HOW TO DRAW A DESSIN?

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Abstract. In this work we present an explicit implementation of an algorithm to draw dessins d'enfants from their combinatorial description as permutation triples. In Seciton 1 we review some background about dessins d'enfants, Belyi maps, and some of the reasons the reader might be interested in these objects. In Section 2 we review the existing methods for drawing pictures of dessins d'enfants and the available catalogs in the literature. In Section 3 we present our technique for explicitly drawing dessins along with explanations of the choices made during this process. We then include a database of all dessins up to degree 11 and conclude in Section 9 with some observations about the data.

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