# Field Maps

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The acquisition and reconstruction of frequency, or field, maps is important for both the acquisition of MRI data, and for its reconstruction. Many of the imaging methods that we will consider later, such as spiral and EPI, are sensitive to the local resonant frequency. Knowledge of the field map can be necessary for the reconstruction of high-fidelity images. Measuring field maps is also important for system tuning. For example, many lipid suppression methods are sensitive to resonant frequency. By measuring the field map, and then adjusting the acquisition parameters, lipid suppression can be made much more robust.

In this section we describe how field maps are measured and reconstructed. Of particular importance are the estimation of the constant and gradient (linear) components of the field map. This is due to the fact that these can easily be changed during the acquisition of the MRI data, and that simple and robust methods exist for correcting for these terms during image reconstruction.

## 1 Field Map Acquisition

There are many different ways that field maps can be acquired. A simple method is to collect images at two different echo times, as shown in Fig. **??**. Assume image $m_1(x, y)$ is acquired at time $T_{E,1}$ and image $m_2(x, y)$ is acquired at echo time $T_{E,2}$. The phase accrued between the two is due to the different precession frequencies at different points in space

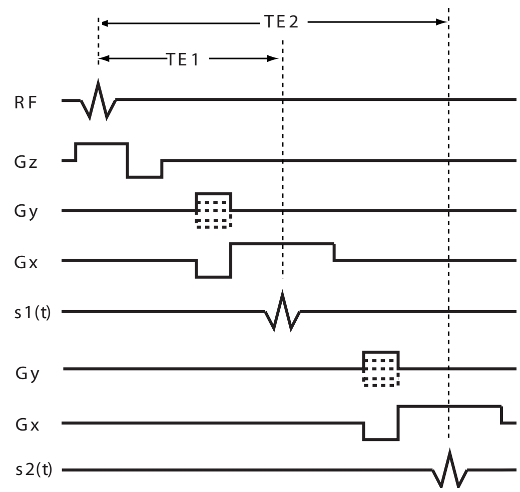


**Figure 1:** Simple field map pulse sequence. Images are acquired with two different echo times, $T_{E,1}$ and $T_{E,2}$.

$$\Delta\phi(x, y) = \angle\{m_1^*(x, y)m_2(x, y)\}. \tag{1}$$

If the change in echo times is $\Delta T_E = T_{E,2} - T_{E,1}$, the estimate of local resonant frequency is the change in phase divided by the difference in echo times,

$$\omega(x, y) = \frac{\Delta\phi(x, y)}{\Delta T_E}. \tag{2}$$

where $\omega(x, y)$ is in radians/second.

There are two main concerns with this approach. One is that the phase difference can exceed $\pm\pi$ if the frequency $\omega(x, y)$ is beyond $\pm\frac{1}{2\Delta T_E}$. Another concern is the presence of multiple chemical species with different resonant frequencies, particularly water and lipids. In this case, chemical shift can be confused with field shift, resulting in errors in the field map.

If the water/lipid issue were not a problem, the simple approach is to choose a $\Delta T_E$ such that the entire range of expected frequencies can be unambiguously resolved. For example, at 1.5T, a $\Delta T_E$ of 1 ms allows frequencies from -500 Hz to 500 Hz to be unambiguously resolved. This corresponds to $\pm$ 7.8 ppm, which covers the entire range of spectral shifts, plus the degree of $B_0$ inhomogeneity that could be expected from a superconducting magnet.

There are several cases where lipids are not a problem. One is when lipids are being suppressed anyway, such as spiral imaging using a spectral-spatial pulse that only excites water. Another is functional imaging, where long echo times are used to create the $T_2^*$ contrast that is used to infer oxygen saturation. The relatively short $T_2$'s of lipids results in little lipid signal in this case.

