begin{pmatrix} 12 & 6 & 2 \\ -23 & 25 & 14 \end{pmatrix}$

4 a) i) 65

  ii) 38

b) i) $\begin{pmatrix} 63 & 92 \\ 134 & 133 \end{pmatrix}$

  ii) The total number of visitors from each group in the day.

c) i) $\begin{pmatrix} 13 & 12 \\ 16 & 3 \end{pmatrix}$

  ii) How many more visitors of each type there were in the afternoon than in the morning.

Exercise 38.3

1 a) Not possible

  b) Not possible

  c) Possible; order of result is 2×3

  d) Not possible

  e) Not possible

  f) Possible; order of result is 1×2

2 a) $\begin{pmatrix} 2 & 15 \end{pmatrix}$

  b) $\begin{pmatrix} 8 & -1 \\ 12 & -12 \end{pmatrix}$

  c) $\begin{pmatrix} 9 \\ -3 \end{pmatrix}$

d) $\begin{pmatrix} 12 & 0 \\ 1 & 1 \\ 0 & 3 \end{pmatrix}$

3 a) $\begin{pmatrix} 12 & -9 \\ -7 & 11 \end{pmatrix}$

  b) $\begin{pmatrix} 18 & -3 \\ -7 & 5 \end{pmatrix}$

  c) $\begin{pmatrix} 9 & 0 \\ -8 & 1 \end{pmatrix}$

  d) $\begin{pmatrix} 13 & -27 \\ 9 & 22 \end{pmatrix}$

4 a) i) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

  ii) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

  b) Any matrix multiplied by the zero matrix will give the zero matrix.

5 a) $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$

  b) $\begin{pmatrix} 285 \\ 470 \end{pmatrix}$

  c) The total amount paid in one day by boys and by girls.

Exercise 38.4

1 a) $\begin{pmatrix} 1 & -\frac{2}{3} \\ 0 & \frac{1}{3} \end{pmatrix}$

b) $\begin{pmatrix} \frac{5}{4} & -2 \\ -\frac{1}{2} & 1 \end{pmatrix}$

c) $\begin{pmatrix} \frac{2}{3} & -1 \\ -1 & 2 \end{pmatrix}$

d) $\begin{pmatrix} \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{4} & \frac{5}{4} \end{pmatrix}$

e) $\begin{pmatrix} 2 & \frac{1}{2} \\ -1 & 0 \end{pmatrix}$

f) $\begin{pmatrix} -\frac{3}{2} & 2 \\ -\frac{1}{2} & 1 \end{pmatrix}$

g) $\begin{pmatrix} -5 & -2 \\ 3 & 1 \end{pmatrix}$

h) $\begin{pmatrix} -\frac{2}{5} & \frac{1}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{pmatrix}$

2 a) 0

  b) Because the determinant is 0 and you cannot

  divide by 0.

3 a) $\begin{pmatrix} \frac{1}{6} & \frac{1}{2} \\ \frac{2}{3} & 1 \end{pmatrix}$

  b) i) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

ii) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

c) i) $\begin{pmatrix} -6 & 3 \\ 4 & -1 \end{pmatrix}$

  ii) $\begin{pmatrix} -6 & 3 \\ 4 & -1 \end{pmatrix}$

Exercise 38.5

1 a) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

  b) $\begin{pmatrix} 4 & 2 \\ 3 & -2 \end{pmatrix}$

  c) $\begin{pmatrix} 7 & 2 \\ 3 & 1 \end{pmatrix}$

  d) $\begin{pmatrix} 5 & 2 \\ 3 & -1 \end{pmatrix}$

  e) $\begin{pmatrix} \frac{1}{11} & \frac{2}{11} \\ \frac{3}{11} & -\frac{5}{11} \end{pmatrix}$

2 a) $\begin{pmatrix} 1 & -1 \\ 3 & 6 \end{pmatrix}$

  b) $\begin{pmatrix} -6 & 18 \\ 5 & -15 \end{pmatrix}$

3 $\begin{pmatrix} 3 & -2 \end{pmatrix}$

4 $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

5 $\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$

Exercise 39.1

1
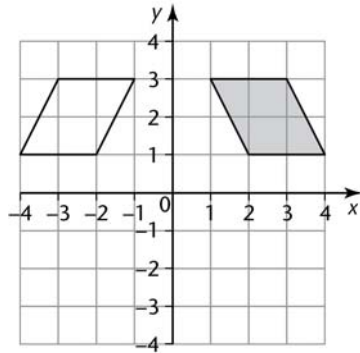
2
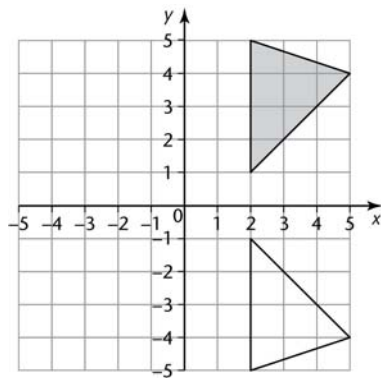
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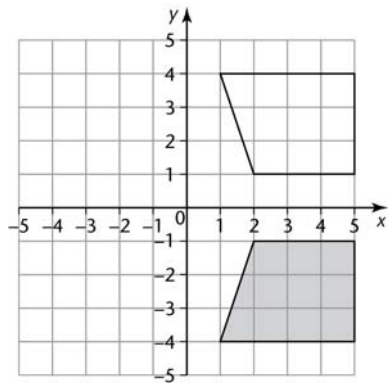
4
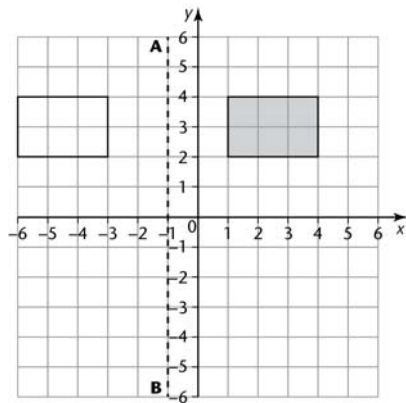
5
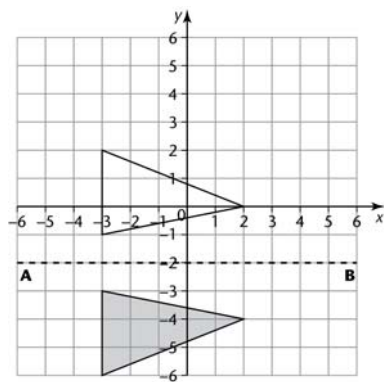
6
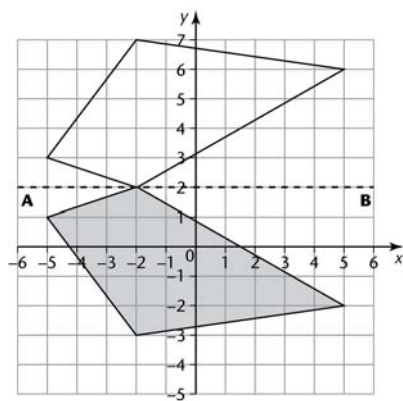
7 a) - c)
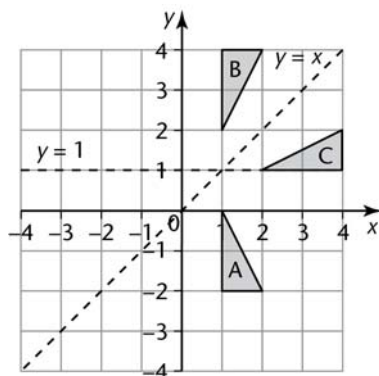
8 **a) - c)**
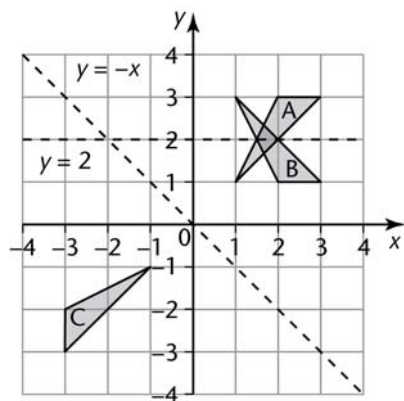
Exercise 39.2

