HOW TO DRAW A DESSIN?

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ABSTRACT. In this work we present an explicit implementation of an algorithm to draw dessins d'enfants from their combinatorial description as permutation triples. In Section 1 we review some background about dessins d'enfants, Belyi maps, and some of the reasons the reader might be interested in these objects. In Section 2 we review the existing methods for drawing pictures of dessins d'enfants and the available catalogs in the literature. In Section 3 we present our technique for explicitly drawing dessins along with explanations of the choices made during this process. We then include a database of all dessins up to degree 11 and conclude in Section 9 with some observations about the data.

Contents

1.	Background	1
1.1.	Equivalent categories	1
1.2.	. Dessins	1
1.3.	. Clean dessins	1
1.4.	Ribbon graphs	1
2.	Existing catalogs	2
3.	Drawing techniques	2
4.	Degree ≤ 4	2
5.	Degree 5	2
6.	Degree 6	2
7.	Degree 7	2
8.	Degree 8	2
9.	Conclusions	2
References		2

1. Background

- 1.1. Equivalent categories.
- 1.2. Dessins.
- 1.3. Clean dessins.
- 1.4. Ribbon graphs.

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- 2. Existing catalogs
- 3. Drawing techniques
 - 4. Degree < 4
 - 5. Degree 5
 - 6. Degree 6
 - 7. Degree 7
 - 8. Degree 8
 - 9. Conclusions

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