PULSE SEQUENCES IN K-SPACE & SAMPLING REQUIREMENTS

MORE ON THE FOURIER INTERPRETATION OF THE SIGNAL EQN.

LAST TIME, WE DERIVED THE MR SIGNAL EQUATION:

$$s(t) = \int_x \int_y m(x,y) e^{-j2\pi[k_x(t)x + k_y(t)y]} \, dx \, dy$$

WHERE:

$$\boxed{\begin{aligned} k_x(t) &= \frac{\gamma}{2\pi}\int_0^t G_x(\tau)\,d\tau \\ k_y(t) &= \frac{\gamma}{2\pi}\int_0^t G_y(\tau)\,d\tau \end{aligned}}$$

VERY IMPORTANT!

WE MENTIONED THAT:

$$s(t) = M[k_x(t), k_y(t)] \quad \text{WHERE:}$$

$$M(k_x, k_y) \text{ IS THE 2D FT OF } m(x,y).$$

MR IMAGE ACQUISITION CAN BE THOUGHT OF AS <u>TRAVERSING K-SPACE</u> USING OUR GRADIENTS $G_x(t)$ AND $G_y(t)$, GATHERING SAMPLES OF $M(k_x, k_y)$ FROM OUR TIME SIGNAL $s(t)$.

<u>HOW</u> WE TRAVERSE K-SPACE IS ENTIRELY A FUNCTION OF <u>WHAT WE DO WITH OUR GRADIENTS OVER TIME</u>. OUR POSITION IN K-SPACE AT ANY POINT IN TIME IS SIMPLY A FUNCTION OF THE TIME INTEGRAL OF OUR GRADIENTS!

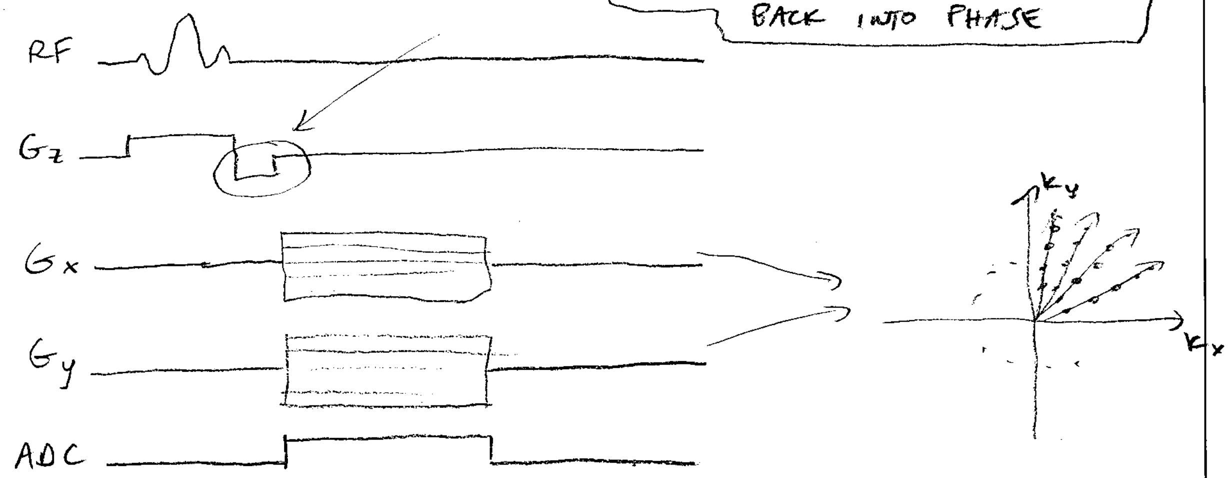
DESIGNING THESE GRADIENT TRAJECTORIES TO GIVE US A FULL SET OF SAMPLES OF $M(k_x, k_y)$ IS A FUNDAMENTAL PROBLEM IN MRI PULSE SEQUENCE DESIGN.

LET'S TAKE A LOOK AT SOME COMMON K-SPACE TRAJECTORIES.

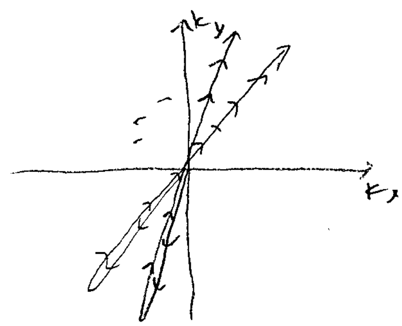
2D PROJECTION RECONSTRUCTION

– SIMILAR TO X-ray CT

EXPLAIN WHY WE NEED THIS: IT BRINGS THE WHOLE SLICE BACK INTO PHASE



ALTERNATELY, WE COULD DO THE FOLLOWING:



THIS IS A <u>NON-CARTESIAN</u> TRAJECTORY ⇒ REQUIRES EITHER <u>GRIDDING</u> OR <u>BACKPROJECTION</u> FOR RECON!