If lipids are present and are producing significant signal levels, one effective solution is using an echo time that is a multiple of the fat-water difference frequency. Both fat and water come into phase at these times. At 0.5 T, the fat water difference frequency is 72 Hz, so this means that the two field map images would be at 13.9 ms and 27.8 ms. At higher field strengths, these times become proportionately shorter, and later rephasing times may be used.

An example of a field map acquisition at 0.5T is shown in Fig. **??**. These are actually two of the three images from a gradient recalled three-point Dixon acquisition that we will be discussing next time. Each of the individual images has an additional linear phase component that is suppressed in the phase difference. The phase difference wraps, as is seen in the lower left. The range of frequencies in this case fits within the $\pm 36$ Hz unambiguous range if we assume a center frequency shift of $-24$ Hz. In general, the range of frequencies can exceed the unambiguous range and require phase unwrapping to produce the field map. Phase unwrapping will be described in the next class.

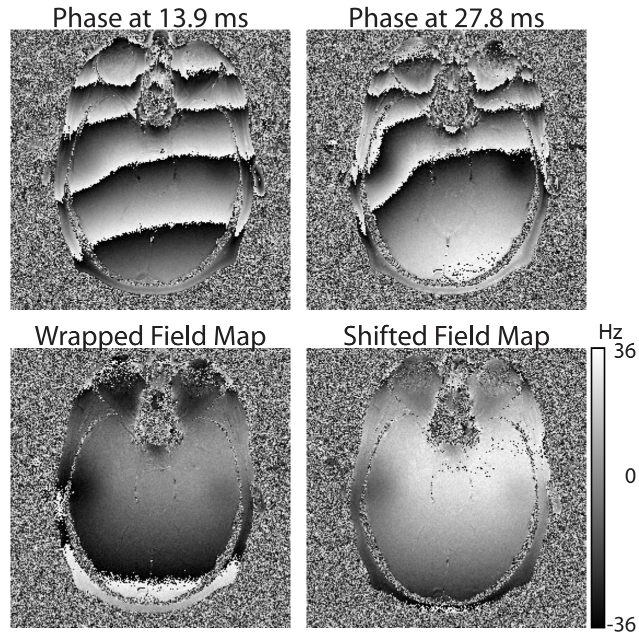


**Figure 2:** Field map acquired at 0.5T using echo times of 13.9 and 27.8 ms, which correspond to the first two times that fat and water are in phase. Both of the individual images (top) show a linear phase term, which is eliminated in the phase difference (bottom left). However, the relatively limited range of frequencies that can be unambiguously resolved results in aliasing. In this case, simulating a constant frequency shift of -24 Hz allows almost the entire frequency map to be resolved unambiguously. In general, phase unwrapping will be required, which will be discussed next time in the context of Dixon image reconstruction.

In this example an additional complete image acquisition was used to determine the field map. In many applications, such as real-time and high-resolution spiral imaging, a lower resolution field map acquisition is used in order to minimize the scan time penalty. In practice, the field map is acquired using two single shot acquisitions that are collected during the magnetization transient at the start of the scan, or when the scan plane changes, and hence the field map is likely to change.

## 2 Linear Fit

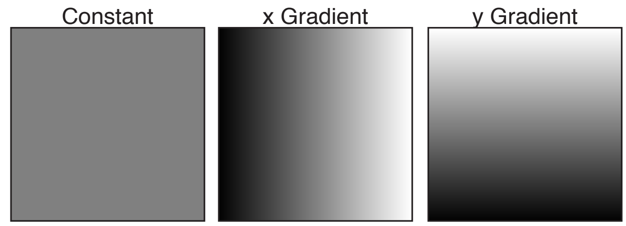


**Figure 4:** Constant and linear basis functions for the field map fit.

One of the most important uses of a field map is to estimate the constant and linear components of the field inhomogeniety [1]. There are two reasons for this. One is that these terms are easily corrected in the acquisition. The constant term is a center frequency shift, and the linear terms are constant biases to the gradient waveforms. Each of these can be changed on a $T_R$ by $T_R$ basis by the pulse sequence. The other reason is that these terms are easy to correct in reconstruction. If these terms are known, it is easy to make corrections for these errors during reconstruction with almost no cost in computation time.