1
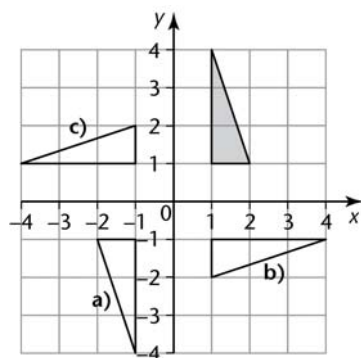
2
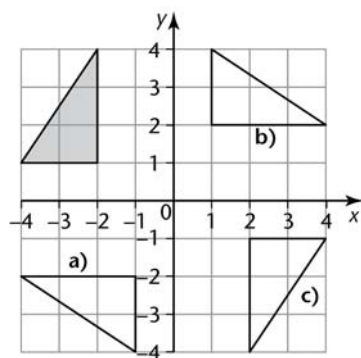
3
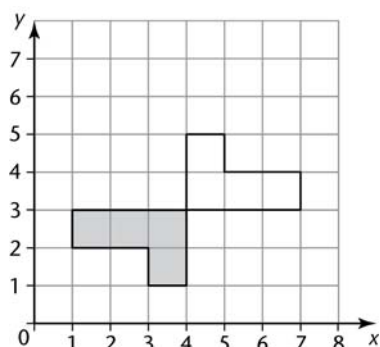
4
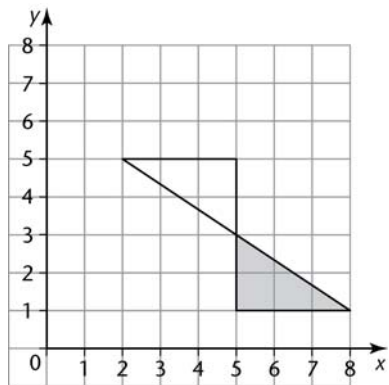
5
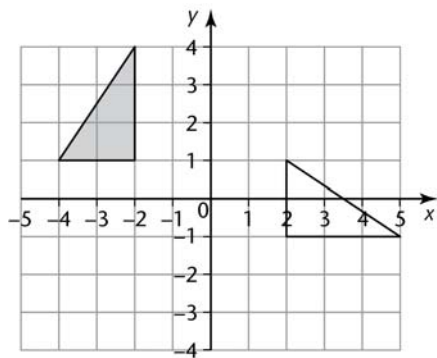
Exercise 39.3

**1** **a) – e)**
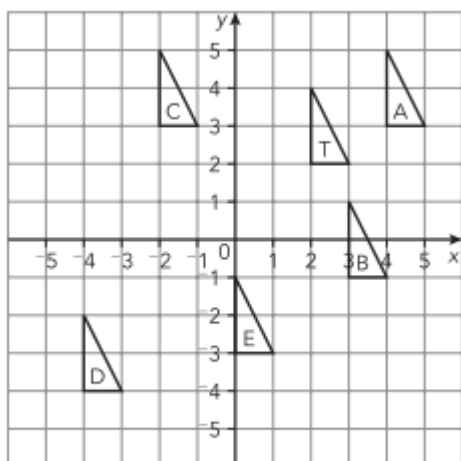
2 a) – c)
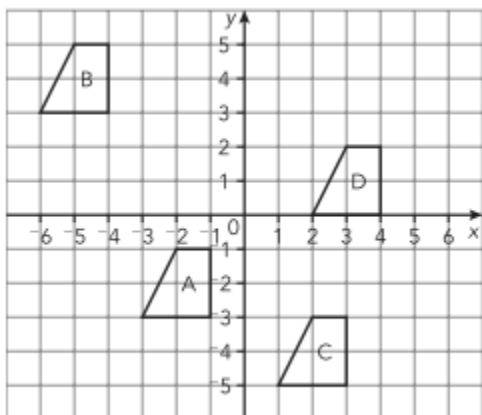
d) Shape D is mapped back on to shape A after translation by $\begin{pmatrix} -5 \\ -3 \end{pmatrix}$

3   a) – c)
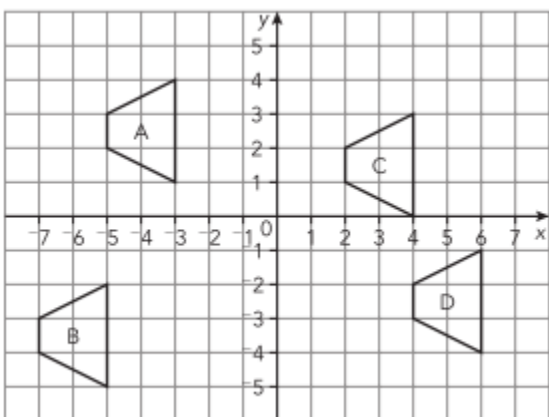
d) Translate D by $\begin{pmatrix} -9 \\ 5 \end{pmatrix}$

