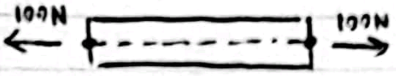
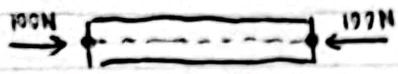
3 Principles of Engineering
1) $F = ma$
2) You can't push on a rope
3) To get the answer you must know the answer

CIV 102 Notes:

① Tension / Compression (+/-) ~

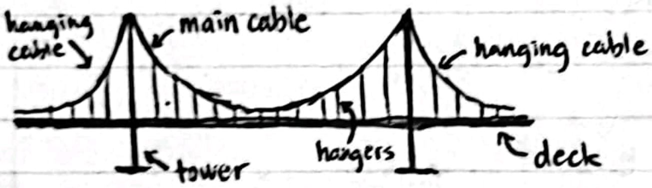


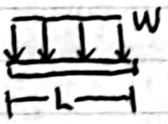
Tension (away from a point)



Compression (into a point)

② Suspension Bridge ~



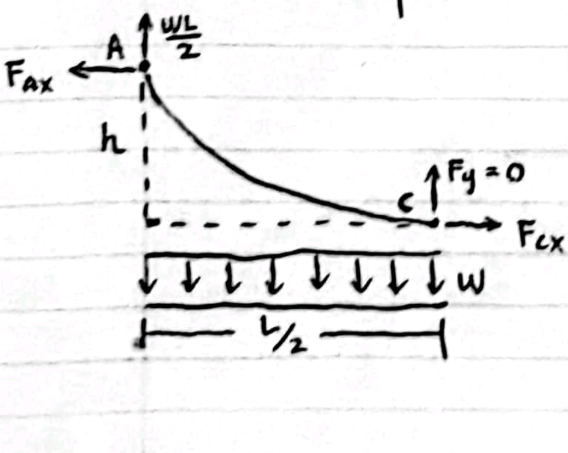
Uniform Load:


~ $F = wL$
~ CM at $L/2$

Non-Uniform Load: (triangle)


~ $F = \frac{1}{2}wL$
~ CM at $L/3$ from long length

Forces in Suspension Bridge:



F_y decreases from $\frac{wL}{2}$ at A to zero at the middle (C).

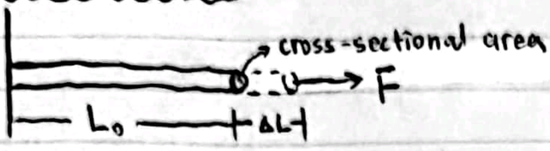
F_x is constant ... $\left[F_x = \frac{wL^2}{8h} \right]$

$$F_{max} = \sqrt{F_{y_{max}}^2 + F_x^2}$$

Have a open mind but not too open that your mind falls out...

③ Stress and Strain ~

Hooke's Law: $F = k \Delta L$, $k = \frac{EA}{L_0}$



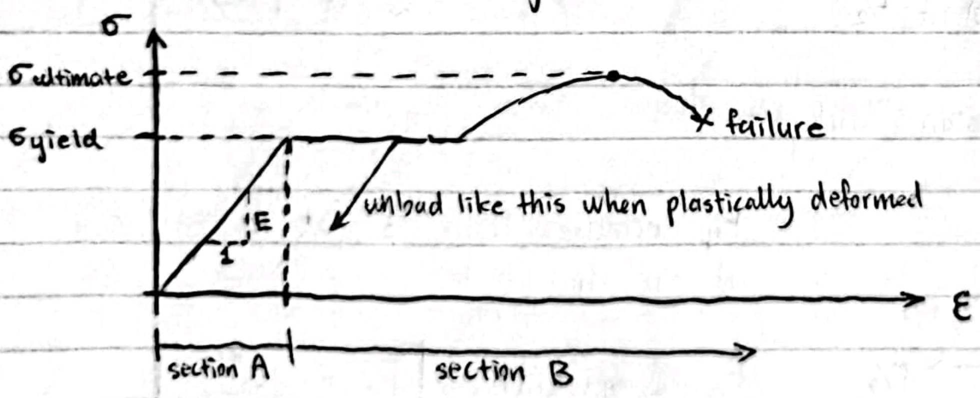
Stress: $\sigma = \frac{F}{A}$ $\left[\frac{N}{mm^2}\right] = MPa$

Strain: $\varepsilon = \frac{\Delta L}{L_0}$ unitless !

Stress + Strain: $\sigma = E\varepsilon$

Necking: tip of wire that breaks get smaller

Stress - Strain Diagram:

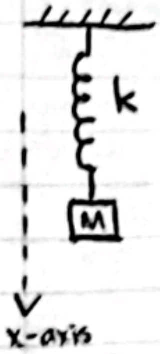


Section A: elastic strain energy, recoverable Hooke's Law Space, *resilience*

Section B: plastic deformation

section A + B: *toughness*, area = normalize strain-energy density
$= \frac{\text{Total Strain Energy}}{\text{Volume}}$

③ Oscillating Systems:~



free vibration with self-weight:

$$x(t) = A\sin(\omega_n t + \phi) + \Delta_0$$

← elastic deformation due to mg

$= \frac{mg}{k}$

natural frequency in rad/sec

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$ ⇒ natural frequency in cycle/sec

$$\left[f_n \approx \frac{15.76}{\sqrt{\Delta_0}} \text{ cycle/sec} \right]$$

④ SAFETY!!

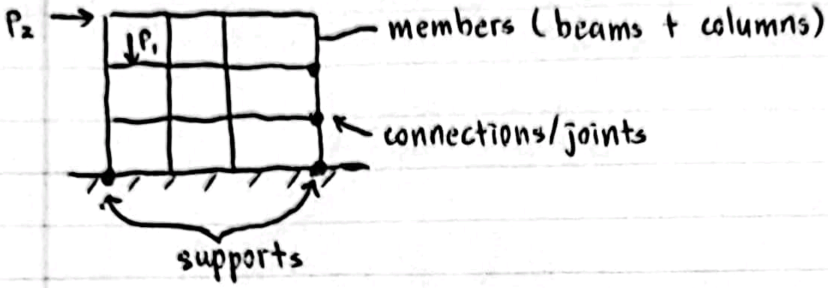
$$\left[\text{Factor of Safety} = \frac{\sigma_{yield}}{\sigma_{allowable}} \right]$$

Check for safety : $\sigma_{applied\ max} \le \sigma_{allowable}$

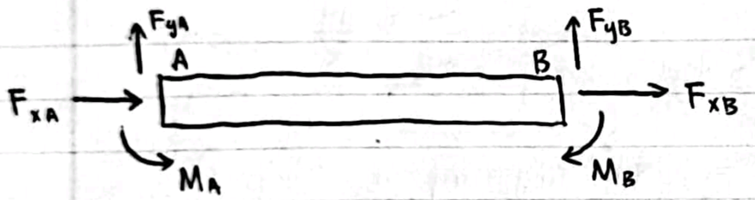
$$\left[\sigma_{applied\ max} \le \frac{\sigma_{yield}}{FoS} \right]$$

⑤ Truss Design ~

A: structures in general



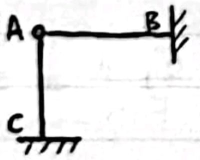
Member Forces:



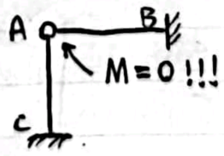
F_{xA} and F_{xB} : Axial Forces
F_{yA} and F_{yB} : Shear Forces
M_A and M_B : Internal Bending Moments

Connections:

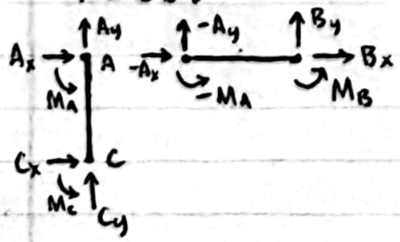
Continuous Connection



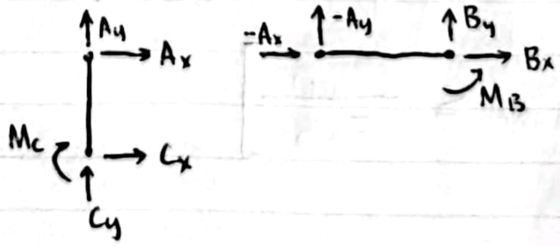
Internal Hinge (INTERNAL BENDING MOMENT = ZERO!!)



FBDs:

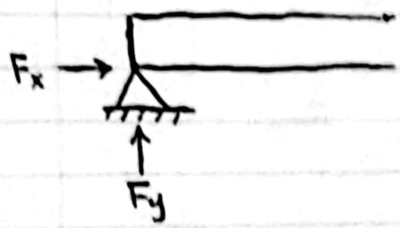


FBDs:

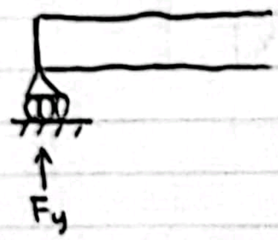


Supports:

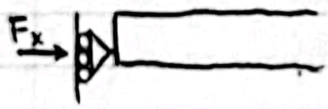
α) pin support $F_x \neq 0$, $F_y \neq 0$, $M = 0$



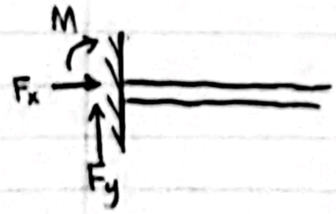
β) roller $F_x = 0$, $F_y \neq 0$, $M = 0$