We start by assuming that an unwrapped phase difference map $\Delta\phi(x, y)$ has already been computed. We want to approximate this as a constant plus linear terms in x and y, and perhaps higher order terms. As an example, we use a field map acquired during an fMRI study on a 3T system. The magnitude, phase, and frequency maps are shown in Fig. **??**

The basic idea is that we want to approximate the field map $\omega(x, y)$ by a linear combination of a constant term, the linear terms, and higher order terms if desired. We can write this as

$$\Delta\phi(x, y) \approx a_0 P_0 + a_1 P_1 + a_2 P_2 + \cdots \qquad (3)$$

where $P_0$ is a matrix that is a constant $2\pi$ over the FOV, $P_1$ is a ramp from $-\pi$ to $\pi$ in the $x$ direction, and constant in the $y$ direction, and $P_2$ is a constant in $x$, and a ramp in $y$. These are illustrated in Fig. **??**. The next three terms would be the quadratic terms $x^2$, $y^2$, and $xy$. The coefficient $a_0$ gives the offset frequency, in cycles over $\Delta T_E$. The coefficients $a_1$ and $a_2$ give the gradient terms in cycles per FOV over $\Delta T_E$.

We wish to determine the coefficients $a_i$ such that the approximation is optimum in a least squares sense. The basis set $\{P_i\}$ consists of two-dimensional matrices. For convenience, we define $\mathbf{p}_i$ to be a one dimensional vector that contains all the columns of $P_i$ sequentially in one long column vector. In addition, $\mathbf{f}$ is a column vector corresponding to $\Delta\phi(x, y)$. In this case we want to approximate

$$\begin{aligned} \mathbf{f} &\approx a_0\mathbf{p}_0 + a_1\mathbf{p}_1 + a_2\mathbf{p}_2 + \cdots && (4) \\ &\approx H\mathbf{a} && (5) \end{aligned}$$

where $H$ is the model matrix with columns

$$H = (\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \cdots) \qquad (6)$$

and $\mathbf{a}$ is a column vector of coefficients of the least squares fit. We wish to minimize the error

$$\mathbf{e} = \mathbf{f} - H\mathbf{a} \qquad (7)$$

in the least squares sense. This is done by minimizing

$$Q = \mathbf{e}^*\mathbf{e} = (\mathbf{f} - H\mathbf{a})^*(\mathbf{f} - H\mathbf{a}). \qquad (8)$$

If we expand this expression, compute the gradient with respect to the vector $\mathbf{a}$, and then set the result to zero

$$\begin{aligned} \frac{\partial Q}{\partial \mathbf{a}} &= \frac{\partial}{\partial \mathbf{a}}\left(\mathbf{f}^*\mathbf{f} - \mathbf{a}^*H^*\mathbf{f} - \mathbf{f}^*H\mathbf{a} + \mathbf{a}^*H^*H\mathbf{a}\right) \\ &= -2H^*\mathbf{f} + 2H^*H\mathbf{a} \\ &= 0. \end{aligned}$$

