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Macro Midterm Fall 2020

2-period Commitment model

Question 1

The planner maximizes total utility, subject to the resource constraints (which will hold w/ equality)

$$\max_{c_1, c_2, n, K} \ln(c_1 + c_2) + \ln(1-n)$$

$$\text{s.t.} \quad c_1 + K \leq w$$

$$c_2 + g \leq n + RK$$

From our resource constraints we find:

$$c_1 = w - K, \quad c_2 = n + RK - g$$

$$\Rightarrow \max_{n, K} \ln(w - K + n + RK - g) + \ln(1-n)$$

$$\Rightarrow \max_{n, K} \ln(w + (R-1)K + n - g) + \ln(1-n)$$

FOCs: n: $\Rightarrow w - K + n + RK - g = 1 - n \Rightarrow n = (1 - (R-1)K - w + g)/2$

Because $\ln(c_1 + c_2)$ is how consumption appears in the utility function, consuming in either period is equivalent. The planner will thus set $c_1 = 0$ and $K = w$ to save all resources and get more of the consumption good out of them in the second period.

Then, $n = \frac{(1 - Rw + g)}{2}$ (positive due to parametric assumptions)

Therefore, the planner sets $c_1 = 0$, $c_2 = \frac{1}{2} + \frac{Rw}{2} - \frac{g}{2}$, $n = \frac{1 - Rw + g}{2}$, $K = w$.

Sorry in advance about the mess - don't have time to rewrite clearly.

Case 2: Decentralized Eqm w/ commitment

The government budget constraint is the following:

$$g = \tau n + \delta R K$$

The consumers take τ and δ as given, and g as exogenous.

$$\max_{c_1, c_2, n, K} \ln(c_1 + c_2) + \ln(1-n)$$

$$\text{s.t. } c_1 + K \le W \quad \text{(again these hold with equality)}$$

$$c_2 \le (1-\tau)n + (1-\delta)RK$$

$\Rightarrow c_1 = W - K, \ c_2 = (1-\tau)n + (1-\delta)RK$

$\Rightarrow \max_{n,K} \ln(W + (1-\tau)n + ((1-\delta)R - 1)K) + \ln(1-n)$

FOC w.r.t. n: $\Rightarrow (W + (1-\tau)n + ((1-\delta)R-1)K)/(1-\tau) = 1-n$

$\Rightarrow n = \dfrac{1 - (W + ((1-\delta)R-1)K)/(1-\tau)}{2}$

In this case, the household will choose whether or not to store based on whether or not they get a positive return on investing, i.e., the HH stores everything if $(1-\delta)R - 1 \ge 0 \Rightarrow 1-\delta \ge \frac{1}{R}$. In this case, $\boxed{c_1 = 0, \ K = W}$

$\Rightarrow \boxed{n = \frac{1-(1-\delta)RW/(1-\tau)}{2}}$, $c_2 = (1-\tau)\frac{1-(1-\delta)RW/(1-\tau)}{2} + (1-\delta)RW$

(rewritten more legibly $\Rightarrow$) $\boxed{n = \dfrac{1-((1-\delta)RW/(1-\tau))}{2}}$

$c_2 = \frac{1-\tau}{2} + (1-\delta)RW(1 - \frac{1}{2})$

$\boxed{c_2 = \frac{1-\tau}{2} + (1-\delta)RW/2}$.

If, instead, $(1-\delta)R - 1 < 0 \Rightarrow 1-\delta < \frac{1}{R}$ then the HH will receive a negative ROI and she will not save: $K = 0, \ c_1 = W, \ n = \frac{1}{2} - \frac{W}{2(1-\tau)}$

$\boxed{n = \frac{1}{2} - \frac{W}{2(1-\tau)}}$ $c_2 = (1-\tau)\frac{1-W/(1-\tau)}{2} = \frac{1-\tau}{2} - \frac{(1-\tau)}{2}\frac{W}{(1-\tau)}$

$$\boxed{c_2 = \frac{1-\tau}{2} - \frac{W}{2}}.$$

The government will take n^r, K^r as given and choose τ, δ to max utility:

$$\max_{\tau,\delta} \ln(c_1 + c_2) + \ln(1-n)$$

$$\text{s.t. } c_1 + K^r \le W$$

$$c_2 \le (1-\tau)n^r + (1-\delta)RK^r$$

$$g = \tau n^r + \delta R K^r \leftarrow \boxed{\text{This needs to hold}}$$

$\Rightarrow \max_{\tau,\delta} \ln(W + (1-\tau)n^r + ((1-\delta)R-1)K^r) + \ln(1-n^r)$

The government will want the household to save everything and thus will set δ such that it is the highest value for which this will occur $\Rightarrow 1-\delta = \frac{1}{R} \Rightarrow \delta = 1-\frac{1}{R}$

~~This is 1 equation in one unknown that characterizes the government's solution to their optimization problem.~~

They then need to clear their budget constraint so $g = \tau n^r + \delta R K^r = \tau n^r + (1-\frac{1}{R})K^r$.
This is 1 eqn in 1 unknown (tax τ) that the gov't solves.

Case 3 - No Commitment

Given K and (τ, δ) the household solves the following:

Note: I have already "plugged-in" the budget constraints as they are unchanged from Case 2

$$\max_{n} \ \ln\left(w + (1-\tau)n + ((1-\delta)R - 1)K\right) + \ln(1-n)$$

$$\frac{w + ((1-\delta)R - 1)K}{1-\tau} + n = 1 - n$$

$$\Rightarrow n^{\wedge} = \frac{1}{2} - \frac{w + ((1-\delta)R - 1)K}{2(1-\tau)}$$

Given K and $n^{\wedge}$ the government solves:

$$\max_{\delta,\tau} \ \ln\left(w + (1-\tau)n^{\wedge} + ((1-\delta)R - 1)K\right) + \ln(1-n)$$

The government has no reason not to tax capital, as the capital decision has already been made. Thus, $\delta = 1$. Therefore, the following determines the gov't's τ choice:

GBC clears $\Rightarrow$ $\boxed{g = \tau n^{\wedge}(\tau,\delta,K) + \delta R K = \tau n^{\wedge}(\tau,1,K) + RK}$

This is 1 eqn in 1 unknown (τ)

The ~~consumer~~ HH anticipates the gov't's actions and does not save anything as they are fearful of $\delta = 1$

$\Rightarrow K = 0.$

11) The Ramsey equilibrium cannot be supported a finite number of ~~the~~ times as, in the second period of the final repitition, the government has no incentive not to set $\delta = 1$ $\Rightarrow$ the consumer in the first period of the final repitition anticipates this and sets $K = 0$ $\Rightarrow$ the government anticipates this and has no reason not to set $\delta = 1$ in the penultimate period $\Rightarrow$ the ~~H~~ HH anticipates this and sets $K = 0$ in the penultimate repitition ~~period~~ $\Rightarrow$ .... This process repeats backwards in repititions and the Ramsey equilibrium cannot be ~~[illegible]~~ supported in any period owing to this unfurling process.