Micro HW6

Michael B. Nattinger*

December 8, 2020

1 Question 1

If both play C, they will receive 2 utility in each period. one diverges to D, they will receive 8 utility in that period and 1 utility in all periods moving forwards. Playing (C,C) in all periods can be supported if:

$$\frac{2}{1-\delta} \geq 8 + \frac{\delta}{1-\delta}$$
$$\Rightarrow 2 \geq 8 - 8\delta + \delta$$
$$\Rightarrow \delta \geq \frac{6}{7}.$$

2 Question 2

2.1 Part A

The strategy profile will be a subgame perfect equilibrium iff no player has a profitable deviation from the strategy after any history.

There will not be a deviation from $(C, C), \ldots$ to $(C, D), (P, P), (C, C), \ldots$ if:

$$\frac{2}{1-\delta} \geq 3 + \frac{2\delta^2}{1-\delta}$$
$$\Rightarrow \delta \geq \frac{1}{2}$$

Assume there is a deviation. There will not be a deviation from $(P, P), (C, C), \ldots$ to $(P, D), (P, P), (C, C), \ldots$ if:

*I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, Ryan Mather, and Tyler Welch. I have also discussed problem(s) with Emily Case, Sarah Bass, and Danny Edgel.

$$
\begin{aligned}
\frac{\delta 2}{1-\delta} &\geq 1 + \frac{\delta^2 2}{1-\delta} \\
\delta 2 &\geq 1 - \delta + 2 \\
\Rightarrow \delta &\geq \frac{1}{2}
\end{aligned}
$$

Both will hold for $\delta \in [\frac{1}{2}, 1)$.

2.2 Part B

There will not be a deviation from $(C, C), \dots$ to $(C, D), (P, P), (C, C), \dots$ if:

$$
\begin{aligned}
\frac{2}{1-\delta} &\geq 3 + \frac{1}{2} + \frac{2\delta^2}{1-\delta} \\
\Rightarrow \delta &\geq \frac{2}{3}
\end{aligned}
$$

Assume there is a deviation. There will not be a deviation from $(P, P), (C, C), \dots$ to $(P, D), (P, P), (C, C), \dots$ if:

$$
\begin{aligned}
\frac{1}{2} + \frac{\delta 2}{1-\delta} &\geq 1 + \frac{\delta}{2} + \frac{\delta^2 2}{1-\delta} \\
\delta 2 &\geq 1 - \delta + 2 \\
\Rightarrow \delta &\geq \frac{1}{3}
\end{aligned}
$$

Both will hold for $\delta \in [\frac{2}{3}, 1)$.

2.3 Part C

Increasing the (P,P) payoff increases the relative payoff from deviating in the initial type of deviation, but decreases the relative payoff of deviating in the second type of deviation. This increases the level of δ required in the first type of deviation for the SPE to hold, but decreased the level of δ required in the second type of deviation for the SPE to hold.

2.4 Part D

D strictly dominates C and strictly dominates P, so there are no correlated equilibria possible.

3 Question 3

3.1 Part A

The profile I will construct is to play (I), and play (D,D,D) forever after any deviation.

By symmetry, the SPE will hold if all 3 people are willing to in the first period:

$$
\begin{aligned}
2 + \delta^2(-1) + \delta^3(2 + \delta^2(-2)) + \ldots &\geq 0 \\
\Rightarrow \frac{2 - \delta^2}{1 - \delta^3} &\geq 0 \\
\Rightarrow \delta &\geq 0
\end{aligned}
$$

$$
\begin{aligned}
\delta(-1) + 2\delta^2 + \delta^3(\delta(-1) + 2\delta^2) + \ldots &\geq 0 \\
\Rightarrow \frac{\delta(-1) + 2\delta^2}{1 - \delta^3} &\geq 0 \\
\Rightarrow \delta &\geq \frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
-1 + 2\delta + \delta^3(-1 + 2\delta) + \ldots &\geq 0 \\
\Rightarrow \frac{-1 + 2\delta}{1 - \delta^3} &\geq 0 \\
\Rightarrow \delta &\geq \frac{1}{2}
\end{aligned}
$$

3.2 Part B

A smaller set will allow the SPE to hold, which we will show below.

