Econometrics HW2

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1 Question 1

1.1 Part i

$$\begin{aligned} E[ZX'] &= E\left[\begin{pmatrix} Z_1 \\ X_2 \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix}'\right] \\ &= \begin{pmatrix} E[Z_1X_1] & E[Z_1X_2'] \\ E[X_2X_1] & E[X_2X_2'] \end{pmatrix} \\ E[ZZ'] &= E\left[\begin{pmatrix} Z_1 \\ X_2 \end{pmatrix}\begin{pmatrix} Z_1 \\ X_2 \end{pmatrix}'\right] \\ &= \begin{pmatrix} E[Z_1^2] & E[Z_1X_2'] \\ E[X_2Z_1] & E[X_2X_2'] \end{pmatrix} \end{aligned}$$

Note that $E[X_2X_2']$ must be invertible for either $E[ZX']$ or $E[XX']$ to be invertible.

Block inversion implies that $E[ZX']$ is invertible iff $E[Z_1X_1]-E[Z_1X_2']E[X_2X_2']^{-1}E[X_2X_1] \neq 0$, and similarly $E[ZZ']$ is invertible iff $E[Z_1^2] - E[Z_1X_2']E[X_2X_2']^{-1}E[X_2Z_1] \neq 0$

*I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, and Ryan Mather. I have also discussed problem(s) with Emily Case, Sarah Bass, Katherine Kwok, and Danny Edgel.