Vertical Roller $F_x \neq 0$, $F_y = 0$, $M = 0$

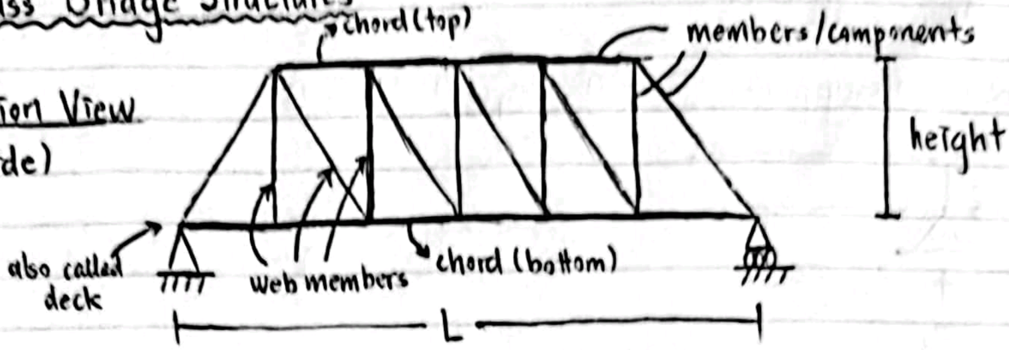


γ) Fixed End $F_x \neq 0$, $F_y \neq 0$, $M \neq 0$

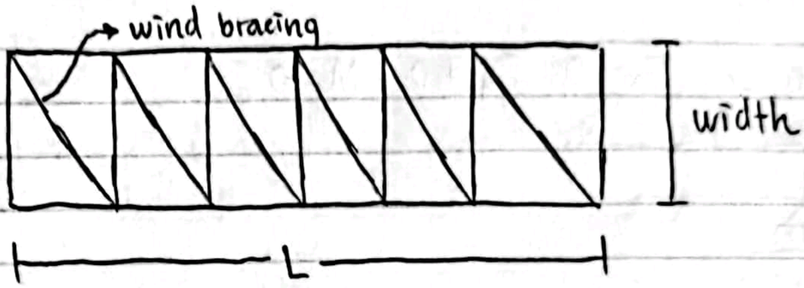


B: Truss Bridge Structures

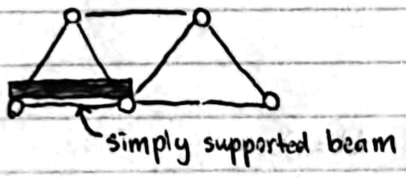
Elevation View
(side)



Plan View
(top)



Deck:

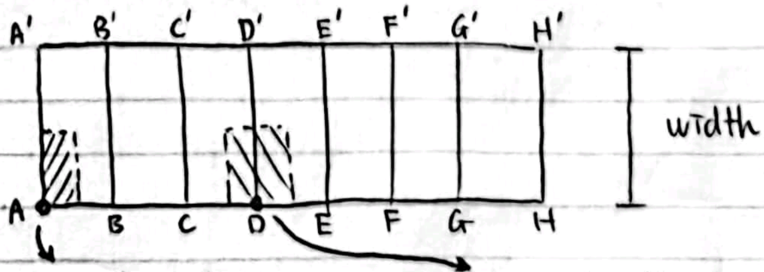


Load Path: you → deck → joints → truss members → support

Assumptions when solving for trusses:
↳ all connections are hinges ($M=0$)
↳ all loads are applied at joints
∴ only axial load, no shear no moment

Joint Loads: (uniform load W)

Plan view



$$F = \left(\frac{\text{width}}{2}\right)\left(\frac{L_{AB}}{2}\right)W \qquad F = \left(\frac{\text{width}}{2}\right)\left(\frac{L_{CD}}{2} + \frac{L_{DE}}{2}\right)W$$

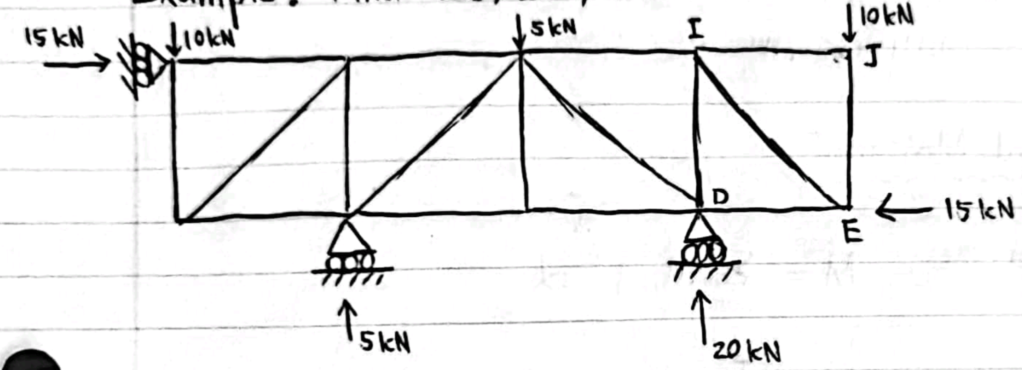
☆ NOTE: If already given joint loads, don't add any more!!!

C: Method of Sections

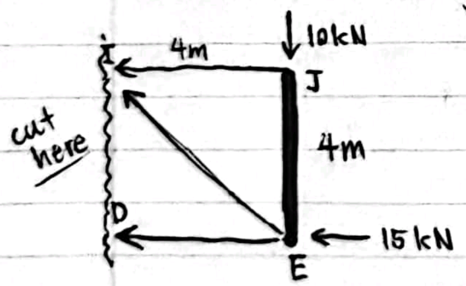
① Solve for reaction forces
② Choose a member to analyze
③ Draw FBD of member with its associated unknown and known forces (cut 3 or fewer)

④ Solve! :)

Example: Find IJ, IE, DE



Choose JE to analyze:



$$\Sigma M_E = 0$$
$$IJ(4) = 0$$
$$IJ = 0 \text{ kN}$$

$$\Sigma M_I = 0$$
$$-10(4) - 15(4) - DE(4) = 0$$
$$DE = -25 \text{ kN}$$

$$\Sigma F_x = 0 \quad @E$$
$$-IE\left(\frac{1}{\sqrt{2}}\right) - (-25) - 15 = 0$$
$$IE = 10\sqrt{2}$$

D: Method of Joints

① Draw FBD at each point
② Solve for member forces

☆ DON'T FORGET JOINT LOAD!!

☆ DRAW EVERY FORCE AS TENSION
↳ (+) is tension
↳ (−) is compression

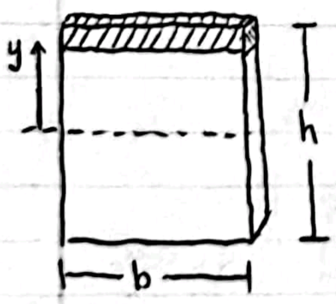
weight a moment...

E: Rotational Motion

For point-masses: $M = \sum_{i=1}^{N} m_i y_i^2 \alpha$

$$M = \underbrace{I_m}_{\text{mass moment of inertia}} \alpha$$

For distributed mass: $I_m = \underset{\text{mass density}}{\rho} \overset{\text{thickness}}{t} \int y^2 \, \underset{b \cdot dy}{dA}$



$I = \int y^2 \, dA \Rightarrow$ second moment of inertia

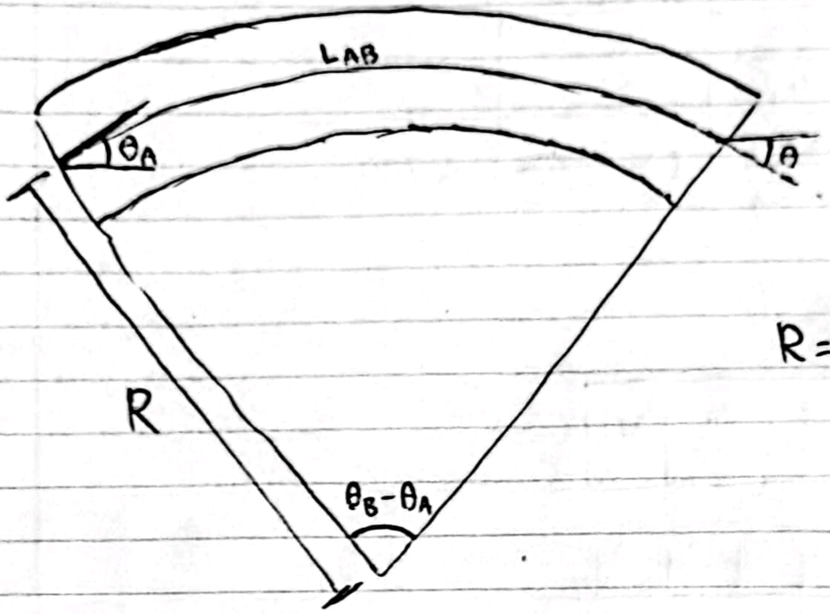
$$I = \int_{-h/2}^{h/2} y^2 \, b \, dy$$

$$I = b \left[\frac{y^3}{3} \right]_{-h/2}^{h/2}$$

$$\boxed{I = \frac{bh^3}{12}}$$

for regular rectangular prism.

