Bio-inspired Modeling of a Hovering Hummingbird with Control Policy Comparisons

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Presentation Layout

1. Modeling a Hovering Hummingbird
    a. Lift force, z-axis torque and x,y-axis torque
2. PID Control
    a. Formally evaluated for stability
3. Model Predictive Control
    a. Experimentally evaluated for stability
4. PID vs. MPC Comparisons
    a. Demonstrate results in simulation

1). Modeling a Hovering Hummingbird

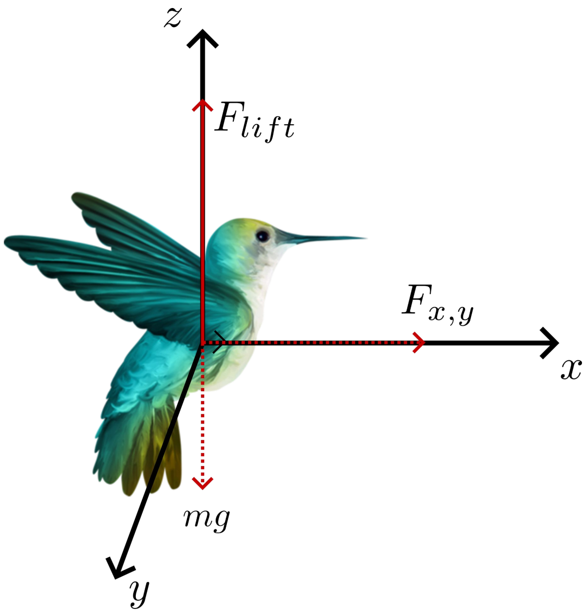
A Hovering Hummingbird Model

When hovering, a hummingbird's dynamics can be characterized by five states:

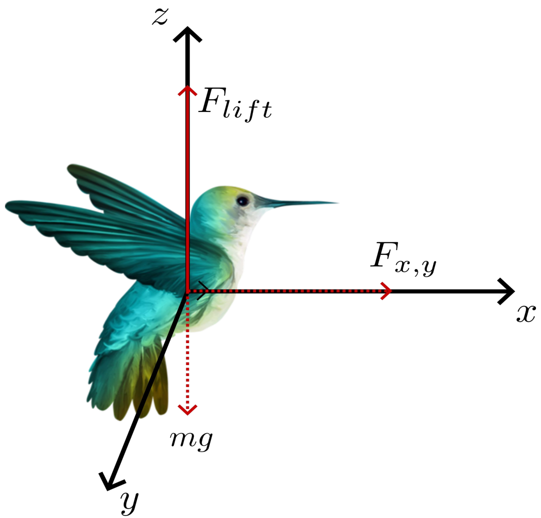
1. x, y, z positions
2. z-axis deflection, x-axis deflection

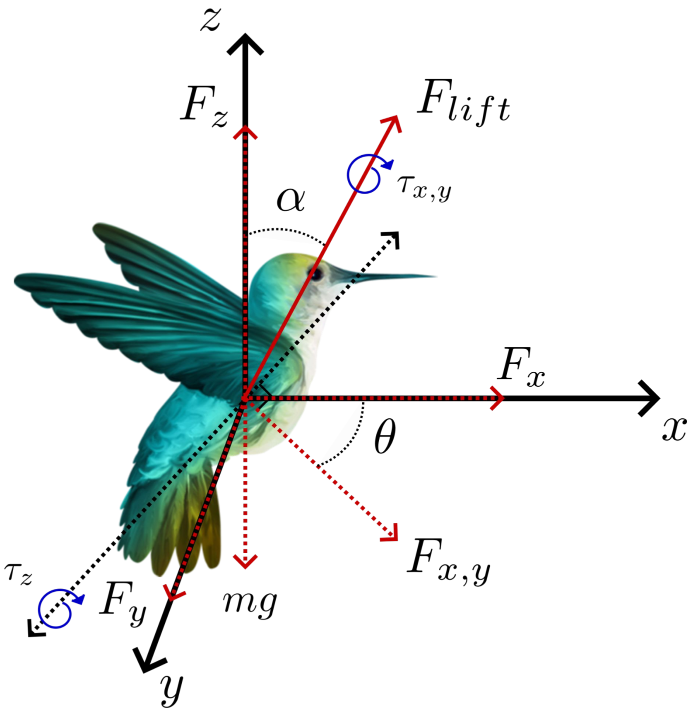
$$x = [x, y, z, \alpha, \theta]^\intercal$$

The inputs will be defined in terms of forces/torques giving us a second-order system.