Exercise 39.4

1
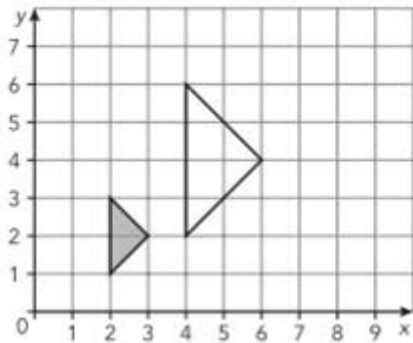
2
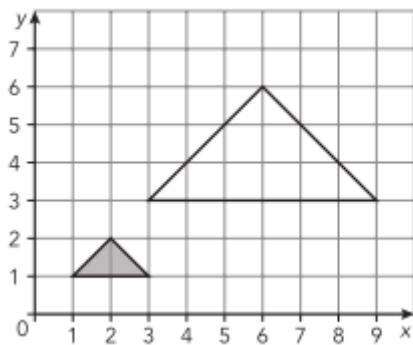
3
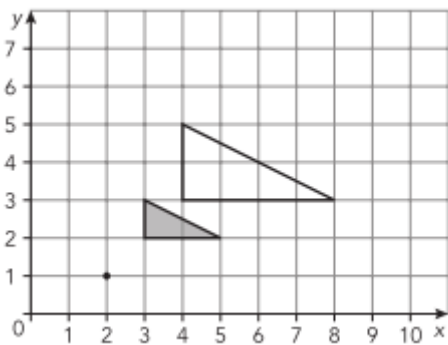
Exercise 39.5

1
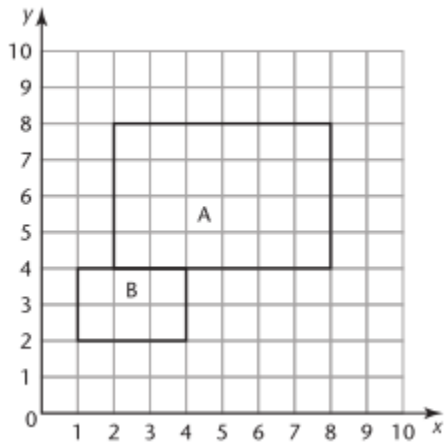
2
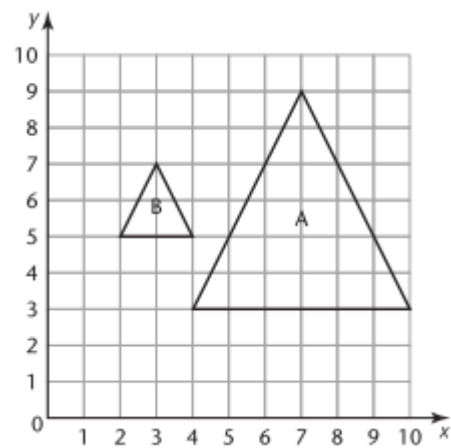
3
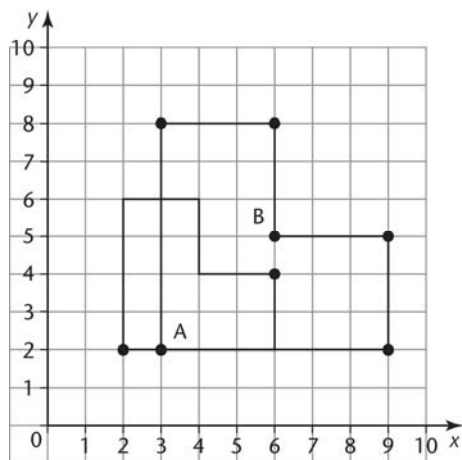
4
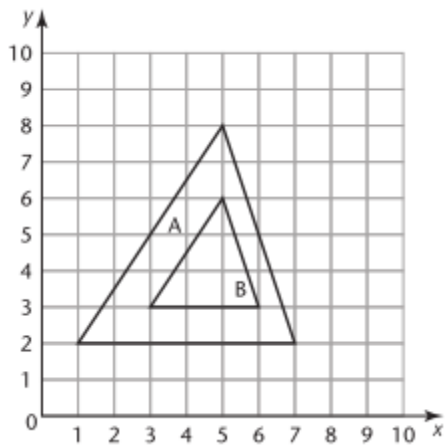
Exercise 39.6

1
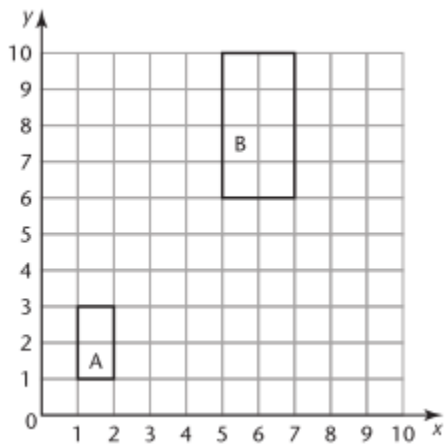
2
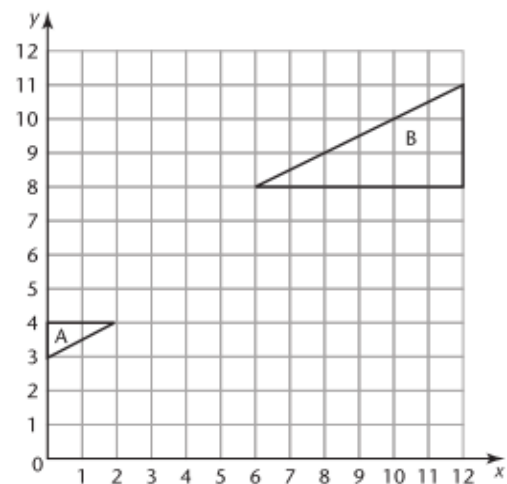
3
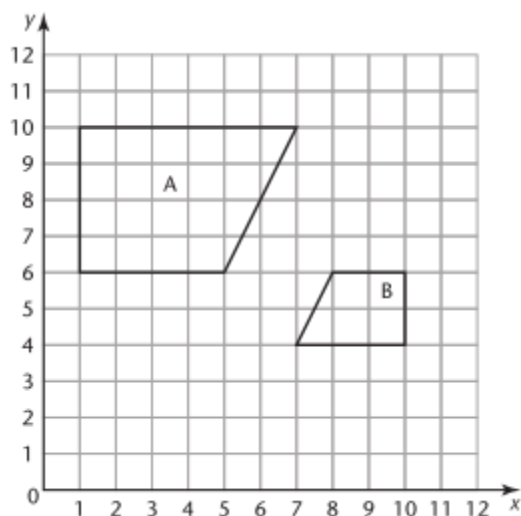
4
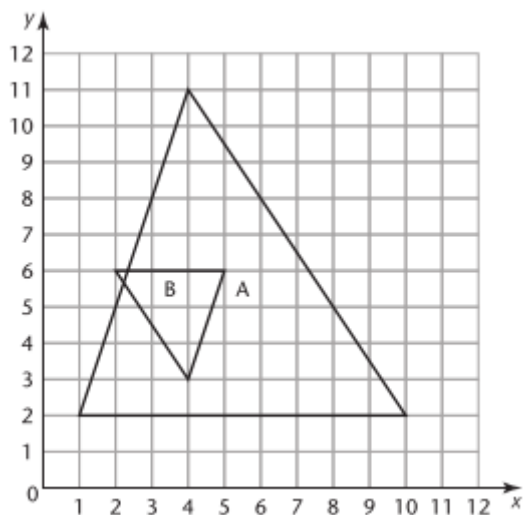
Exercise 39.7