Then, after moving the negative quantity to the other side of the equation,

$$H^*H\mathbf{a} = H^*\mathbf{f} \qquad (9)$$

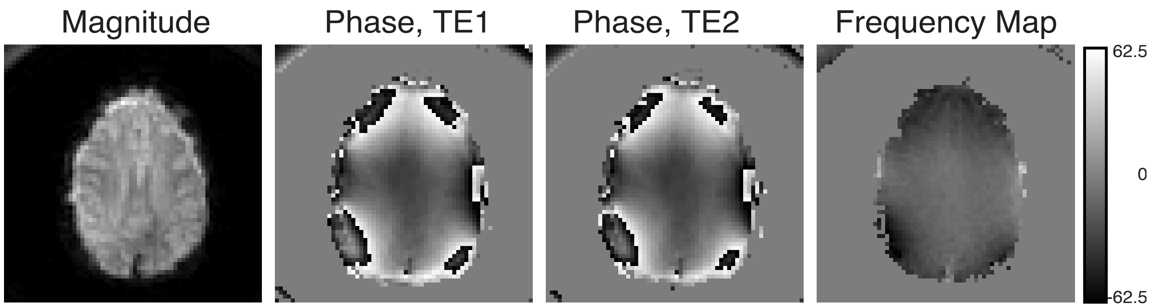


**Figure 3:** Field map for an fMRI study performed at 3T.

we get

$$\mathbf{a} = (H^*H)^{-1}H^*\mathbf{f}, \tag{10}$$

which is the familiar form for a least-squares fit.

In practice, we usually want to limit the area of the image that the fit is performed over. Areas of low signal should be excluded because the phase measurements are dominated by noise. In addition, certain parts of the image are of greater interest than others. We can include a weighting term in the quadratic form

$$Q = (\mathbf{f} - H\mathbf{a})^* W(\mathbf{f} - H\mathbf{a}). \tag{11}$$

where $W$ is a diagonal matrix. In this case the weighted least squares fit is

$$\mathbf{a} = (H^*WH)^{-1}H^*W\mathbf{f}. \tag{12}$$

There are two things that the weighting factor $W$ are typically used for. The first is to indicate the reliability of particular phase measurements. The second is to limit the fit to an area of interest.

Using a weighting that reflects the reliability of the data is reasonable if we are interested in fitting the entire slice, and there is significant variation in signal level in different parts of the image. Since the noise in MRI is constant across the image, the error in the phase estimate is inversely proportional to the magnitude of the image at each pixel. If the noise is normally distributed $N(0, \sigma_n)$, the standard deviation of the phase estimate at a high signal pixel is

$$\sigma_\phi(x, y) \approx \sigma_n/m(x, y) \tag{13}$$

by the small angle approximation, where the angle is measured in radians. Note that only the noise component in quadrature to $m(x, y)$ contributes to the phase error. This is illustrated in Fig. **??**. To minimize the variance of the estimates the diagonal elements of $W$ are then

$$1/\sigma_\phi^2(x, y) = \frac{1}{\sigma_n^2} m^2(x, y). \tag{14}$$

Since $\sigma_n$ is a constant, the optimum weighting is simply the image magnitude squared.

In many respects this does the right thing. Background signal is effectively masked out, and areas of high signal are relied on more heavily in the fit. However, there are many cases where this is undesirable [2]. One very important case is when the receiver coil sensitivity is non-uniform. One example is cardiac imaging using a surface coil, as shown in Fig. **??**. Here the chest wall is by far the brightest area of the image. Weighting by the signal magnitude would preferentially fit the chest wall. Since this is typically shifted several ppm from the heart, this can be a serious problem.

Another approach is to uniformly weight all of the data that falls within a region of interest and is above a noise dependent threshold (a multiple of

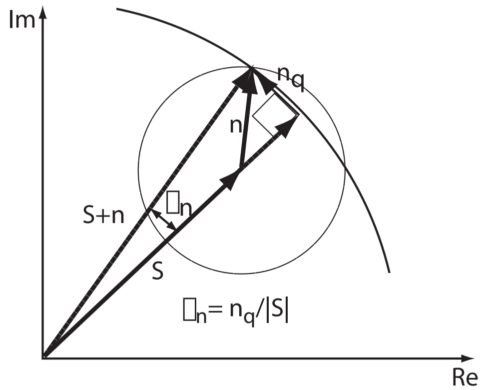


**Figure 5:** For the high SNR case, the error in angle due to noise is the length of the noise component in quadrature to the signal vector, scaled by the signal vector.