State Space Description

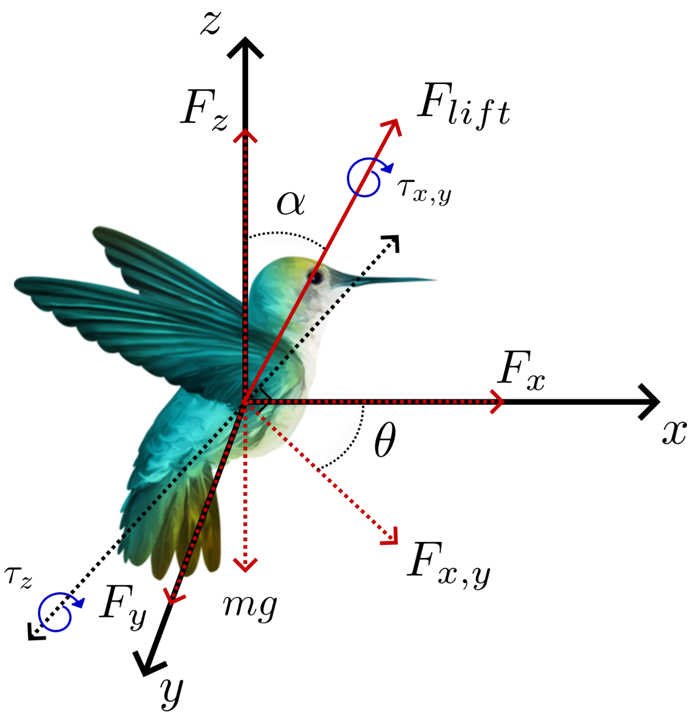




Second-order Dynamics

$$u = \begin{bmatrix} F_{lift} \\ \tau_{x,y} \\ \tau_z \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ \alpha \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\alpha} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{bmatrix} \Rightarrow f(x,u) = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \\ \dot{x}_{10} \end{bmatrix} = \begin{bmatrix} x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ \frac{1}{m}(u_1 \sin(x_4)\cos(x_5) - wx_6) \\ \frac{1}{m}(u_1 \sin(x_4)\cos(x_5) - wx_7) \\ \frac{1}{m}(u_1 \cos(x_4) - wx_8 - mg) \\ u_2 \\ u_3 \end{bmatrix}$$



2). PID Control

PID Controller Design

In order to implement the integrator control, the state space is augmented to incorporate the change rate of error.

$$\dot{y} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \qquad \dot{z} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

$$u = \begin{bmatrix} F_{lift} \\ \tau_{x,y} \\ \tau_z \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{cases} -\alpha_1 z_3 - \beta_1 z_8 - \gamma_1 z_{13} \\ -\alpha_2 z_4 - \beta_2 z_9 - \gamma_2 z_{14} \\ -\alpha_3 z_5 - \beta_3 z_{10} - \gamma_3 z_{15} \end{cases}$$

Evaluating Stability of PD Controller (1)

- In order to simplify the evaluation process, we will only look at the PD controller form…
- We define the objective function V(x) to be…

$$V(x) = \frac{1}{2} x^\intercal P x \quad \text{where } P = \begin{bmatrix} a & 0 & \cdots & 0 \\ 0 & b & \cdots & 0 \\ \vdots, & \vdots, & \ddots, & 0 \\ 0 & 0 & 0 & j \end{bmatrix}$$

- Giving the expanded form to be a positive definite function where each state is squared and has the appropriate coefficient…

Evaluating Stability of PD Controller (2)

- Because V(x) is by definite positive definite, decrescent and continuously differentiable, we can differentiate and see if its derivative is negative definite…

$$\dot{V}(x) = \frac{1}{2}\dot{x}^{\intercal}Px + \frac{1}{2}x^{\intercal}Px = \dot{x}^{\intercal}Px.$$

- Which we will expand and substitute in the earlier defined dynamics to get…

$$\begin{aligned}
\dot{V}(x) = \quad & ax_1\dot{x}_1 + bx_2\dot{x}_2 + \cdots + jx_{10}\dot{x}_{10} \\
= \quad & ax_1x_6 + bx_2x_7 + cx_3x_8 + dx_4x_9 + ex_5x_{10} \\
& + fx_6(u_1\sin(x_4)\cos(x_5) - wx_6) \\
& + gx_7(u_1\sin(x_4)\cos(x_5) - wx_7) \\
& + hx_8(u_1\cos(x_4) - wx_8) \\
& + ix_9u_2 + jx_{10}u_3
\end{aligned}$$

Evaluating Stability of PD Controller (3)

- Moving forward, negative definite terms (safe) will be moved to the top row of the equation.
- That said, we can substitute in the controller functions as previously defined and move the velocity gains to the collection of safe terms.

$$
\begin{aligned}
\dot{V}(x) = \quad & -w(fx_6^2 + gx_7^2 + hx_8^2) - i\beta_2 x_9^2 - j\beta_2 x_{10}^2 \\
& + ax_1x_6 + bx_2x_7 + cx_3x_8 + dx_4x_9 + ex_5x_{10} \\
& - fx_6 \sin(x_4)\cos(x_5)(\alpha_1 x_3 + \beta_1 x_8) \\
& - gx_7 \sin(x_4)\cos(x_5)(\alpha_1 x_3 + \beta_1 x_8) \\
& - hx_8 \cos(x_4)(\alpha_1 x_3 + \beta_1 x_8) \\
& - i\alpha_2 x_4 x_9 - j\alpha_3 x_5 x_{10}
\end{aligned}
$$

Evaluating Stability of PD Controller (4)

- Now, combining like terms we can use our knowledge of the model to define bounds on the z-axis velocity gain and move it to the collection of safe terms…

$$\begin{aligned}
\dot{V}(x) = \quad & -w(fx_6^2 + gx_7^2 + hx_8^2) - i\beta_2 x_9^2 - j\beta_2 x_{10}^2 \\
& - h\beta_1 x_8^2 \cos(x_4) \leftarrow \text{always positive} \\
& + x_4 x_9 (d - i\alpha_2) + x_5 x_{10} (e - j\alpha_3) \\
& + x_6 (a x_1 - f \sin(x_4) \cos(x_5)(\alpha_1 x_3 + \beta_1 x_8)) \\
& + x_7 (b x_2 - g \sin(x_4) \cos(x_5)(\alpha_1 x_3 + \beta_1 x_8)) \\
& + x_3 x_8 (c - h\alpha_1 \cos(x_4))
\end{aligned}$$

- That is, if we bound $-\frac{\pi}{2} < x_4 < \frac{\pi}{2}$ then the z-axis velocity gain is always positive.

