Macro PS2

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1 Question 1

1.1 Part A

The equilibrium is a set of prices R and allocations c^h, c^l such that the allocations solve the agents' problem and markets clear.

Agents maximize utility subject to the budget constraint and the endogenous debt constraint:

$$\begin{aligned}&\max \sum_{t=0}^{\infty} u(c_{it})\\ &\text{s.t.} c_{it} + b_{it+1} = e_{it} + R_t b_{it}\\ &\text{and } b_{it+1} \geq -\phi.\end{aligned}$$

This constraint will bind on the low types, which will determine their consumption via their budget constraint. The high types will follow their Euler equations.

Market clearing is the following: $b_{it} + b_{jt} = 0$. If we let $R = \frac{1}{\beta}\frac{9}{10}, \phi = \frac{5}{1+R}$ then the agents will choose the efficient allocation.

The Euler for the high type is the following:

$$u'(c^h) = \beta R u'(c^l)$$
$$\frac{1}{c^h} = \frac{\beta R}{c^l}$$

Notice that $c^h = 10, c^l = 9$ satisfies the Euler under the interest rate given above.

We also need to check feasibility: From market clearing $c_h = 15 - \phi(1 + R) = 10, c_l = 4 + \phi(1 + R) = 9$. Thus, this satisfies the budget constraints with equality. By Walras' law the aggregate resource constraint also clears. Therefore, the constrained efficient allocation holds as an equilibrium of the environment described.

1.2 Part B

A second outcome arises from this setup: autarky. If $\phi = 0$, $c^h = e^h, c^l = e^l, R = \frac{e^l}{\beta e^h}$ then the high type's Euler is still satisfied, and budget constraints are satisfied trivially, and finally the resource constraint clears by Walras' law.

2 Question 2

2.1 Part A

In this scenario, if the agent defaults then they can still save. They will choose the optimal savings:

$$\max_s log(e^h - s) + \beta log(e^l + Rs) + \beta^2 log(e^h - s) + \beta^3 log(e^l + Rs) + \dots$$
$$\max_s \frac{log(e^h - s) + \beta log(e^l + Rs)}{1 - \beta^2}$$
$$\frac{1}{e^h - s} = \frac{\beta R}{e^l + Rs}$$
$$e^l + Rs = \beta R e^h - \beta Rs$$
$$s = \frac{\beta R e^h - e^l}{R + \beta R}$$
$$\Rightarrow V^d(h) = \frac{log\left(\frac{Re^h + e^l}{R + \beta R}\right) + \beta log\left(\frac{e^l \beta R + \beta R e^h}{1 + \beta}\right)}{1 - \beta^2}$$

2.2 Part B

A competitive equilibrium with not-too-tight constraints is an allocation c^l, c^h and set of prices R and constraint ϕ such that agents optimize, markets clear, and the not-too-tight constraint is satisfied. Agents solve the following:

$$\max_{c_t^i, B_t^i} \sum_{t=1}^{\infty} \beta^t log(c_t^i)$$
$$\text{s.t.} c_t^i + B_{t+1}^i = e_t^i + RB_t^i$$
$$\text{and } B_{t+1}^i \geq -\phi.$$

The not-too-tight constraint is the following:

$$\frac{log(c^h) + \beta log(c^l)}{1 - \beta^2} = V^d(h)$$

Market clearing is the following:

$$c^h + c^l = e^h + e^l$$
$$B^h + B^l = 0.$$

2.3 Part C

The constraint on borrowing is going to bind on the low type, i.e. $c^l = \phi(1 + R) + e^l \Rightarrow c^h = e^h - \phi(1 + R)$. We can plug this into our not-too-tight constraint:

$$\frac{log(e^h - \phi(1 + R)) + \beta log(\phi(1 + R) + e^l)}{1 - \beta^2} = V^d(h) \tag{1}$$

The high type is not constrained so their Euler equation must hold:

$$\frac{1}{e^h - \phi(1 + R)} = \frac{\beta R}{e^l + \phi(1 + R)} \tag{2}$$

Our constraint equation (1) and Euler (2) yield 2 equations in 2 unknowns which we can solve for R, ϕ which yield the equilibrium.

2.4 Part D

We can use (2) to solve for ϕ:

$$\begin{aligned} e^l + \phi(1+R) &= \beta R e^h - \beta R \phi(1+R) \\ \phi &= \frac{\beta R e^h - e^l}{(1+\beta R)(1+R)} \end{aligned}$$

Rewriting (1) we get the following:

$$\begin{aligned} log(e^h - \phi(1+R)) + \beta log(\phi(1+R) + e^l) &= log\left(\frac{Re^h + e^l}{R + \beta R}\right) + \beta log\left(\frac{e^l \beta R + \beta R e^h}{1+\beta}\right) \\ log\left(e^h - \frac{\beta R e^h - e^l}{(1+\beta R)}\right) + \beta log\left(\frac{\beta R e^h - e^l}{(1+\beta R)} + e^l\right) &= log\left(\frac{Re^h + e^l}{R + \beta R}\right) + \beta log\left(\frac{e^l \beta R + \beta R e^h}{1+\beta}\right) \\ log\left(\frac{e^h + e^l}{(1+\beta R)}\right) + \beta log\left(\frac{\beta R e^h + \beta R e^l}{(1+\beta R)}\right) &= log\left(\frac{Re^h + e^l}{R + \beta R}\right) + \beta log\left(\frac{e^l \beta R + \beta R e^h}{1+\beta}\right) \end{aligned}$$

It is clear that $R = 1$ satisfies the above equation. Then, $\phi = \frac{\beta e^h - e^l}{2(1+\beta)}$.

2.5 Part E

In class we solved the autarky case so I will jump to the solution in that case: $(c^h, c^l) = (10, 9)$. For the same calibrations for the model in this question we have $c^h = e^h - \phi(1+R) = 15 - \frac{0.5(15)-4}{1.5} = 12 + \frac{2}{3}, c^l = 19 - c^h = 6 + \frac{1}{3}$. This model yields less consumption smoothing compared to autarky.

2.6 Part F

The larger the punishment, the more consumption smoothing we can sustain.