Macro PS7

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1 Question 1

The planner maximizes utility subject to the resource constraint:

$$\max_{c_t^t, h_t, c_t^{t-1}} ln(c_t^t) + \alpha h_t + \beta c_t^{t-1}$$
$$\text{s.t. } c_t^t + c_t^{t-1} = y$$
$$\text{and } h_t = H^s$$

Clearly we can use the resource constraints to find that $h_t = H^s = 1$, and then solve for $c_t^t = y - c_t^{t-1}$. We then rewrite our optimization problem as:

$$\max_{c_t^{t-1}} ln(y - c_t^{t-1}) + \alpha + \beta c_t^{t-1}$$

Taking FOCs:

$$\frac{du}{dc_t^{t-1}} = 0 \Rightarrow \beta = \frac{1}{y - c_t^{t-1}} \Rightarrow c_t^{t-1} = y - \frac{1}{\beta}$$
$$\Rightarrow c_t^t = y - (y - \frac{1}{\beta}) = \frac{1}{\beta}$$

2 Question 2

2.1 Part A

The young agents face the following optimization problem:

$$\max_{c_t^t, h_t, c_t^{t-1}} ln(c_t^t) + \alpha h_t + \beta c_{t+1}^t$$
$$\text{s.t. } c_t^t + p_t h_t \leq y$$
$$\text{and } c_{t+1}^t \leq p_{t+1} h_t$$

*I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, and Ryan Mather. I have also discussed problem(s) with Emily Case, Sarah Bass, and Danny Edgel.

2.2 Part B

Markets clearing in the goods and housing market implies the following:

$$c_t^t + c_t^{t-1} = y$$
$$h_t = H^s = 1$$

2.3 Part C

A competitive equilibrium is a set of allocations and prices such that agents optimize and markets clear.

2.4 Part D

Utility is strictly increasing in consumption, so the budget constraints will hold with equality and we can substitute $c_t^t = y - p_t h_t, c_{t+1}^t = p_{t+1} h_t$ into the maximization problem and take first order conditions:

$$\max_{h_t} ln(y - p_t h_t) + \alpha h_t + \beta p_{t+1} h_t$$
$$\frac{du}{dh_t} = 0 \Rightarrow \frac{p_t}{y - p_t h_t} = \alpha + \beta p_{t+1}$$
$$\Rightarrow h_t = \frac{y}{p_t} - \frac{1}{\alpha + \beta p_{t+1}}$$
$$\Rightarrow c_t^t = \frac{p_t}{\alpha + \beta p_{t+1}}$$
$$\Rightarrow c_{t+1}^t = \frac{y p_{t+1}}{p_t} - \frac{p_{t+1}}{\alpha + \beta p_{t+1}}$$

We should verify that consumption and housing are nonnegative. This will be the case so long as p_t, p_{t+1} are positive and $\frac{y}{p_t} > \frac{1}{\alpha + \beta p_{t+1}}$.

