Micro HW1

Michael B. Nattinger*

September 14, 2020

1 Question 1

This problem asks a set of questions revolving around the concept of the law of supply. We use this law to investigate the case of a firm which uses goods one and two as inputs and good 2 as input. Formally, $y \in Y$ requires $y_1, y_2 \leq 0$.

The law of supply invokes the following inequality, which will be the basis of the solutions which follow: $\Delta p \cdot \Delta y \geq 0$.

1.1 If p_3 falls and p_1, p_2 stay the same, can y_3 go up?

Let us construct our vectors $\Delta p, \Delta y$. $\Delta p = \begin{pmatrix} \Delta p_1 \\ \Delta p_2 \\ \Delta p_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \Delta p_3 \end{pmatrix}, \Delta y = \begin{pmatrix} \Delta y_1 \\ \Delta y_2 \\ \Delta y_3 \end{pmatrix}$. Then, $\Delta p \cdot \Delta y \geq 0 \Rightarrow \Delta p_1 \Delta y_1 + \Delta p_2 \Delta y_2 + \Delta p_3 \Delta y_3 \geq 0 \Rightarrow \Delta p_3 \Delta y_3 \geq 0$. Since p_3 falls, $\Delta p_3 < 0 \Rightarrow \Delta y_3 \leq 0$ so y_3 cannot increase.

1.2 If p_1 rises and p_2, p_3 stay the same, can y_3 go up?

It can and we will construct an example which demonstrates the possibility. Let the set of possible production vectors be $\{(-5, -1, 30), (-1, -8, 32)\}$. For the price vector $(1, 1, 1)$ the profit maximizing production vector is $(-5, -1, 30)$ while for the price vector $(2, 1, 1)$ the profit maximizing production vector is $(-1, -8, 32)$. In this case, p_1 rises while p_2, p_3 stay the same, yet y_3 rises from 30 to 32.

1.3 If p_1, p_2 both increase and p_3 stays the same, can y_3 go up?

The example in the previous subsection holds also for this case if we make a small adjustment. For the price vector $(1, 1, 1)$, the profit maximizing production vector is $(-5, -1, 30)$ while for the price vector $(2, 1.001, 1)$ the profit maximizing production function is $(-1, -8, 32)$. Thus, in this case, p_1, p_2 increase while p_3 stays the same, and the firm's output, y_3, goes up from 30 to 32.

*I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, Ryan Mather, and Tyler Welch. I have also discussed problem(s) with Emily Case, Sarah Bass, and Danny Edgel.

1.3.1 What if both p_1, p_2 rise by 10%?

Prices are homogenous of degree 0. Therefore, $Y^*(1.1p_1, 1.1p_2, p_3) = Y^*(p_1, p_2, p_3/1.1)$. Thus, this situation is equivalent to the case of p_3 falling while p_1, p_2 stay the same.[1] Thus, by the law of supply, $y_3 \leq 0$ so y_3 weakly decreases and, therefore, cannot go up.

2 Question 2

Let us first observe Dataset 1. Note that in the case of $p = (5, 5)$, the profit obtained from producing $(-20, 40)$ is 100 while the profit obtained from producing $(-50, 60)$ at that price is 50. However, the firm chose to produce $(-50, 60)$ at this price - a clear violation of the weak axiom of profit maximization! Therefore, Dataset 1 is inconsistent with a profit maximizing firm.

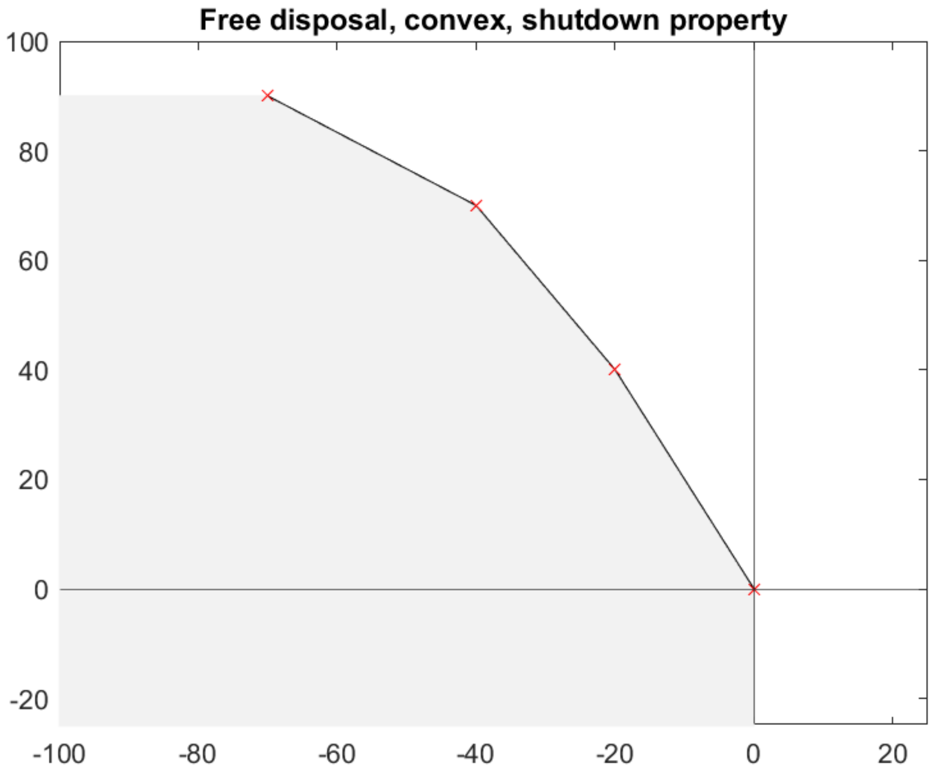
Dataset 2, upon observation, does not suffer from such a violation and thus is consistent with a profit-maximizing firm.

