Macro PS3

Michael B. Nattinger*

November 30, 2020

1 Question 1

We will construct the sequential market structure equilibrium. Each period, we our bond markets contain claims to each "tree".

1.1 Part A

Define q_t^i as the price in period t of a consumption good in period $t+1$ on the condition that consumer i receives an endowment in period $t+1$. $b_t^{i,j}$ is the quantity of that bond demanded by person j.

Each agent maximizes expected utility:

$$\max_{\{c_t^1, b_t^{1,1}, b_t^{2,1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t log c_t^1 \tag{1}$$

$$\text{s.t. } c_t^1 + q_t^1 b_t^{1,1} + q_t^2 b_t^{2,1} \leq e_t^1 + b_{t-1}^{1,1} 1\{e_t^1 = 1\} + b_{t-1}^{2,1} 1\{e_t^2 = 1\}$$

$$\max_{\{c_t^2, b_t^{1,2}, b_t^{2,2}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t log c_t^2 \tag{2}$$

$$\text{s.t. } c_t^2 + q_t^1 b_t^{1,2} + q_t^2 b_t^{2,2} \leq e_t^2 + b_{t-1}^{1,2} 1\{e_t^1 = 1\} + b_{t-1}^{2,2} 1\{e_t^2 = 1\}$$

Market clearing implies the following conditions:

$$b_t^{1,1} + b_t^{1,2} = 0 \tag{3}$$
$$b_t^{2,1} + b_t^{2,2} = 0 \tag{4}$$
$$c_t^1 + c_t^2 = e_t^1 + e_t^2 \tag{5}$$

The competitive equilibrium is a set of prices $\{q_t^1, q_t^2\}_{t=0}^{\infty}$ and allocations $\{b_t^{1,1}, b_t^{2,1}, b_t^{1,2}, b_t^{2,2}, c_t^1, c_t^2\}_{t=0}^{\infty}$ such that agents optimize (1),(2) and markets clear (3),(4),(5).

---

*I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, and Ryan Mather. I have also discussed problem(s) with Emily Case, Sarah Bass, and Danny Edgel.

1.2 Part B

Our bellman equation takes the following form:

Once the future endowment has shifted, it will remain shifted forever. So,

1.3 Part C

2 Question 2

3 Question 3

3.1 Part A

Our bellman equation takes the following form:

$$\begin{aligned}
V(a,l) &= \max_{a'} \frac{(wl+(1+r)a-a')^{1-\gamma}}{1-\gamma} + \beta E[V(a',l')] \\
&= \max_{a'} \frac{(wl+(1+r)a-a')^{1-\gamma}}{1-\gamma} + \beta(V(a',l_h)P(l'=l_h|l) + V(a',l_h)P(l'=l_h|l))
\end{aligned}$$

Taking FOCs and applying the envelope conditions,

$$\begin{aligned}
(wl+(1+r)a-a')^{-\gamma} &= \beta(V'(a',l_h)P(l'=l_h|l) + V'(a',l_h)P(l'=l_h|l)) \\
V'(a,l) &= (1+r)(wl+(1+r)a-a')^{-\gamma} \\
\Rightarrow c^{-\gamma} &= \beta(1+r)((c'_h)^{-\gamma}P(l'=l_h|l) + (c'_l)^{-\gamma}P(l'=l_h|l))
\end{aligned}$$

The above equation forms our optimality conditions.

