NOISE CONSIDERATIONS:

- RECALL: $SNR \equiv \dfrac{\text{SIGNAL AMPLITUDE}}{\text{STD. DEV. OF NOISE}}$

$$CNR \equiv \frac{\text{SIGNAL DIFFERENCE}}{\text{STD. DEV. OF NOISE}}$$

SOURCES OF NOISE:

- UNLIKE CT, WE MODEL AS GAUSSIAN-DISTRIBUTED AND ADDITIVE.
- MAIN SOURCE IS THERMAL NOISE ARISING FROM THE BROWNIAN MOTION OF ELECTRONS IN A CONDUCTOR, WHICH GENERATES RANDOM ELECTRICAL FLUCTUATIONS.
  - THIS IS CALLED "JOHNSON NOISE" OR "RESISTIVE NOISE".
- TWO SOURCES:

  IDEALLY ONLY INDUCTIVE COUPLING, SO INDUCTIVE LOSSES DUE TO "MAGNETIC RESISTANCE"

  (1) RESISTANCE R_c OF THE RECEIVER COIL

  (2) RESISTANCE R_s OF THE SAMPLE AS SEEN BY THE RECEIVER COIL.
- POWER SPECTRAL DENSITY IS GIVEN BY:

$$N(f) = 4k\underset{\text{ABSOLUTE TEMP.}}{T}R \Leftarrow \text{RESISTANCE}$$

k: BOLTZMANN'S CONSTANT

TOTAL NOISE POWER WITHIN A BANDWIDTH Δf IS THEN GIVEN BY:

$$4kTR\Delta f$$

- IN MRI, "BODY" NOISE TYPICALLY DOMINATES OVER RECEIVER COIL NOISE.

NOISE IN MRI (CONTINUED):

- IMAGE NOISE IN MRI IS A BI-VARIATE (COMPLEX-VALUED) ZERO-MEAN GAUSSIAN RANDOM PROCESS WITH REAL AND IMAGINARY COMPONENTS EACH POSSESSING VARIANCE σ_n^2.

- BECAUSE THE RECONSTRUCTED IMAGE IS NOT GENERALLY REAL VALUED, THE DISPLAYED IMAGE IS TYPICALLY THE ABSOLUTE VALUE OF THE COMPLEX IMAGE.
    - IN BACKGROUND REGIONS: GAUSSIAN $\Rightarrow$ RAYLEIGH

$$\sigma^2 \Rightarrow \left(2-\frac{\pi}{2}\right)\sigma_n^2$$

$$\mu \Rightarrow \sigma\sqrt{\pi/2}$$

    - IN REGIONS OF SIGNAL: GAUSSIAN $\Rightarrow$ RICIAN
        - IF SNR IS HIGH, GAUSSIAN CAN APPROXIMATE RICIAN
        - NOT SO IF SNR IS LOW $\Rightarrow$ NOISE THRESHOLDING THAT DEGRADES SNR

GETTING PRACTICAL ABOUT NOISE:

$$SNR \propto \underbrace{(\Delta x)(\Delta y)(\Delta z)}_{\text{VOXEL VOLUME}} \sqrt{\text{TOTAL READOUT TIME}}\; \underbrace{f(\rho, T_1, T_2)}_{\text{CONTRAST (PROTON DENSITY, } T_1, T_2)}$$

EFFECT OF ACQUISITION TIME:

SIGNAL AVERAGING:

- SIGNALS ADD
- VARIANCES ADD

READOUT TIME:

- NOISE VARIANCE PER SAMPLE IS PROPORTIONAL TO Δf ANTI-ALIASING FILTER BANDWIDTH

$$\sigma_n^2 \propto \Delta f = \frac{1}{\ldots}$$

- DOUBLING THE SAMPLING PERIOD CUTS THE NOISE VARIANCE PER SAMPLE IN HALF, SO STANDARD DEVIATION IS REDUCED BY A FACTOR OF $\sqrt{2}$.

SO:

$$SNR \propto \sqrt{N_{AVERAGES} T_{READOUT}} = \sqrt{\text{TOTAL ACQUISITION TIME}}$$

$$= \sqrt{N_{AVERAGES} N_{PHASE\ ENCODES} T_{READOUT}}$$

EXAMPLE:

CONSIDER 2 2DFT SEQUENCES:

SEQ 1: 256 SAMPLES PER READOUT
SEQ 2: 512 SAMPLES PER READOUT

} GRADIENTS ARE THE SAME, SAME READOUT TIME

WHAT IS RELATIVE SNR?

SPATIAL RESOLUTION:

- VOXEL VOLUME IS DIRECTLY PROPORTIONAL TO SIGNAL.
- ARE WE JUST AS WELL OFF SCANNING AT TWICE THE RESOLUTION AND AVERAGING??

OTHER FACTORS:

$$f(\rho, T_1, T_2) \Rightarrow \text{CONTRAST!}$$

2D vs. 3D?