Micro Exam Notesheet

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1 Rationalizability

- A rational player will never play a strictly dominated strategy. A rational player may still play a weakly dominated strategy (context-dependent).
- A k-rationalizable strategy survives k rounds of iterated strict dominance.
- In a two-player game, a strategy survives iterated strict dominance if and only if it is rationalizable.

2 Nash Equilibrium

- A strategy profile is a nash equilibrium if everyone is best-responding to everyone else.
- 1: Eliminate pure strategies which are not rationalizable.
- 2: For each strategy profile whose supports survive step 1, check if for all $i \in P, \sigma_i \in B_i(\sigma_{-i})$.
- 1': Using ISD, eliminate all strategies which are not rationalizable.
- 2': Find all "closed rationalizable cycles"
- 3: Look for Nash Equilibria on the support of each cycle.
  - For pure strategy profiles, just check for pure best response cycles.
  - For mixed strategies, solve using indifference between all pure strategies in the support.

3 Nash Equilibrium - continuum of players

- With a continuum of players, we don't need to worry about how each player is choosing each strategy to play. We only need to choose the randomization probabilities to make everyone indifferent between all strategies being played.

- Large timing games: continuum of identical players on unit interval, measure $Q(t)$ of players stopping at time $\tau \leq t$ is the quantile function. Payoffs are a function of stopping time t, and the stopping quantile q. A Nash equilibrium is a quantile function Q whose support contains only maximum payoffs, where quantile $q = Q(t)$ stops at time t.

4 Supermodular and Submodular games

- The game $(S_1, \ldots, S_n; u_1, \ldots, u_n)$ is a supermodular game if $\forall i \in \{1, \ldots, n\}$, S_i is a compact subset of $\mathbb{R}$, u_i is upper semi-continuous in s_i, s_{-i}, and u_i has increasing differences in s_i, s_{-i}.
- Apply topkis: each player's best response is increasing in the actions of other players.
- Thm (maximum and minimum eqm) Consider a supermod game with continuous payoff functions on a compact domain for all individuals. Then there exists a maximum and minimum equilibrium.
- $f(x, \theta)$ has decreasing differences if $f(x, -\theta)$ has increasing differences. A submodular game is one whose payoffs have decreasing differences for all individuals.

5 Bayesian Nash Equilibrium

- An agent knows their type, but not their opponents' types. It is important to set up the expected utility, then follow the steps to solve.
- Player i knows their own type, so the condition probability of the joint type vector is $p(\theta_{-i}, \theta_i)$.
- Each player optimizes conditional on their own type, and knowing the strategy profiles of all other players (as functions of their types). Beliefs are based on the prior probabilities p and updated using Bayes rule.
- Steps to solve:
  - Write down utility as a function of b_i, b_j, with your type (valuation) given.
  - Write down expected utility by assuming that the other player's bid is a function of their valuation ($b_j = b(v_j)$). Make sure to rearrange objects such as $P(b(v_j) < b_i)$ as $P(v_j < b^{-1}(b_i))$ and then integrate over v_j appropriately to calculate expectation.
  - If integral is not possible, can keep in integral form and apply leibniz differentiation later.
  - Take first-order conditions.

    - $\frac{\partial}{\partial b_i} b^{-1}(b_i) = \frac{1}{b'(b^{-1}(b_i))}$
    - $\frac{\partial}{\partial b_i} \int_0^{b^{-1}(b_i)} (v_i - b(v_j))dv_j = \frac{(v_i - b(b^{-1}(b_i)))}{b'(b^{-1}(b_i))}$
  - Plug in form of linear reaction function (if that is the type of bid function we are asked to assume/find). Specifically, plug in derivative of b where necessary, but do not necesssarily plug in linear form for b_i in all cases (can solve then for $b_i = (RHS)$ and compare original form with (RHS) to solve for parameters that hold for all v_i).
  - Solve for parameters via first order conditions. If approach before does not hold, another option is to plug in $v_i = 1, v_i = 0$ and solve 2 equations in 2 unknowns.

6 Correlated Equilibria

- A randomizing device privately suggests to each player which strategy to play. She does not know which strategies of her opponents are suggested. However, the distribution over outcomes is known, so each agent can update their beliefs via bayes rule.
- If a joint distribution is such that every agent has no incentive to disobey the suggestion, then it defines a correlated equilibrium.