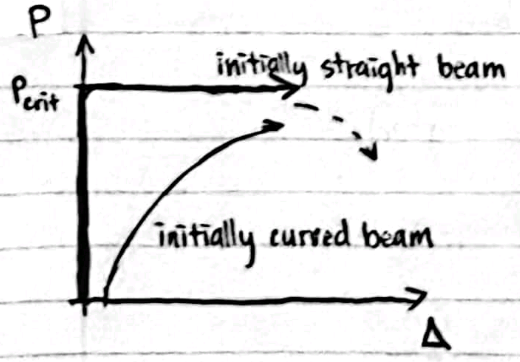
Bentness: $\boxed{\phi = \dfrac{(\theta_B - \theta_A)}{L_{AB}} = \dfrac{d\theta}{dx}}$

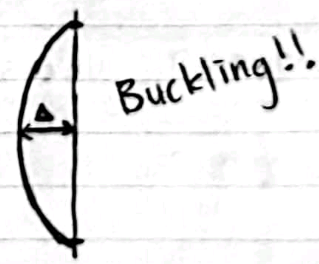


$$R = \frac{L_{AB}}{\theta_B - \theta_A} = \frac{1}{\phi}$$

$\boxed{M = E\phi I} \rightarrow$ Moment at all points on beam

Buckling: $\boxed{P_{crit} = \dfrac{\pi^2 EI}{L^2}} \Rightarrow$ force required to start buckling



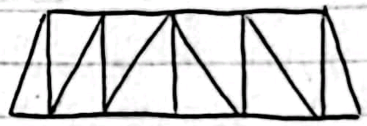


E: Hss Structures

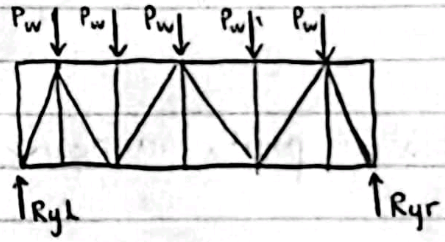
	Area A	Inertia I	L/r
TENSION	$A \geq \frac{2F}{\sigma_{yield}}$	X	< 200
COMPRESSION	$A \geq \frac{2F}{\sigma_{yield}}$	$I \geq \frac{3FL^2}{\pi^2 E}$	< 200

F: Wind Design

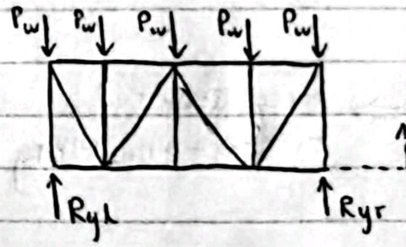
Side View:



Plan View:
(bottom)



Plan View:
(top)

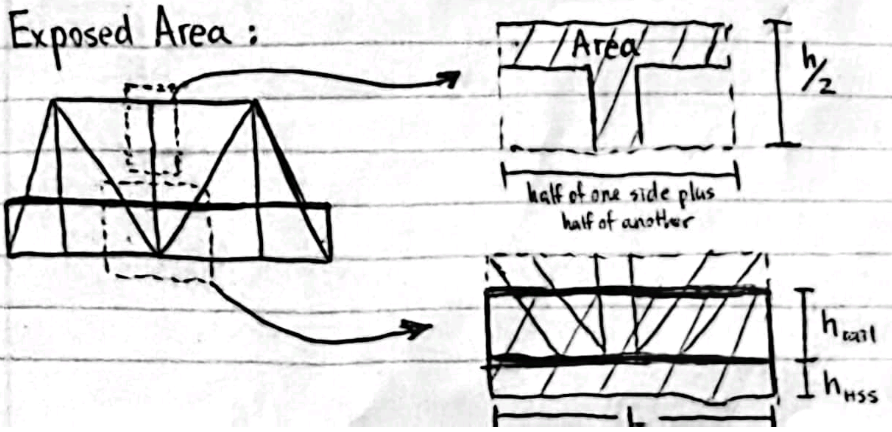


(reaction force is technically here... but stiff frame connection transfers it)

Wind Force = Pressure x Area

Pressure = 2.0kPa (unless otherwise stated)

Exposed Area:



$A \approx L(h_{rail} + h_{HSS})$

G: Virtual Work

① Solve for forces in real system $F_1, F_2, F_3 \ldots F_n$
② Find ΔL for each member in real system $\Delta_1, \Delta_2, \Delta_3 \ldots \Delta_n$
③ Add EXTERNAL virtual force at joint of displacement; find internal virtual forces
④ Solve for displacement.

... info...

~ $\Delta_i = \dfrac{F_i L_i}{E A_i}$

~ set virtual force $\overset{F^*}{\vee}$ as 1 N

~ $\boxed{\Delta = \dfrac{\sum F_i^* \Delta_i}{F^*}}$

Method of Virtual Work Table:

Member ID	(mm) L_o	(mm²) A	(N) F	(mm) $\Delta = \frac{FL}{EA}$	(N) F^*	(N mm) $F^* \Delta$
↓	↓	↓	↓	↓	↓	↓ } sum this

H: Where Has the Soldiers Gone ?

Natural Frequency $\boxed{f_n = \frac{17.76}{\sqrt{\Delta_0}}}$ for uniformly distributed load

Forced Vibration :

Dynamic Amplification Factor $DAF = \dfrac{1}{\sqrt{(1-(\frac{f}{f_n})^2 + (\frac{2\beta f}{f_n})^2}}$

f = forced vibration frequency
f_n = natural frequency
β = damping ratio

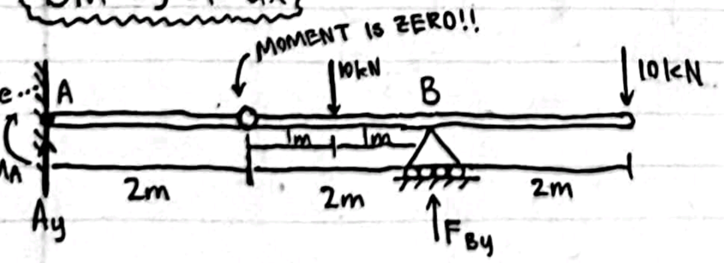
$$\boxed{\text{Max Apparent Force} = \underbrace{F_{stationary}}_{\text{from truss analysis}} + DAF(\underbrace{F_{oscillation}}_{\text{usually given}})}$$

aka 'bad mark diagram'

⑥ SFD and BMD ... How it works is literally in the name... you stoopid if you can't rember.

$$\boxed{BM = \int SF\, dx}$$

Just an example...



Take ΣM @ HINGE (must equal zero... either side)

Reaction Forces

Right Side: $\Sigma M = 0$

$$-10(1) + F_{By}(2) - 10(4) = 0$$

$$F_{By} = 25kN$$

$\Sigma F_y = 0$

$A_y - 10 + 25 - 10 = 0$

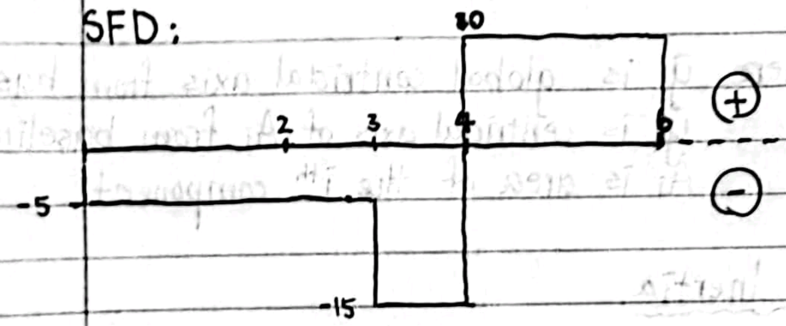
$A_y = -5\ kN$

$\Sigma M = 0$ @ HINGE (Left side now...)

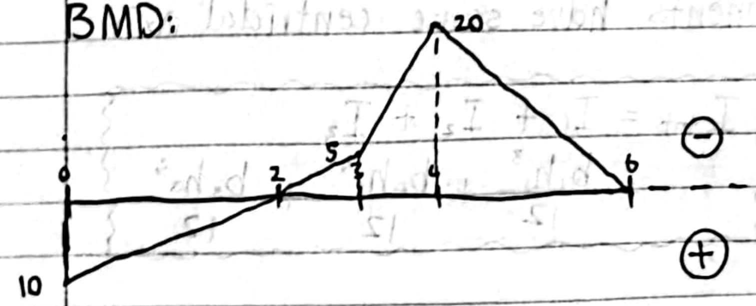
$-M_A - (-10)(2) = 0$

$M_A = 10\ kNm$

SFD:

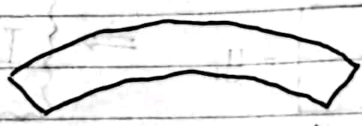


BMD:

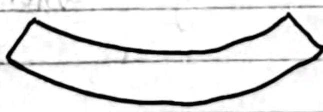


Special Notes:

~ Negative Moment

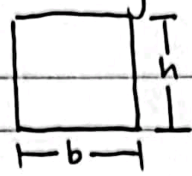


~ Positive Moment

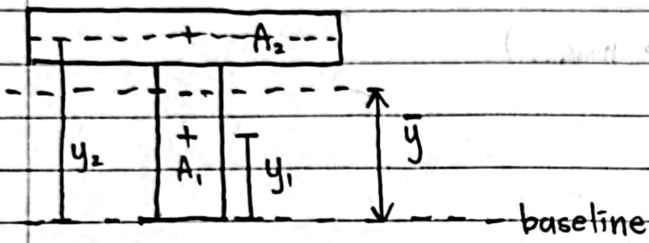


☆ ~ Max tensile stress and max compressive stress are the same.
(if one side is tensile other side is compressive

⑦ "I" → for rectangle... $I = \frac{bh^3}{12}$



A: Global Centridal Axis

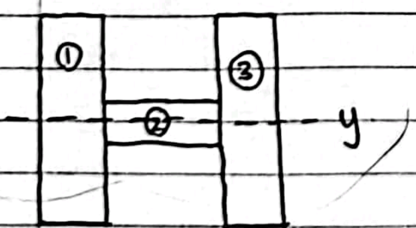


$$\bar{y} = \frac{\Sigma y_i A_i}{\Sigma A_i},$$

where $\bar{y}$ is global centridal axis from baseline
y_i is centridal axis of A_i from baseline
A_i is area of the i^{th} component

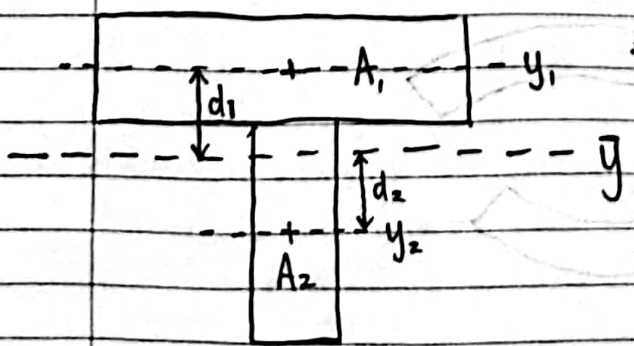
B: Second Moment of Inertia

i) Easy ~ all components have same centridal axis



$$I_{tot} = I_1 + I_2 + I_3 = \frac{b_1 h_1^3}{12} + \frac{b_2 h_2^3}{12} + \frac{b_3 h_3^3}{12}$$

ii) Harder ~ components don't have same centridal axis



$$\Rightarrow I_{tot} = \sum_{i=1}^{N} \left(I_i + A_i d_i^2 \right)$$

where ~I_i is I of component i
~A_i is area of component i
~d_i is distance from GCA to local centridal axis of component i

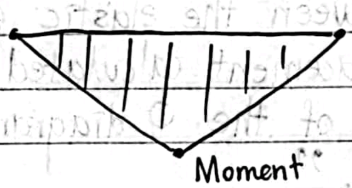
⑧ Beam Displacement ~

A: Curvature again...