1 Reflection in the line $y = x$

2 Reflection in the line $x = 3$

3 Translation by the vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

4 a) Translation by the vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

b) Translation by the vector $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$

c) Translation by the vector $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

d) Translation by the vector $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$

5 a) Reflection in the line $y = 2$

b) Reflection in the line $x = 3$

c) Reflection in the line $y = x$

6 a) Translation by the vector $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$

b) Enlargement, scale factor 2, centre (0, 4)

c) Translation by the vector $\begin{pmatrix} -8 \\ -3 \end{pmatrix}$

d) Enlargement, scale factor $2\frac{1}{2}$, centre (0, 0)

e) Translation by the vector $\begin{pmatrix} -6 \\ 4 \end{pmatrix}$

f) Enlargement, scale factor $\frac{1}{3}$, centre (5, 3)

7 Enlargement, scale factor -3, centre (1, 8)

8 Enlargement, scale factor -2, centre (0, 3)

9 Enlargement, scale factor $-\frac{1}{2}$, centre (2, 3)

Exercise 39.8

1 a) and b)
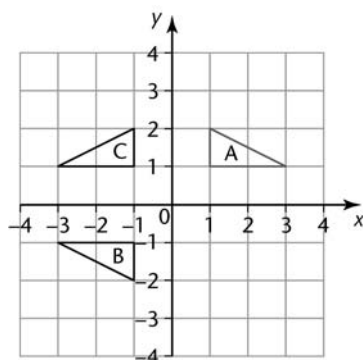
    c) Reflection in the *y*-axis

2 a), b)
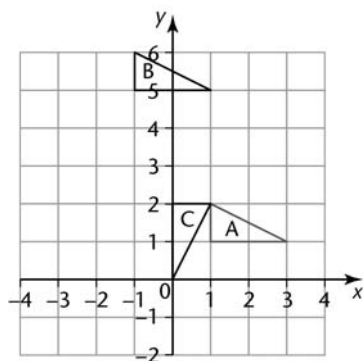
    c) Rotation 90° about (1, 2)

3 Reflection in the line $y = x$

4 Rotation 180° about (3, −2)

5 Translation through the vector $\begin{pmatrix} 0 \\ -8 \end{pmatrix}$

6 Reflection in the line $y = x + 1$

7 Enlargement, scale factor −2, centre (0, 2)

8 a) Rotation 90° clockwise about (1, 2)

    b) Rotation 90° anticlockwise about (1, 2)

9 Reflection in the line $y = -1$

10 Reflection in the line $y = x - 3$

Exercise 39.9

1 a) and b)
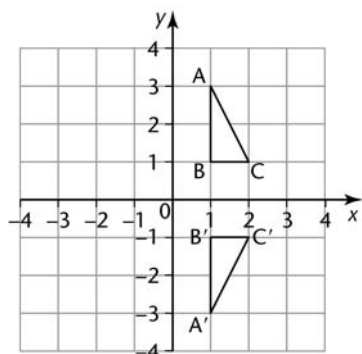
c) Reflection in the *x*-axis

2 a)
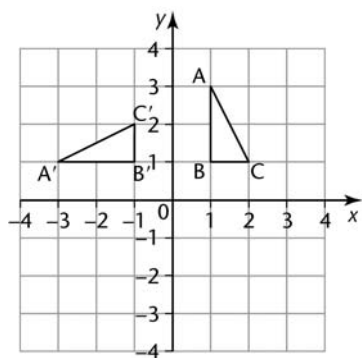
b) Rotation 90° anticlockwise about the origin.

3 a)
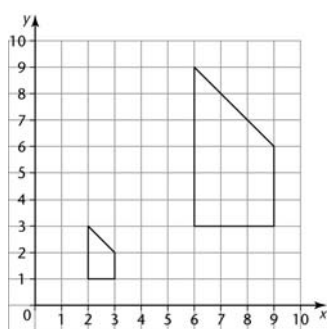
b) Enlargement, scale factor $\frac{1}{3}$, centre the origin

Exercise 39.10

1 a) and b)
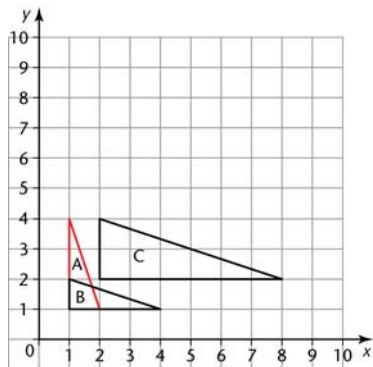
c) $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$

d) $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 8 \\ 2 & 4 & 2 \end{pmatrix}$; yes, coordinates are the same.

2 a) and b)
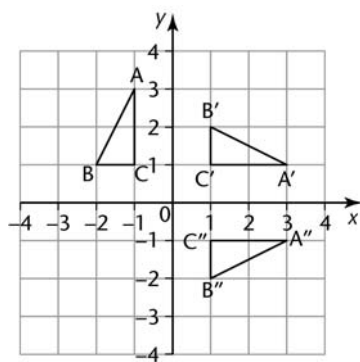
c) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

d) Reflection in the line $y = x$

Exercise 39.11

1 $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

2 $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

3 $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

4 $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$

5 $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

6 $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

7 $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

8 $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$

9 a) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

  b) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

10 $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Exercise 40.1

1 a) $\dfrac{1}{6}$

  b) $\dfrac{1}{2}$

  c) $\dfrac{2}{3}$

2 a) $\dfrac{2}{5}$

  b) $\dfrac{3}{5}$

3 a) $\dfrac{7}{20}$

  b) $\dfrac{3}{20}$

  c) $\dfrac{1}{2}$

4 a) $\dfrac{1}{3}$

  b) $\dfrac{2}{9}$

5 a) $\dfrac{3}{20}$

  b) $\dfrac{3}{8}$

  c) $\dfrac{23}{40}$

6 a) $\dfrac{2}{7}$

b) $\dfrac{8}{35}$

Exercise 40.2

1 $\dfrac{5}{8}$

2 0.999

3 0.3

4 $\dfrac{3}{5}$ or 0.6

5 $\dfrac{1}{3}$

6 0.7

7 $\dfrac{1}{5}$

8 0.11

Exercise 40.3

1 125

2 50

3 18

4 50

5 20

Exercise 40.4

1 a) 0.154

  b) 0.255

2 a) i) $\dfrac{103}{500}$

  ii) $\dfrac{96}{500}$

  b) Yes, all frequencies are close to the expected value of $500 \div 5 = 100$

3 a) 0.41

  b) 0.59

4 a) 0.27

  b) 0.19

5 a) $\dfrac{7}{20}$

  b) 420

6 a) $\dfrac{1}{5}$

  b) 60

Exercise 40.5

1 a)