$\sigma_n$ for example). In general, since we are only estimating a few parameters from a large number of samples, the variance of the estimated parameters is not a major concern. Far more important is whether the estimate accurately represents the features of interest in the image.

As an example, we return to the fMRI data set shown previously in Fig. **??**. This is the lowest slice in a multislice set. A volume shim was done first. This produces a shim that is globally optimum over the entire volume. However, additional linear shims on each slice can still be significant. First we compute the constant and linear terms over the entire slice where the signal level is greater than 20% of the peak signal. This excludes background noise, as well as spiral image artifacts. The initial shim had an RMS error of 37 Hz, which is reduced to 18.3 Hz by correcting for the constant and linear terms. The initial shim, the fitted linear correction, and the final field map are shown in Fig. **??**.

Often in fMRI studies only part of the brain is of interest. In this case the correction can be improved by using the weighting function to reduce the region being fit even further. In Fig. **??** only

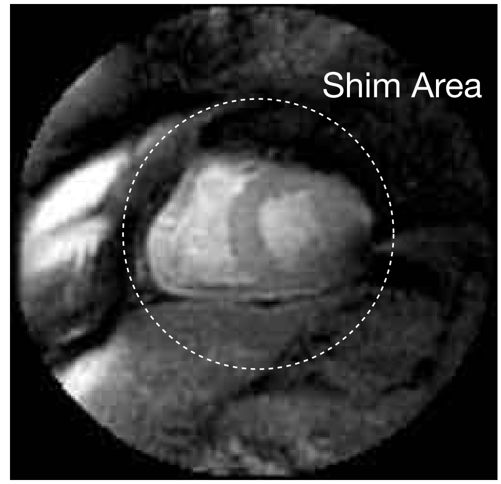


**Figure 6:** A short axis view of the heart acquired at 10 images/s. Because a surface coil is used for receive, the chest wall is very much brighter than the heart. Typically the chest wall can be shifted several ppm from the heart. A linear shim using signal intensity weighting would focus on the chest wall. A better option is to shim over the central part of the image (here shown at 60%) of the FOV, and uniformly weight all pixels above a noise-based threshold.

the back half of the brain has been used for the fit. The original shim map, the linear correction fit, and the corrected map are shown. Over the back half of the brain the RMS frequency error has been reduced to 12 Hz. The price for this reduction is an increase in the overall RMS frequency error, which has increased to 33 Hz.

# 3 Matlab Implementation

In implementing the fitting methods described here, it is useful to be able to convert between a vector representation of an image and a matrix representation. Any ordering of the vector representation will work as long as it is used consistently. A particularly convenient ordering is to append all the columns of an `nx` by `ny` image `m` into one long `nx*ny` column vector `mv`. This can

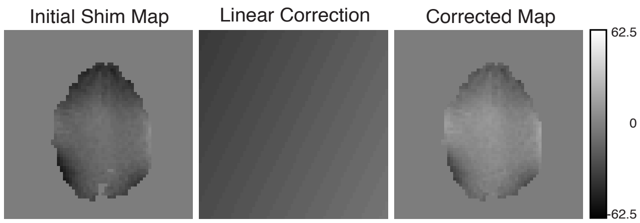


**Figure 7:** A field map acquired using a single-shot spiral sequence at 3T (left). This is the lowest slice in a multislice set. A volume shim had been performed. Since this is a compromise over the entire volume, the linear shims on individual slices can be improved. The fit of constant and linear terms is shown in the middle, and the corrected field map on the right. The RMS frequency error has been reduced from 37 Hz to 18.3 Hz over the slice.

