

Problem Set¹ #1

Problem 1: A first look at difference equations in economics

Here we are interested in how the stock market may react to a future FOMC announcement of an increase in short term interest rates. To do so, consider the following simple perfect foresight model of stock price dynamics given by the following equation

$$p_t = \frac{d + p_{t+1}}{(1 + r)} \quad (1)$$

where p_t is the price of a share at the beginning of period t before constant dividend d is paid out, and r is the short term risk free interest rate. The left hand side of (1) is the cost of buying a share while the right hand side is the benefit of buying the share (the owner receives a dividend and capital gain or loss from the sale of the share). Assume $r > 0$.

1. Solve for the steady state stock price $p^* = p_t = p_{t+1}$.
2. Assume the initial price, p_0 , is given. Solve the closed form solution to the first order linear difference equation in (1). Explain how price evolves over time (i.e. what if $p_0 > p^*$, $p_0 < p^*$, $p_0 = p^*$) using both a phase diagram (i.e p_{t+1} against p_t) as well as a graph of p_t against time t . If the initial stock price is away from the steady state, does it converge or diverge from the steady state. Explain why.
3. **[Matlab]** Suppose the risk free rate is $r = 1\%$ and the stock pays constant dividend $d = 1$ per share per period. Open the Matlab code we provided. The code generates and plots the price dynamics given the first-order difference equation in part 2 given initial share price $p_0 = 100$ at time $t = 0$. Modify the code (i.e. simply replace 100) with three different initial prices which respectively are below, at, and above the steady state price level implied by these parameters to plot the price dynamics over 100 periods.

¹Based on previous problem sets by Anton Babkin, Fu Tan and Eirik Brandsås

4. **[Analytical and Matlab]** Suppose the Federal Reserve announces at $t = 20$ to raise the federal funds rate from 1% to 2% at $t = 50$ and remain at the new level forever. Using (1) with $d = 1$, how does the price respond to the policy announcement and the interest rate change over time? Plot the price dynamics from $t = 0$ to $t = 99$. Please assume that we rule out rational bubbles (i.e., agents know and believe that prices will be at its fundamental value, the steady state, at $t=50$).