Dice

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10

Spinner

b) $\dfrac{1}{12}$

c) $\dfrac{5}{12}$

d) $\dfrac{1}{2}$

2 a)

Dice

	1	**2**	**3**	**4**	**5**	**6**
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

Coin

b) i) $\dfrac{1}{12}$

ii) $\dfrac{1}{4}$

3 a)

First spin

	1	**2**	**3**	**4**
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

Second spin (row labels at left)

b) i) $\dfrac{3}{16}$

ii) $\dfrac{1}{4}$

iii) $\dfrac{1}{4}$

4 a)

First spin

	2	**4**	**6**	**8**	**10**
2	4	6	8	10	12
4	6	8	10	12	14
6	8	10	12	14	16
8	10	12	14	16	18
10	12	14	16	18	20

Second spin (row labels at left)

b) $\dfrac{3}{5}$

c) $\dfrac{7}{25}$

Exercise 40.6

1 $\dfrac{9}{100}$ or 0.09

2 0.24

3 a) $\dfrac{1}{3}$

  b) $\dfrac{1}{9}$

4 $\dfrac{1}{49}$

5 a) 0.12

  b) 0.42

6 $\dfrac{1}{125}$

Exercise 40.7

1

  a) 0.49

  b) 0.21

  c) 0.21

  d) 0.42

2 a)

| | **1st day** | **2nd day** |

Late
0.2 — Late
0.8 — Not late

Not late
0.2 — Late
0.8 — Not late

0.2 (Late), 0.8 (Not late)

  b) i)   0.04

   ii)   0.32

3 a) $\dfrac{3}{5}$

  b)

1st throw   2nd throw

PU
$\frac{3}{5}$ PU
$\frac{2}{5}$ PD

PD
$\frac{3}{5}$ PU
$\frac{2}{5}$ PD

$\frac{3}{5}$, $\frac{2}{5}$

  c) i) $\dfrac{9}{25}$

   ii) $\dfrac{12}{25}$

4

PU
  PU
    $\frac{3}{5}$ PU
    $\frac{2}{5}$ PD
  PD
    $\frac{3}{5}$ PU
    $\frac{2}{5}$ PD
PD
  PU
    $\frac{3}{5}$ PU
    $\frac{2}{5}$ PD
  PD
    $\frac{3}{5}$ PU
    $\frac{2}{5}$ PD
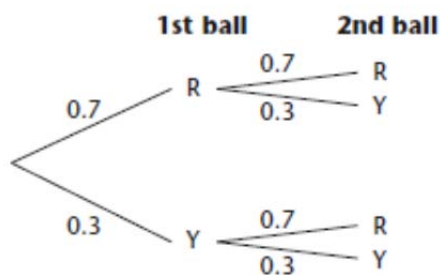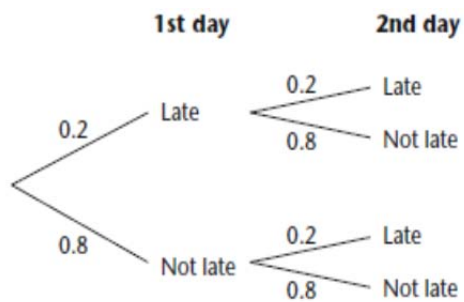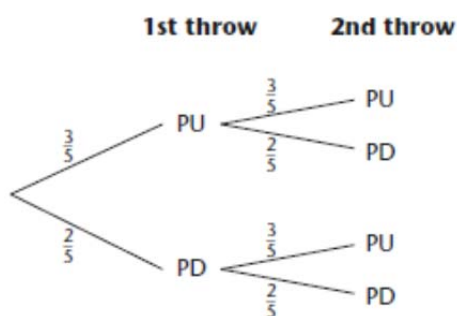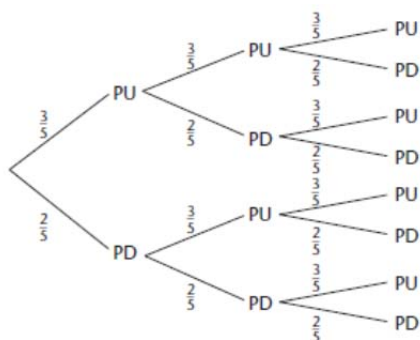
  a) $\dfrac{27}{125}$

b) $\dfrac{36}{125}$

5 a)
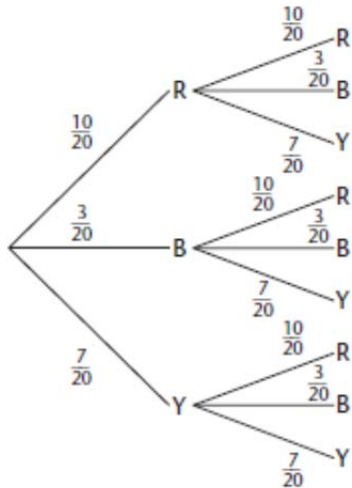
b) i) $\dfrac{9}{400}$

ii) $\dfrac{79}{200}$

iii) $\dfrac{121}{200}$

6 a) 0.343

b) 0.09

c) 0.441

Exercise 40.8

1 $\dfrac{5}{8}$

2 a)
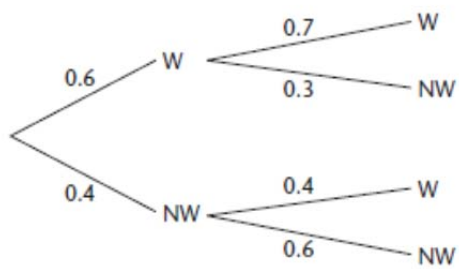
b) 0.34

c) 0.76

3 a)
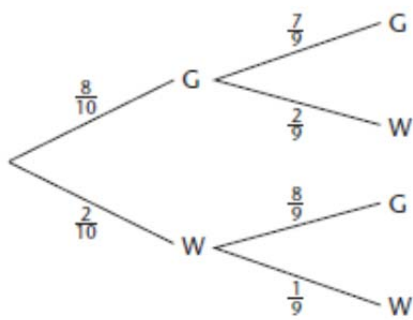
b) $\dfrac{2}{90} = \dfrac{1}{45}$

c) $\dfrac{32}{90} = \dfrac{16}{45}$

4 0.76

5 $\dfrac{25}{28}$

Exercise 41.1

1   a)

Number of letters	Frequency
0	16
1	19
2	21
3	10
4	5
5	3
6	3
7	2
8	1

b) 10

2 Possible answers include the following:

a) Those not in the phone book have no chance of being included.

b) Those working on Saturday would not be included.