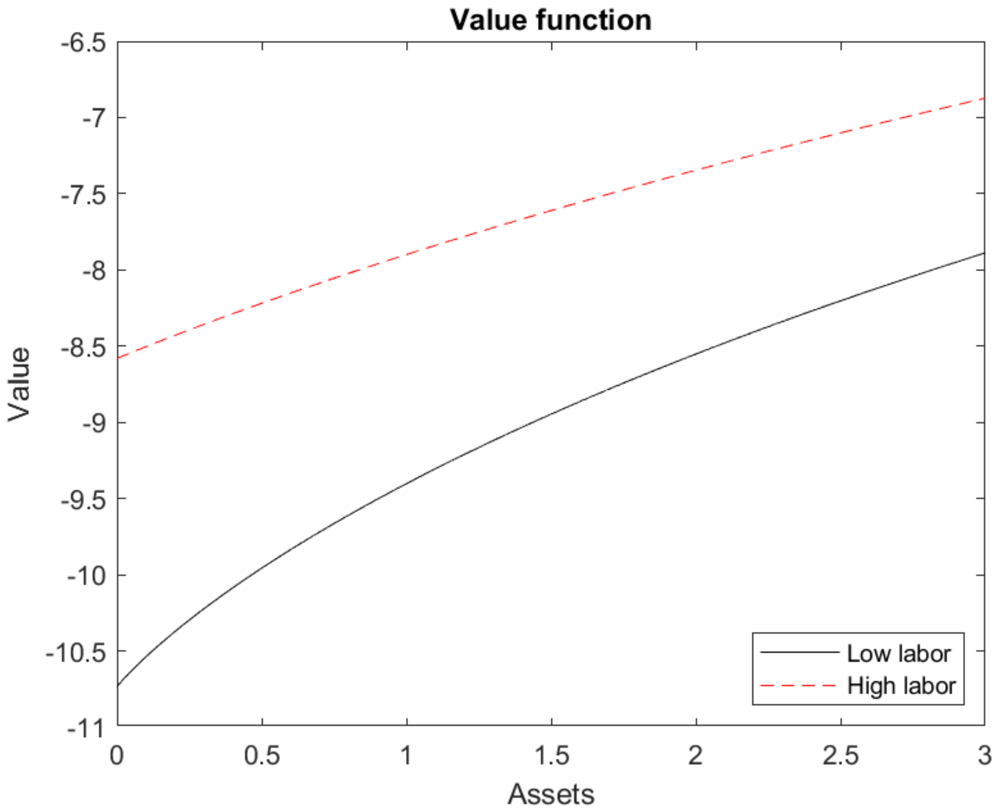
3.2 Part B

In the stationary distribution, $PQ = P$ where $P = [P_l P_h]$. We can easily solve for this distribution numerically from an arbitrary starting point. As this is a very simple markov process, markov-chain monte carlo methods will converge to the stationary distribution by standard ergodic properties which Q easily satisfies. We can therefore start with an arbitrary initial distribution P_0 and iterate through the transition matrix Q, with product $P := P_0 Q$ becoming next iteration's P_0, until P and P_0 have converged to some tolerance.

