Micro HW6

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1 Question 1

1.1 Part A

For the first three data points, $p \cdot x = 100$, and for the final data point $p \cdot x = 150$. Thus the data is consistent with Walras law.

1.2 Part B

$x^3 > x^4 > \{x^1, x^2\}$ which means that x^3 was not affordable at p^4, p^2, p^1. We also know that $\{x^1, x^2, x^3\}$ were affordable at p^3 but not chosen, so $x^3 \succ \{x^1, x^2, x^4\}$ and there were no conflicting revealed preferences. By similar logic $x^4 \succ \{x^1, x^2\}$ with no conflicting revealed preferences. Now, $p^1 \cdot x^2 = 85 < 100$ so $x_1 \succ x_2$. $p^2 \cdot x^1 = 120 > 100$ so we have no conflicting revealed preferences. Thus, $x^3 \succ x^4 \succ x^1 \succ x^2$ so GARP is satisfied, and the data is rationalizable.

2 Question 2

2.1 Part A

Using Roy's Identity, $x^i(p, w_i) = -\frac{\partial v^i}{\partial p} / \frac{\partial v^i}{\partial w_i} = -\frac{a_i'(p)+b'(p)w_i}{b(p)}$.

2.2 Part B

Again, applying Roy's identity: $X(p, W) = -\frac{\partial V}{\partial p} / \frac{\partial V}{\partial W} = -\frac{\sum_{i=1}^n (a_i'(p))+b'(p)W}{b(p)} = -\frac{\sum_{i=1}^n (a_i'(p)+b'(p)w_i)}{b(p)} = \sum_{i=1}^n -\frac{a_i'(p)+b'(p)w_i}{b(p)}$

*I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, Ryan Mather, and Tyler Welch. I have also discussed problem(s) with Emily Case, Sarah Bass, and Danny Edgel.

3 Question 3

3.1 Part A

Let preferences be homothetic. Then, for any $x(p, tw) = \arg\max_{x \in B(p,tw)} u(x)$, $x(p, w) = \arg\max_{x \in B(p,w)} u(x)$, notice that $p \cdot x(p, w) \leq w \Rightarrow p \cdot tx(p, w) \leq tw$ so $tx(p, w)$ is affordable for wealth level tw. Next, let $ty \in B(p, tw)$ be arbitrary. Then, note that $y \in B(p, w) \Rightarrow x(p, w) \succeq y \Rightarrow tx(p, w) \succeq ty$ so $tx(p, w) = \arg\max_{x \in B(p,tw)} u(x) = x(p, tw)$

3.2 Part B

We will use the same utility representation as in class, where $u(x) = \alpha$ is the value of α for which $x \sim (\alpha, \ldots, \alpha)$. We showed in class that, if preferences were continuous and monotone, $u(x)$ represents the preference relation. We will now show that this is homogenous of degree 1 for homothetic preferences:

Let $u(x) = \alpha$. This means that $x \sim (\alpha, \ldots, \alpha) \Rightarrow tx \sim t(\alpha, \ldots, \alpha)$ because $tx \succeq t(\alpha, \ldots, \alpha), t(\alpha, \ldots, \alpha) \succeq tx$. Thus, $u(tx) = t\alpha$ so $u(.)$ is homogenous of degree 1.

3.3 Part C

$v(p, w) = u(x(p, w)) = wu(x(p, 1)) = wb(p)$ where $b(p) = u(x(p, 1))$.

4 Question 4

4.1 Part A

Due to our utility function strictly increasing in x_1, our preferences are LNS so our budget constraint will hold with equality, $w = x_1 + \sum_{i=2}^{k} p_i x_i \Rightarrow x_1 = w - \sum_{i=2}^{k} p_i x_i$. We then have the following:

$$
\begin{aligned}
X(p, w) &= \arg\max_{x \in B(p,w)} u(x) = \arg\max_{x \in B(p,w)} x_1 + U(x_2, \ldots, x_k) = \arg\max_{x} w - \sum_{i=2}^{k} p_i x_i + U(x_2, \ldots, x_k) \\
&= \arg\max_{x} - \sum_{i=2}^{k} p_i x_i + U(x_2, \ldots, x_k) = x_{2,\ldots,k}(p),
\end{aligned}
$$

so Marshallian demand for goods $2 - k$ does not depend on wealth.

4.2 Part B

$$v(p,w) = u(X(p,w)) = u\left(\left(\left(\left(w - \sum_{i=2}^{k} p_i x_i\right) \; x_{2,\ldots,k}^T(p)\right)^T\right)\right.$$
$$= w - \sum_{i=2}^{k} p_i x_i + U(X_{2,\ldots,k}) = w - g(X_{2,\ldots,k}) + U(X_{2,\ldots,k})$$
$$= w + \tilde{v}(X_{2,\ldots,k})$$

4.3 Part C

$$e(p,u) = \min_{u(x)\leq u} p \cdot x = \min_{u(x)\leq u} x_1 + \sum_{i=2}^{k} p_i x_i.$$

If the constraint were to hold without equality, we could reduce our spending on x_1 until the constraint held with equality and reduce costs. Thus, the constraint must hold with equality. Thus, $u = x_1 + U(x_2,\ldots,x_k) \Rightarrow x_1 = u - U(x_2,\ldots,x_k)$. Plugging this constraint into the objective function,

$$e(p,u) = \min_x u - U(x_2,\ldots,x_k) + \sum_{i=2}^{k} p_i x_i = u - \min_x -U(x_2,\ldots,x_k) + \sum_{i=2}^{k} p_i x_i$$
$$= u - f(p).$$

4.4 Part D

$$h(p,u) = \arg\min_{u(x)\leq u} p \cdot x = \arg\min_{u(x)\leq u} x_1 + \sum_{i=2}^{k} p_i x_i = \arg\min_x u - U(x_2,\ldots,x_k) + \sum_{i=2}^{k} p_i x_i$$
$$= \arg\min_x -U(x_2,\ldots,x_k) + \sum_{i=2}^{k} p_i x_i = h(p).$$

4.5 Part E

Compensating variation is $\int_{p_i^1}^{p_i^0} h_i(p,u^0)dpi = \int_{p_i^1}^{p_i^0} h_i(p)dpi = \int_{p_i^1}^{p_i^0} h_i(p,u^1)dpi$, which is equivalent variation. Consumer surplus is $\int_{p_i^1}^{p_i^0} x_i(p,w)dpi = \int_{p_i^1}^{p_i^0} x_i(p)dpi = \int_{p_i^1}^{p_i^0} h_i(p)dpi$ as both Hicksian and Marshallian demand for good i are functions only of price, so at each price they must be equal.