# Micro Midterm

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1) (a) $u(x) = \min\{x_1, x_2\}^\alpha (x_3 + x_4)^{1-\alpha}$

~~We will spend equal amounts as goods 1 and 2~~

$(x_1 = x_2)$ We will consume the same amount of goods 1 and 2. Thus, we can reduce the opt. problem to:

$\max x_1^\alpha (x_3 + x_4)^{1-\alpha}$ s.t. $(p_1+p_2)x_1 + p_3 x_3 + p_4 x_4 \le w$, and preferences are LNS so Walras' law shows that the constraint will hold with equality. Also note that we will only consume the cheaper of goods 3 and 4 so we have

$\max x_1^\alpha y^{1-\alpha}$ s.t. $(p_1+p_2)x_1 + p_y y = w$ where $p_y = \min\{p_3, p_4\}$, and $y$ is the cheaper good between 3 and 4.

We will consume a nonzero amount of $x_1, x_2, y$ as the marginal utility from consuming an epsilon of each goes to infinity in the limit as their consumption goes to 0. Thus $\mu_i = 0$ in our Lagrangian.

$$\mathcal{L} = x_1^\alpha y^{1-\alpha} + \lambda((p_1+p_2)x_1 + p_y y - w)$$

FOC wrt $x_1$: $\alpha x_1^{\alpha-1} y^{1-\alpha} + \lambda(p_1+p_2) = 0 \Rightarrow -\lambda = \frac{\alpha}{p_1+p_2}(x_1/y)^{\alpha-1}$

" $y$: $(1-\alpha)x_1^\alpha y^{-\alpha} + \lambda p_y = 0 \Rightarrow -\lambda = \frac{1-\alpha}{p_y}(x_1/y)^\alpha$

$$\Rightarrow \frac{\alpha}{(p_1+p_2)}\frac{p_y}{(1-\alpha)} = x_1/y \Rightarrow x_1 = \frac{\alpha}{1-\alpha}\frac{p_y}{p_1+p_2} y$$

$$\Rightarrow w = p_y y + \frac{\alpha}{1-\alpha} p_y y = \left(\frac{1-\alpha+\alpha}{1-\alpha}\right) p_y y = \frac{1}{1-\alpha} p_y y$$

$$\Rightarrow y = (1-\alpha)\frac{w}{p_y}, \quad x_1 = \alpha\frac{w}{(p_1+p_2)}$$

$$\Rightarrow V(p,w) = \left(\alpha\frac{w}{p_1+p_2}\right)^\alpha \left((1-\alpha)\frac{w}{p_y}\right)^{1-\alpha} \text{ where } p_y = \min\{p_3, p_4\}$$

$x_1 = x_2 = \alpha\frac{w}{p_1+p_2}$, the cheaper of goods 3 and 4 is $y = (1-\alpha)\frac{w}{p_y}$, the more expensive of 3 and 4 is not consumed.

$\frac{dx_1}{dw} = \frac{\alpha}{p_1+p_2}$ so good 1 is normal.

b Fix $u$. $u = V(p, e(p,u)) = \left(\frac{\alpha}{p_1+p_2}\right)^\alpha \left((1-\alpha)\frac{1}{p_y}\right)^{1-\alpha} e(p,u)$

$$\Rightarrow e(p,u) = u\left(\frac{p_1+p_2}{\alpha}\right)^\alpha \left(\frac{p_y}{1-\alpha}\right)^{1-\alpha}$$

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(b) $e(p,u) = u\left(\frac{p_1+p_2}{\alpha}\right)^{\alpha}\left(\frac{p_4}{1-\alpha}\right)^{1-\alpha}$

Shepard's Lemma: $h_1(p,u) = \frac{de}{dp_1}(p,u) = \cancel{\alpha} u\left(\frac{p_1+p_2}{\alpha}\right)^{\alpha-1}\frac{1}{\cancel{\alpha}}\left(\frac{p_4}{1-\alpha}\right)^{1-\alpha}$

$\alpha - 1 < 0$ so $h_1$ is decreasing in $p_1, p_2$ and increasing in the price of the cheaper of goods 3 and 4. Thus, good 1 is a substitute for ~~good~~ the cheaper of good 3 and 4, and good 1 is a complement for good 2.

(c) If $p_3 > p_4$ the consumer will sell all of their initial endowment of good 3 and will not consume any of good 3, because they can instead consume the cheaper good 4. So, the consumer is a net seller of good 3. Wealth increases with $p_3$ so demand for good 1 increases with $p_3$ purely because of a wealth effect.

(d) If we dedine $\overline{w} = p_3e_3 + p_4e_4$ then the consumer will be a net buyer of good 3 if

$e_3 < (1-\alpha)\frac{\overline{w}}{p_3} \Rightarrow p_3e_3 < (1-\alpha)(p_3e_3 + p_4e_4)$

$\Rightarrow p_3e_3(\cancel{1} + (\cancel{1}+\alpha)) < (1-\alpha)p_4e_4 \Rightarrow p_3e_3 < \frac{1-\alpha}{\alpha}p_4e_4$.

