

CS 2731 / ISSP 2230

# Introduction to Natural Language Processing

Session 3: Bag of words, tf-idf, PPMI

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# About Jayden (TA)

- **6th year at Pitt! (2nd semester Masters)**
- **Research interests:**
  - Privacy in NLP
  - Computational Musicology (Music and language are more alike than you'd think!)
- **Office Hours:**
  - Tuesday: 10:30am – 12:30pm or by appt.
  - Email: [jserenari@pitt.edu](mailto:jserenari@pitt.edu)
  - Location TBD

# Course logistics

- Reading quiz due **11:59pm** tonight (and after every class session)
  - Main reading quizzes are graded
  - "Muddiest point" quizzes are ungraded, optional. Helps me know what is confusing to people
- Readings for the next few weeks are updated to the newest version of the textbook (the Aug 20 version)
- Homework and project schedule is current, on website
- HW1 will be released by tomorrow, due **Thu Sep 19**
- Project idea submission will be released by Sep 11, due **Thu Sep 19**

# Overview: Bag of words, tf-idf, PPMI

- Term-document and term-term matrices
- Cosine similarity
- Tf-idf weighting
- PPMI weighting

*Review activity:*

Define a term from last session  
about text preprocessing

# Core tasks and applications of NLP

## CORE TASKS

text  
classification

representation  
learning

language  
models

conditional  
language  
models

sequence  
labeling

syntactic  
parsing

### MODULE 2



## APPLICATIONS

machine  
translation

speech recognition  
& synthesis

chatbots

information retrieval

# Text classification

*"My dear Mr. Bennet," said his lady to him one day, "have you heard that Netherfield Park is let at last?"*

ROMANCE

*Pride and Prejudice*

DIALOG





# Bag of words document representation

I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun... It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet!



it	6
I	5
the	4
to	3
and	3
seen	2
yet	1
would	1
whimsical	1
times	1
sweet	1
satirical	1
adventure	1
genre	1
fairy	1
humor	1
have	1
great	1
...	...

# Documents Can Be Represented as Bags of Words

A BAG OF WORDS is a BAG or MULTISSET of the words in a document.

- Like a set, except that identical elements can appear multiple times
- You could also think of it like a `Counter` object in Python

```
bow = {'and': 23502,  
       'or': 12342',  
       'the': 54939508,  
       ...  
       'hippopotamus': 1}
```

- If you take out sequencing information, any document can be viewed as a bag of words.

# Bags of Words Can Be Represented as Sparse Vectors

- So far, we've represented bags of words as dense MAPS, but sometimes it is useful to represent them as sparse vectors
- Let  $V$  be the set of all words in the document collection  $D$ . Then our BoW vectors for documents in  $D$  will have  $|V|$  dimensions.
- Each dimension corresponds to a word **type** and the value at this dimension corresponds to the number of **TOKENS** of that type in the document that the vector is representing

# Term-document and term-term matrices

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# Term-document matrix

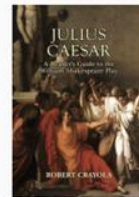
- Each cell is the count of term  $t$  in a document  $d$  ( $tf_{t,d}$ ).
- Each document is a **count vector** in  $\mathbb{N}^V$ , a column below.



