Micro HW3

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1 Question 1

The TA must be paid M dollars. For a given class size with n students, the total utility is:

$$U = \begin{cases} nm, \text{no TA} \\ na + nm - M, \text{TA} \end{cases}$$

It is optimal to pay for the TA if $m \leq a + m - M/n \Rightarrow 0 \leq a - M/n \Rightarrow n \geq M/a$. Therefore, it is optimal to pay for the TA for $n \geq N$ where $N = M/a$.

2 Question 2

The social planner's problem is represented by the following Lagrangian:

$$\mathcal{L} = x_L^2/2 + x_H^2/2 + (H - b - x_H)x_H + (L - b - x_L)x_L - \beta\bar{x} + \lambda_H(\bar{x} - x_H) + \lambda_L(\bar{x} - x_L)$$

Our Kuhn-Tucker conditions are the following:

$$\begin{aligned} H - x_H - b &= \lambda_H \\ L - x_L - b &= \lambda_L \\ \lambda_H + \lambda_L &= \beta \\ x_H \leq \bar{x}, \lambda_H \geq 0, \lambda_H(\bar{x} - x_H) &= 0 \\ x_L \leq \bar{x}, \lambda_L \geq 0, \lambda_L(\bar{x} - x_L) &= 0 \end{aligned}$$

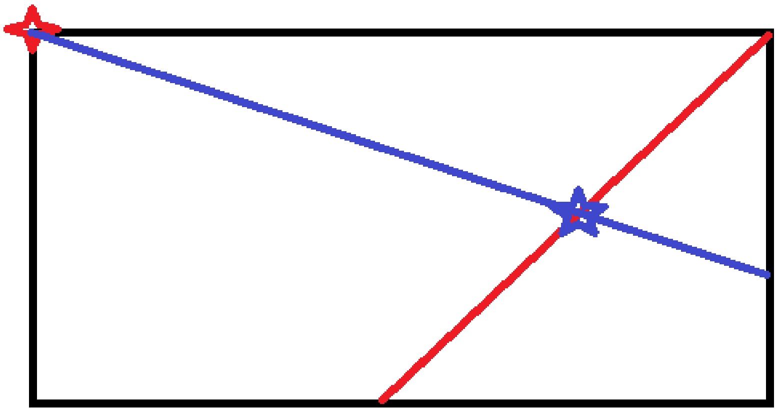
If $\lambda_L = 0, \lambda_H = \beta = h - x_H - b, x_L = L - b$. Moreover, $\bar{x} = x_H$, and since $x_L = L - b < \bar{x} = H - b - \beta \rightarrow \beta < H - L$. Therefore, if $\beta < H - L$, then our efficient prices are $p_H^* = H - x_H = b + \beta, p_L^* = L - x_L = b$.

If $\lambda_L, \lambda_H > 0, x_L = \bar{x} = x_H$. Then, $\beta = H - x_H - b + L - x_L - b = H + L - 2b - 2\bar{x} \Rightarrow \bar{x} = \frac{H+L-\beta}{2} - b \Rightarrow p_H = \frac{H-L+\beta}{2} + b, p_L = \frac{L-H+\beta}{2} + b$, if $\beta \geq H - L$

[*]I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, Ryan Mather, and Tyler Welch. I have also discussed problem(s) with Emily Case, Sarah Bass, Katherine Kwok, and Danny Edgel.

3 Question 3

The Edgeworth box is drawn below, with the endowment labeled with a red star. The contract curve is also drawn in red on the graph, with the result being a line that sets consumer B's consumption in periods 1 and 2 to be equal, due to that agent's utility curve being Leontief. We further plot the results from part (c) in blue.



We can now solve for the interest rate and endowment.

$$\mathcal{L}^A = c_1^1 c_2^2 - \lambda_A(p_1 c_1^1 + p_2 c_2^1 - 100p_2),$$
$$c_1^2 = c_2^2, p_1 c_1^2 + p_2 c_2^2 = 200p_1,$$
$$c_1^1 + c_1^2 = 200, c_2^1 + c_2^2 = 100.$$

Taking FOCs of the Lagrangian,

$$\begin{aligned}
c_2^1 &= \lambda p_1 \\
c_1^1 &= \lambda p_2 \\
c_2^1 &= c_1^1 p_1/p_2, \\
c_1^2 &= \frac{200}{1 + p_2/p_1}, \\
c_1^1 &= 50p_2/p_1, \\
50p_2/p_1 + \frac{200}{1 + p_2/p_1} &= 200 \\
(1/4)(p_2/p_1) + 1/(1 + p_2/p_1) &= 1 \\
(1/4)(p_2/p_1)^2 + (1/4)(p_2/p_1) + 1 - 1 - p_2/p_1 &= 0 \\
(1/4)(p_2/p_1) &= 3/4, \\
p_2/p_1 &= 3.
\end{aligned}$$

Given the interest rate $p_2/p_1 = 3$,

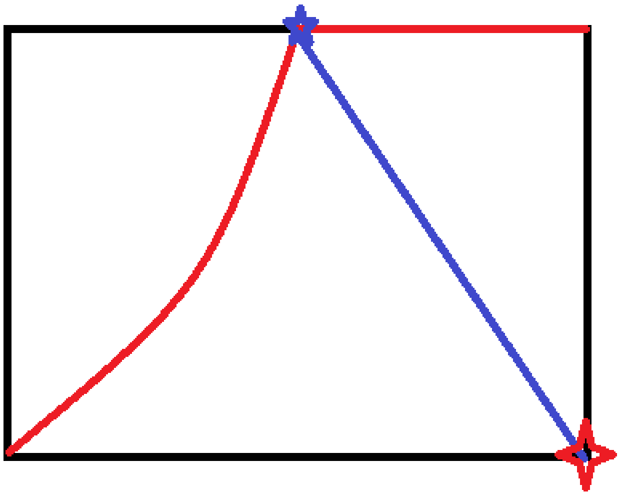
$$\begin{aligned}
c_1^1 &= 150, \\
c_2^1 &= 50, \\
c_1^2 &= 50, \\
c_2^2 &= 50.
\end{aligned}$$

4 Question 4

The contract curve equates the marginal rates of substitutions between the two:

$$\begin{aligned}
\frac{Y_A}{X_A} &= \frac{20}{X_B} \\
Y_A &= \frac{20x_A}{30 - x_A}
\end{aligned}$$

We draw the Edgeworth box, including the contract curve and our results from part (c) below.



We now solve for the equilibria.

Our optimality for the first consumer is the following:

$$\begin{aligned}
&\mathcal{L}^A = x^1 y^1 - \lambda_A(x^1 + y^1/(p_x/p_y) - 30) \\
&y^1 = \lambda_A \\
&x^1 = \lambda_A/(p_x/p_y) \\
&y^1 = x^1(p_x/p_y) \\
&x^1 = 15 \\
&x^2 = 15
\end{aligned}$$

For the second consumer we have the following:

$$\begin{aligned}
\mathcal{L}^B &= y^2 + 20log(x^2) - \lambda_B(x^2 + y^2/(p_x/p_y) - x_0^2 - y_0^2/(p_x/p_y)) \\
20/x^2 &= \lambda_B \\
p_x/p_y &= \lambda_B \\
20/x^2 &= p_x/p_y \\
4/3 &= p_x/p_y \\
y^1 &= 20 \\
y^2 &= 0.
\end{aligned}$$