Evaluating Stability of PD Controller (5)

o remove the remaining nonlinear terms, we define the system within a small ball around in. This leads to $\cos(x_i) \approx 1$ and $\sin(x_i) \approx 0$ for i = 4,5, and after some rearranging the final equation...

$$
\begin{aligned}
\dot{V}(x) = & \\
& - w(f x_6^2 + g x_7^2 + h x_8^2) \\
& - h\beta_1 x_8^2 - i\beta_2 x_9^2 - j\beta_2 x_{10}^2 \\
& \left\{ \begin{array}{l} + x_3 x_8 (c - h\alpha_1) \\ + x_4 x_9 (d - i\alpha_2) \\ + x_5 x_{10} (e - j\alpha_3) \\ + a x_1 x_6 + b x_2 x_7 \end{array} \right\} \qquad \text{unresolved terms}
\end{aligned}
$$

Evaluating Stability of PD (6 - final)

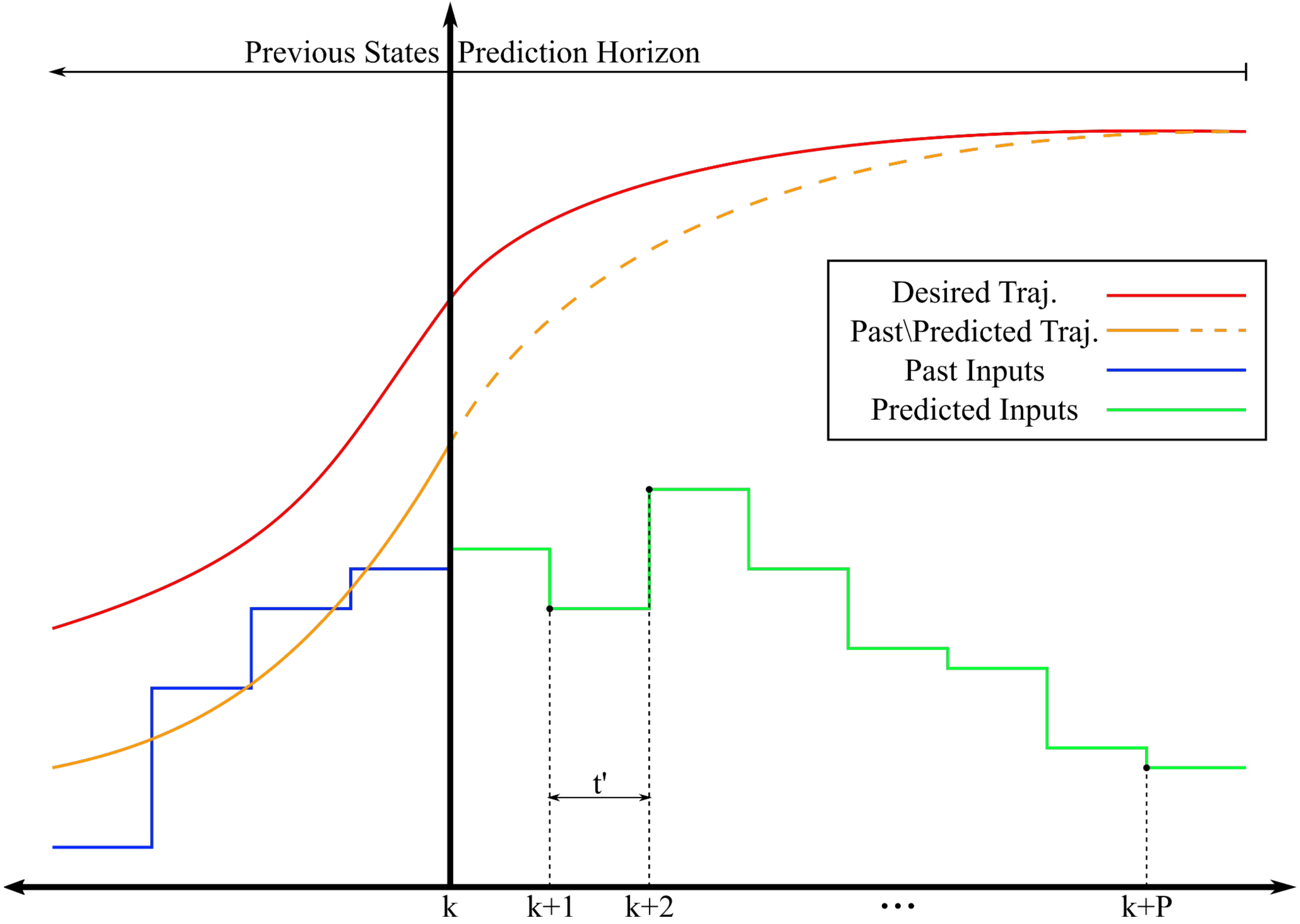
- With the intent to use the Lasalle's Principle to identify asymptotic stability, we use the proportional controller gains to remove sign-indefinite terms…

$$\alpha_1 = \frac{c}{h} \quad \alpha_2 = \frac{d}{i} \quad \alpha_3 = \frac{e}{j} \quad \Rightarrow \quad \dot{V}(x) = \begin{array}{ll} -w(fx_6^2 + gx_7^2 + hx_8^2) & \\ -h\beta_1 x_8^2 - i\beta_2 x_9^2 - j\beta_2 x_{10}^2 & \\ \{+ax_1x_6 + bx_2x_7\} & \text{unresolved terms} \end{array}$$

- Leaving us with an equation where we cannot make claims on the stability of the origin…
- That said, we can identify that the velocity terms are all negative definite and while we cannot make claims on local/global stability because of the x,y-state terms, **we will experimentally show that the origin is stable for z and the angle-based states only if we ignore the x,y-axis positions**.

