Micro HW1

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1 Question 1

Consider the matching problem described in the following table:

	W1	W2	W3
M1	10,5	8,3	6,12
M2	4,10	5,2	3,20
M3	6,15	7,1	8,16

Note that not matching results in 0 utility so all individuals would rather match than not match.

We apply the DAA algorithm. First let men propose.

- M1 proposes to W1, M2 proposes to W2, M3 proposes to W3.
  - W1, W2, and W3 accept the proposals from M1, M2, and M3, respectively.

Next, let women propose.

- W1 proposes to M3, W2 proposes to M1, W3 proposes to M2.
  - M1, M2, and M3 accept the proposals from W2, W3, and W1, respectively.

The outcomes are different depending on whether men or women propose.

Now, we change the grid to the following:

	W1	W2	W3
M1	10,10	8,3	6,12
M2	4,5	5,2	3,16
M3	6,15	7,1	8,20

First let men propose:

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- M1 proposes to W1, M2 proposes to W2, M3 proposes to W3.
  - W1, W2, and W3 accept the proposals from M1, M2, and M3, respectively.

Next, let women propose.

- W1 proposes to M3, W2 proposes to M1, W3 proposes to M3.
  - M1 and M3 accept the proposals from W2 and W3, respectively.
  - M3 rejects the proposal from W1. W1 is unmatched.
- W1 proposes to M1.
  - M1 accepts the proposal from W1.
  - M1 leaves W2. W2 is unmatched.
- W2 proposes to M2.
  - M2 accepts the proposal from W2.

In this scenario, the final matches are the same regardless of whether men propose or women propose.

2 Question 2

Consider the following matching market:

	M1	M2	M3
W1	1,2	4,3	3,2
W2	1,3	2,4	3,2
W3	2,2	2,2	4,4

Note that not matching results in 0 utility so all individuals would rather match than not match.

2.1 Part A

First let us apply the DA algorithm. Assume men propose. Then,

- M1 proposes to W2, M2 proposes to W2, M3 proposes to W3.
  - W2 and W3 accept the proposals from M2 and M3, respectively.
  - W2 rejects the offer from M1. M1 is unmatched.
- M1 could then either propose to W1 or W3 as they are indifferent between the two. Assume they propose to W3. (If they propose to W1 instead, skip the following line)

- W3 rejects the proposal from M1. M1 remains unmatched.

- M1 proposes to W1.
  - W1 accepts the proposal from M1.

Next, assume women propose. Then,

- W1 proposes to M2, W2 proposes to M3, W3 proposes to M3.
  - M2 and M3 accept the proposals from W1 and W3, respectively.
  - M3 rejects the proposal from W2. W2 is unmatched.
- W2 proposes to M2.
  - M2 accepts the proposal from W2.
  - M2 leaves W1. W1 is unmatched.
- W1 proposes to M1.
  - M1 accepts the proposal from W1.

The same matches result from both the men and women proposing. When the men propose the matching is male-optimal, while when the women propose the matching is male-pessimal. Since the same stable matching is simultaneously male-optimal and male-pessimal, the stable matching is unique.

2.2 Part B

	M1	M2	M3
W1	3	**7**	5
W2	**4**	6	5
W3	4	4	**8**

We can quite easily determine the efficient matching by summing the values in each entry in the above table and choosing the combination that maximizese the total sum across all matches. The efficient matching, denoted in bold above, is M1-W2, M2-W1, M3-W3.

2.3 Part C

Free entry gives us $w_2 + v_2 \geq 6$, and free exit gives us $w_2 + v_1 \leq 4$. Combining the inequalities via subtraction, we yield the following: $v_2 \geq v_1 + 2$. Since $v_1 \geq 0$ as the outside option is 0, $v_2 \geq 2$

3 Question 3

3.1 Part A

In this case, the utility of each individual is increasing in the type of their partner, so the optimal matches will be determined by positive assortive matching. Each individual will match with an individual of the same value.

3.2 Part B

The cross-partial of the total utility from each pair is positive, so the optimal matches will be determined by positive assortive matching. Therefore, each individual will match with a partner of the same value. The matchmaker maximizes profits:

$$\begin{aligned}
\max \pi &= \max x + 2axy + y - v(x) - w(y) \\
\Rightarrow v'(x) &= 1 + 2ay \\
\Rightarrow v(x) &= x + ax^2 + c_x, \\
\Rightarrow w'(y) &= 1 + 2ax \\
\Rightarrow w(y) &= y + ay^2 + c_y.
\end{aligned}$$

Note that, above, we have used the fact that $x = y$.

We now use the zero profit condition:

$$\begin{aligned}
0 &= x + 2axy + y - x - ax^2 - c_x - t - ay^2 - c_y \\
\Rightarrow c_x + c_y &= 0.
\end{aligned}$$

We know that, for each individual x or y, $v(x) \geq 0$ or $w(y) \geq 0$ so $c_x = c_y = 0$. Thus, the only wage which decentralizes the market is $v(x) = x + ax^2$, $w(y) = y + ay^2$.

4 Question 4

Let the market clearing rate be $3\% + x(0.01\%)$. Then, $x \in (k, k+1]$ for some $k \in \{1, \ldots, 29\}$. The borrower i will agree to borrow if $x \leq i$, so students $i \in \{b, \ldots, 29\} := B$ will agree to borrow (where $b = k + 1 + 1\{k \text{ is odd}\}$). Similarly, lender i will agree to lend if $x \geq 2i$, so students $i \in \{2, 4, \ldots, l\} := L$ will agree to lend (where $l = \text{floor}(k/2) - 1\{\text{floor}(k/2) \text{ is odd}\}$). There are $l/2$ elements of L and $\frac{29-b}{2} + 1$ elements of B.

Assume the number of lenders and borrowers must be the same for the market to clear. Then,

$$\begin{aligned}
l/2 &= \frac{29-b}{2} + 1 \\
\Rightarrow (\text{floor}(k/2) - 1\{\text{floor}(k/2) \text{ is odd}\})/2 &= \frac{29 - (k + 1 + 1\{k \text{ is odd}\})}{2} + 1
\end{aligned}$$

First assume $\mathrm{mod}(k, 4) = 0$. Then,

$$\begin{aligned} k/4 &= \frac{29 - k - 1}{2} + 1 \\ \Rightarrow k &= 20. \end{aligned}$$

This is a valid solution.

Next assume $\mathrm{mod}(k, 4) = 1$.

$$\begin{aligned} (k-1)/4 &= \frac{29 - k - 2}{2} + 1 \\ \Rightarrow k &= 59/3. \end{aligned}$$

This is not an integer, so this solution is invalid.

Next assume $\mathrm{mod}(k, 4) = 2$.

$$\begin{aligned} (k/2 - 1)/2 &= \frac{29 - k - 1}{2} + 1 \\ \Rightarrow k &= 62/3. \end{aligned}$$

This is not an integer, so this solution is invalid.

Next assume $\mathrm{mod}(k, 4) = 3$.

$$\begin{aligned} ((k-1)/2 - 1)/2 &= \frac{29 - k - 2}{2} + 1 \\ \Rightarrow k &= 61/3. \end{aligned}$$

This is not an integer, so this solution is invalid.

Therefore, the market clearing rate can be any rate $3\% + x(0.01\%)$, $x \in [20, 21]$.[1] At this rate, there will $20/4 = 5$ borrowers and 5 lenders. There will, therefore, be 5 transactions.

[1] Note that the derived solution 'misses' the possibility of $x = 20$, which here I assert is also a valid solution as it does not break any of the inequalities necessary for 5 borrowers and 5 lenders to agree to the transaction.