2.1 Describe the smallest production set that can rationalize the data.

The smallest production set that can rationalize the data is the set containing only the realized choices: $\{(-20, 40), (-40, 70), (-70, 90)\}$.

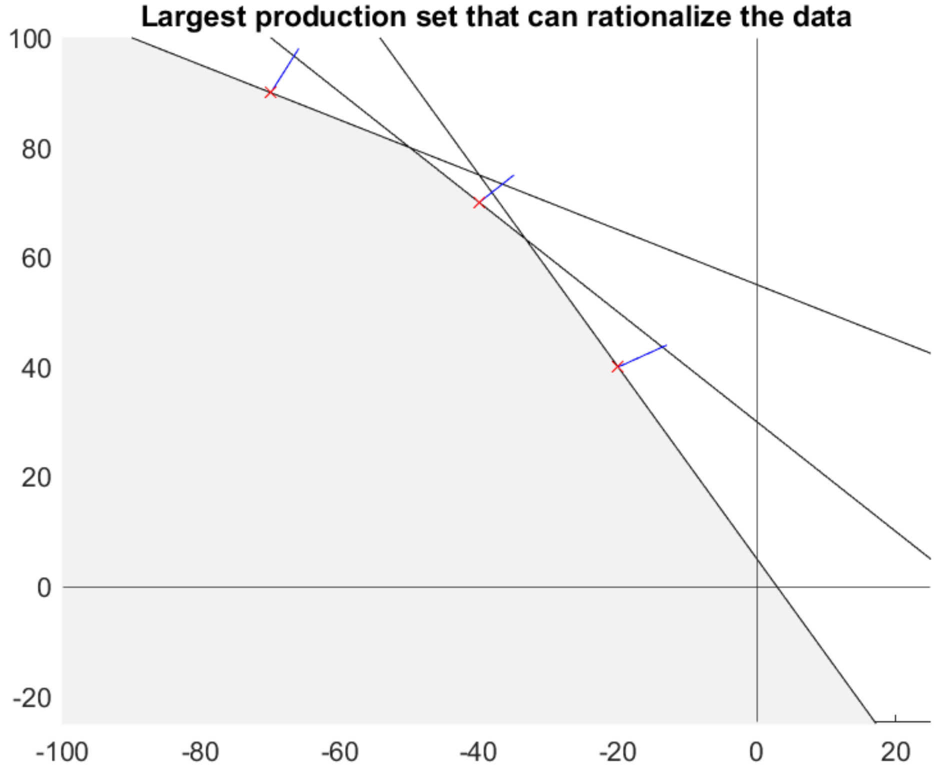
[1]Note: this is true if $p_3 > 0$. However, if $p_3 = 0$, then $Y^*(p_1, p_2, p_3/1.1) = Y^*(p_1, p_2, p_3)$. Then there is a possible case where the firm is indifferent between two production vectors of equal profit and, thus, may switch between those production vectors. For example, if a firm's possible production vectors are $\{(-1, -1, 4), (-1, -1, 5)\}$ and the firm produces $(-1, -1, 4)$ at the price vector of $(1, 1, 0)$, then for the price vector of $(1.1, 1.1, 0)$ the firm is still indifferent between production vectors and so may choose to produce $(-1, -1, 5)$.

2.2 Draw the smallest convex production set with free disposal and the shutdown property that can rationalize the data.



The above chart shows the realized datapoints, along with the shutdown production option of producing $(0, 0)$. It also shows the lines connecting the points, which must be in the set as the set is specified to be convex. The shaded area in the chart show regions attainable by producing at one of the realized levels, not producing, or producing along the lines connecting the realized points, and then utilizing free disposal.

2.3 Draw the largest production set that can rationalize the data.



The above chart shows the realized datapoints (in red), along with (black) lines going through those points that are perpendicular to the price vectors (shown in blue). The shaded region, which includes the lines and the points, represents the largest production set that can rationalize the data.

3 Question 3

Define $y(p) = y_1(p) + \cdots + y_n(p)$ as the aggregate level of production for the industry at price level p, and define $\pi(p) = p \cdot y(p)$. We know that each firm in the industry is profit maximizing and price-taking, and therefore individually each firm satisfies the weak axiom. Thus,

$$\begin{aligned}\pi(p) = p \cdot y(p) = p \cdot (y_1(p) + \cdots + y_n(p)) &= p \cdot y_1(p) + \cdots + p \cdot y_n(p) \\ &\geq p \cdot y_1(p^{'}) + \cdots + p \cdot y_n(p^{'}) = p \cdot (y_1(p^{'}) + \ldots y_n(p^{'})) = p \cdot y(p^{'}).\end{aligned}$$

Therefore, the industry must also satisfy the weak axiom. It can, in a sense of maximizating profit via choosing optimal production vectors, be rationalized as if the industry as a whole was acting as a single profit-maximizing firm. (Note, however, that if the industry as a whole truly was a single firm maximizing profit, then that firm may gain market power and not act as a price-taker. So we should be careful not to take the analogy too far.)