

LAST TIME:

- WE TALKED ABOUT GRADIENT FIELDS ALLOWING US TO SPATIALLY VARY FREQUENCY:
 
$$\left. \begin{matrix} G_x \\ G_y \\ G_z \end{matrix} \right\} B_z = B_0 + G_x x + G_y y + G_z z$$
- $B_1$  RF FIELD  $\rightarrow$  LET'S US "EXCITE" OR "TIP" SPINS
- MAIN FIELD  $B_0 \rightarrow$  SETS LARMOR FREQUENCY, ALWAYS IN Z DIRECTION.
- WE ALSO INTRODUCED THE BASIC 2D PULSE SEQUENCE 2DPR AND THE CONCEPT OF "SLICE SELECTION".
- WE INTRODUCED A BASIC "PULSE SEQUENCE DIAGRAM"
  - PULSE SEQUENCE DIAGRAM DESCRIBES WHAT IS HAPPENING WITH THE RF, GRADIENTS, AND ADC (OR SIGNAL SAMPLING) OVER TIME
- TODAY WE RETURN TO SOME BASIC MR PHYSICS.

THE BLOCH EQUATION:

IN GENERAL, THE BEHAVIOR OF THE MAGNETIZATION VECTOR  $\vec{M}$  OF A SPIN SYSTEM (SAY A GIVEN VOXEL) IS DESCRIBED BY THE BLOCH EQUATION:

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} - \frac{M_x \hat{i} + M_y \hat{j}}{T_2} - \frac{(M_z - M_0) \hat{k}}{T_1}$$

$\uparrow$  PRECESSION                       $\uparrow$   $T_2$  DELAY OF SIGNAL                       $\uparrow$   $T_1$  RECOVERY OF SIGNAL

- LET'S LOOK AT EACH TERM.

# PRECESSION:

- AT THERMAL EQUILIBRIUM,  $\vec{M}$  AND  $\vec{B}$  WILL POINT IN THE SAME DIRECTION, SO:

$$\vec{M} \times \vec{B} = 0$$

- IF  $\vec{M}$  IS MADE TO POINT IN A DIFFERENT DIRECTION THAN  $\vec{B}$ , PRECESSIONAL BEHAVIOR OF THE MAGNETIZATION WILL OCCUR.

- FROM CLASSICAL MECHANICS, THE TORQUE APPLIED TO A DIPOLE MOMENT  $\vec{\mu}$  IN THE PRESENCE OF  $\vec{B}$  IS:

$$\begin{aligned} \text{TORQUE} &= \vec{\mu} \times \vec{B} \\ \uparrow \\ &\text{RATE OF CHANGE OF ANGULAR MOMENTUM VECTOR } \vec{\Phi} \end{aligned}$$

$$\frac{d\vec{\Phi}}{dt} = \vec{\mu} \times \vec{B}$$

RECALL THAT  $\vec{\mu} = \gamma \vec{\Phi}$ , SO WE CAN WRITE:

$\uparrow$   
DIPOLE  
(MAGNETIC  
MOMENT)

$$\gamma \frac{d\vec{\Phi}}{dt} = \gamma \vec{\mu} \times \vec{B}$$

OR:

$$\frac{d\vec{\mu}}{dt} = \vec{\mu} \times \gamma \vec{B}$$

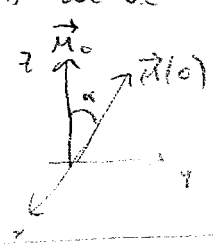
SUMMING EACH DIPOLE OVER THE SPIN SYSTEM ( $\vec{M} = \sum_{n=1}^{N_s} \vec{\mu}_n$ ):

$$\boxed{\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}}$$

← FIRST TERM IN BLOCH EQUATION ⇒ PRECESSION!