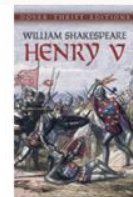
As You Like It



Twelfth Night



Julius Caesar



Henry V

*battle*  
*soldier*  
*fool*  
*clown*

1  
2  
37  
6

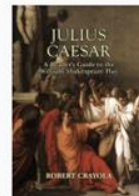
1  
2  
58  
117

8  
12  
1  
0

15  
36  
5  
0

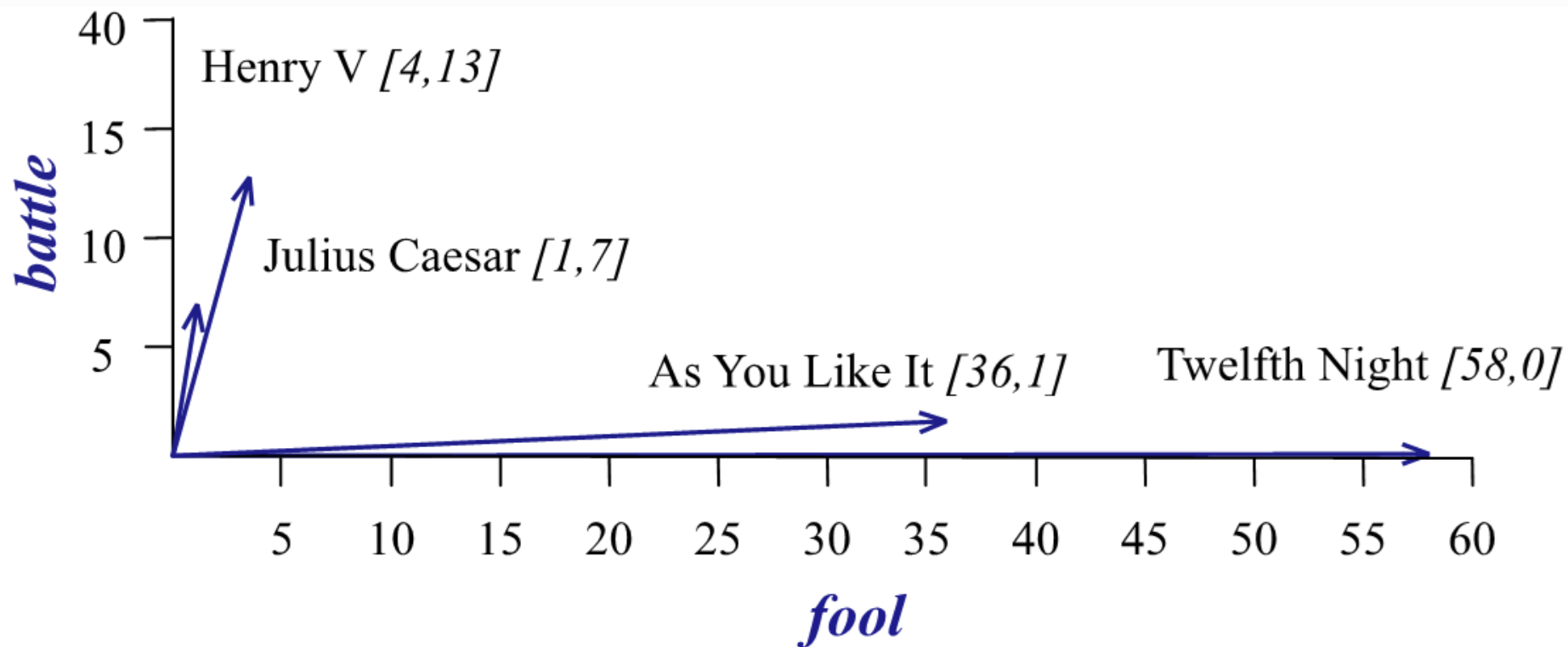
# Term-document matrix

- Two documents are similar if their vectors are similar.



	As You Like It	Twelfth Night	Julius Caesar	Henry V
<i>battle</i>	1	1	8	15
<i>soldier</i>	2	2	12	36
<i>fool</i>	37	58	1	5
<i>clown</i>	6	117	0	0

# Visualizing document vectors



# Vectors are the basis of information retrieval

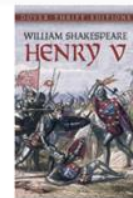
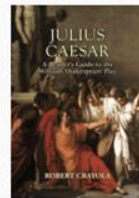
	<i>As You Like It</i>	<i>Twelfth Night</i>	<i>Julius Caesar</i>	<i>Henry V</i>
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

- Vectors for comedies are different from tragedies
- Comedies have more *fools* and fewer *battles*



# Term-document matrix: word vectors

Two words are similar if their vectors are similar.



	As You Like It	Twelfth Night	Julius Caesar	Henry V
<i>battle</i>	1	1	8	15
<i>soldier</i>	2	2	12	36
<i>fool</i>	37	58	1	5
<i>clown</i>	6	117	0	0

- *battle* is "the kind of word that occurs in Julius Caesar and Henry V"
- *fool* is "the kind of word that occurs in comedies, especially Twelfth Night"

# Term-term matrix (or word-word or word-context matrix)

- Instead of entire documents, use smaller contexts
  - Paragraph
  - Window of a few words (e.g. 3, 5, 7):



- A word is now defined by a vector over counts of words in context.
  - If a word  $w_j$  occurs in the context of  $w_i$ , increase  $count_{ij}$ .
- Assuming we have  $V$  words,
  - Each vector is now of length  $V$ .
  - The word-word matrix is  $V \times V$ .

