IO Problem Set 2

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1 Question 1

Note: lots of omitted algebra done by hand on scratch paper.

1.1 Part (a)

We have $Q = \frac{a_0 - P - \nu}{a_1}$. $\epsilon = -\frac{P\partial Q}{Q\partial P} = \frac{P}{Qa_1} = \frac{a_0 - a_1 Q + \nu}{a_1 Q} = \frac{a_0+\nu}{a_1 Q} - 1$. Thus, $\frac{\partial\epsilon}{\partial Q} = -\frac{\nu + a_0}{a_1 Q^2} \leq 0$. Similarly, $\frac{\partial \epsilon}{\partial \nu} = \frac{1}{a_1 Q} \geq 0$. Thus, as Q increases, elasticity of demand decreases; and as ν increases, the elasticity of demand increases.

1.2 Part (b)

We have $c = F + (b_0 + \nu)Q$. The firm takes the quantities of the other firms as given and maximizes profit:

$$\max_q [a_0 - a_1(q + Q_{-i})]q - [F + (b_0 + \eta)q]$$
$$\Rightarrow a_0 - 2a_1 q - a_1 Q_{-i} - b_0 - \eta = 0$$

Applying symmetry, $Q = Nq \Rightarrow q = \frac{a_0+\nu-b_0-\nu}{a_1(N+1)}$. Quantity is $Q = N\frac{a_0+\nu-b_0-\nu}{a_1(N+1)}$; prices are $P = \frac{a_0+\nu+N(b_0+\eta)}{N+1}$.

1.3 Part (c)

Individual firm profits are $Pq - C = \frac{a_0+\nu+N(b_0+\eta)}{N+1}\frac{a_0+\nu-b_0-\nu}{a_1(N+1)} - F - \frac{(b_0+\eta)(a_0+\nu-b_0-\eta)}{a_1(N+1)} = \frac{1}{a_1}\left[\frac{a_0+\nu-b_0-\eta}{N+1}\right]^2 - F$. Firms enter until profits are zero: $0 = \frac{1}{a_1}\left[\frac{a_0+\nu-b_0-\eta}{N+1}\right]^2 - F \Rightarrow N = \frac{1}{\sqrt{Fa_1}}(a_0+\nu-b_0-\eta) - 1$.

1.4 Part (d)

$$\begin{aligned}
L_I = \frac{P - mc}{P} &= \frac{\frac{a_0+\nu+N(b_0+\eta)}{N+1} - (b_0+\eta)}{\frac{a_0+\nu+N(b_0+\eta)}{N+1}} \\
&= \frac{a_0+\nu-b_0-\eta}{a_0+\nu+N(b_0+\eta)} \\
&= \frac{a_0+\nu-b_0-\eta}{a_0+\nu+\left(\frac{1}{\sqrt{Fa_1}}(a_0+\nu-b_0-\eta)-1\right)(b_0+\eta)} \\
&= \frac{\sqrt{Fa_1}}{\sqrt{Fa_1}+b_0+\eta}.
\end{aligned}$$

Firms are identical so the Herfindahl index is $H = \frac{1}{N} = \frac{\sqrt{Fa_1}}{a_0+\nu-b_0-\eta-\sqrt{Fa_1}}$.

$$
\begin{aligned}
\epsilon &= \frac{a_0+\nu-N\left(\frac{a_0+\nu-b_0-\eta}{N+1}\right)}{N\left(\frac{a_0+\nu-b_0-\eta}{N+1}\right)} \\
&= \frac{b_0+\eta+\sqrt{Fa_1}}{a_0+\nu-b_0-\eta-\sqrt{Fa_1}}.
\end{aligned}
$$

1.5 Part (e)

$$
\begin{aligned}
\frac{\partial\epsilon}{\partial F} &= \frac{\sqrt{a_1}(a_0+\nu)}{2\sqrt{F}(a_0+\nu-b_0-\eta-\sqrt{Fa_1})^2} \\
&\geq 0, \\
\frac{\partial\epsilon}{\partial\nu} &= -\frac{b_0+\eta+\sqrt{Fa_1}}{(a_0+\nu-b_0-\eta-\sqrt{Fa_1})^2} \\
&\leq 0, \\
\frac{\partial\epsilon}{\partial\eta} &= \frac{a_0+\nu}{(a_0+\nu-b_0-\eta-\sqrt{Fa_1})^2} \\
&\geq 0.
\end{aligned}
$$