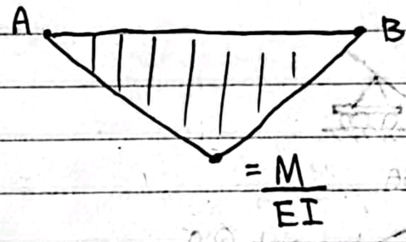
$$\phi = \frac{M}{EI}$$

E and I are constants usually... E depends on material, I depends on the shape of material.

ϕ Diagram:



if this is BMD...

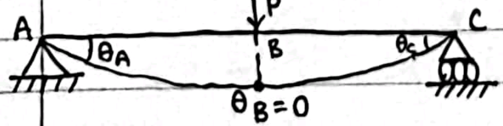


then this is ϕ Diagram...

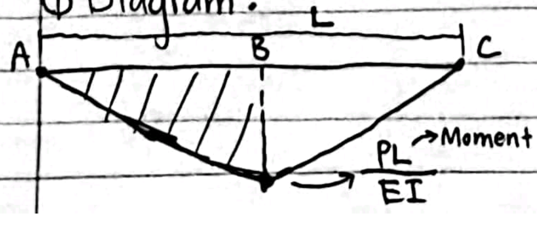
Now we can find Δ...

B: Moment - Area Theorem #1

"The change in slope between points A and B in a beam is equal to the area under the ϕ diagram between A and B."



ϕ Diagram:



We could throw our hands up in despair and go home... but that's not the right answer

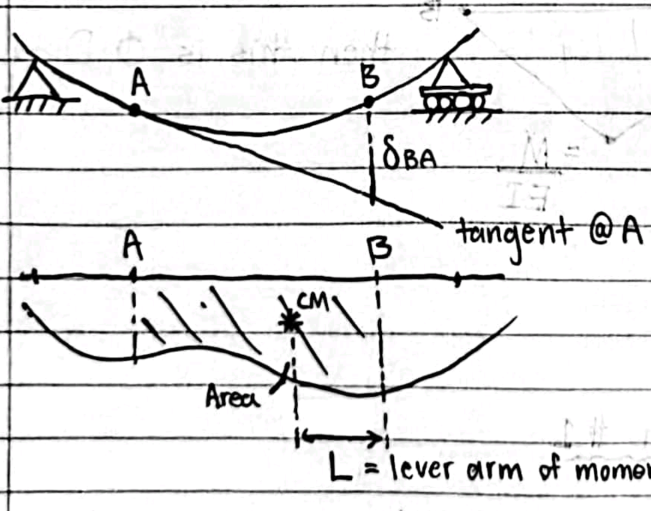
leafing

Since $\theta_B = 0$ and $|\theta_B - \theta_A| = $ Area under ϕ diagram.

$\therefore |\theta_A| = \text{shaded area} = \frac{(\text{length})(\text{height})}{2} = \frac{L}{2}\left(\frac{PL}{EI}\right)\left(\frac{1}{2}\right) = \boxed{\frac{PL^2}{4EI} = \theta_A}$

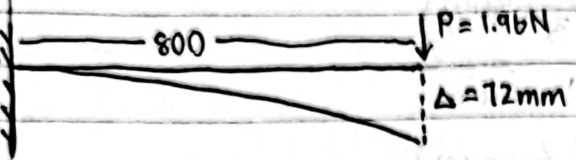
C: Moment - Area Theorem #2:

"The tangential deviation δ_{BA} is the distance (perpendicular to undeformed shape) between the elastic curve and a tangent drawn from A with the displacement calculated @ B and equals the first moment of area of the ϕ diagram between A and B with moment taken at B."



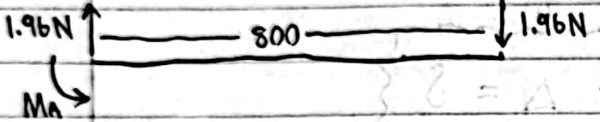
Basically... $\delta_{BA} = (\text{Area})(L)$

3 Examples of calculating stuff with δ:

Ex 1. 

$I = 504mm^4$, $E = ?$

FBD:



$\Sigma M = 0$

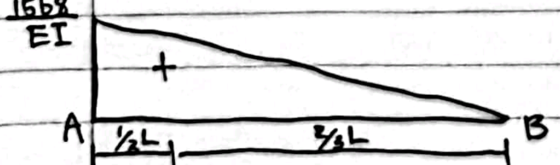
$M_A - 1.96(800) = 0$

$M_A = 1568\ Nmm$

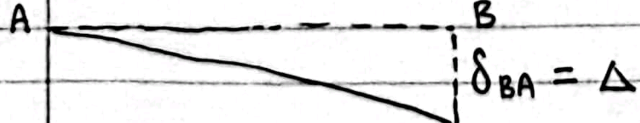
SFD:

BMD:

ΦD:



MAT #2: Use undeformed shape as tangent @ A



$$\delta_{BA} = Ad$$

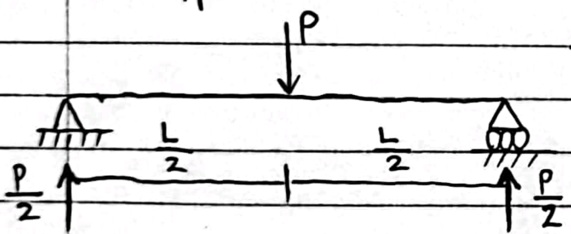
$$\Delta = \underbrace{\left(\frac{1568}{EI}\right)(800)\left(\frac{1}{2}\right)}_{A \text{ of } \phi} \cdot \underbrace{\left(\frac{2}{3}(800)\right)}_{\text{lever arm}}$$

$$72 = \left(\frac{1568}{E(504)}\right)(800)\left(\frac{1}{2}\right) \cdot \left(\frac{2}{3}(800)\right)$$

$$E = 9281 \text{ MPa}$$

Basically... If possible, set $\Delta = \delta$

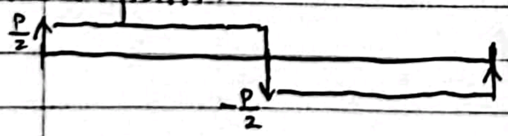
Ex2. Load applied at middle



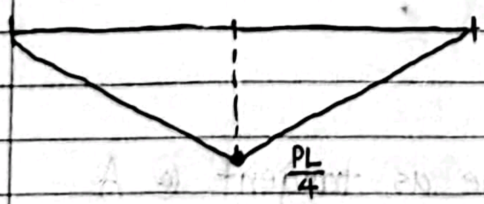
EI = constant

Δmidspan ?

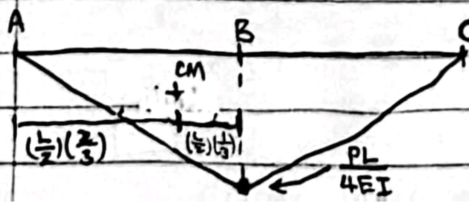
SF Diagram:

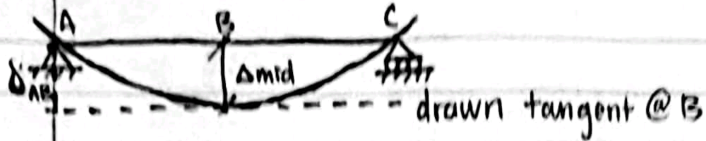


BMD:



ϕD:





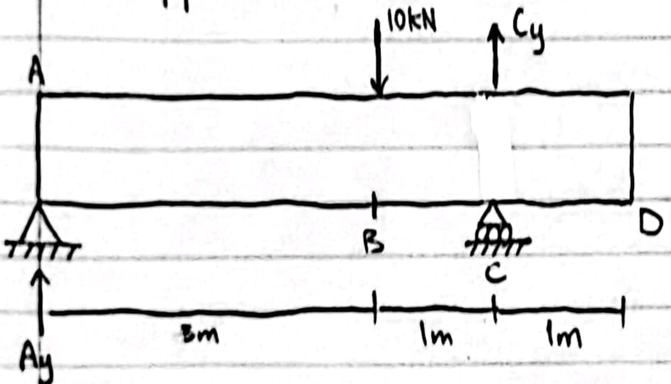
AGAIN... set $\Delta = \delta$

$\delta_{AB} = A \cdot d$

$\Delta_{mid} = \left(\frac{PL}{4EI}\left(\frac{L}{2}\right)\right)\left(\frac{1}{2}\right)\left(\frac{L}{2}\cdot\frac{2}{3}\right)$

$\Delta_{mid} = \frac{PL^3}{48}$

Ex 3. Load applied elsewhere...