# Sample Contexts of $\pm 7$ Words

sugar, a sliced lemon, a tablespoonful of their enjoyment. Cautiously she sampled her first well suited to programming on the digital for the purpose of gathering data and **apricot** **pineapple** **computer.** **information** preserve or jam, a pinch each of, and another fruit whose taste she likened In finding the optimal R-stage policy from necessary for the study authorized in the

	aardvark	computer	data	pinch	result	sugar ...
$\vdots$						
<i>apricot</i>	0	0	0	1	0	1
<i>pineapple</i>	0	0	0	1	0	1
<i>digital</i>	0	2	1	0	1	0
<i>information</i>	0	1	6	0	4	0
$\vdots$						

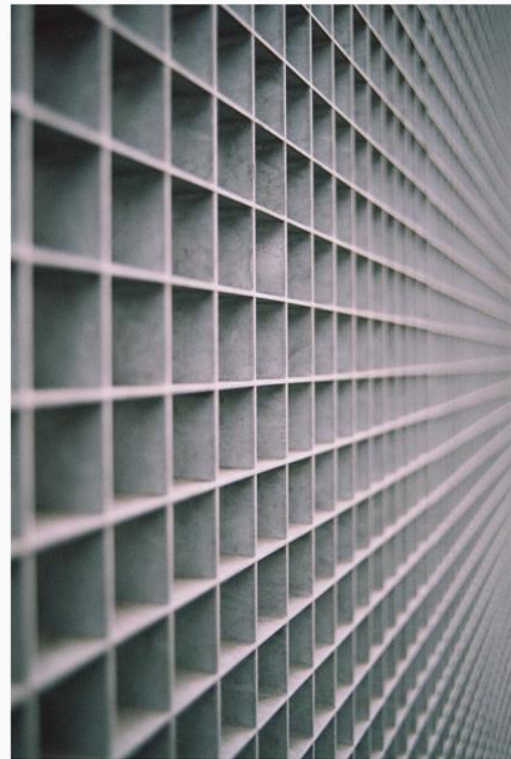
# The Word-Word Matrix

We showed only a  $4 \times 6$  matrix, but the real matrix is  $50,000 \times 50,000$ .

- So it is very sparse: Most values are 0.
- That's OK, since there are lots of efficient algorithms for sparse matrices.

The size of windows depends on the goals:

- The **smaller** the context ( $\pm 1 - 3$ ), the more **syntactic** the representation
- The **larger** the context ( $\pm 4 - 10$ ), the more **semantic** the representation



# Types of Co-occurrence between Two Words

**First-order co-occurrence** (syntagmatic association):

- They are typically nearby each other.
- *wrote* is a first-order associate of *book* or *poem*.

