Micro HW1

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1 Question 1

Consider the matching problem described in the following table:

	W1	W2	W3
M1	10,5	8,3	6,12
M2	4,10	5,2	3,20
M3	6,15	7,1	8,16

Note that not matching results in 0 utility so all individuals would rather match than not match.

We apply the DAA algorithm. First let men propose.

- M1 proposes to W1, M2 proposes to W2, M3 proposes to W3.
  - W1, W2, and W3 accept the proposals from M1, M2, and M3, respectively.

Next, let women propose.

- W1 proposes to M3, W2 proposes to M1, W3 proposes to M2.
  - M1, M2, and M3 accept the proposals from W2, W3, and W1, respectively.

The outcomes are different depending on whether men or women propose.

Now, we change the grid to the following:

	W1	W2	W3
M1	10,10	8,3	6,12
M2	4,5	5,2	3,16
M3	6,15	7,1	8,20

First let men propose:

*I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, Ryan Mather, and Tyler Welch. I have also discussed problem(s) with Emily Case, Sarah Bass, Katherine Kwok, and Danny Edgel.

- M1 proposes to W1, M2 proposes to W2, M3 proposes to W3.
  - W1, W2, and W3 accept the proposals from M1, M2, and M3, respectively.

Next, let women propose.

- W1 proposes to M3, W2 proposes to M1, W3 proposes to M3.
  - M1 and M3 accept the proposals from W2 and W3, respectively.
  - M3 rejects the proposal from W1. W1 is unmatched.
- W1 proposes to M1.
  - M1 accepts the proposal from W1.
  - M1 leaves W2. W2 is unmatched.
- W2 proposes to M2.
  - M2 accepts the proposal from W2.

In this scenario, the final matches are the same regardless of whether men propose or women propose.

2 Question 2

Consider the following matching market:

	M1	M2	M3
W1	1,2	4,3	3,2
W2	1,3	2,4	3,2
W3	2,2	2,2	4,4

Note that not matching results in 0 utility so all individuals would rather match than not match.

2.1 Part A

First let us apply the DA algorithm. Assume men propose. Then,

- M1 proposes to W2, M2 proposes to W2, M3 proposes to W3.
  - W2 and W3 accept the proposals from M2 and M3, respectively.
  - W2 rejects the offer from M1. M1 is unmatched.
- M1 could then either propose to W1 or W3 as they are indifferent between the two. Assume they propose to W3. (If they propose to W1 instead, skip the following line)

- W3 rejects the proposal from M1. M1 remains unmatched.

- M1 proposes to W1.
  - W1 accepts the proposal from M1.

Next, assume women propose. Then,

- W1 proposes to M2, W2 proposes to M3, W3 proposes to M3.
  - M2 and M3 accept the proposals from W1 and W3, respectively.
  - M3 rejects the proposal from W2. W2 is unmatched.
- W2 proposes to M2.
  - M2 accepts the proposal from W2.
  - M2 leaves W1. W1 is unmatched.
- W1 proposes to M1.
  - M1 accepts the proposal from W1.

The same matches result from both the men and women proposing. When the men propose the matching is male-optimal, while when the women propose the matching is male-pessimal. Since the same stable matching is simultaneously male-optimal and male-pessimal, the stable matching is unique.

2.2 Part B

We have not had this lecture yet.

2.3 Part C

We have not had this lecture yet.

3 Question 3

3.1 Part A

The type x will agree to match if $y + axy \geq 0 \Rightarrow y \geq -axy \Rightarrow 1 \geq -ax \Rightarrow x \leq \frac{1}{-a}$.

Similarly, y will agree to match if $x + axy \geq 0 \Rightarrow y \leq \frac{1}{-a}$.

The match will occur if both conditions are met, i.e. $x, y \leq \frac{1}{-a}$.

3.2 Part B

4 Question 4

Let the market clearing rate be $3\% + x(0.01\%)$. Then, $x \in (k, k+1]$ for some $k \in \{1, \ldots, 29\}$. The borrower i will agree to borrow if $x \leq i$, so students $i \in \{b, \ldots, 29\} := B$ will agree to borrow (where $b := k + 1 + 1\{k \text{ is odd}\}$). Similarly, lender i will agree to lend if $x \geq 2i$, so students $i \in \{2, 4, \ldots, l\} := L$ will agree to lend (where $l = \text{floor}(k/2) - 1\{\text{floor}(k/2) \text{ is odd}\}$). There are $l/2$ elements of L and $\frac{29-b}{2} + 1$ elements of B.

Assume the number of lenders and borrowers must be the same for the market to clear. Then,

$$\begin{aligned} l/2 &= \frac{29-b}{2} + 1 \\ \Rightarrow (\text{floor}(k/2) - 1\{\text{floor}(k/2) \text{ is odd}\})/2 &= \frac{29 - (k + 1 + 1\{k \text{ is odd}\})}{2} + 1 \end{aligned}$$

First assume $\text{mod}(k, 4) = 0$. Then,

$$\begin{aligned} k/4 &= \frac{29 - k - 1}{2} + 1 \\ \Rightarrow k &= 20. \end{aligned}$$

This is a valid solution.

Next assume $\text{mod}(k, 4) = 1$.

$$\begin{aligned} (k-1)/4 &= \frac{29 - k - 2}{2} + 1 \\ \Rightarrow k &= 59/3. \end{aligned}$$

This is not an integer, so this solution is invalid.

Next assume $\text{mod}(k, 4) = 2$.

$$\begin{aligned} (k/2 - 1)/2 &= \frac{29 - k - 1}{2} + 1 \\ \Rightarrow k &= 62/3. \end{aligned}$$

This is not an integer, so this solution is invalid.

Next assume $\text{mod}(k, 4) = 3$.

$$\begin{aligned} ((k-1)/2 - 1)/2 &= \frac{29 - k - 2}{2} + 1 \\ \Rightarrow k &= 61/3. \end{aligned}$$

This is not an integer, so this solution is invalid.

Therefore, the market clearing rate can be any rate $3\% + x(0.01\%)$, $x \in (20, 21]$. At this rate, there will $20/4 = 5$ borrowers and 5 lenders. There will, therefore, be 5 transactions.