ASSUME  $\vec{B}(t) = B_0 \hat{k}$  (AFTER EXCITATION) AND THAT WE'VE  
STATIC IN  
Z DIRECTION

"TIPPED"  $\vec{M}$  AWAY FROM  $\vec{M}_0$  BY ANGLE  $\alpha$ :



SOLUTION IS:

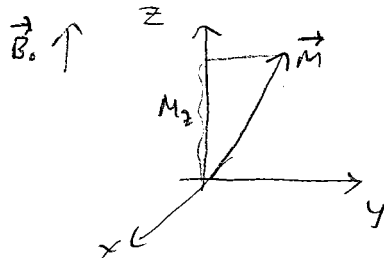
PRECESSION AT  $\omega = -\gamma B_0$ !!

$$\vec{M}(t) = M_0 \sin \alpha [\sin(-\gamma B_0 t) \hat{i} + \cos(-\gamma B_0 t) \hat{j}] + M_0 \cos \alpha \hat{k}$$

RELAXATION: ← WE'RE ONLY GOING TO CONSIDER MAGNETIC DIPOLE-DIPOLE INTERACTIONS.

- NOW THAT WE'VE DESCRIBED PRECESSION, LET'S LOOK AT THE OTHER TERMS OF THE BLOCH EQN.

- FIRST, SOME DEFINITIONS:



$M_z$  = LONGITUDINAL MAGNETIZATION

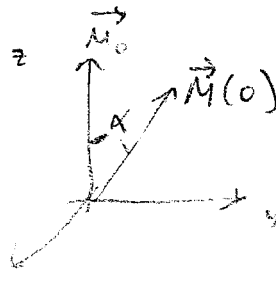
$M_{xy}$  = TRANSVERSE MAGNETIZATION

LONGITUDINAL RELAXATION:

FROM BLOCH EQN: (IGNORING PRECESSION!)

$$\frac{dM_z}{dt} = -\frac{(M_z - M_0)}{T_1}$$

AFTER A TIP BY ANGLE  $\alpha$ :



SOLUTION IS:

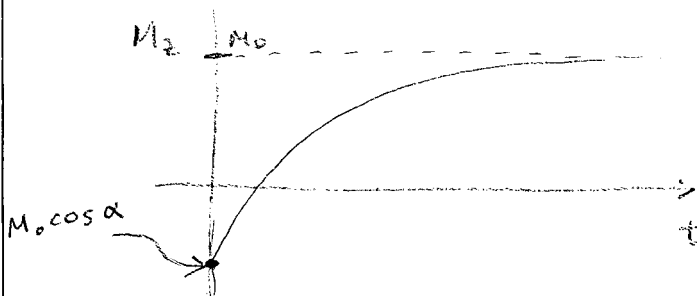
$$M_z = M_0 + \underbrace{[M_0 \cos \alpha - M_0]}_{M_z(0)} e^{-t/T_1}$$

FOLLOWING AN  $\alpha = 90^\circ$  TIP:

$$M_z = M_0 - M_0 e^{-t/T_1} = M_0 (1 - e^{-t/T_1})$$

FOLLOWING AN  $\alpha = 180^\circ$  TIP:

$$M_z = M_0 + [-M_0 - M_0] e^{-t/T_1} = M_0 - 2M_0 e^{-t/T_1}$$



$T_1 \Rightarrow$  "SPIN-LATTICE" TIME CONSTANT

CHARACTERIZES RETURN TO EQUILIBRIUM IN Z DIRECTION

## TRANSVERSE RELAXATIONS:

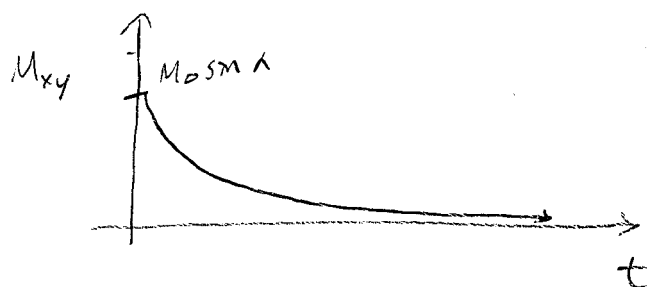
$$\frac{dM_{xy}}{dt} = -\frac{M_{xy}}{T_2} \quad \text{FROM BLOCH EQN, IGNORING PRECESSION!}$$