**Second-order co-occurrence** (paradigmatic association):

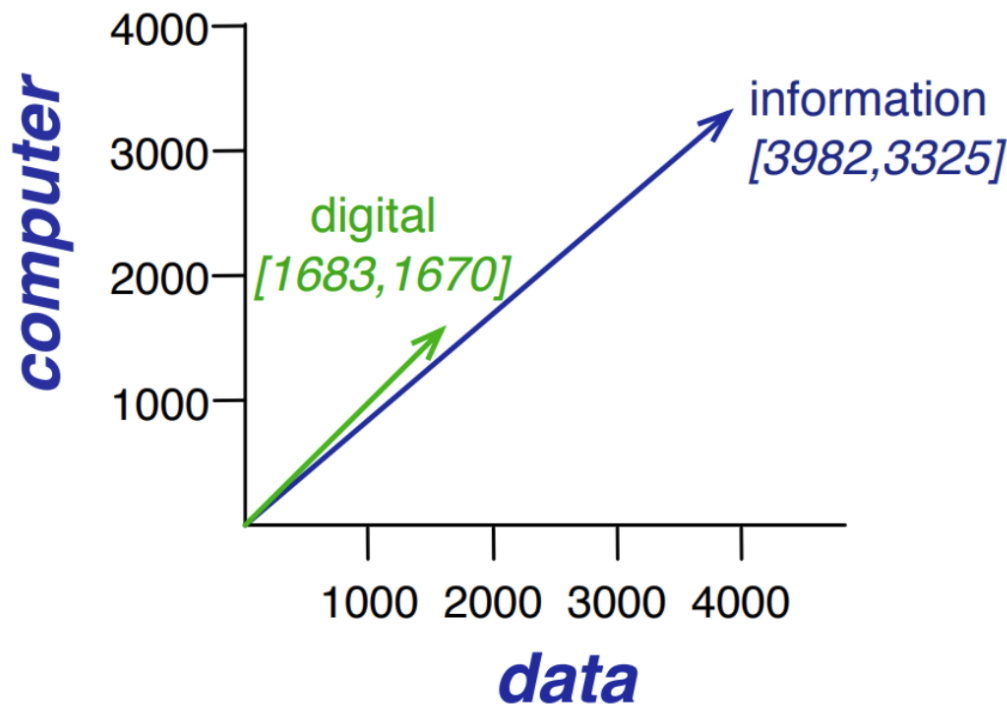
- They have similar neighbors.
- *wrote* is a second-order associate of words like *said* or *remarked*.

# Cosine similarity

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# Measuring similarity between word or document vectors

- Do we care about magnitude/word frequencies? (No)



# Cosine is Used to Measure the Similarity between Word Vectors

- Given two target words represented with vectors  $\mathbf{v}$  and  $\mathbf{w}$ .
- The **dot product** or **inner product** is usually used as the basis for similarity.

$$\mathbf{v} \cdot \mathbf{w} = \sum_{i=1}^N v_i w_i = v_1 w_1 + v_2 w_2 + \cdots + v_N w_N = |\mathbf{v}| |\mathbf{w}| \cos \theta$$

- $\mathbf{v} \cdot \mathbf{w}$  is high when two vectors have large values in the same dimensions.
- $\mathbf{v} \cdot \mathbf{w}$  is low (in fact 0) with zeros in complementary distribution.
- We also do not want the similarity to be sensitive to word-frequency.
- So normalize by vector length and use the cosine as the similarity

$$\cos(\mathbf{v}, \mathbf{w}) = \frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{v}| |\mathbf{w}|}$$

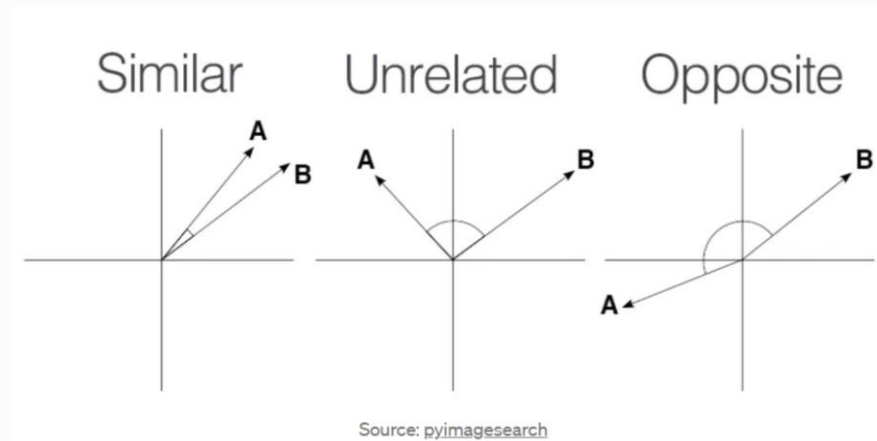


# Cosine as a similarity metric for vectors

-1: vectors point in opposite directions

+1: vectors point in same directions

0: vectors are orthogonal



But since raw frequency values are non-negative, the cosine for term-term matrix vectors ranges from 0–1

# Tf-idf weighting

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# Raw frequency is a bad representation

- The co-occurrence matrices we have seen represent each cell by word frequencies
  - Whether in term-document or term-term matrices
- Frequency is clearly useful; if *sugar* appears a lot near *apricot*, that's useful information.
- But overly frequent words like *the*, *it*, or *they* are not very informative about the context
  - 2 documents that use a lot of *the* are not necessarily similar
- It's a paradox! How can we balance these two conflicting constraints?