FOR HALF AS MANY REPETITIONS.

WHAT WOULD MY GRADIENTS NEED TO LOOK LIKE?

THEORETICALLY, IF $m(x,y)$ IS A REAL FUNCTION, THEN $M(k_x,k_y)$ IS <u>HERMITIAN</u>. THAT IS:

$$M(-k_x,-k_y) = M^*(k_x,k_y)$$

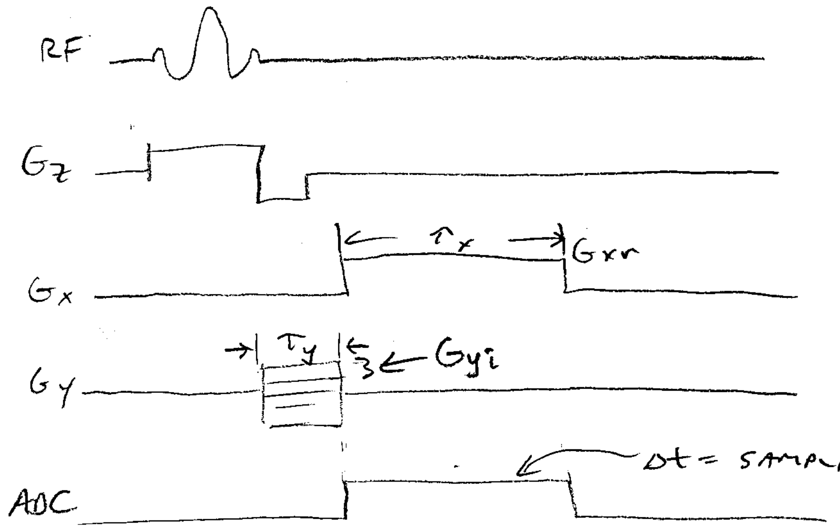
THUS, QUADRANT I CAN BE USED TO FIND QUADRANT III, AND II TO FIND IV.

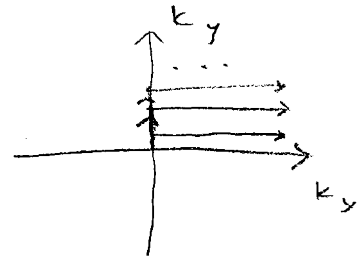
IN PRACTICE, HOWEVER, $m(x,y)$ IS USUALLY NOT REAL VALUED (DUE TO PHASE SHIFTS FROM A VARIETY OF SOURCES) AND THE HERMITIAN ASSUMPTION BREAKS DOWN.

HOWEVER, WE CAN GET AWAY WITH SOME REDUCTIONS IN SAMPLING THAT WE WILL DISCUSS LATER.

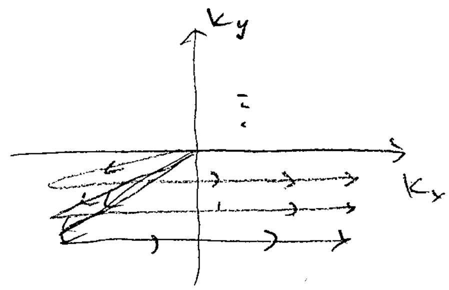
2D FT (Fourier Transform

- CARTESIAN TRAJECTORY





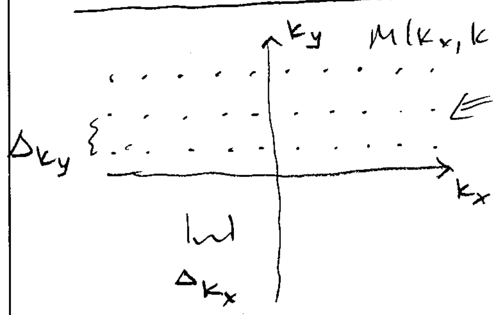
ALTERNATELY, WE TYPICALLY DO THIS:



SHOW SLIDES FOR PHASE ENCODING AND FREQUENCY ENCODING

SAMPLING REQUIREMENTS IN 2D FT IMAGING

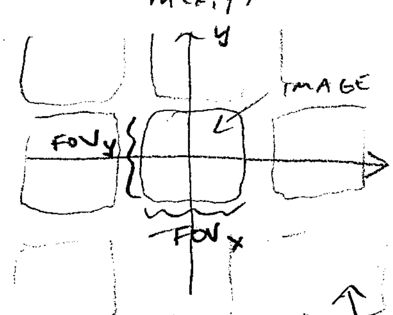
FIELD OF VIEW (FOV):



WE ARE SAMPLING $M(k_x, k_y)$ (IN FOURIER DOMAIN!)