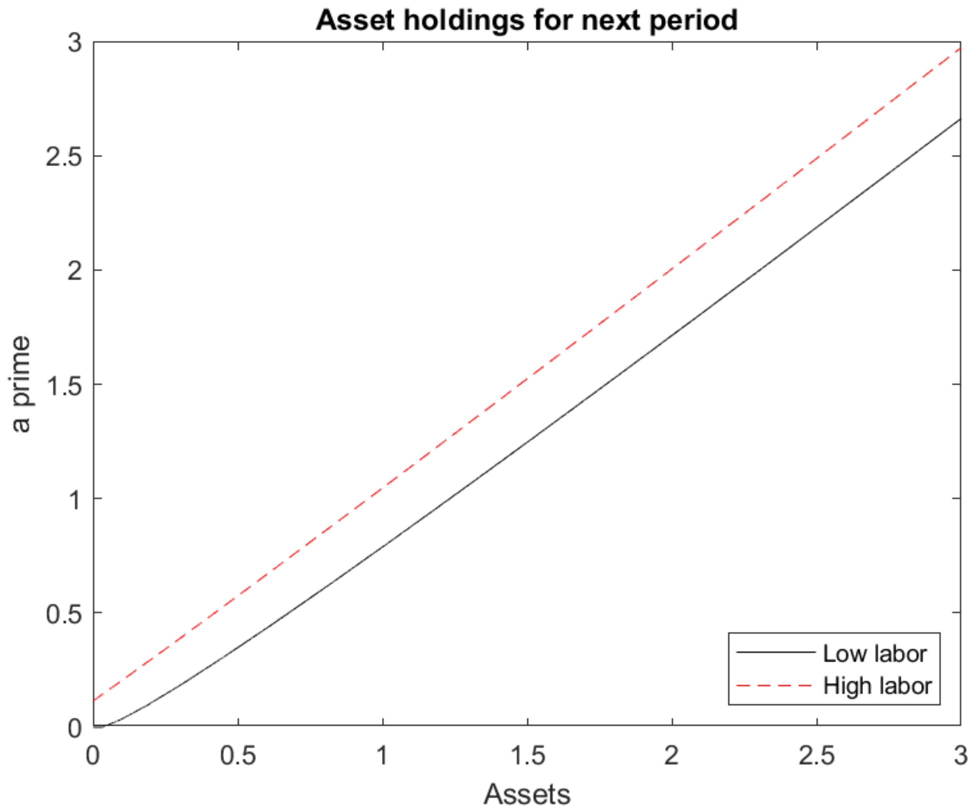
The numerical solution is $P = [0.250.75]$ so the stationary distribution has 3/4 of the weight on the high-labor distribution, and the rest on the low-labor distribution. Therefore, the unconditional mean of the labor endowment is $0.25(0.7) + 0.75(1.1) = 1$.

3.3 Part C

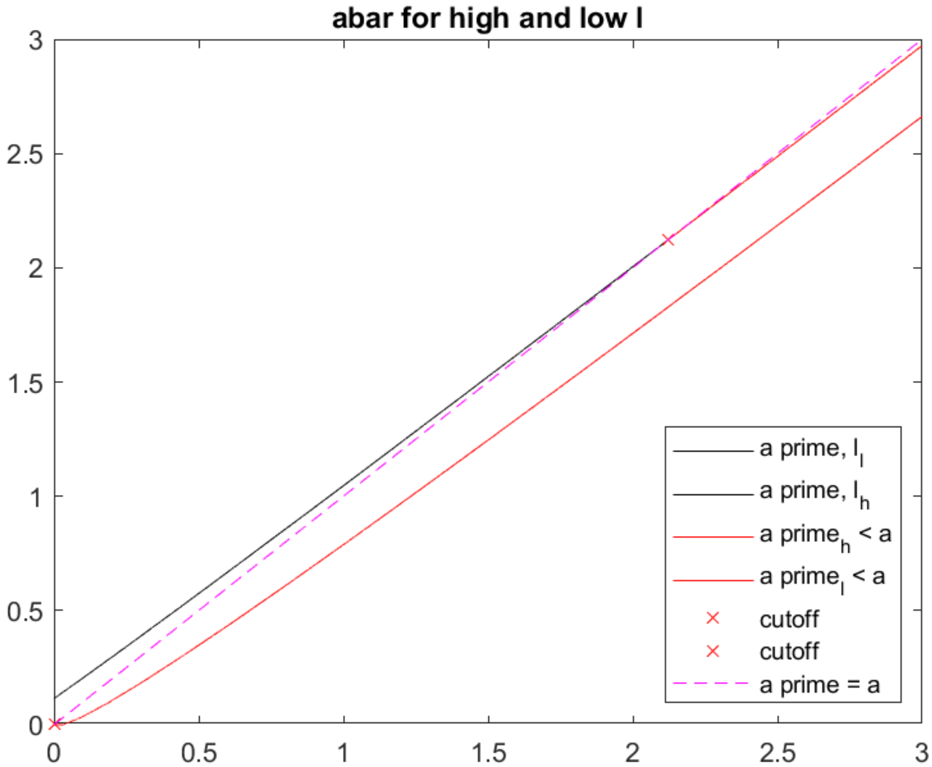
I solved for the value function numerically. Results are plotted below.



