

HOW TO DRAW A DESSIN?

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ABSTRACT. In this work we present an explicit implementation of an algorithm to draw dessins d'enfants from their combinatorial description as permutation triples. In Section 1 we review some background about dessins d'enfants, Belyi maps, and some of the reasons the reader might be interested in these objects. In Section 2 we review the existing methods for drawing pictures of dessins d'enfants and the available catalogs in the literature. In Section 3 we present our technique for explicitly drawing dessins along with explanations of the choices made during this process. We then include a database of all dessins up to degree 11 and conclude in Section 9 with some observations about the data.

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1. BACKGROUND

- 1.1. **Equivalent categories.**
- 1.2. **Dessins.**
- 1.3. **Clean dessins.**
- 1.4. **Ribbon graphs.**

2. EXISTING CATALOGS

3. DRAWING TECHNIQUES

4. DEGREE ≤ 4

5. DEGREE 5

6. DEGREE 6

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9. CONCLUSIONS

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