3 Possible answers include the following:

Make the categories exclusive (in Paul's first draft, for example, 1 hour could go in two categories).

Have more categories, for example, split the groups into classes of 30 minutes rather than 1 hour.

4 Possible faults include the following:

a) The person's favourite sport might be one that isn't listed.

b) This questions is too vague. It needs categories such as hours spent exercising.

c) This is a leading question.

Check students' new questions.

5 Check students' questions.

6 a)

	Japanese	Not Japanese	Total
Red	35	65	100
Not red	72	438	510
Total	107	503	610

b) 610

c) 107

d) 72

e) 100

7 a)

	Existing drug	New drug	Total
Symptoms eased	700	550	1250
No change in symptoms	350	250	600
Total	1050	800	1850

b) 1850

c) 550

8 a)

	Gold	Silver	Bronze	Total
USA	31	18	10	59
Germany	18	16	9	43
China	22	9	11	42
Total	71	43	30	144

  b) USA

  c) China

Exercise 41.2

1   **a)** 6

      b) 5.5

2 a) 4 people

  b) Mode = 194 mm; median = 198 mm

3 a) Mode = $10 000; median = $13 000

  b) Check students' explanations.

4 Harvey: mode = 0; median = 19

  Nick: mode = 9; median = 9

  You would choose Harvey if you wanted the possibility of high scores but Nick if you wanted a more consistent player.

Exercise 41.3

1   **a)** Mean = 6; range = 9

   b) Mean = 6.5; range = 11

   c) Mean = 20.625; range = 19

   d) Mean = 466; range = 756

2

	Data set A	Data set B	Data set C
Range	6	6	12
Mean	3.36	3.36	6.73

   The data in set C are twice those in set A, as are the range and the mean.

   The data in set B are 2 sets of the data in set A. The range and mean of sets A and B are the same.

3 a) 3

   b) i)   Mean = 3; range = 6

   ii)   Mean = 30; range = 60

   iii) Mean = 130; range = 60

4 Mean = $19 500; range = $60 000

5 a) 15.875

   b) 11

6 a) i) $32

   ii) $175.80

   b) i) $92

   ii) $188

7 a) 83 cm

   b) 25 cm

8 a) 63

  b) 7.3

Exercise 41.4

All comments given as answers are only suggestions and any comment that makes sense should be accepted.

1 **a)** Carl: mean = 19; range = 10
         Adam: mean = 19.75; range = 17

   b) Adam has a slightly better (higher) average, but his scores are more spread out.

2 **a)** Resort A: mean = 165 hours; median = 170.5 hours; range = 81 hours
         Resort B: mean = 161.5 hours; median = 168.5 hours; range = 58 hours

   b) Resort A has a higher average number of hours of sunshine, but resort B is more consistent.

3 a) Mean = 31.6; median = 32; mode = 30

   b) The mode because it shows which size sold the most often.

4 a) Mean = 8.88; median = 9.0; range = 1.2

   b) The better average would be the median as it is not affected by extreme scores.
   (Alternatively, the mean is better because it uses every value.)

5 a) 8.825

   b) Slightly better; she would have scored 8.8 if the mean of all the judge's marks had been calculated.

6 The median salary at the two factories was the same but the mean salary at Prothero was higher. However, the range at Prothero was much higher, suggesting that a few of the salaries there were much higher than the rest. Apart from a few high salaries, the rest were probably about the same as those at Jaline.

Exercise 41.5

1   1.96

2 a) 5

    b) 4.57 (i.e. 5 minutes late)

3 a) 30

    b) 40

    c) 1.33

    d) 1

4 $1.07

5 Mean = 7.52; median = 7; mode = 7

Exercise 41.6

1 a) $8 \le t < 10$

    b) $4 \le t < 6$

    c) 8 seconds

    d) 5.18 seconds to 3 s.f.

2 a) $70 \le h < 80$

    b) $70 \le h < 80$

    c) 40 cm

    d) 72.8 cm

3 a) $1.4 \le l < 1.6$

    b) $1.4 \le l < 1.6$

    c) 0.8 m

    d) 1.5 m

4 35.4 cm

5 47.25 g

6 51.5 seconds

7 4.66 m

8 3.48 cm

Exercise 41.7

1 a) $40 \le m < 60$

   b) $40 \le m < 60$

   c) 80

   d) 55.3

2 a) 13 - 15

   b) 10 – 12

   c) 12

   d) 11.3

3 a) $12\,000 \le a < 16\,000$]

   b) $8000 \le a < 12\,000$]

   c) 16 000

   d) 11 067

4 17.2

Exercise 42.1

1 a) Monday 30; Tuesday 35; Wednesday 25; Thursday 40; Friday 55

  b) Friday; people want books to read over the weekend.

2 a) 7

  b) 3

  c) 30

3

4

Exercise 42.2

1 Students should draw a pie chart with angles of 36°, 120°, 84°, 72° and 48°.

2 a) A

  b) i) 20

    ii) 9

3 a) $\dfrac{140}{360} = \dfrac{7}{18}$

  b) 72

Exercise 42.3

1
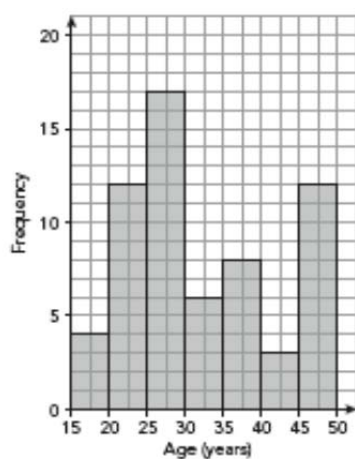
2 a)
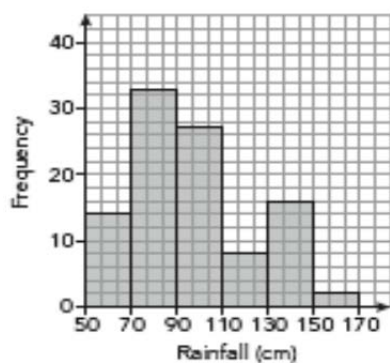
  b) $70 \leq r < 90$

  c) $90 \leq r < 110$

3 a)

Age (*y* years)	Frequency
$20 \leq y < 30$	1
$30 \leq y < 40$	4
$40 \leq y < 50$	8
$50 \leq y < 60$	10
$60 \leq y < 70$	7
$70 \leq y < 80$	5
$80 \leq y < 90$	2

b) $50 \leq y < 60$

c) The table shows that the youngest is somewhere in the range $20 \leq y < 30$, but we do not know their exact age.