2.5 Part E

From the market clearing conditions,

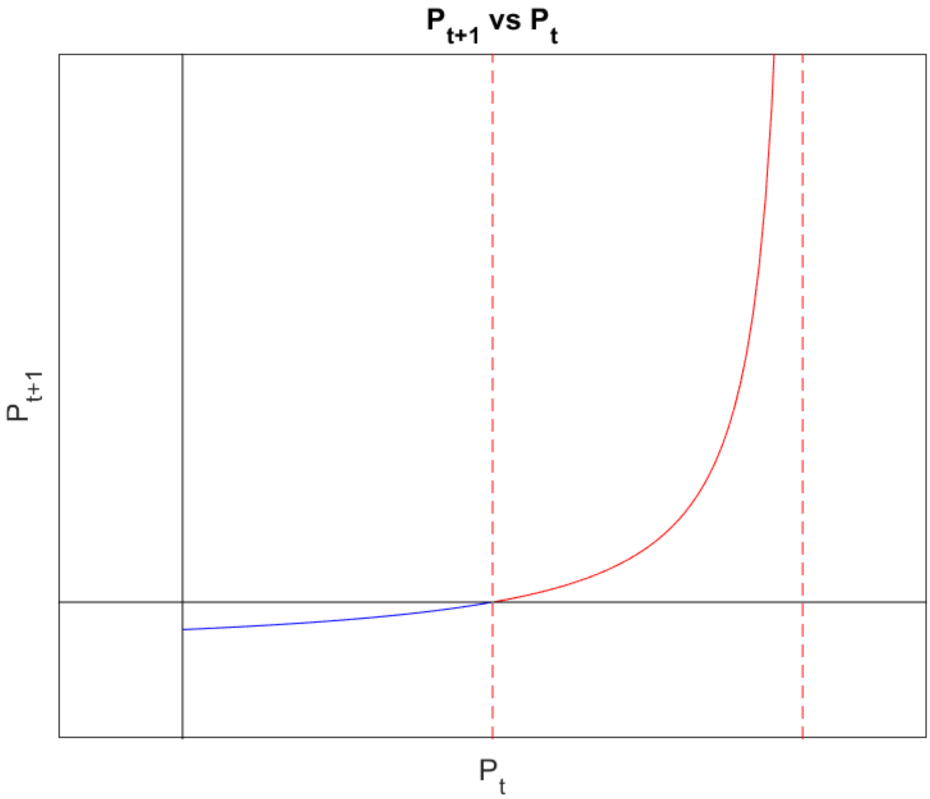
$$\frac{y}{p_t} - \frac{1}{\alpha + \beta p_{t+1}} = 1 \Rightarrow p_{t+1} = \frac{p_t}{\beta(y - p_t)} - \frac{\alpha}{\beta}$$
$$\Rightarrow c_t^t = y - p_t, c_{t+1}^t = p_{t+1}$$

Note that this also implies the second condition for consumption to be nonnegative. Now we should check the price condition:

$$0 \leq p_{t+1} = \frac{p_t}{\beta(y - p_t)} - \frac{\alpha}{\beta} \Rightarrow p_t \geq \frac{\alpha}{1 + \alpha} y.$$

We are given that $p_t < y$ so $\frac{\alpha}{1+\alpha} y \leq p_t \leq y$.

Below is the graph of p_{t+1} vs p_t.



In the above figure, we can see p_{t+1} as a function of p_t. Due to the nonnegativity constraints, the allowed values of p_{t+1} are positive, and are drawn in red. The positive region is within the dashed red lines, indicating $\frac{\alpha}{1+\alpha}y \leq p_t \leq y$.

2.6 Part F

In the steady state, $\bar{p} = p_t = p_{t+1} \Rightarrow \bar{p} = \frac{\bar{p}}{\beta(y-\bar{p})} - \frac{\alpha}{\beta}$

$$\begin{aligned}
&\Rightarrow 0 = \bar{p} - \alpha(y - \bar{p}) - \bar{p}\beta(y - \bar{p}) \\
&\Rightarrow 0 = \beta\bar{p}^2 + (1 + \alpha - \beta y)\bar{p} - \alpha y \\
&\Rightarrow \bar{p} = \frac{-(1 + \alpha - \beta y) \pm \sqrt{(1 + \alpha - \beta y)^2 + 4\beta\alpha y}}{2\beta}
\end{aligned}$$

The nonnegativity constraint rules out the negative $\bar{p}$ value so $\bar{p} = \frac{-(1+\alpha-\beta y)+\sqrt{(1+\alpha-\beta y)^2+4\beta\alpha y}}{2\beta}$.

2.7 Part G

Since in the steady state $p_t = \bar{p} = \frac{-(1+\alpha-\beta y)\pm\sqrt{(1+\alpha-\beta y)^2+4\beta\alpha y}}{2\beta}$, $\bar{c}_t^t = \bar{p} = \frac{-(1+\alpha-\beta y)\pm\sqrt{(1+\alpha-\beta y)^2+4\beta\alpha y}}{2\beta}$, $\bar{c}_{t+1}^t = y - \bar{p} = y - \frac{-(1+\alpha-\beta y)\pm\sqrt{(1+\alpha-\beta y)^2+4\beta\alpha y}}{2\beta}$ which is not the planner's solution of

$c_t^t = \frac{1}{\beta}, c_{t+1}^t = y - \frac{1}{\beta}$. However, in both equilibria the housing is the same due to market clearing in the housing market: $\bar{h} = 1 = h_t$.