7 Knowledge and Common Knowledge

- We have a set of finitely many states of nature with prior probability $p > 0$. Player $i's$ information set is a partition of the overall set, where i cannot distinguish between any two states within any section of the partition.
- Whenever somebody knows E then some state of E is the true state (the decision-maker does not know anything that is false).
- Event E is mutual knowledge at ω if $\omega \in K_I(E), \bigcap_{i \in I} K_I(E)$. "Everyone knows E."
- Event E is common knowledge at ω if $\omega \in K_I^n(E) \forall n = 1, 2, \ldots, m, \ldots$. "Everyone knows that everyone knows that ... everyone knows event E"
- Theorem (Aumann(1976)) If two people have the same priors, and their posteriors for an event E are common knowledge, then these posteriors are equal.

8 Extensive form game, perfect information

- Each player, when choosing an action, knows all actions chosen previously, and moves alone.

- Solve: Zermelo Algorithm. Start at the end of the tree, and work back up the tree by solving for optimal behavior at each node (Backwards Induction).
- Finite games w/ perfect information w/ finite # nodes, Zermelo Algorithm yields a Nash Equilibrium outcome.

9 Extensive-form games

- If a game is played a finite number of times, then it can be expressed as a perfect-information extensive-form game and solved via backwards induction. Having the game instead be played infinitely (or having the end of the game be stochastic) prevents us from doing this and admits subgame perfect equilibria which otherwise would not exist. (e.g., repeated prisoners' dilemma).
- Consider minmax value $\underline{v}_i = \min_{\alpha_{-i}} \max_{\alpha_i} u_i(\alpha_i, \alpha_{-i})$.
- The set of strictly individually rational payoffs is then given by $F^* = \{v \in F : v_i \geq \underline{v}_i \forall i \in P\}$
- If there are exactly 2 players, and no two players have identical preferences (up to affine transformations), then for all δ close enough to 1, there is a subgame perfect equilibrium with payoffs $v \in F^*$.
- In this class we will generally assume pure strategies, 2 players, and stick-and-carrot strategies used to support the higher payoffs.
- Useful property (Finite punishment periods): $\sum_{t=1}^{L} a\delta^t = \frac{a\delta(1-\delta^L)}{1-\delta}$ (can derive by manipulating infinite geo sums).

10 Repeated games

- A subgame is a subest of a game that contains a decision node and all subsequent edges and nodes without tearing information sets.
- We require that players form beiliefs about where they are in each of their information sets, and then require that behaviors are optimal under these beliefs at each information set.
- if the assessment (actions, beliefs) is optimal for all players at all information sets, then the action set is sequentially rational given the set of beliefs.
- The strategy-beliefs pair (β, μ) is a WEAK sequential equilibrium if μ is bayesian given β, and β is sequentially rational given μ.
- Weak sequential equilibrium places no restrictions on beliefs in unreached information sets, which may lead to unreasonable predictions.

- μ is consistent given β if there exists a sequence of completely mixed strategy profiles $\{\beta^k\}_{k=1}^{\infty}$ such that $\lim_{k\to\infty} \beta^k = \beta$ and $\lim_{k\to\infty} \mu^k = \mu$
- Consistency forces players to entertain "correct" beliefs even in the unreached information sets.
- The strategy-beliefs pair (β, μ) is a sequential equilibrium if μ is consistent given β, and if β is sequentially rational given μ.

11 Absent-minded-type games

- Find pure strategy equilibria (where will you end up if you always go straight/always turn off?)
- Calculate expected payoff from mixing, and maximize with respect to the mixing probability.
- Note: this is calculated from the "bird's eye view" with a naive agent blindly following the mixing strategy.
- Time-consistent belief version: your likelihood of being in a set is dependent on your mixing probability.
- Let α be your chance of mixing. (Given absent-minded driver setup:) You have a $\frac{1}{2-\alpha}$ chance of being in the first node when you turn/don't turn, and a $\frac{1-\alpha}{2-\alpha}$ chance of being in the second note when you turn/don't.
- Set up expected payoffs: $\max_\beta \frac{1}{2-\alpha}$(payoff (bird's eye) of mixing conditional on currently being in the $\frac{1}{2-\alpha}$(payoff (bird's eye) of mixing conditional on currently being in the second state, w/ turning prol
- Take FOC wrt β, and in the FOC impose $\beta = \alpha$.

12 Signaling games

- Sender receives a private signal (his type) from nature and chooses a message. The receiver sees the signal but does not know which type sent the signal.
- Weak sequential equilibria apply naturally in this environment, and are equivalent to sequential equilibria in this setting.
- Separating eq'm: different types choose different actions. Pooling: all types choose the same actions.
- "intuitive criterion" - (definition)

13 Bayesian persuasion

- Verifiable free communication: Signal-sender will send the signal to maximize their own utility
- Send signal with frequencey that makes signal receiver indifferent in their actions
- If sender is mixing in some case, remember that they, too, must be indifferent in those cases - since receiver is indifferent they can mix such that sender is also indifferent.