3). Model Predictive Control (MPC)

Main Idea



Previous States
Prediction Horizon
Desired Traj.
Past\Predicted Traj.
Past Inputs
Predicted Inputs
t'
k
k+1
k+2
...
k+P

MPC Set Definitions

- Defining a discrete model function (as opposed to continuous) we get…

$$x_{k+1} = F(x_k, u_k) \quad F : \mathbb{R}^n \to \mathbb{R}^n$$

- Then we can defined the **action** sets for a prediction horizon of length P…

$$X = \{x_0 \wedge x_{k+1} : x_{k+1} = F(x_k, u_k)\ \forall k < P : k \in \mathbb{N}\}$$
$$U = \{u_k : u_k \in \mathbb{R}^m\ \forall k < P : k \in \mathbb{Z}\}$$

MPC Set Definitions

- With the action sets in mind we can define cost functions that act on the state and input…
- A **cost** set is also written as the evaluating of the cost functions at each state…

$$g(x_k, u_k) = \text{period cost}$$
$$g_P(x_P) = \text{terminal cost}$$

$$G = \{g_k(x_k, u_k) \wedge g_P(x_P) : \forall x_k \in X, u_k \in U\}$$

MPC Optimization Problem

- The optimization problem then becomes a minimization of all costs over the prediction horizon...

$$X^*, U^* = \min_{X,U} \left(\sum_{k=0}^{P-1} g_k(x_k, u_k) + g_P(x_P) \right)$$

- It is important to note that by definition we will be optimization the entire action set through the control inputs, but at the end of the optimization we only care about the control policy found.

Applying MPC to the Hummingbird

- First we must discretize the model function…

$$F(x_k, u_k) = x_k + Tf(x_k, u_k)$$

- We then choose our cost function as a strictly-state minimizing form - i.e. we do not set limits on the magnitude of the inputs…

$$g(x_k, u_k) = x_k^\intercal Q x_k \ \text{ where } Q = \text{diag}\left(\begin{bmatrix} 50 \\ 50 \\ 120 \\ 10 \\ 10 \\ 5 \\ 5 \\ 5 \\ 1 \\ 1 \end{bmatrix}\right)$$

Applying MPC to the Hummingbird

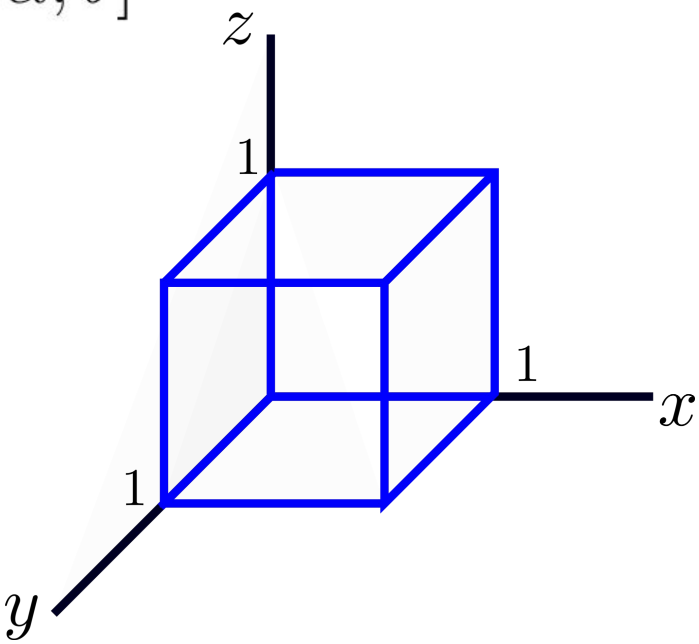
- Finally, since the initial state does not need to be incorporated into the cost function (as it cannot change) we can write the optimization as....

$$X^*, U^* = \min_{X,U} \sum_{k=0}^{P-1} f(x_k, u_k)^\intercal f(x_k, u_k)$$
$$\text{s.t. } x_0 = x(0)$$

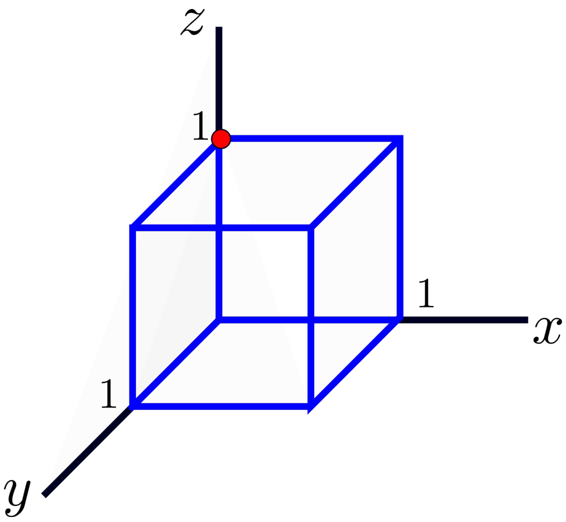
4). PID vs. MPC Comparisons

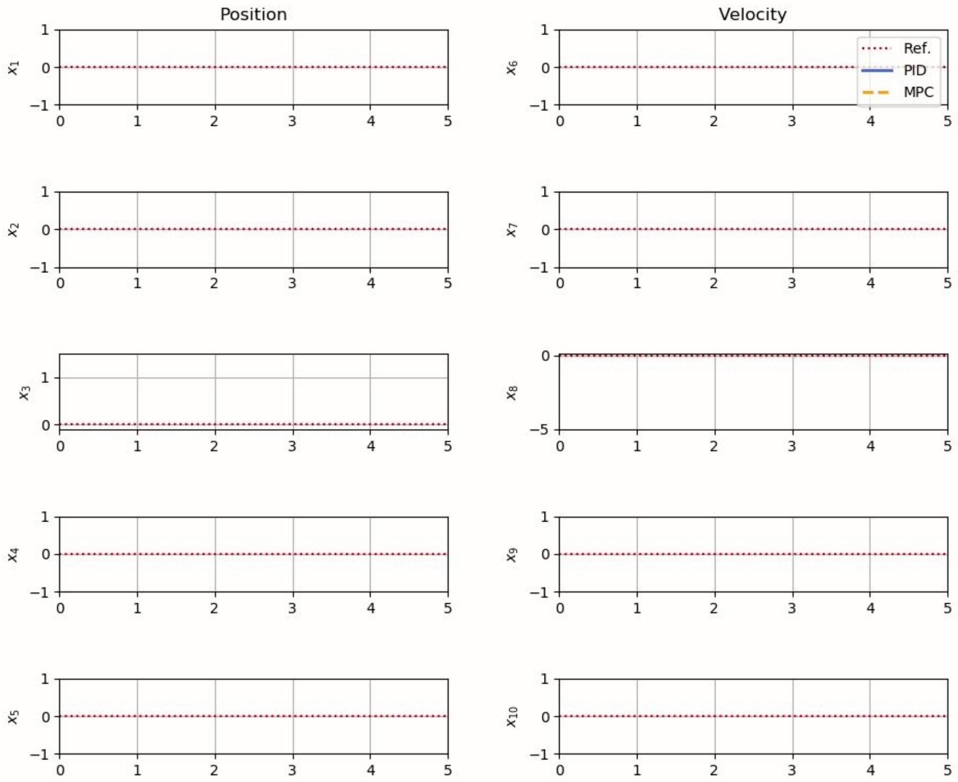
Disturbance Testing: $[x, y, z, \alpha, \theta]$