4 a)
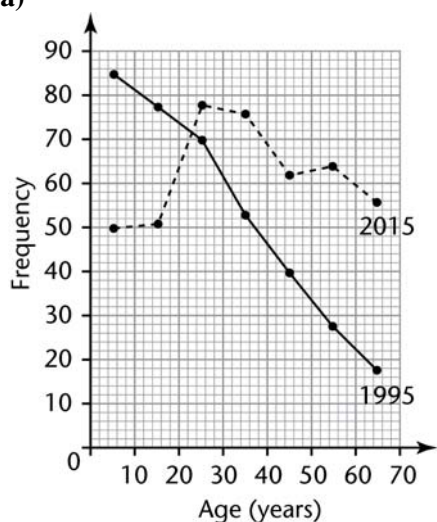
b) Answers should include the following comments:

- In 1995 there were more younger people/fewer older people
- In 2015 there were fewer younger people/ more middle-aged and older people
- For 1995 the modal group is $0 \leq a < 10$, for 2015 it is $20 \leq a < 30$

Exercise 42.4

1 a), c)
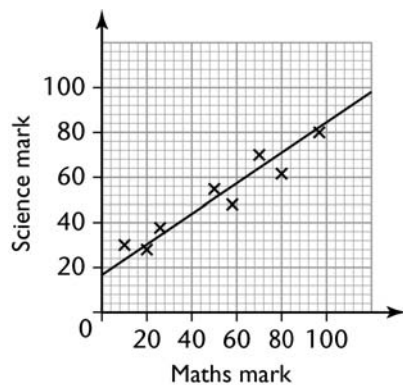
b) Reasonably strong positive correlation

d) i)   44

   ii)   85

2 a), c)
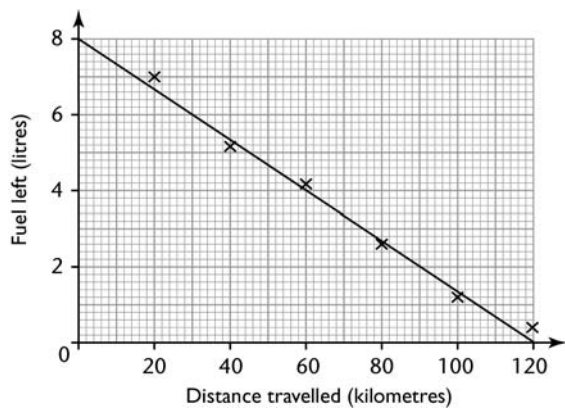
b) Strong negative correlation

d) About 3.4 litres

3 a), c)
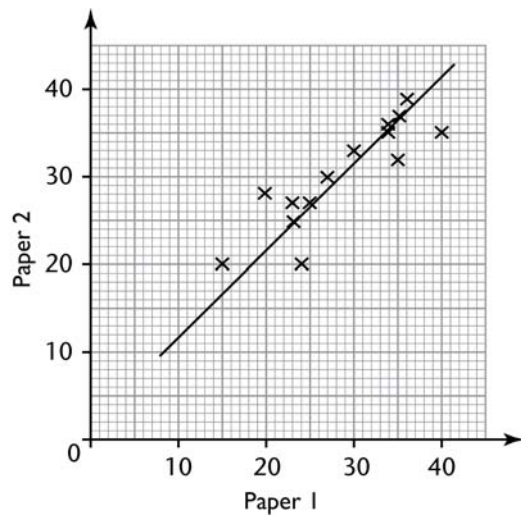
b) Strong positive correlation

d) About 33

4 a), c)
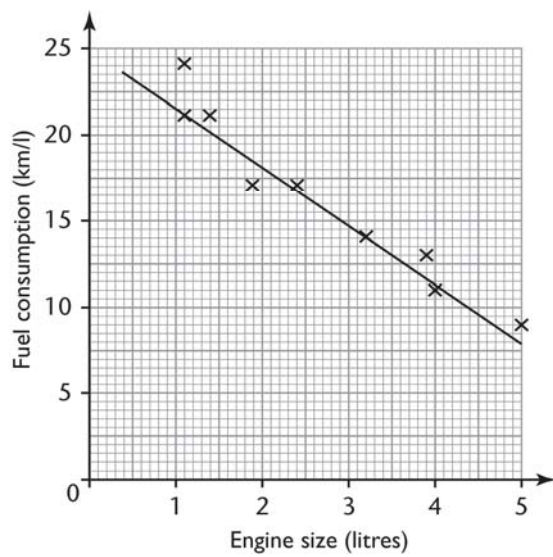
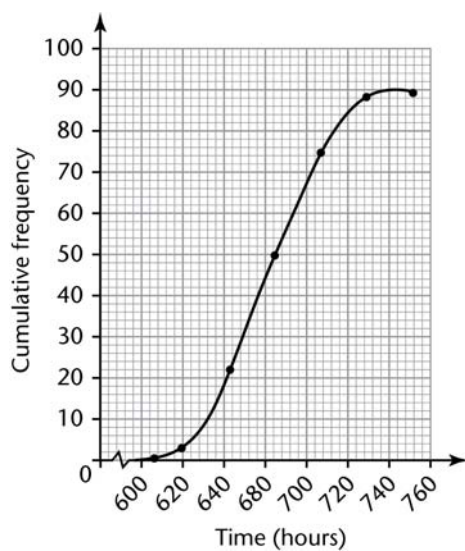**b)** Strong negative correlation

d) About 15 km/litre

Exercise 42.5

1 a)

Time in minutes (*t*)	$t \le 60$	$t \le 80$	$t \le 100$	$t \le 120$	$t \le 140$	$t \le 160$	$t \le 180$	$t \le 200$
Cumulative frequency	0	10	47	119	174	192	199	200

b)
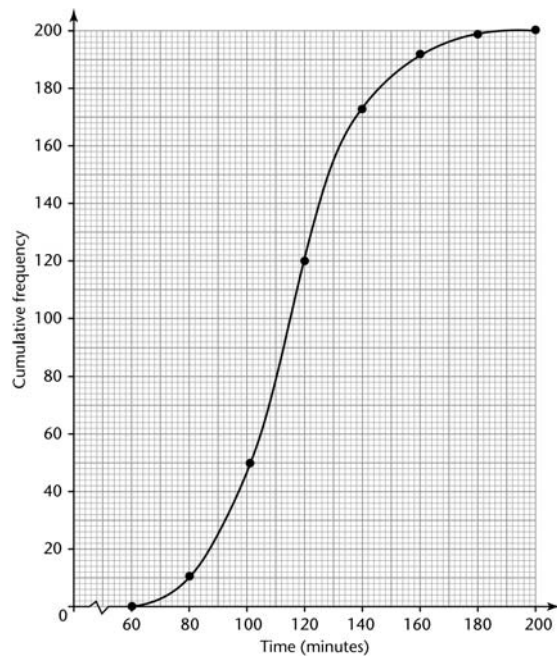
c) i)   24 (±1)

ii)   15 (±1)

