Micro HW5

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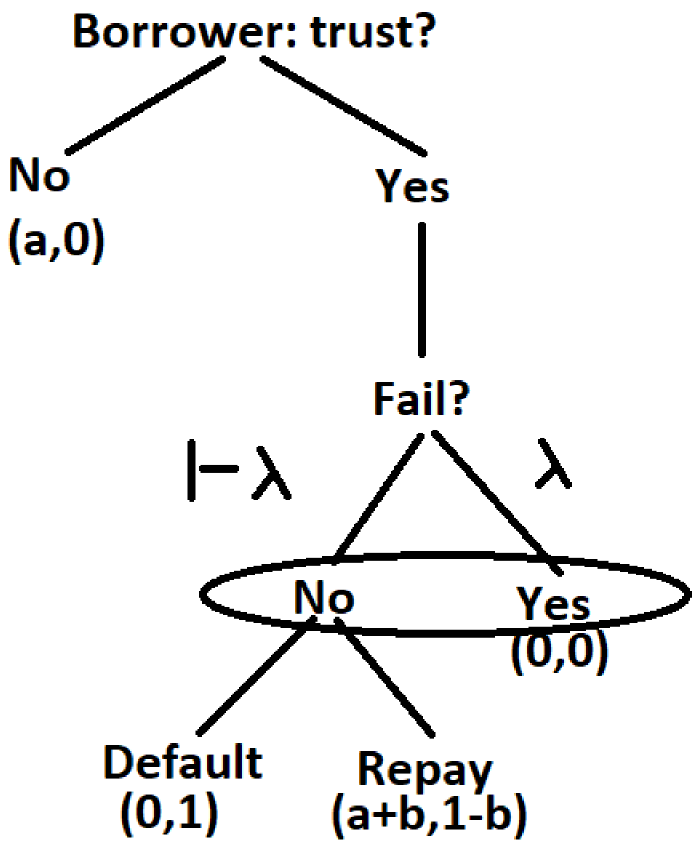
1 Question 1

Colin will take the larger of the two pieces of cake. Therefore, Rowena will always be stuck with the smaller of the two pieces of cake. Rowena, therefore, will maximize his utility by making the smallest piece of cake as large as possible. The largest that the smallest piece of cake can be is half the size of the cake, lest it becomes the largest piece of cake. Therefore, Rowena will split the cake exactly in half, and Colin will take either of the two evenly sized slices.

*I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, Ryan Mather, and Tyler Welch. I have also discussed problem(s) with Emily Case, Sarah Bass, and Danny Edgel.

2 Question 2



From the above game tree, we can deduce the subgame perfect equilibrium by backwards induction. If the project occurs and does not fail, then the lender does not have any reason to repay their investments. They are strictly better off by not repaying. So, they will not repay. Therefore, when the investor looks at the game tree in front of them, they see an expected utility from investing of 0, and an expected utility from not investing of a. They will choose not to invest. So, the subgame perfect equilibrium is for the investor to not invest.

3 Question 3

3.1 Part A

For $n = 1$, once the coin is flipped then the offer should be accepted, as their payoff for accepting the offer is (weakly) greater than their payoff for rejecting the offer (0). Therefore, whomever wins the coin toss knows that their offer will be accepted, so they should offer 0 to the other person and 1 for themselves. Each person thus has a probability 1/2 to win the coin flip, and a payoff of 1 if they win the coin flip and 0 if they lose, so their expected utility is 1/2.

3.2 Part B

For $n = 2$, if the offer is declined in the first round then the expected payoff of each person is $\delta(1/2)$. Therefore, in the first round, the person that loses the coin toss will accept any offer (weakly) greater than $\delta(1/2)$. The person who wins the coin toss can gain $1 - \delta(1/2) > \delta(1/2)$ if they offer $\delta(1/2)$ to the second person so they will offer that amount and the second person will accept. Their expected utility, therefore, is $(1/2)(1 - \delta(1/2)) + (1/2)(\delta(1/2)) = 1/2$.