- Each Test will move the initial position to one of the corners of the unit-cube.
- We will also test the response from a disturbance on each of the angle-states.

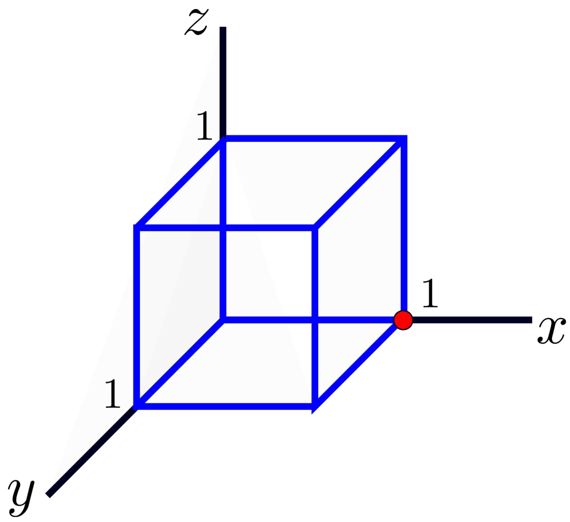


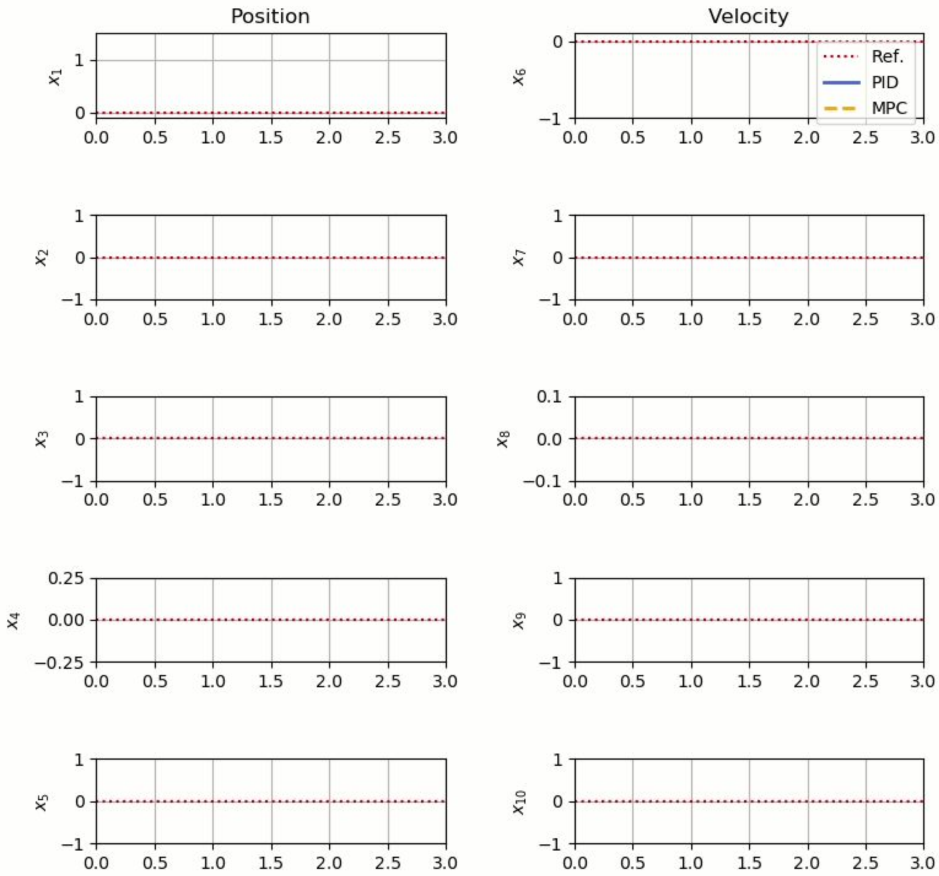
Disturbance: [0,0,1,0,0]