2 a)

Time in hours (*t*)	$t \leq 600$	$t \leq 625$	$t \leq 650$	$t \leq 675$	$t \leq 700$	$t \leq 725$	$t \leq 750$
Cumulative frequency	0	3	21	50	75	88	90

b)

c)   35

Exercise 42.6

1 a)

Height in cm (*h*)	$h \leq 130$	$h \leq 140$	$h \leq 150$	$h \leq 160$	$h \leq 170$	$h \leq 180$	$h \leq 190$	$h \leq 200$
Cumulative frequency	0	5	17	43	78	101	116	120
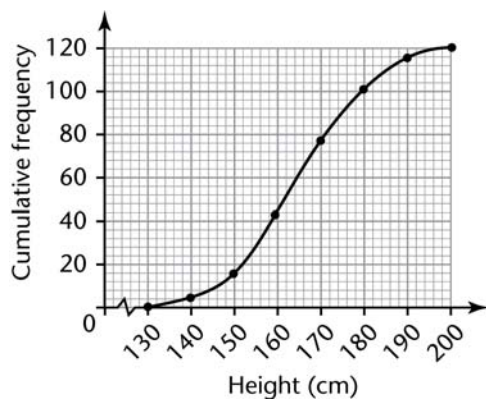
b) i)   164 cm

   ii)   155 cm

   iii)   175 cm

   iv)   20 cm

2 a)
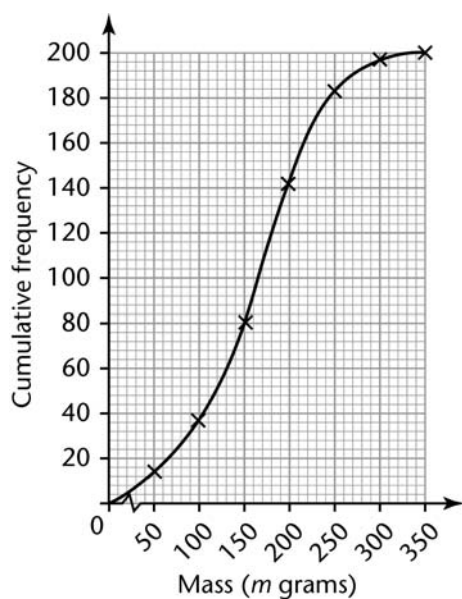
b) Median = 165 g; IQR = 90 g

c) 215 g

3 a)
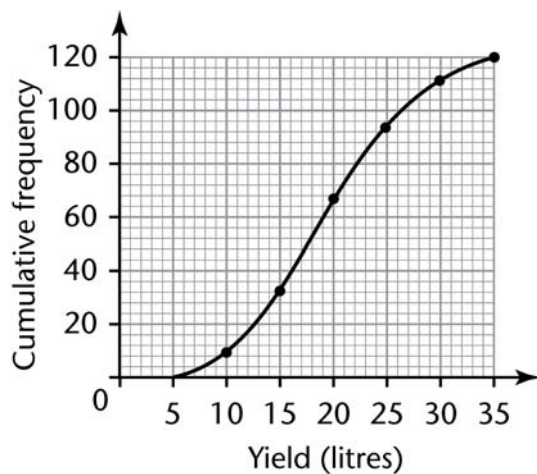
b) Median = 18.5 litres; IQR = 9.5 litres

c) 15 cows

d) 14 litres

4 a) Road B; the curve is further to the right so the median is higher.

b) Road B; the curve is steeper so the interquartile range is lower and the speeds are more consistent.

5 a) i) 62 kg

ii) 79 kg

b) i) 19 kg

ii) 23 kg

c) The masses of the girls are lower on average because their median is lower.
The masses of the girls are more consistent because their interquartile range is lower.

Exercise 42.7

1
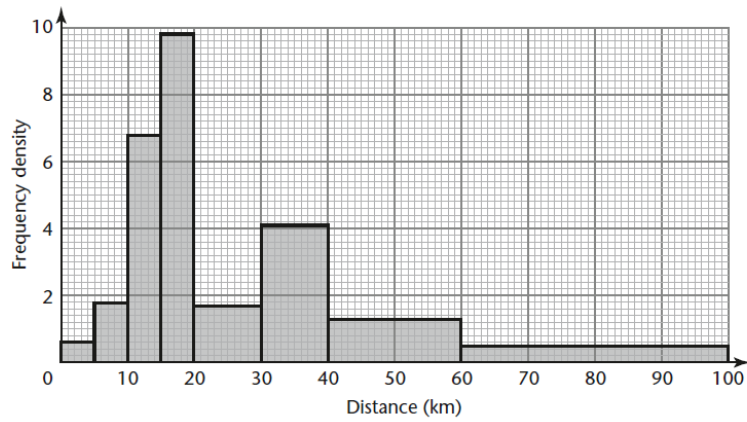
2
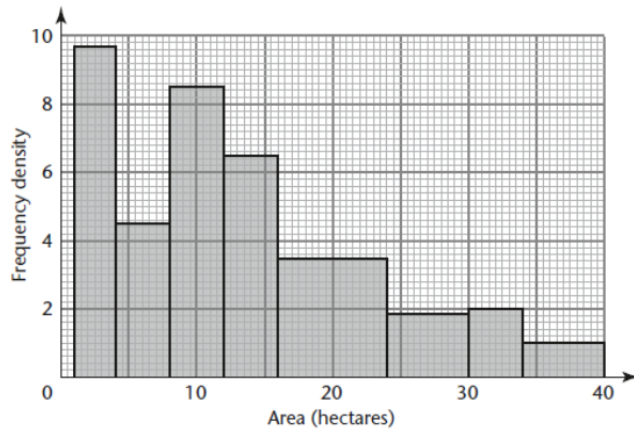
3
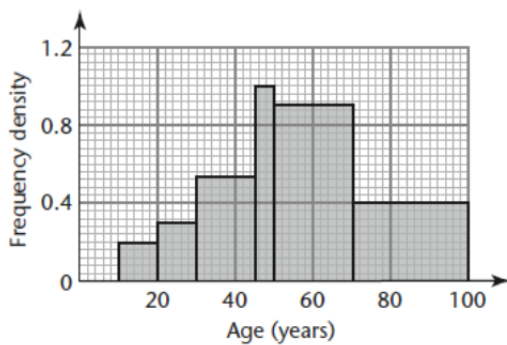
4 a) 90 students

   b) 490 students

   c) 140 students

5 a) 100 gym members

   b) 81.2 kg to 1 d.p.