Q1 Macro Study Guide

2020 Entering Cohort*

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Overlapping Generations

Social Planner's Problem

Objective function

- Maximize utility function for current old and current young (c_t^t, c_t^{t-1})

$$\max U(c_t^t, c_t^{t-1})$$

Constraints

- Sum of consumption across generations = sum of young and old endowments
- Other assets = initial stock + interest

$$c_t^t + c_t^{t-1} = w_1 + w_2$$

Other notes

- Planners don't take into account price level, wages, or rental rate of capital
- Planners account for population size in the constraints, but not in the objective function

Competitive Equilibrium

Objective function

- Maximize utility function for one generation while they're young and old (c_t^t, c_{t+1}^t)

$$\max U(c_t^t, c_{t+1}^t)$$

*Contributions made by Sarah Bass, (add names here)

Constraints

- Time t consumption + assets = time t endowment ***adjust for price level at t
- Time t+1 consumption = time t+1 endowment + assets ***adjust for price level at t+1

$$p_t c_t^t + M_{t+1}^t = p_t w_1$$
$$p_{t+1} c_{t+1}^t = M_{t+1}^t + p_{t+1} w_2$$

- When $M_{t+1}^t > 0$, this can be consolidated into one budget constraint:

$$p_t c_t^t + p_{t+1} c_{t+1}^t = p_t w_1 + p_{t+1} w_2$$

- Introducing lump sum Social Security taxes/payments:

$$p_t c_t^t + p_{t+1} c_{t+1}^t = p_t (w_1 - \tau) + p_{t+1} (w_2 + b)$$

- Solve for steady states by combining HHBC w/ FOC

Market Clearing

- Competitive equilibrium occurs when agents optimize and markets clear
- Market clearing conditions are basically the constraints from the planner's problem (subscripts must match)
- Population size doesn't factor into constraints, but does factor into MCC
- Use total production for MCC supply = demand
- MCC doesn't take into account price level, wages, or rental rate of capital

$$c_t^t + c_t^{t-1} = w_1$$
$$M_{t+1}^t = \bar{M}$$

Welfare Theorems

- **First Welfare Theorem:** Any competitive equilibrium is pareto optimal
- **Second Welfare Theorem:** Any pareto optimal allocation can be achieved by a competitive equilibrium with the right transfers (taxes)
- There are multiple CEs that are all pareto optimal

Labor Choice

Social Planner's Problem

- Planner chooses $\{c, n\}$ to maximize utility of representative agent:

$$\max u(c) - g(n)$$
$$\text{s.t. } c \leq y = f(n)$$

Competitive Equilibrium

- Firms are owned by households, so firm profits are returned to households
- Firm Problem:

$$\max F(n) - wn$$
$$\rightarrow w = F_n$$

- Household Problem:

$$\max u(c) - g(n)$$
$$\text{s.t. } c = wn + \pi$$

- Market Clearing:

$$\text{Labor Market: } n^d = n^s$$
$$\text{Goods Market: } c = f(n)$$

Ramsey Problem (With Commitment)

Timing

1) Government chooses tax rate τ
2) Households choose investment

- Households solve for x, taking τ as given.
- Consistency: $x^r(\tau) = X^r(\tau)$
- Goods markets clear: $c + g = w + (R-1)X^r(\tau)$
- Government solves for max of utility given household X.

First Solve Social Planner Problem

$$\begin{aligned}
&\max u(c) \\
&\text{s.t. } x + m = w \\
&\text{and } c + g = m + Rx \\
&\rightarrow \text{Invest in productive technology}
\end{aligned}$$

Second Solve HH Problem

$$\begin{aligned}
&\max u(c) \\
&\text{s.t. } x + m = w \\
&\text{and } c = m + (1 - \tau)Rx
\end{aligned}$$

- A CE is when HHs solve this problem, the government budget constraint clears, and markets clear.
- If $(1 - \tau)R > 1$, same solution as planners problem.
- Laffer curve: Maps out the government tax revenue as a function of tax rate. Peak revenue occurs at $(1 - \tau)R = 1$.

Nash Equilibrium (No Commitment)

Timing

1) Households choose investment
2) Government chooses tax rate τ

- Government solves for τ, taking X as given.
- Households solve for x, no household makes an impact individually, but they all make the same decision.

Same social planner as Ramsey

HH problem

- Households know the government will set tax rate $\tau = 1$, so they will not invest at all. No need for math

Capital-based Model

Other notes

- write MCC in terms of production function, K, L (show firm side = household side)

Idiosyncratic Model

- Households are going to be assigned a shock of high or low employment
- **Duong is going to cover this in OH 5/18**

Other Definitions

- Pareto-Optimal: no one situation can be improved w/o making someone worse off
- Autarkic equilibrium: no trading, no one wants money in the money market