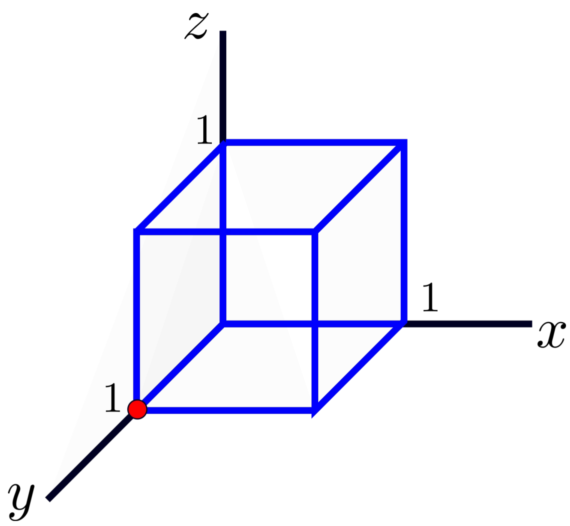


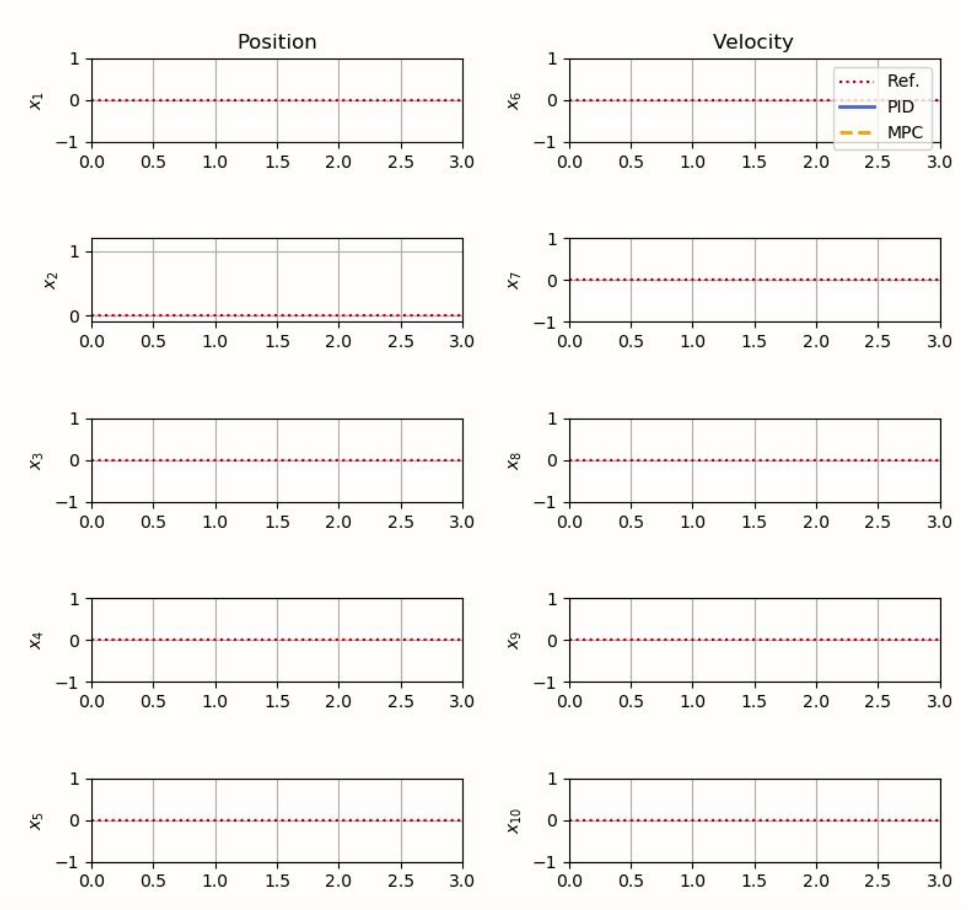
Disturbance: [1,0,0,0,0]



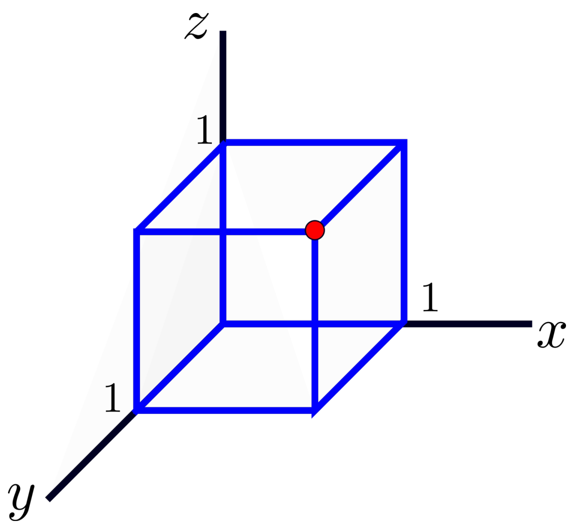


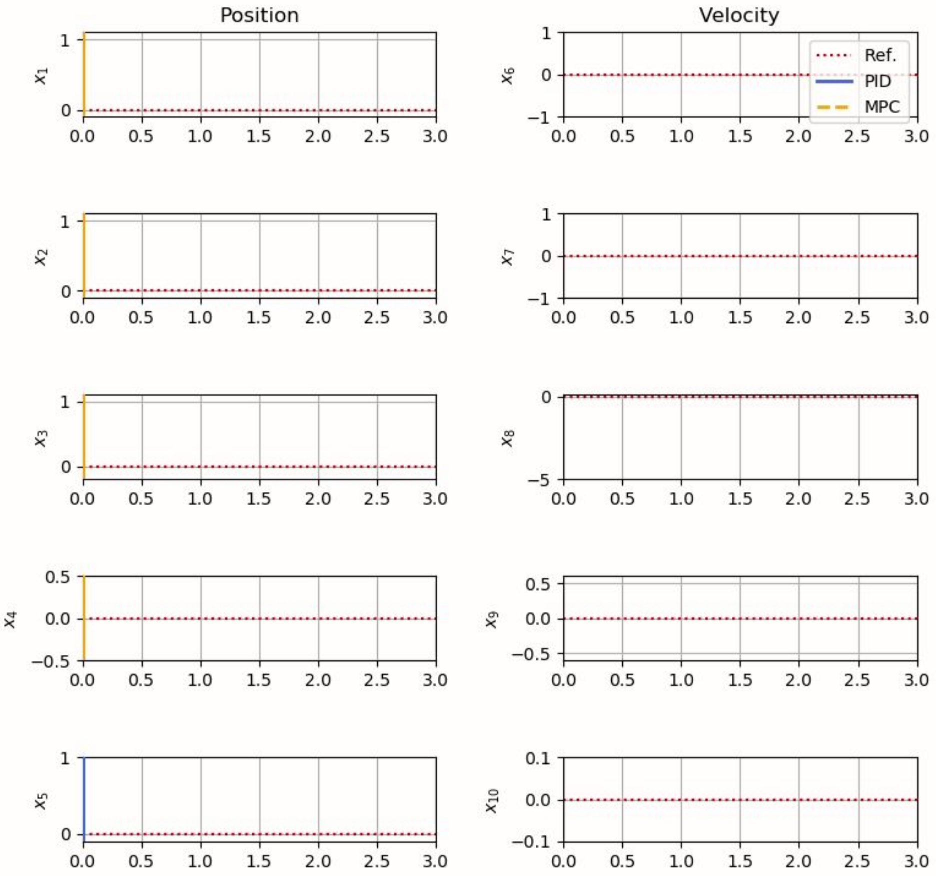
Disturbance: [0,1,0,0,0]