3.3 Part C

For $n \geq 3$, we work backwards from the last period. Their expected utility of entering into that last period is $\delta^{n-1}(1/2)$. In the period prior, the discounted expected utility of entering into that last period is the bid that would be accepted, so $\delta^{n-2}(1/2)$ is the lowest amount that the person who lost the coin toss would accept. In each period proceeding backwards, the accepted bids proceeds in the same fashion, with one fewer exponent of the geometric decay factor at a time, until the first period. In this first period, following the trend, the discounted expected utility of entering into that last period is $\delta(1/2)$, and so this is the lowest amount that the individual that loses the coin toss will accept, and the payoffs thus are the same as iin Part B, $1 - \delta(1/2)$ for the winner of the coin toss and $\delta(1/2)$ for the loser.

4 Question 4

The two coffee shops will choose locations to maximize revenue, given the location of the other firm and their pricing functions. We can work backwards to find the subgame perfect nash equilibrium.

Consumers are indifferent between coffee shop 1 and 2 if $c(w-x_1)^2 p_i = c(w-x_2)^2 + p_j$. If $x_1 \neq x_2$,

$$
\begin{aligned}
c(w^2 - 2x_1 w + x_1^2) + p_1 &= c(w^2 - 2x_2 w + x_2^2) + p_2 \\
\Rightarrow w^* &= \frac{c(x_2^2 - x_1^2) + p_2 - p_1}{2c(x_2 - x_1)}
\end{aligned}
$$

Without loss of generality, let coffee shop 1 in this case be the coffee shop that is (weakly) to the left of the other coffee shop.

Coffee shop 1 then serves consumers to the left of w^* and coffee shop 2 serves consumers to the right of w^*. The quantity of customers for either firm is then $q_1 = \frac{c(x_2^2-x_1^2)+p_2-p_1}{2c(x_2-x_1)}, q_2 = 1 - \frac{c(x_2^2-x_1^2)+p_2-p_1}{2c(x_2-x_1)}$. If the locations chosen are the same, then the cheaper firm will serve everyone. In this case, the pricing decision would be to pick a lower price than the other firm, so both firms will choose a price of 0. For the moment, we will return to the other case.

For Firm 1, they choose prices to maximize revenue (we assume cost is 0), taking x_1, x_2, and p_2 as given.

$$\begin{aligned}
&\max_{p_1} p_1 \frac{c(x_2^2-x_1^2)+p_2-p_1}{2c(x_2-x_1)} \\
&\Rightarrow \frac{c(x_2^2-x_1^2)+p_2-2p_1}{2c(x_2-x_1)} = 0 \\
&\Rightarrow p_1^* = \frac{c(x_2^2-x_1^2)+p_2}{2}
\end{aligned}$$

Firm 2 chooses prices to maximize revenue, taking x_1, x_2, and p_1 as given.

$$\begin{aligned}
&\max_{p_2} p_2 - p_2 \frac{c(x_2^2-x_1^2)+p_2-p_1}{2c(x_2-x_1)} \\
&\Rightarrow \frac{c(x_2^2-x_1^2)+2p_2-p_1}{2c(x_2-x_1)} = 1 \\
&\Rightarrow c(x_2^2-x_1^2)+2p_2-p_1 = 2c(x_2-x_1) \\
&\Rightarrow p_2^* = \frac{2c(x_2-x_1)-c(x_2^2-x_1^2)+p_1}{2}
\end{aligned}$$

Both firms anticipate that the other firms are best responding, so:

$$\begin{aligned}
p_1^* &= \frac{c(x_2^2-x_1^2)+p_2^*}{2} \\
&= \frac{c(x_2^2-x_1^2)+\frac{2c(x_2-x_1)-c(x_2^2-x_1^2)+p_1^*}{2}}{2} \\
\Rightarrow p_1^* &= \frac{c(x_2^2-x_1^2)}{3} + \frac{2c(x_2-x_1)}{3} \\
\Rightarrow p_2^* &= \frac{2c(x_2-x_1)-c(x_2^2-x_1^2)+\frac{c(x_2^2-x_1^2)}{3}+\frac{2c(x_2-x_1)}{3}}{2} \\
\Rightarrow p_2^* &= \frac{4c(x_2-x_1)}{3} - \frac{c(x_2^2-x_1^2)}{3}
\end{aligned}$$

Now, we move backwards into the location decision.

