Macro PS5

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1 Question 1

Working-age agents maximize their value subject to their budget constraint:

$$V_j(k) = \max_{k',l}\{u_j^W(c,l) + \beta V_j(k_{t+1})\}\text{s.t. } c = (1-\tau)wel + (1+r)k - k'$$

We will take first order conditions to solve for l as desired.

$$\begin{aligned}
\frac{\partial V_j}{\partial l} &= 0 \\
\Rightarrow (c^\gamma(1-l)^{1-\gamma})^{-\sigma}\left(\gamma c^{1-\gamma}\frac{\partial c}{\partial l}(1-l)^{1-\gamma} - c^\gamma(1-l)^{-\gamma}(1-\gamma)\right) &= 0 \\
\Rightarrow \gamma\left(\frac{1-l}{c}\right)^{1-\gamma}(1-\tau)we &= \left(\frac{1-l}{c}\right)^{-\gamma}(1-\gamma) \\
\Rightarrow \frac{\gamma}{1-\gamma}(1-l)(1-\tau)we_j &= c = (1-\tau)we_jl + (1+r)k - k' \\
\Rightarrow \frac{\gamma}{1-\gamma}(1-\tau)we_j &= \left(\frac{\gamma}{1-\gamma}+1\right)(1-\tau)we_jl + (1+r)k - k' \\
\Rightarrow \frac{\gamma}{1-\gamma}(1-\tau)we_j - [(1+r)k-k'] &= \left(\frac{1}{1-\gamma}\right)(1-\tau)we_jl \\
\Rightarrow \frac{\gamma(1-\tau)we_j - (1-\gamma)[(1+r)k-k']}{(1-\tau)we_j} &= l
\end{aligned}$$

*I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, and Ryan Mather. I have also discussed problem(s) with Emily Case, Sarah Bass, and Danny Edgel.