THE MR SIGNAL EQUATION AND K-SPACE:

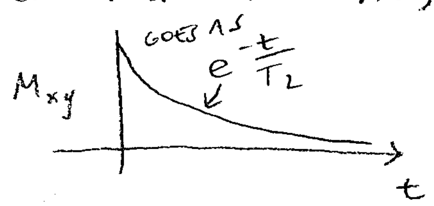
WE HAVE NOW TALKED SEVERAL TIMES ABOUT THE BLOCH EQN:

$$\frac{d\vec{M}}{dt} = \underbrace{\vec{M} \times \gamma \vec{B}}_{\text{PRECESSION}} - \underbrace{\frac{M_x \hat{i} + M_y \hat{j}}{T_2}}_{T_2 \text{ DECAY}} - \underbrace{\frac{(M_z - M_0)\hat{k}}{T_1}}_{T_1 \text{ RECOVERY}}$$

AND WE HAVE REVIEWED HOW THE FIRST TERM DESCRIBES PRECESSION, THE SECOND T_2 DECAY, AND THE THIRD T_1 RECOVERY.

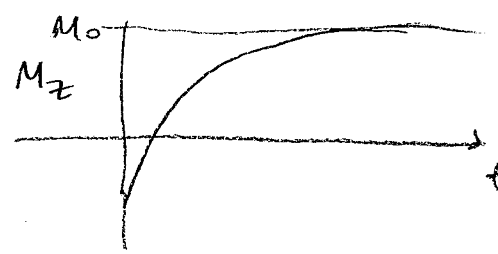
T_2 DECAY

(DECAY OF NMR SIGNAL)



T_1 RECOVERY

(HOW SOON CAN I TIP AGAIN?)



> THE TRANSVERSE MAGNETIZATION M_{xy} DETERMINES THE TIME SIGNAL WE RECORD.

IT IS CONVENIENT TO THINK OF THE XY PLANE AS THE COMPLEX PLANE, AND DESCRIBE THE PROJECTION OF $\vec{M}$ ONTO THAT PLANE AS A COMPLEX NUMBER (A KIND OF PHASOR NOTATION).

LET'S DEFINE:

$$M \equiv M_x + iM_y = M^\circ \cos\theta + M^\circ i \sin\theta = M^\circ e^{i\theta}$$

$\Rightarrow M^\circ$ IS INITIAL LENGTH OF M_{xy} AFTER EXCITATION

$\Rightarrow$ (NOT THE SAME AS M_0 UNLESS TIPPED BY 90°)

AFTER A TIP, WE HAVE:

$$M = \underset{\text{DENSITY, TIP ANGLE}}{M^\circ} \underbrace{e^{-t/T_2}}_{\text{T2 DECAY}} \underbrace{e^{-i\omega_0 t}}_{\text{PRECESSION}}$$

$\boxed{\text{ASSUMING A UNIFORM FIELD } B_0,\ \omega_0 = \gamma B_0}$

NOW LET'S LOOK AT M AS A FUNCTION OF POSITION $\vec{r}$ (DIVIDE OBJECT UP INTO VOXELS):

$$M(\vec{r}) = M^{\circ}(\vec{r})\, e^{-t/T_2(\vec{r})}\, e^{-i\omega_0 t}$$

WHAT IF ω_0 VARIES WITH $\vec{r}$?? (LIKE WHEN WE APPLY GRADIENTS OR HAVE CHEMICAL SHIFT??)

<u>THEN:</u>

$$M(\vec{r}) = M^{\circ}(\vec{r})\, e^{-\frac{t}{T_2(\vec{r})}}\, e^{-i\boxed{\omega(\vec{r})t}}$$

PHASE $\phi(\vec{r},t)$

LET'S IGNORE CHEMICAL SHIFT FOR A MOMENT AND THINK ABOUT OUR GRADIENT FIELDS G_x, G_y, AND G_z.

IN A PREVIOUS LECTURE, WE WROTE:

$$B_z = B_0 + G_x x + G_y y + G_z z$$

$$\omega = \gamma B_z = \gamma\left(B_0 + G_x x + G_y y + G_z z\right)$$

IF WE LET OUR GRADIENTS VARY W/ TIME, WHAT IS $\phi(\vec{r},t)$??

$$\omega(t) = \gamma B_0 + \gamma\left(G_x(t)\,x + G_y(t)\,y + G_z(t)\,z\right)$$

$$\phi(t) = \gamma B_0 t + \gamma x\int_0^t G_x(\tau)\,d\tau + \gamma y\int_0^t G_y(\tau)\,d\tau + \gamma z\int_0^t G_z(\tau)\,d\tau$$

$$M(\vec{r}) = M^{\circ}(\vec{r})\, e^{-\frac{t}{T_2(\vec{r})}}\, e^{-i\overset{\omega_0}{\gamma B_0} t}\, e^{-i\gamma\int_0^t \vec{G}(\tau)\cdot\vec{r}\,d\tau}$$

OUR SIGNAL WILL BE:

$$S_r(t) = \iiint M(\vec{r})\,d\vec{r} = \iiint M^{\circ}(\vec{r})\, e^{-t/T_2(\vec{r})}\, e^{-i\omega_0 t}\, e^{-i\gamma\int_0^t \vec{G}(\tau)\cdot\vec{r}\,d\tau}\, dx\,dy\,dz$$

NOW:

⇒ IGNORE RELAXATION

⇒ DEMODULATE AT $\omega_0 = \gamma B_0$

⇒ 2D

$$s(t) = \int_x \int_y \underbrace{m(x,y)}_{\substack{\Uparrow \\ \text{DENSITY OF OBJECT} \\ M^0}} e^{-i\gamma \int_0^t \vec{G}(\tau)\cdot\vec{r}\,d\tau}\,dx\,dy$$

DEFINE $k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)\,d\tau$

$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)\,d\tau$

$$\boxed{s(t) = \int_x \int_y m(x,y)\, e^{-i\gamma\,(k_x(t)x + k_y(t)y)}\,dy\,dx}$$

MR SIGNAL EQUATION

GRADIENTS LET US "WALK" THROUGH K-SPACE!! THEN WE CAN SAMPLE SIGNAL $s(t)$ AT VARIOUS POINTS IN K-SPACE, GIVING US THE <u>FOURIER-DOMAIN</u> DATA FOR OUR IMAGE!

⇒ THE WAY WE NAVIGATE THROUGH K-SPACE IS CALLED A <u>K-SPACE TRAJECTORY</u>.

⇒ TIE THIS BACK TO PULSE SEQUENCE DIAGRAMS FOR 2DFT AND 2DPR.