$E = 200000\,MPa$
$I = 4.04 \times 10^8\,mm^4$

$\Delta_B = ?$

Reaction Forces: $\Sigma M_A = 0$

$-10(3) + C_y(4) = 0$

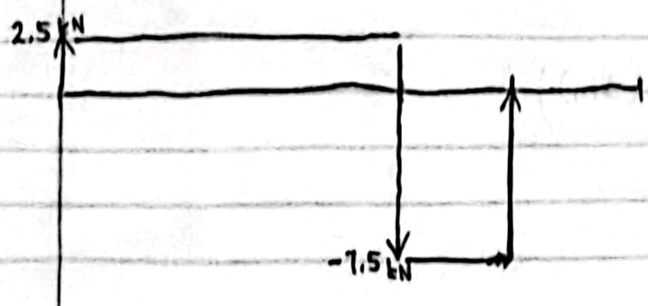
$C_y = 7.5\,kN$

$\Sigma F_y = 0$

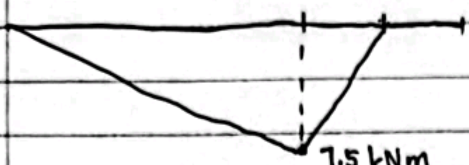
$A_y - 10 + 7.5 = 0$

$A_y = 2.5\,kN$

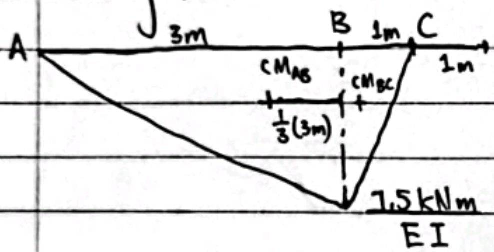
SFD:



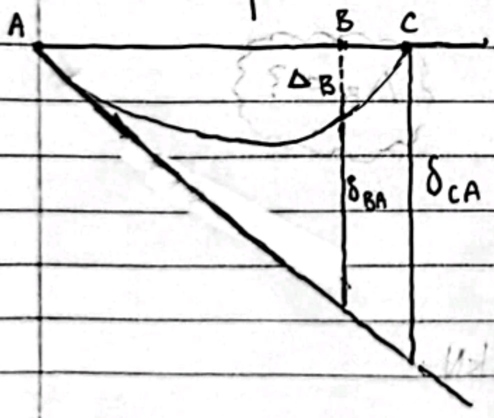
BMD:



Φ Diagram:



Deform Shape:



$$\frac{\delta_{CA}}{L_{CA}} = \frac{\delta_{BA} + \Delta B}{L_{AB}}$$

KEY!! ⇒ $$\boxed{\Delta_B = \frac{L_{BA}}{L_{CA}} \delta_{CA} - \delta_{BA}}$$

$$\delta_{BA} = Ad$$

$$= \left(\frac{7.5\,kNm}{EI}\right)(3m)\left(\frac{1}{2}\right)\left(\frac{1}{3}(3m)\right)$$

$$= \frac{11.25\,kNm^3}{EI}$$

$$\delta_{CA} = A_1 d_1 + A_2 d_2$$

$$= \left(\frac{7.5\,kNm}{EI}\right)(3m)\left(\frac{1}{2}\right)(2m) + \left(\frac{7.5\,kNm}{EI}\right)(1m)\left(\frac{1}{2}\right)\left(\frac{2}{3}(1m)\right)$$

$$= \frac{25\,kNm^3}{EI}$$

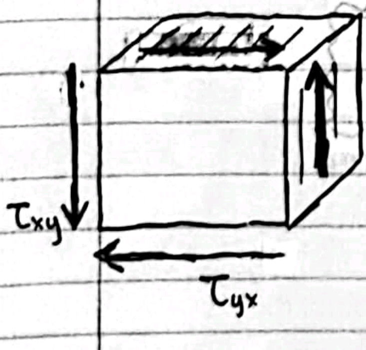
$$\Delta_B = \left(\frac{3m}{4m}\right)\left(\frac{25\,kNm^3}{EI}\right) - \frac{11.25\,kNm^3}{EI}$$

$$\Delta_B = \frac{7.5\,kNm^3}{EI}$$

$$\Delta_B = \frac{7500000000000\,Nm^3}{200000\,MPa\,(4.04\times10^8\,mm^4)}$$

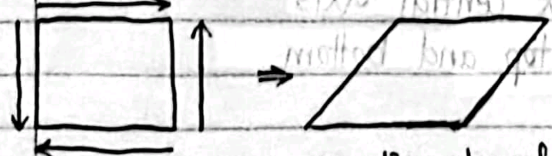
$$\Delta_B = 0.092\,mm$$

9 Shear Stresses ~

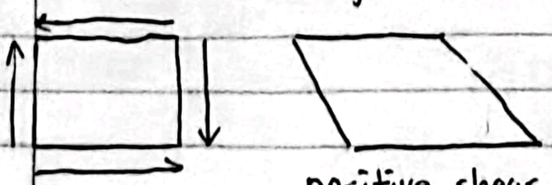


$$\boxed{\text{Shear Stress, } \tau_{xy} = \frac{\text{Shear Force}}{\text{parallel area}} = \frac{V}{A}}$$

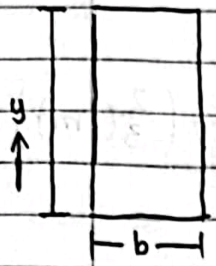
$$\boxed{\tau_{xy} = \tau_{yx}}$$



negative shear force



positive shear force



~ Cross Section Area of Beam

$$\boxed{\tau_{xy} = \frac{V\,Q(y)}{Ib}}$$

V = shear force from SFD
I = second moment of area
b = width of beam where Q(y) is calculated at

Q (mm³) = 1st moment of area where

$$\boxed{Q = \sum_{i=1}^{N} A_i d_i}$$

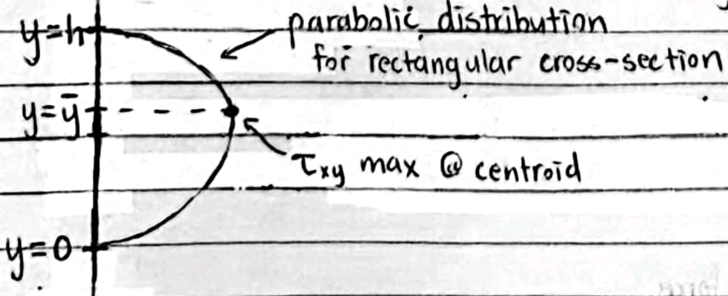
A_i: area of cross section from stress free surface (usually top) to depth τ_{xy} is calculated

d_i: centroid of A_i to neutral axis of beam

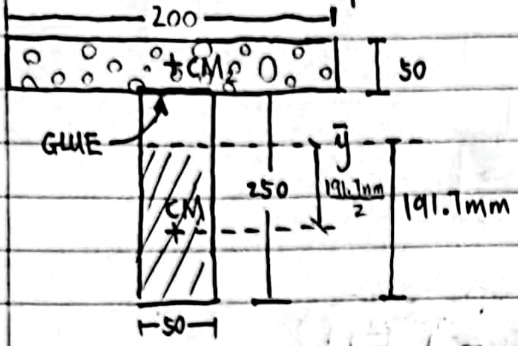
Important Notes:

☆ Q is maximum at centroid $\bar{y}$.
☆ Q is minimum (Q = 0) at top and bottom.

Shear Stress Distribution Diagram:



Shear Stress Example:



$\bar{y} = 191.7mm$

$I = 192.2 \times 10^6 mm^4$

$V = 40kN$

$\tau_{xy\,max} = ?$

$\tau_{xy\,glue} = ?$

$$\tau_{xy\,max} = \text{@ centroid} = \frac{VQ}{Ib}$$

$$= \frac{V}{Ib}\left(A_{shade}\, d_{CM \to \bar{y}}\right)$$

$$= \frac{40kN}{(192.2 \times 10^6 mm^4)(50mm)}\left((50mm)(191.7mm)\left(\frac{191.7m}{2}\right)\right)$$

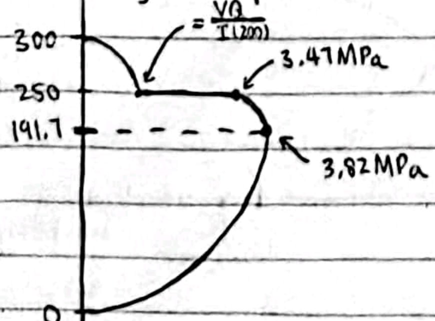
$$= 3.82\,MPa$$

$$\tau_{xy\,glue} = \frac{V}{Ib}\left(A_{bubble}\, d_{CM_2 \to \bar{y}}\right)$$

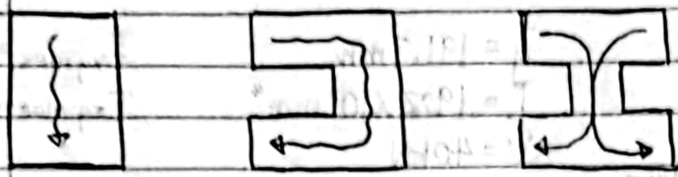
$$= \frac{40000N}{(192.2 \times 10^6 mm^4)(200mm)}\left((200mm)(50mm)(300mm - 25mm - 191.7mm)\right)$$

$$= 3.47\,MPa$$

τ_{xy} Graph:



Shear Flow:



⑩ Timber ~
(aka wood...)

$$\boxed{\text{USE FOS} = 1.5!!}$$

~ Anistropy : different properties in different directions.

~ Weakness is usually shear!

~ Size Effect : larger members have different failure stresses than smaller.

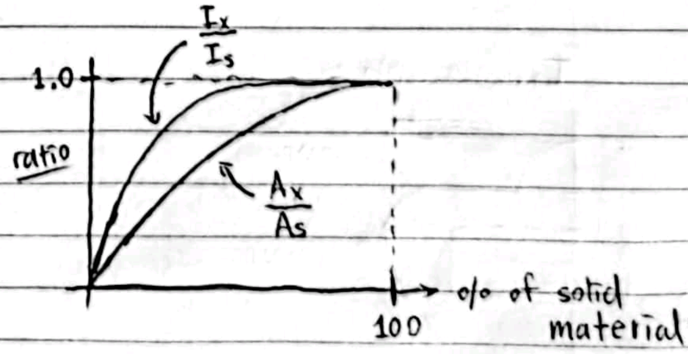
E_{50} ~ average E of many trees (use for Δ calculations)
E_{95} ~ E exceeded by 95% of all experiments (use for buckling calculation)

⑪ Thin - Plate Buckling ~

A: Thin-Walled Member

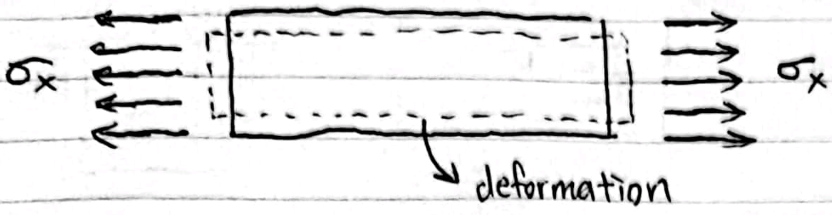
$$\frac{I_{hollow}}{I_{solid}} = 1 - \left(1 - \frac{2t}{b}\right)^4$$

$$\frac{A_{hollow}}{A_{solid}} = 1 - \left(1 - \frac{2t}{b}\right)^2$$



$$\boxed{\text{Hollow thin wall section weigh less but still have high I value.}}$$