$$
\begin{aligned}
log(L_I) &= (1/2)log(Fa_1) - log(\sqrt{Fa_1}+b_0+\eta), \\
log(H) &= (1/2)log(Fa_1) - log(a_0+\nu-b_0-\eta-\sqrt{Fa_1}).
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial log(L_I)}{\partial F} &= \frac{1}{2F} - \frac{\sqrt{a_1}}{(\sqrt{Fa_1}+b_0+\eta)2\sqrt{F}}, \\
\frac{\partial log(H)}{\partial F} &= \frac{1}{2F} + \frac{\sqrt{a_1}}{(-\sqrt{Fa_1}+a_0+\nu-b_0-\eta)2\sqrt{F}}, \\
&\neq \frac{\partial log(L_I)}{\partial F}. \\
\frac{\partial log(L_I)}{\partial\nu} &= 0, \\
\frac{\partial log(H)}{\partial\nu} &= -\frac{1}{a_0+\nu-b_0-\eta-\sqrt{Fa_1}}, \\
&\neq \frac{\partial log(L_I)}{\partial\nu}. \\
\frac{\partial log(L_I)}{\partial\eta} &= -\frac{1}{\sqrt{Fa_1}+b_0+\eta}, \\
\frac{\partial log(H)}{\partial\eta} &= \frac{1}{a_0+\nu-b_0-\eta-\sqrt{Fa_1}}, \\
&\neq \frac{\partial log(L_I)}{\partial\eta}.
\end{aligned}
$$

Clearly, $log(L_I), log(H)$ do not change at the same rate in responses to F, ν, η.

1.6 Part (f)

Colluding firms have:

$$\begin{aligned}
\max_Q (a_0 - a_1 Q + \nu)Q - (b_0 + \eta)Q - F \\
\Rightarrow a_0 - 2a_1 Q + \nu - b_0 - \eta = 0 \\
\Rightarrow Q = \frac{a_0 + \nu - b_0 - \eta}{2a_1}, \\
\Rightarrow P = (1/2)(a_0 + \nu + b_0 + \eta), \\
\Rightarrow \pi = \frac{(a_0 + \nu - b_0 - \eta)^2}{4Na_1} - F.
\end{aligned}$$

Firms enter until profits are zero $\Rightarrow N = \frac{(a_0+\nu-b_0-\eta)^2}{4Fa_1}$. $L_I = \frac{P-mc}{P} = \frac{a_0+\nu-b_0-\eta}{a_0+\nu+b_0+\eta}$. $H = \frac{1}{N} = \frac{4Fa_1}{(a_0+\nu-b_0-\eta)^2}$. $\epsilon = -\frac{P\partial Q}{Q\partial P} = \frac{a_0+\nu+b_0+\eta}{a_0+\nu-b_0-\eta}$.

1.7 Part (g)

We now have $Q = exp((1/c_1)(c_0 - log(P) + \xi))$. $\epsilon = (P/Q)(exp((1/c_1)(c_0 - log(P) + \xi))) = (1/Q)exp((1/c_1)(c_0 - log(P) + \xi))$.

$$\begin{aligned}
\frac{\partial \epsilon}{\partial Q} &= -\frac{1}{Q^2} exp((1/c_1)(c_0 - log(P) + \xi)) \\
&\leq 0 \\
\frac{\partial \epsilon}{\partial Q} &= (1/Q)exp((1/c_1)(c_0 - log(P) + \chi)) \\
&\geq 0.
\end{aligned}$$

Under the new inverse supply curve, the firm maximizes profit:

$$\begin{aligned}
\max_q exp(c_0 - c_1 log(q + Q_{-i}) + \xi)q - (b_0 + \eta)q - F \\
\Rightarrow exp(c_0 - c_1 log(q + Q_{-i}) + \xi) - exp(c_0 - c_1 log(q + Q_{-i}) + \xi)\frac{qc_1}{Q} - b_0 - \eta = 0
\end{aligned}$$

Applying symmetry across firms,

$$\begin{aligned}
exp(c_0 - c_1 log(Nq) + \xi) = \frac{N}{N - c_1}(b_0 + \eta) \\
\Rightarrow q = \frac{1}{N} exp((1/c_1)(c_0 + \xi - log((N/(N - c_1)(b_0 + \eta))))), \\
Q = exp((1/c_1)(c_0 + \xi - log((N/(N - c_1)(b_0 + \eta))))), \\
P = exp(c_0 - c_1 log(exp((1/c_1)(c_0 + \xi - log((N/(N - c_1)(b_0 + \eta)))))) + \xi), \\
\Rightarrow P = \frac{N}{N - c_1}(b_0 + \eta).
\end{aligned}$$

$$
\begin{aligned}
L_I &= \frac{\frac{N}{N-c_1}(b_0+\eta)-(b_0+\eta)}{\frac{N}{N-c_1}(b_0+\eta)} \\
&= \frac{c_1}{N}. \\
H &= \frac{1}{N}. \\
\epsilon &= (1/Q)exp((1/c_1)(c_0-log(P)+\xi)) \\
&= -\frac{1}{c_1}.
\end{aligned}
$$