If the consumer is a net buyer of good 3, as $p_3$ increases it costs more for the consumer to buy the additional good 3 that they would like to buy, which reduces the amount of ~~extra~~ ~~money~~ money they have to spend on good 1. At the same time, the substitution abbect ~~push~~ works in the opposite direction, as good 1 is now ~~the~~ cheaper relative to good 3. The overall direction is thus indeterminate due to offsetting wealth/sub effects.

If the consumer is a net seller of good 3, as $p_3$ increases the consumer is wealthier and will substitute towards good 1 so both the wealth/sub effects increase the cons of good 1.

2) (a)

| $\pi$ | $y$: 1 | 2 | 3 |
|---|---|---|---|
| $p$: 1 | 3 | 1 | 2 |
| 2 | 13 | 16 | 10 |
| 3 | -1 | -7 | 1 |

It is clear that, for each price level, the firm acted in a way which ~~maximized profits given~~ is rationalizable given the production data we are given.

$Y = \{(10,-3,-4), (15,-6,-8), (8,-5,-1)\}$ rationalizes the data.

(b) I cannot find any ~~the~~ convex combinations which yield higher profit than the chosen production levels at ~~the~~ each price. Thus, we must conclude that the data is consistent with a $\pi$-maximizing firm whose production set is convex.

$Y^c = \{\lambda y_1 + (1-\lambda) y_2 ; \lambda \in [0,1]; y_1, y_2 \in Y\}$ rationalizes the data and is convex.

(c) The price of the first good rises between $p_1$ and $p_2$ with everything else remaining constant. $q, z_1, z_2$ all increase. This would be rationalizable potentially, ~~[illegible]~~. However the price of the third good increases while $q$ and $z_2$ decrease and ~~$z_1$~~ $z_1$ increases. This is not ~~rationalizable by a firm with~~ consistent with a firm with supermodular production function as $z_1, z_2$ move in opposite directions.

Changing $y^3$ to $(8,-2,-1)$ would be consistent with a supermod $f$. This would cause $z_1, z_2$ to move in the same direction in response to an increase in $p_3$.

Let $L, L', L''$ be lotteries with worst-case outcomes $L_*, L'_*, L''_*$

3) (a) • Either $L_* \geq L'_*$ or $L_* \leq L'_*$ (or both) so $\succsim_{max}$ is complete.

• Let $L \succsim_{max} L'$, $L' \succsim_{max} L'' \Rightarrow L_* \geq L'_* \geq L''_*$ so $L \succsim L''$ so $\succsim_{max}$ is transitive.

$\succsim_{max}$ is continuous because the set of $\lambda \in [0, 1]$ s.t. $pL_* + (1-p)L'_* \geq L''_*$
$\Leftrightarrow pL \oplus (1-p)L' \succsim L''$ is closed.

• Let $L \succsim_{max} L'$, $p \in (0,1)$. Then $L_* \geq L'_*$. Then,
$pL_* \geq pL'_* \Rightarrow pL_* + (1-p)L''_* \geq pL'_* + (1-p)L''_*$
$\Rightarrow pL \oplus (1-p)L'' \succsim_{max} pL' \oplus (1-p)L''$ so $\succsim_{max}$ is independent.

Yes, it can. It is represented by

$$u(x_i) = \begin{cases} \frac{1}{p_i} x_i, & x_i = L_* \\ 0, & \text{otherwise} \end{cases}$$

Then, $U(L) = \sum_{1 \leq x_i \leq X} p_i u(x_i) = L_*$

Also, von Neumann and Morgenstern's prop shows that $\succsim_{max}$ can be rep'd by a $U(L)$.

(b) Let $L, L'$ be s.t. $L \succ_{max} L'$. Then, $L_* > L'_*$
$U(L) = \sum_{1 \leq x_i \leq X} p_i (1 - e^{-(cx_i)})$, $U(L') = \sum_{1 \leq x_i \leq X} p_i (1 - e^{-cx_i})$
For $c$ sufficiently high, $U(L) \cong 1$, $U(L') \cong 1 - p_{L'_*}$
and as $c \uparrow \infty$, $U(L)$ will remain higher than $U(L')$.

(c) $U(L) = \frac{2}{5}(1 - e^{-10c}) + \frac{3}{5}(1 - e^{-100c}) = 1 - \frac{2}{5}e^{-10c} - \frac{3}{5}e^{-100c}$
$U(L') = \frac{1}{5}(1 - e^{-10c}) + \frac{4}{5}(1 - e^{-20c}) = 1 - \frac{1}{5}e^{-10c} - \frac{4}{5}e^{-20c}$
$\Rightarrow U(L) - U(L') = -\frac{1}{5}e^{-10c} - \frac{3}{5}e^{-100c} + \frac{4}{5}e^{-20c}$
The $\frac{1}{5}e^{-10c}$ term will dominate as $c \to \infty$ so $U(L) - U(L') < 0$
$\Rightarrow U(L) < U(L')$ in the limit.
So, CARA prefers $L'$ for $c$ sufficiently large.

In the limit as $c \to \infty$, CARA utility corresponds to preferences which are minmax with a tiebreaker for equal worst-case outcomes, this tiebreaker prefers the lottery w/ a less likely worst-case outcome.