AFTER A TIP BY  $\alpha$ :

$$M_{xy} = M_0 \sin \alpha e^{-t/T_2}$$

- REMEMBER THAT OUR MR SIGNAL IS PROPORTIONAL TO  $M_{xy}$  (SINCE OUR RF COIL IS ORIENTED SO IT IS SENSITIVE IN THE XY PLANE, AND PRECESSION OCCURS AT OUR RF FREQUENCY IN THE XY PLANE ABOUT THE Z AXIS!)

SO:  $T_2$  IS A SIMPLE EXPONENTIAL SIGNAL DECAY!



$T_2 \Rightarrow$  "SPIN-SPIN" TIME CONSTANT

CHARACTERIZES DECAY OF TRANSVERSE MAGNETIZATION.

$T_1$  INCREASES w/  
FIELD STRENGTH!

## SOME BIOLOGICAL TISSUES:

|              | $T_2$ (ms) | $T_1$ (AT 1.5 T) (ms) |
|--------------|------------|-----------------------|
| GRAY MATTER  | 100        | 920                   |
| WHITE MATTER | 92         | 780                   |
| MUSCLE       | 47         | 870                   |
| FAT          | 85         | 270                   |
| KIDNEY       | 58         | 650                   |
| LIVER        | 43         | 495                   |

CHEMICAL SHIFT:

- SMALL DISPLACEMENT OF RESONANT FREQUENCY DUE TO SHIELDING CREATED BY THE ORBITAL MOTION OF SURROUNDING ELECTRONS IN RESPONSE TO  $\vec{B}_0$ .

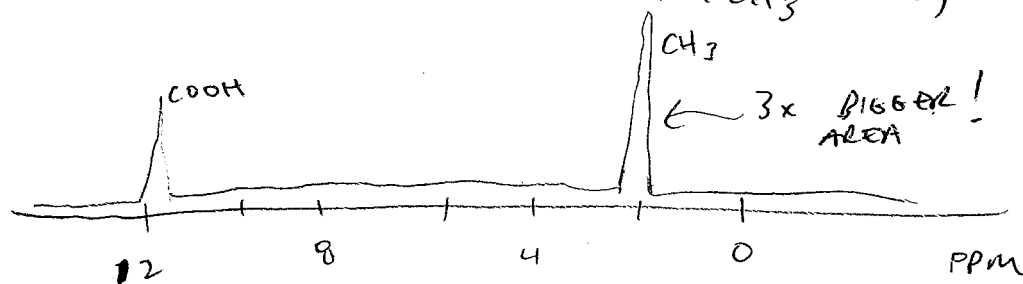
FOR ANY GIVEN NUCLEUS:

$$B_{\text{eff}} = B_0(1 - \sigma)$$

↑  
SHIELDING CONSTANT  
SPECIFIC TO CHEMICAL  
ENVIRONMENT OF NUCLEUS.

- BY CONVENTION IN NMR, THE FREQUENCY AXIS IS REVERSED.
- WE OFTEN GIVE SHIFTS IN PARTS PER MILLION (PPM)

### NMR SPECTRUM OF ACETIC ACID ( $\text{CH}_3\text{COOH}$ )



- IN HUMANS, THE MOST ABUNDANT  $^1\text{H}$  SPECIES ARE WATER ( $\text{H}_2\text{O}$ ) AND FAT (LIPIDS  $\Rightarrow$  MANY  $\text{CH}_2$  GROUPS)

FAT IS  $\approx 3.5$  ppm LOWER THAN  $\text{H}_2\text{O}$