$\Longleftrightarrow$

IN SPATIAL DOMAIN $m(x, y)$



ALIASED COPIES OF IMAGE!

$$FOV_x = \frac{1}{\Delta k_x} = \text{SAMPLING RATE ALONG } k_x$$

$$FOV_y = \frac{1}{\Delta k_y} = \text{SAMPLING RATE ALONG } k_y$$

IN THE SPECIFIC CASE OF 2DFT IMAGING:

$$FOV_x = \frac{1}{\Delta k_x} = \frac{1}{\frac{\gamma}{2\pi} G_{xr} \Delta t}$$

A/D SAMPLING RATE (Δt)

READOUT GRADIENT AMPLITUDE (G_{xr})

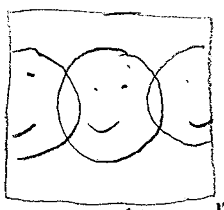
$$FOV_y = \frac{1}{\Delta k_y} = \frac{1}{\frac{\gamma}{2\pi} G_{yi} \tau_y}$$

LENGTH OF TIME PHASE ENCODING GRADIENT IS ON (τ_y)

INCREMENTAL GRADIENT AMPLITUDE BETWEEN PHASE ENCODES. (G_{yi})

WHAT IF WE UNDER SAMPLE?

SHOW ALIASED IMAGE FROM POWER POINT.



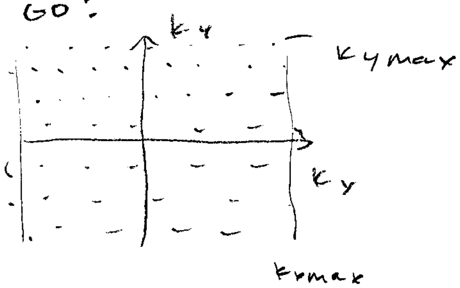
x

y ⇐ UNDERSAMPLED BY A FACTOR OF 2 IN X-DIRECTION

- WHAT IF WE APPLY ANTI-ALIASING FILTER IN k_x?
- CAN WE DO THAT IN k_y?

SPATIAL RESOLUTION:

- SPATIAL RESOLUTION CORRESPONDS TO HOW FAR OUT IN K-SPACE WE GO:



$$\delta_x = \frac{1}{2 k_{xmax}} = \frac{1}{\frac{\gamma}{2\pi} G_{xr} T_x}$$

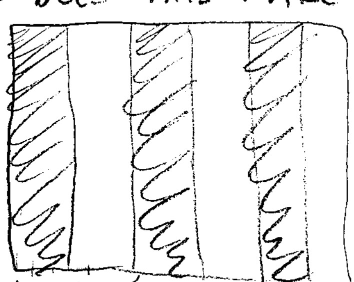
TOTAL A/D TIME (T_x)

$$\delta_y = \frac{1}{2 k_{ymax}} = \frac{1}{\frac{\gamma}{2\pi} 2 G_{yp} T_y}$$

MAX PHASE ENCODE AMPLITUDE (G_{yp})

— RESOLUTION IS THE HALF-CYCLE WIDTH OF THE HIGHEST SPATIAL FREQUENCY RECORDED IN EACH DIRECTION!

WHY DOES THIS MAKE SENSE?



HALF CYCLE WIDTH IS PIXEL WIDTH!

PERIOD IN SPATIAL FREQUENCY

EXAMPLE:

WE WANT: $FOV_x = FOV_y = 25.6$ cm.

AND: $\delta_x = \delta_y = 0.1$ cm (1 mm)

$\Rightarrow$ THAT MEANS OUR K-SPACE DATA MATRIX WILL BE 256×256 POINTS.

SAMPLING RATE?

IF $G_{xr} = 0.3 \ \frac{G}{cm}$, WHAT IS Δt?

$$FOV_x = \frac{1}{\frac{\gamma}{2\pi} G_{xr} \Delta t}$$

$$\boxed{\begin{aligned} \Delta t &\Rightarrow 30.58 \ \mu s \\ \frac{1}{\Delta t} &\Rightarrow 32.7 \ kHz \end{aligned}}$$

READOUT TIME

WHAT IS τ_x?

$$\delta_x = \frac{1}{\frac{\gamma}{2\pi} G_{xr} \tau_x}$$

$$\boxed{\tau_x = 7.83 \ ms}$$

IS THIS OKAY FROM A T_2 DECAY PERSPECTIVE??

IF $\tau_y = 4$ ms, WHAT IS G_{yi}?

$$\boxed{G_{yi} = 2.3 \ \frac{mG}{cm}}$$

WHAT IS G_{yp}?

$$\boxed{G_{yp} = 0.29 \ \frac{G}{cm}}$$