Micro HW1

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1 Question 1

The first firm to enter the market will be the firm with the lowest fixed cost, i.e. firm 1. This firm will choose to enter when profits are nonnegative:

$$pq - 1 - q - q^2 \geq 0$$

The firm maximizes profit:

$$\begin{aligned} &\max_q pq - 1 - q - q^2 \\ &\Rightarrow p = 1 + 2q \\ &\Rightarrow q = \frac{p-1}{2} \end{aligned}$$

Our nonnegative profit condition then yields:

$$\begin{aligned} \frac{p^2 - p}{2} - 1 - \frac{p-1}{2} - \left(\frac{p-1}{2}\right)^2 &\geq 0 \\ p^2 - 2p - 1 - \frac{p^2 - 2p + 1}{2} &\geq 0 \\ p^2 - 2p - 3 &\geq 0 \\ (p-3)(p+1) &\geq 0 \end{aligned}$$

The p^* which is the minimum price to have production is the positive price which satisfies the above expression with equality. Therefore, $p^* = 3$.

*I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, Ryan Mather, and Tyler Welch. I have also discussed problem(s) with Emily Case, Sarah Bass, Katherine Kwok, and Danny Edgel.

2 Question 2

2.1 Part A

Given a tax level τ, a firm will choose quantity to produce q which maximies profits, and will enter the market if their profits are nonnegative.

$$\begin{aligned}\max_x (1-\tau)2\theta x - x^2 - 1\\ \Rightarrow (1-\tau)2\theta = 2x\\ \Rightarrow x = (1-\tau)\theta.\end{aligned}$$

The firms enter if their profits are nonnegative:

$$\begin{aligned}(1-\tau)2\theta x - x^2 - 1 \geq 0\\ (1-\tau)^2 2\theta^2 - (1-\tau)^2\theta^2 - 1 \geq 0\\ \theta \geq \frac{1}{1-\tau}\end{aligned}$$

The aggregate supply of the firms is the integral across their distribution. Note that we are provided with a functional form for the mass of firms with ideas above a value of theta. We can convert this to a density by taking the negative of the derivative of this function: $f(\theta) = \beta\theta^{-\beta-1}$. Then, our supply is the following:

$$\begin{aligned}Q_s(\tau) &= \int_{\frac{1}{1-\tau}}^{\infty} 2(1-\tau)\theta^2 f(\theta)d\theta\\ &= \int_{\frac{1}{1-\tau}}^{\infty} 2(1-\tau)\theta^2 \beta\theta^{-\beta-1}d\theta\\ &= \frac{1(1-\tau)\beta}{2-\beta}[\theta^{2-\beta}]_{\frac{1}{1-\tau}}^{\infty}\\ &= \frac{2\beta(1-\tau)^{\beta-1}}{\beta-2}\end{aligned}$$

2.2 Part B

When Apple raises its tax rate, the cutoff moves up (i.e. fewer firms are developers). Furthermore, the amount of code each producer produces falls.

2.3 Part C

Apple maximizes their revenue:

$$\max_{\tau} \tau \frac{2\beta(1-\tau)^{\beta-1}}{\beta-2}$$
$$\Rightarrow \frac{2\beta(1-\tau^*)^{\beta-1}}{\beta-2} = \tau^* \frac{2\beta(1-\tau^*)^{\beta-2}}{\beta-2}(\beta-1)$$
$$\Rightarrow \tau^* = \frac{1}{\beta}.$$

2.4 Part D

If β rises then the firms become more concentrated at lower levels of ideas. Fewer firms produce, and total production falls.

3 Question 3

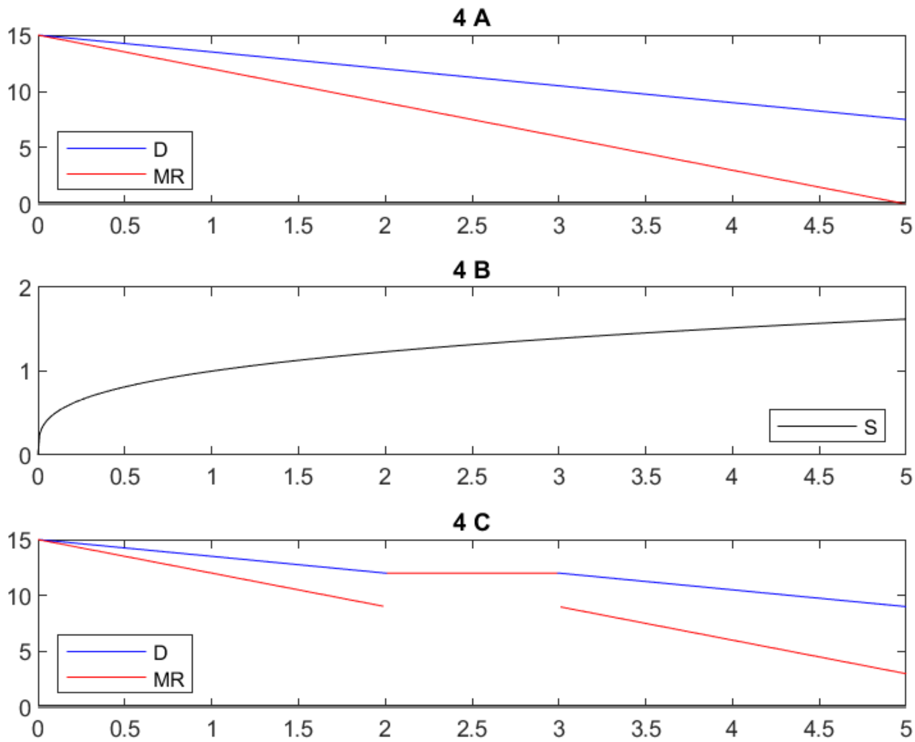
Our two inverse demand curves satisfy $P_i(Q) = a_i - b_i Q$ for $i \in \{3\mathrm{C}, \mathrm{FB}\}$.

Ben and Jerry maximize profits:

$$\max_{Q_i} a_i Q_i - b_i Q_i^2 - MQ_i$$
$$\Rightarrow a_i - 2b_i Q_i - M = 0$$
$$\Rightarrow Q_i = \frac{a_i - M}{2b_i}$$
$$\Rightarrow P_i = \frac{a_i + M}{2}$$

Therefore, Fruit Bowl will be set to the higher price as it has the higher intercept.

4 Question 4



5 Question 5

For the initial part of this question, we have that $P_i(Q_i) = a_i - Q_i$

$$\max_{Q_i} a_i Q_i - Q_i^2 - Q_i$$
$$\Rightarrow a_i - 2Q_i - 1 = 0$$
$$\Rightarrow Q_i = \frac{a_i - 1}{2}$$
$$\Rightarrow P_i = \frac{a_i + 1}{2}$$

This implies that $P_1 = 2, P_2 = 3$. The store can make this happen by offering the higher price online and the lower price in person. Type 1 people will come in to the store and get the lower price, while Type 2 people will order online and be charged the higher price.

When the marginal cost varies by total quantity sold, the two problems become linked by the cost function and, therefore, we have a single optimization to solve.

$$MR = MC,$$
$$3 - 2q_1 = q_1 + q_2,$$
$$5 - 2q_2 = q_1 + q_2.$$

We solve this system: $q_1 = 1/2, q_2 = 3/2, p_1 = 5/2, p_2 = 7/2$.