In this case, since we are told to keep N as exogenously determined, all of the partial derivatives are zero:

$$\frac{\partial\epsilon}{\partial F}=\frac{\partial\epsilon}{\partial\nu}=\frac{\partial\epsilon}{\partial\eta}=\frac{\partial log(L_I)}{\partial F}=\frac{\partial log(L_I)}{\partial\nu}=\frac{\partial log(L_I)}{\partial\eta}=\frac{\partial log(H)}{\partial F}=\frac{\partial log(H)}{\partial\nu}=\frac{\partial log(H)}{\partial\eta}=0.$$

Therefore, $log(L_I), log(H)$ do change at the same rate in response to changes in $F, \nu, \eta : 0$.

2 Question 2

I computed in Matlab. Results follow.

Figure 1: Regression tables and F test results for price given by equation (3).

	Pooled	Collusion	No Collusion
cons	-0.469***	-0.857***	-0.106***
	(0.0536)	(0.0828)	(0.003)
log(H)	0.543***	0.0714	1***
	(0.0322)	(0.0498)	(0.0018)
F	201	348	0.000646
p(f>F)	0	0	0.98

In our initial regressions, we see that results differ greatly across groups. For cities with no collusion, the coefficient is tightly identified to be near 1. In fact, we cannot reject the F test with the null hypothesis being that this coefficient is exactly 1. With collusion, however, the coefficient is not statistically different from zero, and we can thoroughly reject the null hypothesis that the coefficient is 1. The pooled case, unsurprisingly, is somewhere in the middle, but we do still reject that the coefficient is equal to one.

Note that, without collusion, the H index and true L_I index are identical. This is why the regression coefficients are 1 in this case.

Figure 2: Regression tables and F test results for price given by equation (1).

	Pooled	Collusion	No Collusion
cons	-0.777***	-0.973***	-0.594***
	(0.0255)	(0.0373)	(0.00668)
log(H)	0.294***	0.0568**	0.524***
	(0.0153)	(0.0224)	(0.00402)
F	2.11e+03	1.77e+03	1.4e+04
p(f>F)	0	0	0

With the different price equation, note that we no longer have identical H index and true L_I index. Thus, none of the coefficients are close to 1, and the F test rejects that the true coefficient is 1 for all cases. However, we do still have a positive relationship between H and true L_I. Our formula for $L_I = \frac{a_0+\nu-b_0-\eta}{a_0+\nu+N(b_0+\eta)}$ without collusion compared to $H = \frac{1}{N}$. It is clear that $Cov(log(H), log(L_I)) > 0$ from the functional form.

What can we do that is positive from what we have found? In both cases, the OLS coefficients are higher without collusion than with. If we want to show that a subset of markets features collusion then we can run these regressions on those markets and the magnitude of the coefficient can be indicative of likely presence of collusion. I would hesitate to extrapolate this fact to more general model.

3 Question 3

	$\nu \sim U[-1,1]$	$\eta \sim U[-1,1]$
cons	-0.652***	-2.23***
	(1.5e-08)	(0.0121)
log(H)	-8.57e-17	-1.51***
	(1.37e-08)	(0.011)

Above are the results from the simulated regressions. The differences are quite stark, and in line with theory. Our theory predicts that $H = \frac{\sqrt{Fa_1}}{a_0+\nu-b_0-\eta-\sqrt{Fa_1}}$, $L_I = \frac{\sqrt{Fa_1}}{\sqrt{Fa_1}+b_0+\eta}$. The only differences across markets are in ν in the first case, and in η in the second case. In the first case, as ν varies, H is affected but L_I is unaffected as it is independent of ν. Therefore, we expect to see no statistically significant relationship between H, L_I In the second hand, in contrast, H is increasing in η and L_I is decreasing in η, so we expect a negative sign on the regression coefficient - and our results do show a negative relationshop.

The elasticities are increasing in η and decreasing in ν, so we would expect ϵ would positively correlate with $log(H)$ in both experiments, and negatively correlate with L_I only in the experiment where η varies and ν is fixed.