Firm 1 chooses location to maximize revenue, taking as given the location of the other firm and the price decisions:

$$\max_{x_1} p_1^* \frac{c(x_2^2 - x_1^2) + p_2^* - p_1^*}{2c(x_2 - x_1)}$$
$$\Rightarrow \max_{x_1} \left(\frac{c(x_2^2 - x_1^2)}{3} + \frac{2c(x_2 - x_1)}{3} \right) \frac{c(x_2^2 - x_1^2) + \frac{4c(x_2-x_1)}{3} - \frac{c(x_2^2-x_1^2)}{3} - \left(\frac{c(x_2^2-x_1^2)}{3} + \frac{2c(x_2-x_1)}{3} \right)}{2c(x_2 - x_1)}$$
$$\Rightarrow \max_{x_1} \left(\frac{c(x_2^2 - x_1^2)}{3} + \frac{2c(x_2 - x_1)}{3} \right) \frac{\frac{2c(x_2-x_1)}{3} + \frac{c(x_2^2-x_1^2)}{3}}{2c(x_2 - x_1)}$$
$$\Rightarrow \max_{x_1} \frac{1}{3} \left(\frac{c(x_2^2 - x_1^2)}{3} + \frac{2c(x_2 - x_1)}{3} \right) + \frac{(x_2^2 - x_1^2)}{6(x_2 - x_1)} \left(\frac{c(x_2^2 - x_1^2)}{3} + \frac{2c(x_2 - x_1)}{3} \right)$$
$$\Rightarrow \max_{x_1} \frac{2c(x_2^2 - x_1^2)}{9} + \frac{2c(x_2 - x_1)}{9} + \frac{c(x_2^2 - x_1^2)^2}{18(x_2 - x_1)}$$

Firm 2 chooses location to maximize revenue, taking as given the location of the other firm and the price decisions:

$$\max_{x_2} p_2^* - p_2^* \frac{c(x_2^2 - x_1^2) + p_2^* - p_1^*}{2c(x_2 - x_1)}$$
$$\Rightarrow \max_{x_2} \left(\frac{4c(x_2 - x_1)}{3} - \frac{c(x_2^2 - x_1^2)}{3} \right) \frac{c(x_2^2 - x_1^2) + \frac{4c(x_2-x_1)}{3} - \frac{c(x_2^2-x_1^2)}{3} - \frac{c(x_2^2-x_1^2)}{3} - \frac{2c(x_2-x_1)}{3}}{2c(x_2 - x_1)}$$
$$\Rightarrow \max_{x_2} \frac{1}{3} \left(\frac{4c(x_2 - x_1)}{3} - \frac{c(x_2^2 - x_1^2)}{3} \right) + \frac{(x_2^2 - x_1^2)}{6(x_2 - x_1)} \left(\frac{4c(x_2 - x_1)}{3} - \frac{c(x_2^2 - x_1^2)}{3} \right)$$
$$\Rightarrow \max_{x_2} \frac{4c(x_2 - x_1)}{9} + \frac{c(x_2^2 - x_1^2)}{9} - \frac{c(x_2^2 - x_1^2)^2}{18(x_2 - x_1)}$$

What we see here is that, fixing the positions of one of the firms, the optimal choice of the other firm is to move to the furthest corner. The knife-edge case is that one of the firms picks the middle, 0.5, in which case the firm would be indifferent on which corner to move to. The firms would then choose to be in the corners. Locating in opposite corners, then, are the nash equilibria. With these locations, with one firm at 0 and the other at 1, our prices are the same: $p_1 = p_2 = c$. Therefore, the equilibria are $\{((1,c),(0,c)),((0,c),(1,c))\}$