By symmetry, the SPE will hold if all 3 people are willing to in the first period:

$$
\begin{aligned}
2 + \delta(-1) + \delta^3(2 + \delta(-1)) + \ldots &\geq 0 \\
\Rightarrow \frac{2 + \delta(-1)}{1 - \delta^3} &\geq 0 \\
\Rightarrow \delta &\geq 0
\end{aligned}
$$

$$
\begin{aligned}
2\delta - \delta^2 + \delta^3(2\delta - \delta^2) + \ldots &\geq 0 \\
\Rightarrow \frac{2\delta - \delta^2}{1 - \delta^3} &\geq 0 \\
\Rightarrow \delta &\geq 0
\end{aligned}
$$

$$
\begin{aligned}
-1 + 2\delta^2 + \delta^3(-1 + 2\delta^2) + \ldots &\geq 0 \\
\Rightarrow \frac{-1 + 2\delta^2}{1 - \delta^3} &\geq 0 \\
\Rightarrow \delta &\geq \frac{1}{\sqrt{2}}
\end{aligned}
$$

This is a different condition than Part A. Intuitively, the third person gets hit with the C payoff immediately and have to wait two periods to get the A payoff where in Part B they only had to wait one period to get the A payoff. They will only agree to this payoff if they are relatively patient.

4 Question 4

4.1 Part A

$$
\begin{aligned}
\beta_1 &= ((\beta_1(A), \beta_1(B)), (\beta_1(E), \beta_1(F), \beta_1(G))) \\
&= ((1/2, 1/2), (0, 1/2, 1/2)) \\
\beta_2 &= (\beta_2(C), \beta_2(D)) \\
&= (2/3, 1/3)
\end{aligned}
$$

The above behavior strategy is equivalent to the mixed strategy given.

4.2 Part B

$$
\begin{aligned}
\sigma_1 &= (\sigma_1(AE), \sigma_1(AF), \sigma_1(AG), \sigma_1(BE), \sigma_1(BF), \sigma_1(BG)) \\
&= (a/3, b/3, (1 - a - b)/3, c/3, c/6, c/6)
\end{aligned}
$$

For $a, b, c \in [0, 1]$, the above mixed strategies are equivalent to the behavior strategy.

5 Question 5

Person 3 will play L if they think the probability of being at D satisfies $3D + 1d \geq 2D + 3d \Rightarrow D \geq 2d \Rightarrow D \geq 2/3$. In this scenario, person 2 will play a, and person 1 will play A. The information set is unreached, so the specific beliefs for person 3 are consistent. This is a sequential equilibrium.

Person 3 will play R if they think the probability of being at $D \leq 2/3$. Person 2 will choose d, so person 1 will choose 1. This outcome is inconsistent with person 3's belief that $D \leq 2/3$.

Person 3 can mix if their belief of being at D is 2/3. Let β_3 be the probability of person 3 playing L. We will now focus on person 2.

Person 2 will play a if $3(1-\beta_3) \leq 2 \Rightarrow \beta_3 \geq 1/3$. If this occurs, player 1's outcomes are $(1-\beta_3)$ from playing D but 2 from playing A, so player 1 will play A. This is a sequential equilibrium.

Person 2 will play d if $\beta_3 \leq 1/3$. Then, player 1's payoffs are $(1-\beta_3)$ from playing D and 0 from playing A, so they will play A. Then, player 1's belief that they $D = 2/3$ is incorrect, so this is not a sequential equilibrium.

Person 2 will be indifferent if $\beta_3 = 1/3$. Let β_2 be the probability that player 2 plays d. We will now focus on player 1.

If player 1 always chooses D, then $D = 1 > 2/3$ which cannot be a sequential equilibrium. If they instead choose always A, then player 2 must always choose a so this is also a sequential equilibrium. The final possibility is that person 1 mixes. They must be indifferent between choosing A and D. Player 3 is choosing L with probability 1/3, so for person 1 to be indifferent, $(2/3) = (1-\beta_2)2 \Rightarrow \beta_2 = 2/3$. For this to hold as a sequential equilibrium, person 3 must have a 2/3 chance at being at D, so $\beta_1 = 2(1-\beta_1)(\beta_2) \Rightarrow \beta_1 = (1-\beta_1)(4/3) \Rightarrow \beta_1 = 4/7$.

Therefore, the sequential nash equilibrium are (summarizing the above) $(A, a, L), (A, a, \beta_3 \geq 1/3), (A, a, 1/3), (4/7, 2/3, 1/3)$ (which we can further reduce to $(A, a, \beta_3 \geq 1/3), (4/7, 2/3, 1/3)$).

6 Question 6

7 Question 7

8 Question 8