The value function solution does appear to be continuous, increasing, concave, and differentiable. The value from having a high labor draw is higher than the value from having a low labor draw. All of these features are as predicted by theory.

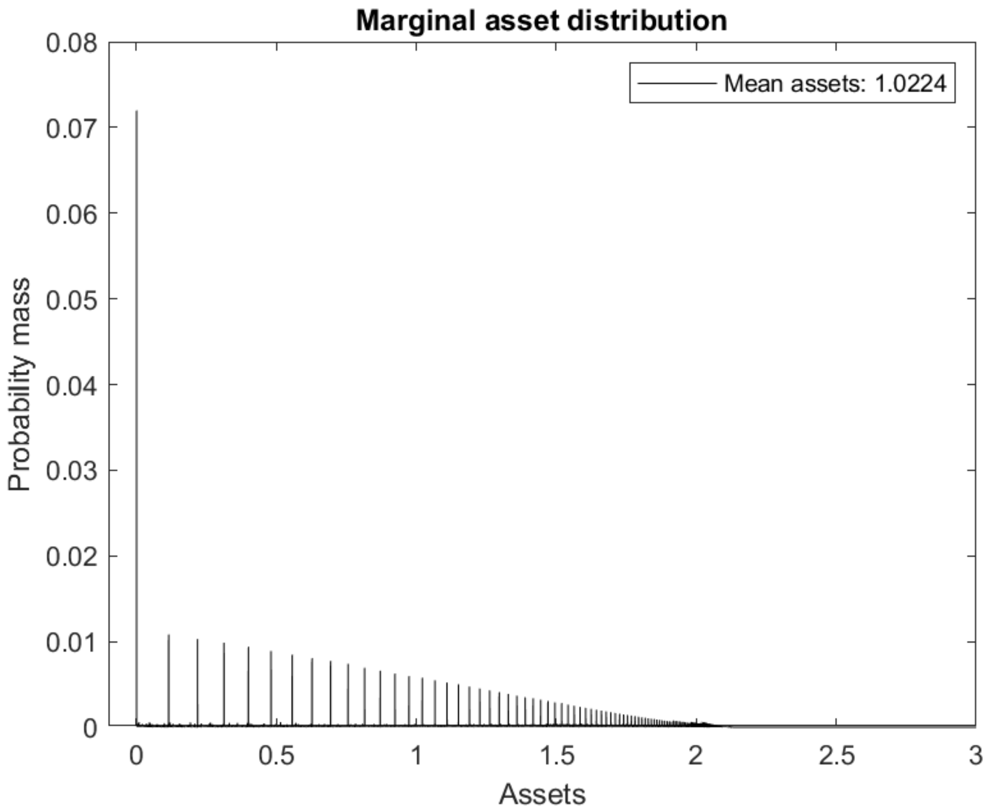


Above we see the asset holding decision rules for the tro labor shocks. We can see the value $\bar{a}$ such that $a' < a \forall a \geq \bar{a}$ on the graph that follows.



Above we see the value $\bar{a}$ such that $a' < a \forall a \geq \bar{a}$. Since a' is decreasing such that $\bar{a}$ exists and is finite, the long term asset holding decisions are finite and will converge to a level $\leq \bar{a}_h$ where $\bar{a}_h$ is the highest $\bar{a}$ level across labor shocks.

3.4 Part D



Above we see the marginal asset distribution. It has some interesting features. First, it has a large mass at 0. This is because drawing a low labor level at a low asset position results in no savings so there is a large proportion of individuals that end up having no savings. Next we can see ridges of high density pop up at regular intervals. Each such ridge represents the savings of those that were at the previous ridge in the previous turn, and received a high labor draw, with the first ridge being asset savings of 0.