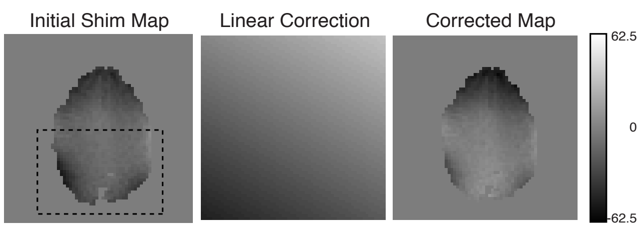


**Figure 8:** This is the same data as in Fig. **??**, where the linear correction has been calculated over the data included in the dashed box. A different correction term results (center) which reduces the RMS frequency error to 12 Hz over the box, but increases the overall RMS error to 33 Hz.

be accomplished by

```
>> mv = m(:);
```

The original matrix representation can be restored by

```
>> m = reshape(mv,nx,ny);
```

The basis set for the linear fit consists of a constant and ramps in $x$ and $y$. These can be generated by

```
>> P0 = 2*pi*ones(nx,ny);
>> PX = 2*pi*([-(nx/2):(nx/2-1)]'/nx)*ones(1,ny);
>> PY = 2*pi*(ones(nx,1)*[-(ny/2):(ny/2-1)]/ny);
```

and the model matrix `H` by

```
>> H = [P0(:)  PX(:)  PY(:)];
```

The phase difference is computed from two images `m1` and `m2` by

```
>> pdm = angle(conj(m1).*m2)
```

and converted to a column vector

```
>> pdv = pdm(:);
```

The unweighted least squares fit is then computed by

```
>> a = inv(H'*H)*H'*pdv;
```

which you probably never want to use, because of the phase noise in the background. One possible weighting matrix is simply the image squared. Another is a unit amplitude mask that includes all pixels over a threshold

```
>> wm = abs(m1)>0.2*max(max(abs(m1)));
```

which is one for all pixels greater than 20% of the maximum. In computing the weighted estimate, the weighting matrix is a huge diagonal matrix (`nx * ny by nx*ny`) with `wv = wm(:)` on the diagonal. However it never needs to be stored this way. If we look at Eq. **??** we can replace the product $W\mathbf{f}$ with an element-by-element product with the column vector `wv`. Similarly, the product $H^*WH$

can be replaced by first computing the element-by-element product of wv with the columns of H. This can be done by creating wv*ones(1,3), a matrix that replicates three columns with wv. The element-by-element product with H is the same as diag(wv)*H which would otherwise be required. Then $H^*WH$ can be computed as

```
>> HpWH = H'*((wv*ones(1,3)).*H);
```

and the weighted fit is then computed by

```
>> aw = inv(HpWH)*H'*(wv.*pdv);
```

Another alternative would be to use the matlab left matrix divide operator, which I can't figure out how to typeset in LaTeX!

```
>> aw = HpWH \ (H'*(wv.pdv));
```

The estimated fit is then H*aw, or reshape(H*aw,nx,ny) in image representation. The basis functions have been scaled so that a is in cycles per $\Delta T_E$, which will be dte in matlab. The center frequency shift is then

```
>> d_omega = a(1)/dte;
```

where d_omega is in kHz if dte is in ms. The linear terms can be converted into gradient amplitudes by realizing that the linear basis images have been scaled to one cycle over the FOV. The gradient amplitude can be computed by scaling a(2) and a(3) for dte in ms, the field of view fov in cm, and gamma in kHz/G. The result, in G/cm, is

```
>> Gx = a(2)*(1/dte)*(1/fov)*(1/gamma)
```

and similarly for Gy.

# 4 References


1. P. Irarrazabal, C.H. Meyer, D.G. Nishimura, and A. Macovski, Inhomogeneity Correction Using an Estimated Field Map,*Magn. Reson. Med.***35**(2):278–282, Feb 1996.

2. A.B. Kerr, J.M. Pauly, B.S. Hu, K.C.P. Li, C.J. Hardy, C.H. Meyer, D.G. Nishimura, and A. Macovski. Real-Time Interactive MRI on a Conventional Scanner. *Magn. Reson. Med.*, **38**(3):355–67, Sep 1997.