Micro HW3

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1 Question 1

1.1 Prove that the firm's objective function has strictly increasing differences in q and $-\tau$. Prove that this implies a monotone selection rule.

The firm's objective function is $g(q,t) := (1-\tau)pq - c(q)$ where $t = -\tau$. For $q^{'} > q, t^{'} = (-\tau^{'}) > (-\tau) = t$,

$$\begin{aligned} g(q^{'},t^{'}) - g(q,t^{'}) &= (1-\tau^{'})pq^{'} - c(q^{'}) - (1-\tau^{'})pq + c(q) \\ &= (1+(-\tau^{'}))p(q^{'}-q) - (c(q^{'}) - c(q)) \\ &> (1+(-\tau))p(q^{'}-q) - (c(q^{'}) - c(q)) \\ &= g(q^{'},t) - g(q,t) \end{aligned}$$

So g has increasing differences in $q, -\tau$. Now, let $q, q^{'}$ be optimal at $t = -\tau, t^{'} = -\tau^{'}$. Then, if $q > q^{'}$,

$$0 \geq g(q,t^{'}) - g(q^{'},t^{'}) > g(q,t) - g(q^{'},t) \geq 0$$

which violates our increasing differences. So, our optimal choice at $t^{'}$ must be at least as large as our choice at t.

Why is this stronger? This stronger because all possible optimal choices for production at $t^{'}$ must be at least as large as all possible optimal choices for production at t. This is not necessarily guaranteed by baby Topkis if there are more than one possible choices for production at $t^{'}, t$.

1.2 Suppose the firm is not a price-taker in the output market. It faces an inverse demand function $P(q)$, where $P(q)$ is the price the firm can sell q units of output. Show that the firm's objective function no longer has increasing differences in $q, -\tau$.

The firm's new objective function is $h(x,\tau) = (1-\tau)P(q)q - c(q)$. $\frac{\partial h}{\partial -\tau} = P(q)q$. If price is decreasing in q, which is not unreasonable for this scenario, then $P^{'}(q) < 0$ so h does

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not necessarily have increasing differences in $q, -\tau$.

1.3 Show that if c is strictly increasing, the firm's objective function still has strictly single-crossing differences; prove that an increase in τ cannot result in an increase in output.

Let $h(q^{'}, t) - h(q, t) > 0$ for $t = -\tau < -\tau^{'} = t^{'}$. Then,

$$\begin{aligned} 0 < (1+t)P(q^{'})q^{'} - c(q^{'}) - (1+t)P(q)q + c(q) &= (1+t)(P(q^{'})q^{'} - P(q)q) - (c(q^{'}) - c(q)) \\ &< (1+t^{'})(P(q^{'})q^{'} - P(q)q) - (c(q^{'}) - c(q)) \\ &= h(q^{'}, t^{'}) - h(q, t^{'}) \end{aligned}$$

so h has strictly single-crossing differences, and $h(q^{'}, t^{'}) > h(q, t^{'})$. If q is optimal at t and $q^{'}$ is optimal at $t^{'}$, and $t < t^{'}$ and if $q < q^{'}$, then

$$0 \geq h(q, t^{'}) - h(q^{'}, t^{'}) \geq h(q, t) - h(q^{'}, t) \geq 0$$

is a contradiction so q cannot decrease from an increase in t, so q cannot increase from a decrease in τ.

2 Question 2

The firm maximizes $\pi = p(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^z - w_l l - w_m m - w_r r - w_e e$ with $z = 1.1$

2.1 What effect does a reduction in w_e have on the firm's demand for each input?

The partials of the firm's objective function with regards to each input are as follows:

$$\begin{aligned} \frac{\partial \pi}{\partial l} &= 1.1p(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^{0.1}(0.5m^{0.3}l^{-0.5}) - w_l, \\ \frac{\partial \pi}{\partial m} &= 1.1p(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^{0.1}(0.3l^{0.5}m^{-0.7}) - w_m, \\ \frac{\partial \pi}{\partial r} &= 1.1p(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^{0.1}(0.7r^{-0.3}e^{0.1}) - w_r, \\ \frac{\partial \pi}{\partial z} &= 1.1p(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^{0.1}(0.1r^{0.7}e^{-0.9}) - w_e. \end{aligned}$$

Each partial is increasing in the other inputs so the objective function is supermodular in (l, m, r, e). Also, each input is weakly increasing in $-w_e$ so a decrease in w_e will lead to an increase in the use of all inputs via Topkis' theorem.

2.2 Over time, z changes to 0.9. With this new value for z, what effect will the wage subsidy have on the firm's demand for each input?

With $z = 0.9$ the following are the partials to the firm's objective function:

$$\frac{\partial \pi}{\partial l} = 0.9p(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^{-0.1}(0.5m^{0.3}l^{-0.5}) - w_l,$$
$$\frac{\partial \pi}{\partial m} = 0.9p(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^{-0.1}(0.3l^{0.5}m^{-0.7}) - w_m,$$
$$\frac{\partial \pi}{\partial r} = 0.9p(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^{-0.1}(0.7r^{-0.3}e^{0.1}) - w_r,$$
$$\frac{\partial \pi}{\partial z} = 0.9p(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^{-0.1}(0.1r^{0.7}e^{-0.9}) - w_e.$$

Let us focus first on $\frac{\partial \pi}{\partial l}$. This is decreasing in r, e. However, it is stil increasing in $m : \frac{\partial \pi}{\partial l} = 0.9p(l^{0.5} + r^{0.7}e^{0.1}m^{-0.3})^{-0.1}(0.5(m^{0.3})^{0.9}l^{-0.5}) - w_l$. We can similarly manipulate the other partials and it is evident that our objective function is supermodular in $(l, m, -r, -e)$ and $(l, m, -r, -e)$ are all weakly increasing in w_e so Topkis' theorem implies that a decrease in w_e will lead to a reduction in l, m and an increase in e, r.

2.3 If the supply of managers is fixed in the short-run, would the subsidy's effect on unskilled labor be larger in the short-run or long-run?

Since we found in the previous section that our objective function is supermodular in $(l, m, -r, -e)$, then we can apply Le Chatelier's principle and find that the effect on unskilled labor is larger in the long-run than the short-run.