# Weighting words in term-document and term-term matrices

- **tf-idf** (term frequency-inverse document frequency)
  - For representing documents with their most unique words (for text classification, information retrieval)
  - Term-document matrix
- **PPMI** (positive pointwise mutual information)
  - For finding associations between words (which appear more often together than chance?)
  - Term-term matrix

# Term frequency (tf)

$$tf_{t,d} = \text{count}(t,d)$$

Instead of using raw count, we squash a bit:

$$tf_{t,d} = \log_{10}(\text{count}(t,d)+1)$$

# Document frequency (df)

$df_t$  is the number of documents term  $t$  occurs in.

(note this is not collection frequency, which is the total count across all documents)

"Romeo" is very distinctive for one Shakespeare play:

	<b>Collection Frequency</b>	<b>Document Frequency</b>
Romeo	113	1
action	113	31

# Inverse document frequency (idf)

$$\text{idf}_t = \log_{10} \left( \frac{N}{\text{df}_t} \right)$$

- N is the total number of documents in the collection
- Documents can be whatever you want! (Full documents, paragraphs, etc)

Word	df	idf
Romeo	1	1.57
salad	2	1.27
Falstaff	4	0.967
forest	12	0.489
battle	21	0.246
wit	34	0.037
fool	36	0.012
good	37	0
sweet	37	0

## tf-idf Controls for Frequent but Uninformative Words

Some words are very common in a given document because they are common across all documents (e.g., *the*). They are not discriminative. **tf-idf** (product of term frequency and inverse document frequency) addresses this:

$$\text{tf}_{t,d} = \log_{10}(\text{count}(t, d) + 1)$$

$$\text{idf}_f = \log_{10} \frac{N}{\text{df}_t}$$

$$\text{tf-idf}(t, d) = \text{tf}_{t,d} \cdot \text{idf}_t$$



# Final tf-idf weighted values

Raw counts

	As You Like It	Twelfth Night	Julius Caesar	Henry V
<b>battle</b>	1	0	7	13
<b>good</b>	114	80	62	89
<b>fool</b>	36	58	1	4
<b>wit</b>	20	15	2	3

tf-idf:

	As You Like It	Twelfth Night	Julius Caesar	Henry V
<b>battle</b>	0.074	0	0.22	0.28
<b>good</b>	0	0	0	0
<b>fool</b>	0.019	0.021	0.0036	0.0083
<b>wit</b>	0.049	0.044	0.018	0.022

# Positive pointwise mutual information (PPMI)

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## The Problem with Raw Counts

The **to** in **to walk** doesn't tell us as much about **walk** as the **slowly** in **walk slowly**.



# Problem with Raw Counts

- Raw word frequency is not a great measure of association between words.
- It is very skewed: “the” and “of” are very frequent, but maybe not the most discriminative.
- We would rather have a measure that asks whether a context word is **particularly informative** about the target word.

## Positive Pointwise Mutual Information (PPMI)

# Pointwise mutual information

- Do events  $x$  and  $y$  co-occur more than if they were independent?

$$\text{PMI}(X, Y) = \log_2 \frac{P(x, y)}{P(x)P(y)}$$

- PMI between 2 words [Church+Hanks 1989]
  - Do words  $x$  and  $y$  co-occur more than if they were independent?

$$\text{PMI}(\text{word}_1, \text{word}_2) = \log_2 \frac{P(\text{word}_1, \text{word}_2)}{P(\text{word}_1)P(\text{word}_2)}$$

- In computational linguistics, PMI has been used for finding collocations and associations between words.