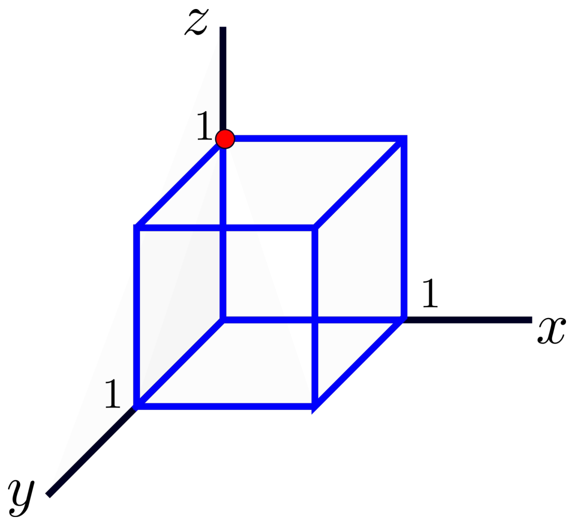


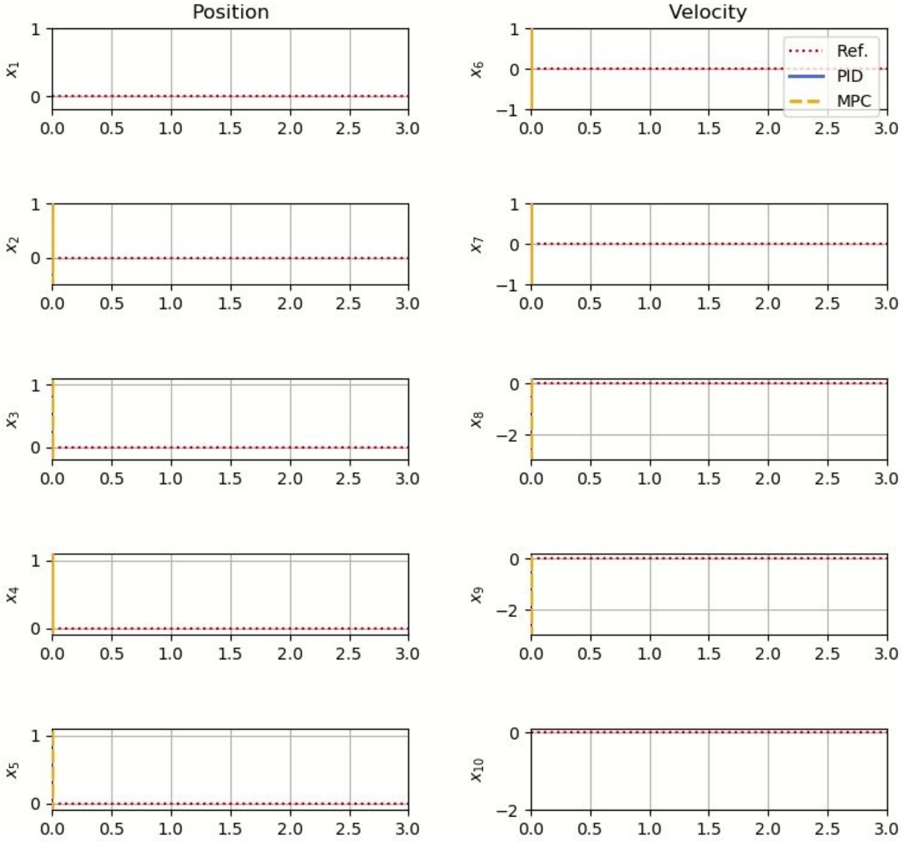
Disturbance: [1,1,1,0,0]



