$$\lambda_{1,2} = \frac{1}{2\beta}\left(1 + \beta + \alpha\kappa^2 \pm \sqrt{(1+\beta+\alpha\kappa^2)^2 - 4\beta}\right),$$

$$\lambda_1 \lambda_2 = \frac{1}{4\beta^2}\left(1+\beta+\alpha\kappa^2 + \sqrt{(1+\beta+\alpha\kappa^2)^2 - 4\beta}\right)\left(1+\beta+\alpha\kappa^2 - \sqrt{(1+\beta+\alpha\kappa^2)^2 - 4\beta}\right)$$

$$\lambda_1\lambda_2 = \frac{1}{4\beta^2}\left[(1+\beta+\alpha\kappa^2)^2 - (1+\beta+\alpha\kappa^2)^2 + 4\beta\right]$$

$$\lambda_1\lambda_2 = \frac{1}{4\beta^2}\cdot 4\beta$$

$$\lambda_1\lambda_2 = \frac{1}{\beta}$$

$$-\beta(1-\lambda_1 L)(1-\lambda_2 L)L^{-1}\hat{p}_t = u_t,$$

$$(\beta\lambda_1 L - \beta)(1-\lambda_2 L)L^{-1} p_t = u_t$$

$$(\beta\lambda_1 L - \beta - \beta\lambda_1\lambda_2 L^2 + \beta\lambda_2 L)L^{-1} p_t = u_t$$

$$(\beta\lambda_1 - \beta/L - \beta\lambda_1\lambda_2 L + \beta\lambda_2)\, p_t = u_t$$

$$\beta\lambda_1 p_t - \beta/L\, p_t - \beta\lambda_1\lambda_2 L p_t + \beta\lambda_2 p_t = u_t$$

$$\beta\lambda_1 p_t - \beta E_t p_{t+1} - \beta\lambda_1\lambda_2 p_{t-1} + \beta\lambda_2 p_t = u_t$$

$$-\beta E_t p_{t+1} + \beta(\lambda_1+\lambda_2) p_t - \beta\lambda_1\lambda_2 p_{t-1} = u_t$$

Note,

$$-\beta\mathbb{E}_t\hat{p}_{t+1} + [1+\beta+\alpha\kappa^2]\hat{p}_t - \hat{p}_{t-1} = \tilde{u}_t.$$

$\beta(\lambda_1+\lambda_2) = 1+\beta+\alpha\kappa^2$ $\quad \frac{1}{2\beta}(1+\beta+\alpha\kappa^2+\sqrt{\cdots}) + \frac{1}{2\beta}(1+\beta+\alpha\kappa^2-\sqrt{\cdots})$

$\beta\lambda_1\lambda_2 = 1$ $\quad \lambda_1\lambda_2 = 1/\beta$