<b>word 1</b>	<b>word 2</b>	<b>count word 1</b>	<b>count word 2</b>	<b>count of co-occurrences</b>	<b>PMI</b>
puerto	rico	1938	1311	1159	10.0349081703
hong	kong	2438	2694	2205	9.72831972408
los	angeles	3501	2808	2791	9.56067615065
carbon	dioxide	4265	1353	1032	9.09852946116
prize	laureate	5131	1676	1210	8.85870710982
san	francisco	5237	2477	1779	8.83305176711
nobel	prize	4098	5131	2498	8.68948811416
ice	hockey	5607	3002	1933	8.6555759741
star	trek	8264	1594	1489	8.63974676575
car	driver	5578	2749	1384	8.41470768304
it	the	283891	3293296	3347	-1.72037278119
are	of	234458	1761436	1019	-2.09254205335
this	the	199882	3293296	1211	-2.38612756961
is	of	565679	1761436	1562	-2.54614706831
and	of	1375396	1761436	2949	-2.79911817902
a	and	984442	1375396	1457	-2.92239510038
in	and	1187652	1375396	1537	-3.05660070757
to	and	1025659	1375396	1286	-3.08825363041
to	in	1025659	1187652	1066	-3.12911348956
of	and	1761436	1375396	1190	-3.70663100173

# Positive Pointwise Mutual Information

- PMI ranges from  $-\infty$  to  $+\infty$
- But the negative values are problematic:
  - Things are co-occurring less than we expect by chance
  - Unreliable without enormous corpora
    - Imagine  $w_1$  and  $w_2$  whose probability is each  $10^{-6}$ .
    - Hard to be sure  $p(w_1, w_2)$  is significantly different than  $10^{-12}$ .
  - Furthermore it's not clear people are good at “unrelatedness”.
- So we just replace negative PMI values by 0.

$$\text{PPMI}(w, c) = \max\left(\log_2 \frac{p(w, c)}{p(w)p(c)}, 0\right)$$

# Computing PPMI on a Term-Context Matrix

- We have matrix  $F$  with  $V$  rows (words) and  $C$  columns (contexts) (in general  $C = V$ )
- $f_{ij}$  is how many times word  $w_i$  co-occurs in the context of the word  $c_j$ .

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^V (\sum_{j=1}^C f_{ij})}$$

$$p_{i*} = \frac{\sum_{j=1}^C f_{ij}}{\sum_{i=1}^V (\sum_{j=1}^C f_{ij})}$$

$$p_{*j} = \frac{\sum_{i=1}^V f_{ij}}{\sum_{i=1}^V (\sum_{j=1}^C f_{ij})}$$

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*} p_{*j}}$$

$$ppmi_{ij} = \max(pmi_{ij}, 0)$$



## Worked Example: Computing PPMI from Term-Context Matrix (Part I)

	computer	data	pinch	result	sugar	
<i>apricot</i>	0	0	1	0	1	2
<i>pineapple</i>	0	0	1	0	1	2
<i>digital</i>	2	1	0	1	0	4
<i>information</i>	1	6	0	4	0	11
	3	7	2	5	2	19

$$p(w = \text{information}, c = \text{data}) = \frac{6}{19} = 0.32$$

$$p(w = \text{information}) = \frac{11}{19} = 0.58 \quad p(c = \text{data}) = \frac{7}{19} = 0.37$$

$$\text{pmi}(\text{information}, \text{data}) = \log_2 \frac{0.32}{0.37 \cdot 0.58} \approx 0.58$$

## Worked Example: Computing PPMI from Term-Context Matrix (Part II)

	$PPMI(w, c)$				
	computer	data	pinch	result	sugar
<i>apricot</i>	-	-	2.25	-	2.25
<i>pineapple</i>	-	-	2.25	-	2.25
<i>digital</i>	1.66	0.00	-	0.00	-
<i>information</i>	0.00	0.32	-	0.47	-

- PMI is biased toward infrequent events.
- Very rare words have very high PMI values.
- Two solutions:
  - Give rare words slightly higher probabilities
  - Use add-one smoothing (which has a similar effect)

# Conclusion

- **Bag of words** representations of documents
  - Counts of terms in documents (no order info)
  - Can be represented in vector form as...
- **Term-document matrices**
- **Term-term (word-word) matrices**
  - How many times words are used in contexts of other words
- **Cosine** to measure similarity between vectors for documents or words
- Downweighting words that appear frequently
  - **tf-idf** for document representations
    - Downweight terms that appear across many documents
  - **PPMI** for word associations
    - Downweight words that appear with many other words



*Questions?*