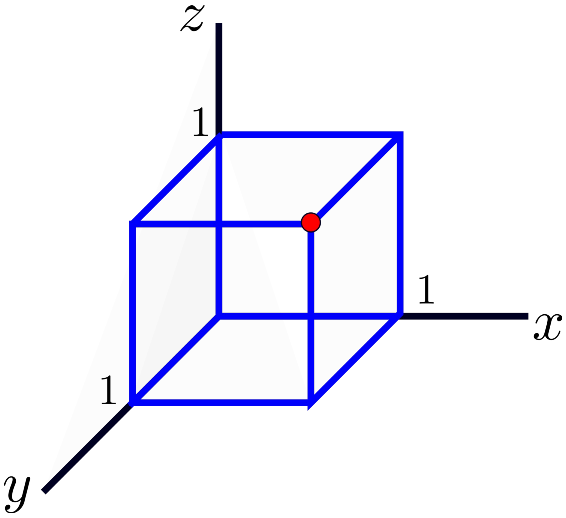


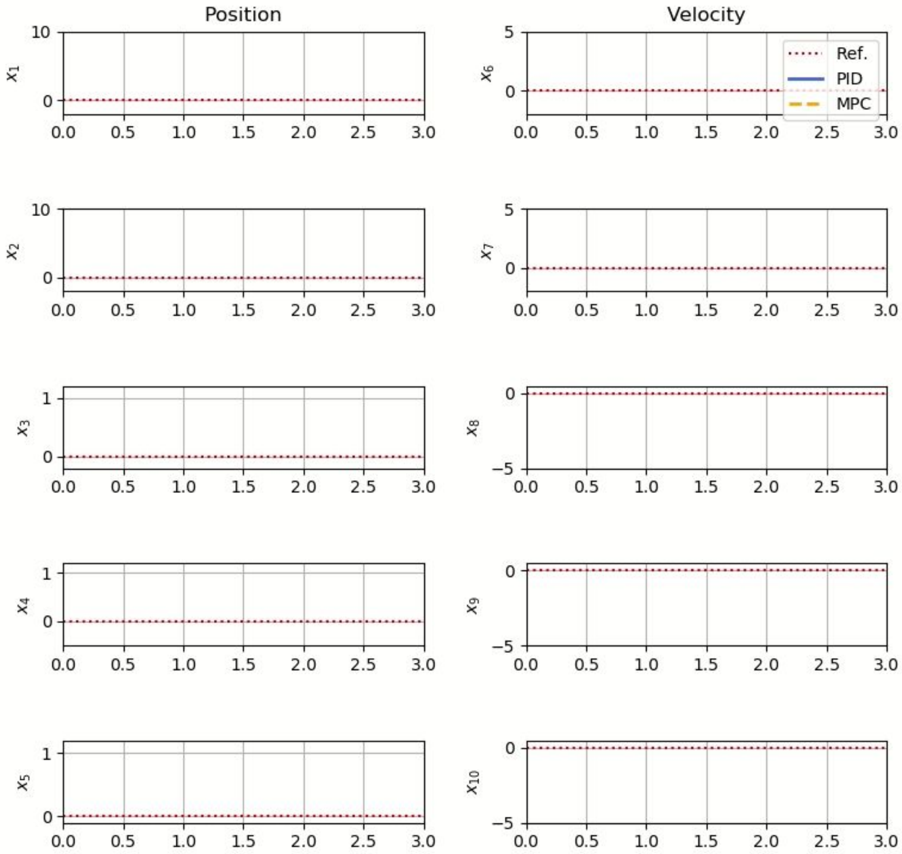
Disturbance: [0,0,1,1,1]





Disturbance: [1,1,1,1,1]





Conclusions

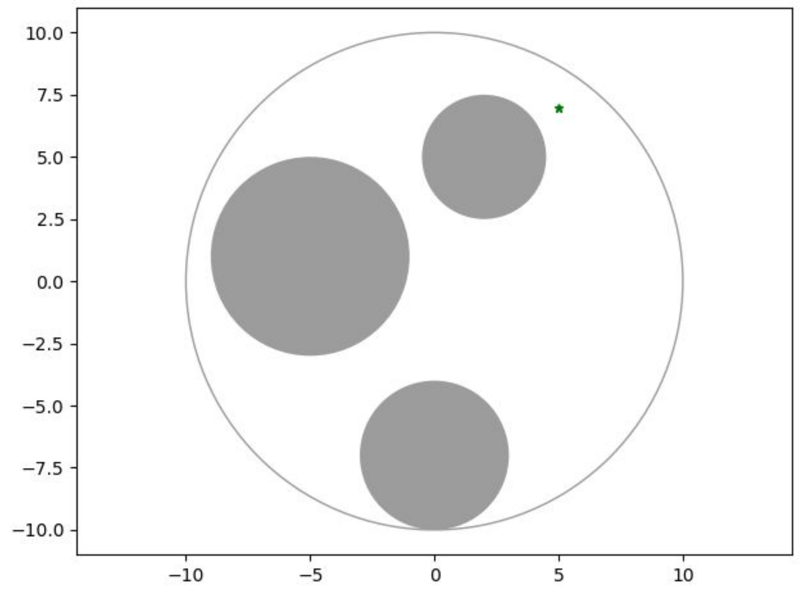
PID:

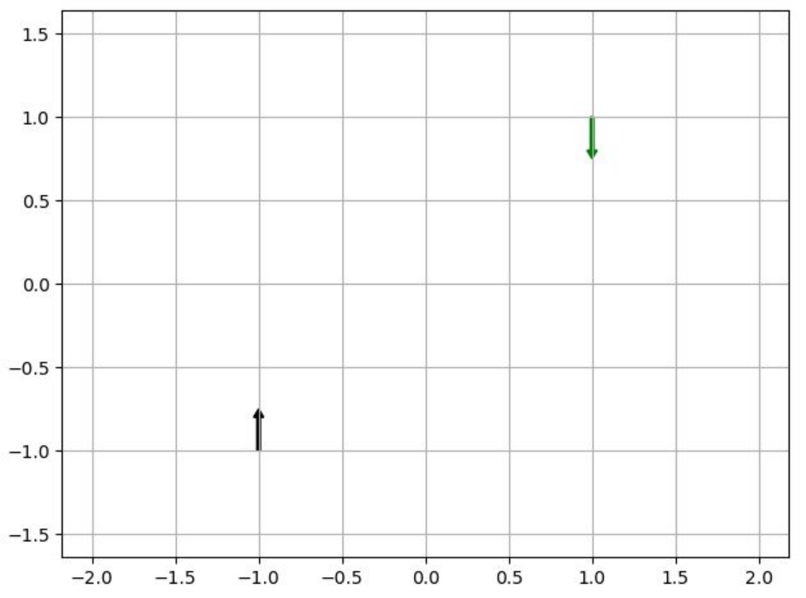
1. Fast, reliable
2. Incapable of controlling x,y-axis positions in current form
3. Finds stable equilibria if x,y-positions are ignored

MPC:

1. Slow, computationally intensive
2. Not guaranteed to find global minimum
3. Adds control to the x-axis

The End.


10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−10
−5
0
5
10


1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0