B: stress-strain in 2D



ε_x caused by σ_x
ε_y caused by σ_x

Poisson's Ratio $\mu = \frac{\varepsilon_y}{\varepsilon_x}$

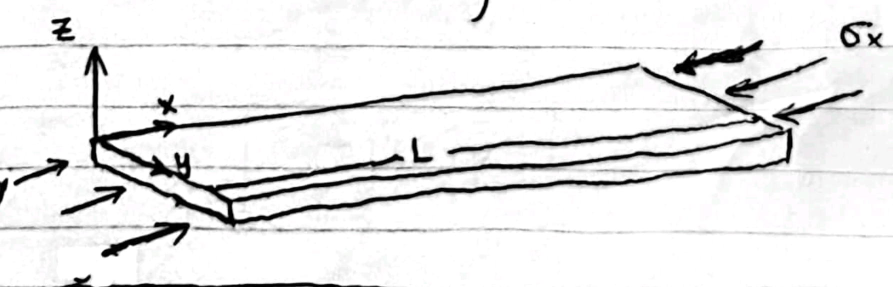
$\varepsilon_{total} = \varepsilon_{stress} + \varepsilon_{poisson}$

☆ $\varepsilon_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E}$

☆ $\varepsilon_y = -\mu \frac{\sigma_x}{E} + \frac{\sigma_y}{E}$

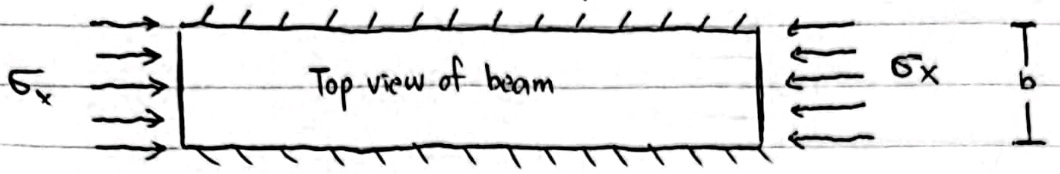
$$\boxed{\begin{aligned} \sigma_x &= \frac{E}{1-\mu^2}(\varepsilon_x + \mu \varepsilon_y) \\ \sigma_y &= \frac{E}{1-\mu^2}(\mu \varepsilon_x + \varepsilon_y) \end{aligned}}$$

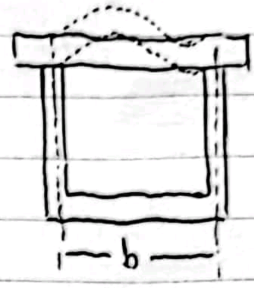
C: Plate Buckling



$\boxed{\text{Buckling Shape: } Z(x,y) = A\sin\left(\frac{m\pi}{b}x\right)\sin\left(\frac{n\pi}{L}y\right), \quad m,n \in \text{integers}}$

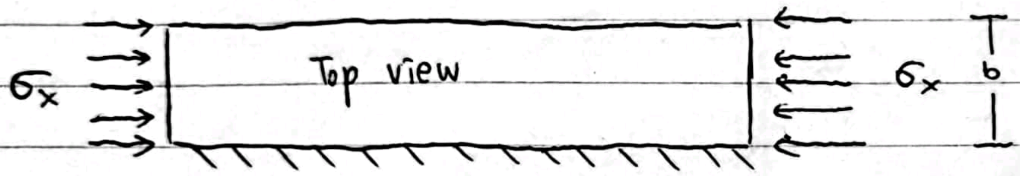
Case 1: uniform compression restrained on all sides

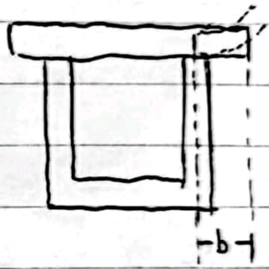




$$\sigma_{crit} = \frac{4\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$$

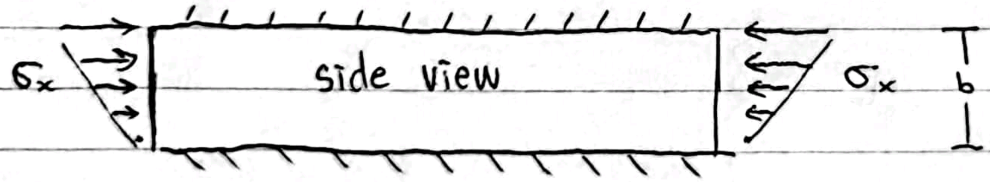
Case 2: uniform σ_x but restrained on ONE side

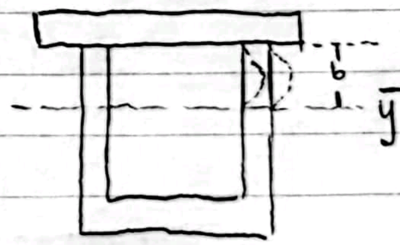




$$\sigma_{crit} = \frac{0.425\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$$

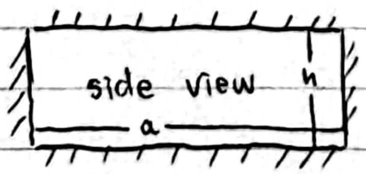
Case 3: non-uniform stress restrained on all sides





$$\sigma_{crit} = \frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$$

Case 4: shear buckling

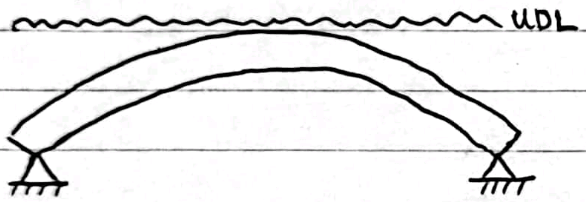


$$\tau_{cr} = \frac{5\pi^2 E}{12(1-\mu^2)}\left[\left(\frac{t}{a}\right)^2 + \left(\frac{t}{h}\right)^2\right]$$

⑫ Stone ~

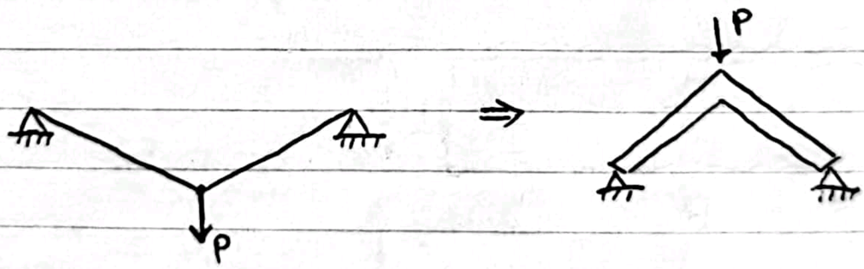
Properties
- low tensile strength, high compressive strength
- linearly elastic
- heavy
- durable

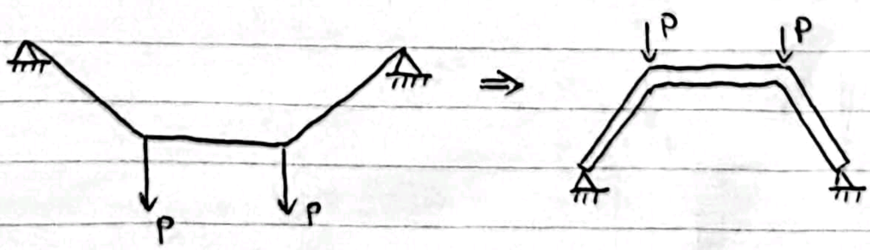
Stone bridges are usually arches, since arches only experience compression iff geometry is optimal... ie. parabolic.

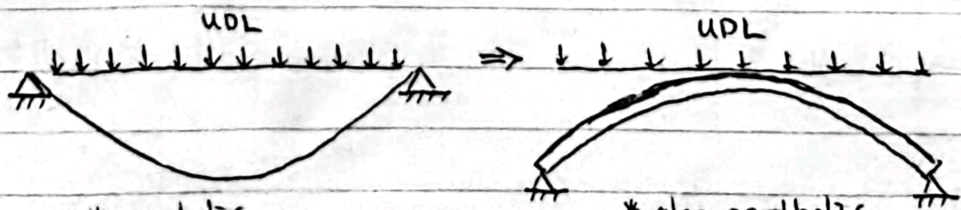


"As hangs a cable, so inverted should the touching parts of an arch stand." - Hooke

ie...





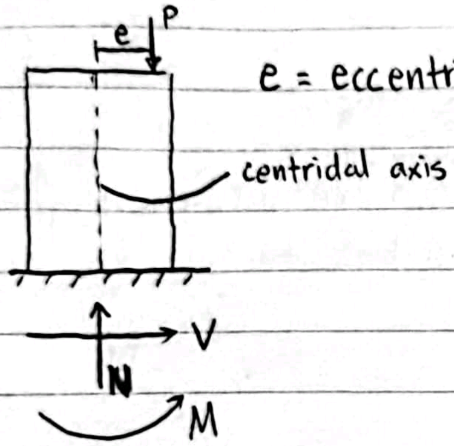


*parabolic

* also parabolic

(12*) Axial Load and Flexural Stress ~

When thrustline is not at cross-section centroid, there will be moment and axial load in member.



e = eccentricity

$$\Rightarrow \quad \Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M = 0$$

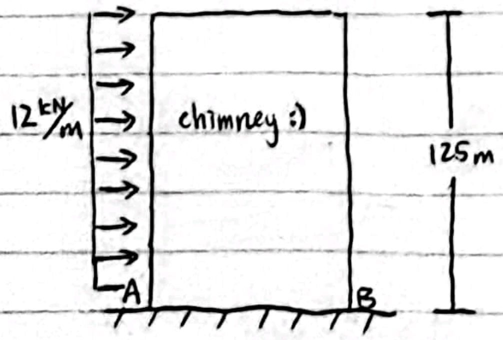
$$V = 0 \qquad N = P \qquad M - (P)(e) = 0$$

$$M = (P)(e)$$

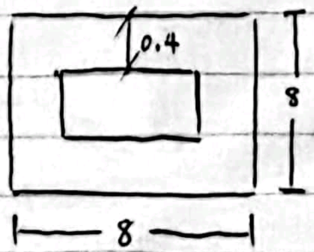
$$\Rightarrow \quad \boxed{\sigma_{applied} = \frac{N}{A} + \frac{My}{I}}$$

$$\sigma_{applied} = \frac{P}{A} + \frac{(Pe)y}{I}$$

Example:



Cross Section:

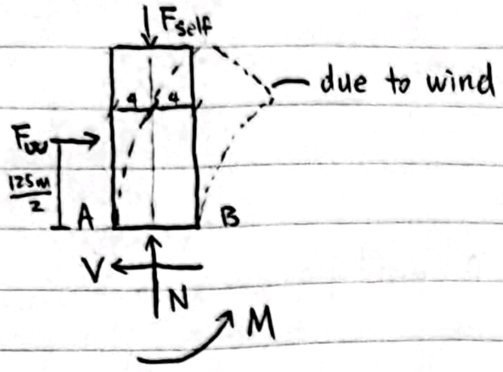


$A = 12.16\ m^2$

$I = 117.4\ m^4$

$\gamma = 26\ kN/m^3$ (density)

FBD:



$\Sigma F_x = 0$

$F_w - V = 0$

$V = F_w$

$V = (12\,kN/m)(125m)$

$V = 1500\,kN$

$\Sigma F_y = 0$

$N - F_{self} = 0$

$N = F_{self}$

$N = (12.16m^2)(125m)(26\,kN/m^3)$

$N = 39520\,kN$

$\Sigma M = 0$

$M - F_w\left(\frac{125m}{2}\right) = 0$

$M = F_w\left(\frac{125m}{2}\right)$

$M = (12\,kN/m)(125m)\left(\frac{125m}{2}\right)$

$M = 93750\,kNm$

☆ $\sigma_A = \frac{N}{A} + \frac{My}{I}$ ☆ $\sigma_B = \frac{N}{A} + \frac{My}{I}$

* Treat compression as positive!

$$\sigma_A = \frac{39520\,kN\left(1000\,N/kN\right)}{12.16m^2\left(\frac{1000000\,mm^2}{m^2}\right)} + \frac{-(93750\,kNm)(4m)}{117.4\,m^4}\left(\frac{1000N}{kN}\right)\left(\frac{m^2}{1000000mm^2}\right)$$

$\sigma_A = 0.06\,MPa$ (still in compression!)

$$\sigma_B = \frac{39520\,kN\left(1000\,N/kN\right)}{12.16m^2\left(\frac{1000000\,mm^2}{m^2}\right)} + \frac{(93750kNm)(4m)}{117.4\,m^4}\left(\frac{1000N}{kN}\right)\left(\frac{m^2}{1000000mm^2}\right)$$

$\sigma_B = 6.44\,MPa$ (ofc this is in compression...)

'you can't pull on concrete...'

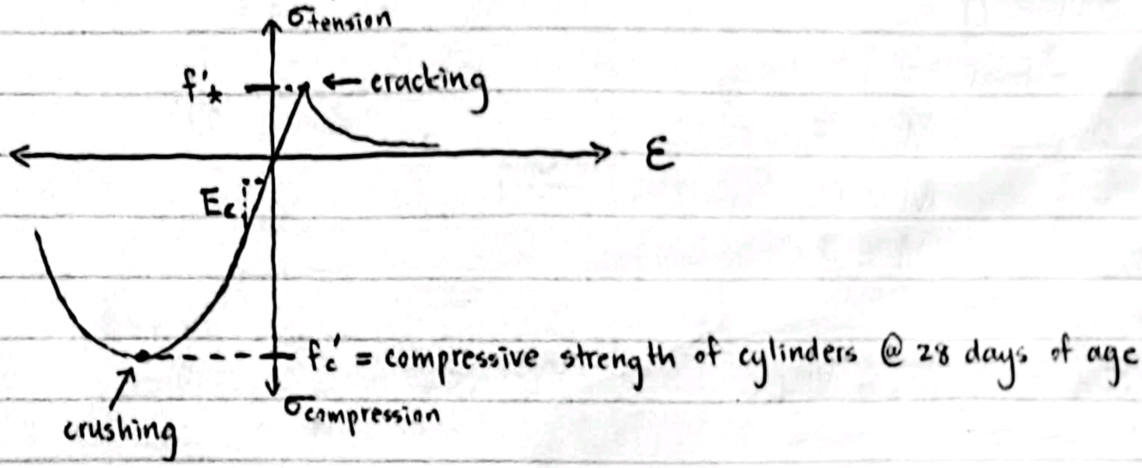
(13) Concrete ~ cement + aggregate (gravel) + sand + water

A: Basic Info

- cheap :)
- durability is good if constructed properly
- locally produced
- CO_2 intensive :<

Kinds
- plain concrete
- reinforced concrete
- prestressed concrete

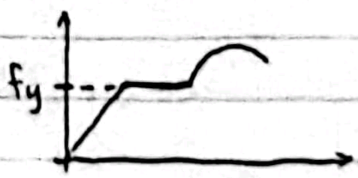
B: Material Properties



☆ $f'_t \approx 0.33\sqrt{f'_c}$

★ $E_c \approx 4500\sqrt{f'_c}$ ★ f'_c is compressive strength of concrete

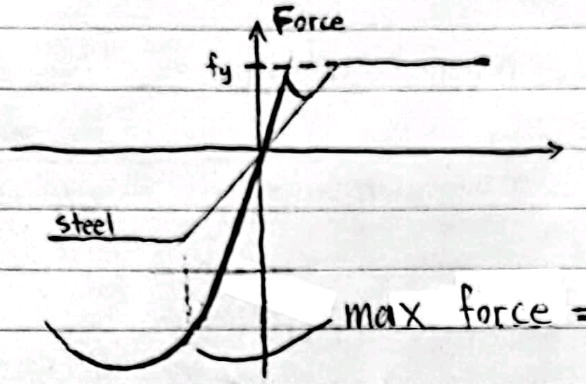
side note about steel bars...



☆ $E = 200{,}000$ MPa

★ f_y = tensile strength of steel

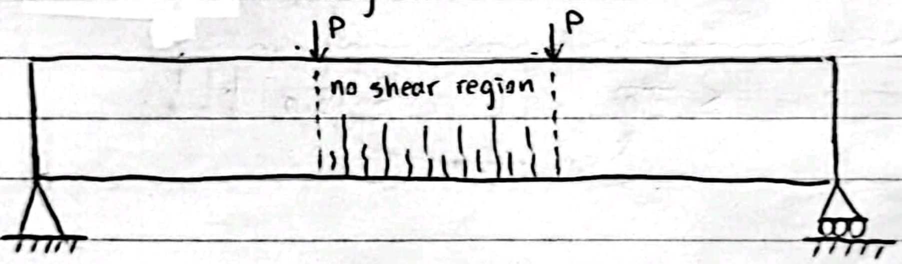
C: Axial Stuff...

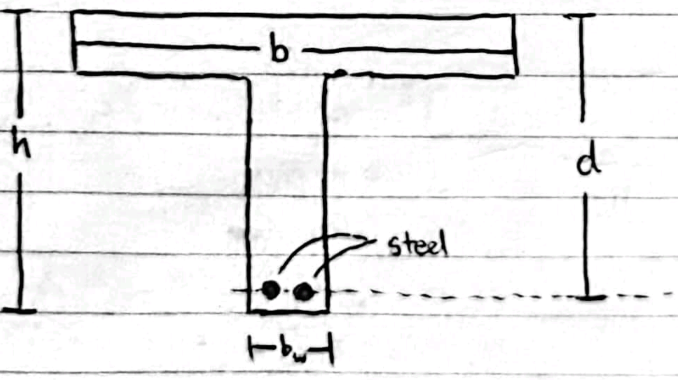


max force = $A_{cross} f'_c + A_{steel} f_y$

In tension, after cracking steel is all that's left.

All about bending...





d = effective depth
b_w = web width

} usually given

VERY IMPORTANT NUMBERS!!!

★ $k = \sqrt{(np)^2 + 2np} - np$ ... n = modular ratio = $\frac{E_{steel}}{E_{concrete}}$

... p = reinforcement % = $\frac{A_{steel}}{bd}$

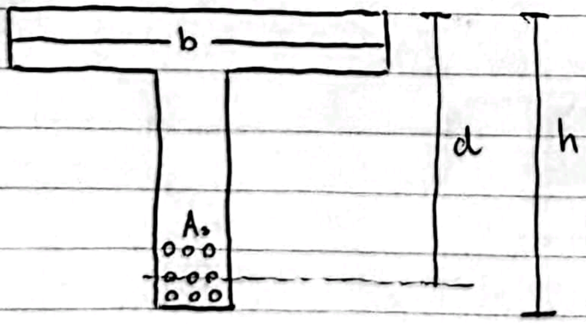
★ $f_{steel} = \frac{M}{A_{steel}(1-\frac{k}{3})d}$ ... can also write $(1-\frac{k}{3})$ as "j"

★ $\sigma_{concrete} = f_c = \frac{k}{1-k}\left(\frac{M}{(1-\frac{k}{3})d\,A_{steel}\,n}\right)$

Make sure $f_{steel} \leq \frac{f_y}{2.0}$ and $f_c \leq \frac{f'_c}{2.0}$!!!

FOS

Example:



$M = 1200\ kNm$

$d = 900\ mm$

$h = 1000\ mm$

$b = 500\ mm$

$A_s = 6300\ mm^2$

$f'_c = 40\ MPa$

$f_y = 400\ MPa$

Safe cross-section?

① Find E_c

$$E_c = 4500\sqrt{f'_c}$$
$$= 4500\sqrt{40}$$
$$= 28500\ MPa$$

② Find n, ρ, k

$$n = \frac{E_s}{E_c}$$

$$n = \frac{200000\ MPa}{28500\ MPa}$$

$$n = 7.03$$

$$\rho = \frac{A_s}{bd}$$

$$\rho = \frac{6300\ mm^2}{(500mm)(900mm)}$$

$$\rho = 0.014$$

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho$$

$$= \sqrt{(7.03(0.014))^2 + 2(7.03)(0.014)} - (7.03)(0.014)$$

$$= 0.356$$

③ Find f_s and f_c

$$f_s = \frac{M}{A_s(1-\frac{k}{3})\,d}$$

$$f_s = \frac{1200\,kNm\,(10^6)}{(6300mm^2)(1-\frac{0.356}{3})(900mm)}$$

$$f_s = 240\ MPa$$

$$f_c = \frac{k}{1-k}\left(\frac{M}{(1-\frac{k}{3})\,d\,A_{steel}\,n}\right)$$

$$f_c = \frac{0.356}{1-0.356}\left(\frac{1200\,kNm\,(10^6)}{(1-\frac{0.356}{3})(900mm)(6300mm^2)(7.03)}\right)$$

$$f_c = 18.89\ MPa$$

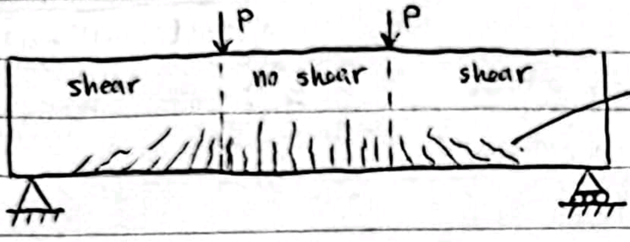
④ Determine Safety

f_s is more then half of f_y!! Not too safe!!

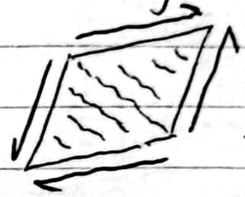
f_c is less then half of f'_c. Good! ✓

D: Shear Stuff...

How it looks...

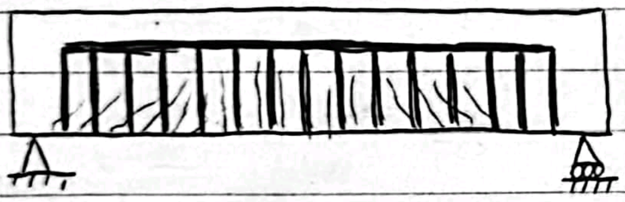


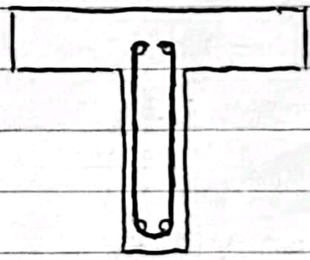
Here is why it is slanted...



shear forces causing this type of deformation

Shear reinforcement (stirrups)





Resisting Shear...

★ V_{max} = shear required to crush concrete $= 0.25 f'_c \, b_w \, d_v$

$\hookrightarrow 0.9d$

i) No shear reinforcement

★ $V_{concrete} = \dfrac{230\sqrt{f'_c}}{1000 + d_v}(b_w)(d_v) \qquad V_s = \emptyset$

ii) With shear reinforcement

★ must satisfy $\dfrac{A_v f_y}{b_w s} \geq 0.06\sqrt{f'_c}$, A_v = area of stirrups, s = spacing of stirrups

★ $V_{concrete} = 0.18\sqrt{f'_c}\, b_w d_v$

★ $V_{stirrup} = \dfrac{A_v f_y d_v}{s} \cot\theta$, $\theta = 35^\circ$ in CIV102

$$\boxed{V_{allowable} = \frac{\min\{V_{max},\ V_c + V_s\}}{2.0}} \Rightarrow \text{SAFE STRENGTH}$$

E: Design for shear (KEY IS FIND SPACING)

* Assume you already know cross-section & A_{steel}

(i) SFD & BMD

↳ max shear from SFD is V

(ii) Check $V \leq V_{max}/2.0$, $V_{max} = 0.25 f'_c b_w d_v$

↳ if not, cross section is too small, change b_w & d_v.

(iii) Check if it is strong enough without shear reinforcement

$$V \leq \frac{1}{2.0} V_c$$

$$\Rightarrow V \leq \frac{1}{2.0}\left(\frac{230\sqrt{f'_c}}{1000 + d_v} b_w d_v\right)$$

iv) Shear reinforcement needed. (if needed)

↳ solve for spacing $s = \dfrac{A_v f_y}{0.06\sqrt{f'_c}\, d_v}$

↳ recalculate $V_c = 0.18\sqrt{f'_c}\, b_w d_v$

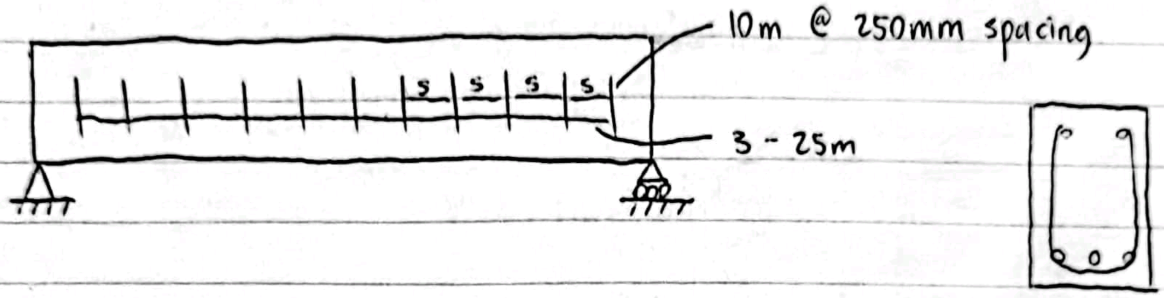
↳ calculate $V_s = \dfrac{A_v f_y d_v}{s} \cot 35°$

↳ check $V \leq \dfrac{V_c + V_s}{2.0}$

(v) Select closer stirrups (if needed)

$$s = \frac{1}{2.0}\left(\frac{A_v f_y d_v \cot 35^\circ}{V - \frac{1}{2.0}(0.18\sqrt{f'_c}\, b_w d_v)}\right) \Rightarrow$$ Round off to smaller 5mm or 10mm

(vi) Sketch (example)



☆ F: Assessment for Shear (KEY IS FIND MINIMUM STRENGTH)

* Ignore FOS

(i) SFD & BMD

(ii) Check if you have minimum shear reinforcement

$$\frac{A_v f_y}{b_w s} \geq 0.06\sqrt{f'_c} \ ?$$

↳ if yes: shear strength $= 0.18\sqrt{f'_c}\, b_w d_v + \frac{A_v f_y d_v}{s} \cot 35^\circ$

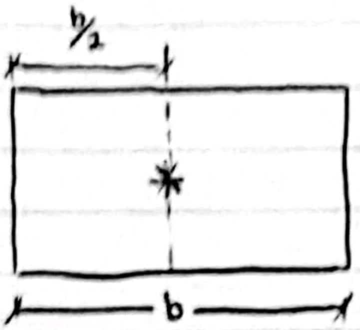
↳ if no: shear strength $= \frac{230\sqrt{f'_c}}{1000 + d_v} b_w d_v + \phi$

(iv) check $V_{max} = 0.25 f'_c b_w d_v$

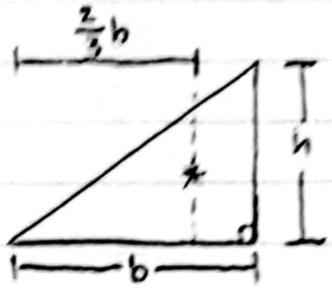
(v) strength $= \min\{V_c + V_s, V_{max}\}$

*VERY IMPORTANT!!

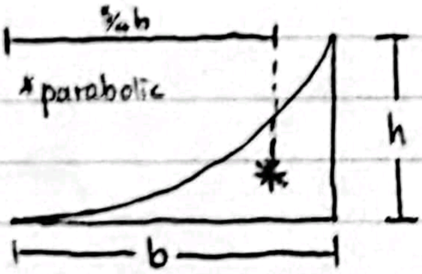
Area and Centroids of Common Shapes:



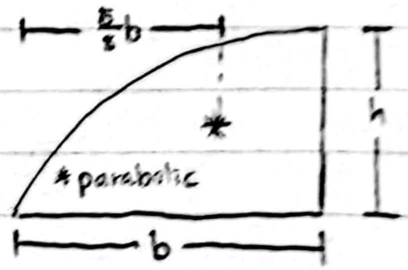
$A = bh$



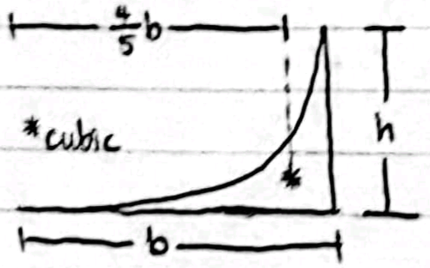
$A = \frac{1}{2}bh$



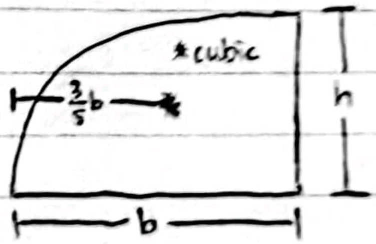
$A = \frac{1}{3}bh$



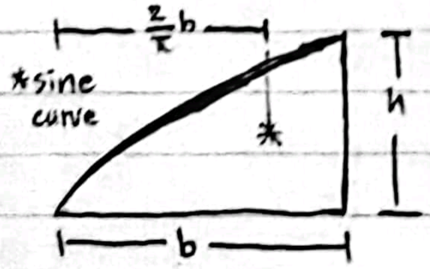
$A = \frac{2}{3}bh$



$A = \frac{1}{4}bh$



$A = \frac{3}{4}bh$



$A = \frac{2}{\pi}bh$