Micro HW2

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1 Question 1

1.1 Prove that if the production set $Y = \{(q, -z) : f(z) \geq q\} \subset \mathbb{R}^{m+1}$ is convex, the production function f is concave.

Let $q_1 = f(z_1), q_2 = f(z_2)$. $(q_1, -z_1), (q_2, -z_2) \in Y$ by definition and by convexity $t(q_1, -z_1) + (1-t)(q_2, -z_2) \in Y, t \in (0, 1)$. By definition, $f(t(z_1) + (1-t)(z_2)) \geq tq_1 + (1-t)q_2 = tf(z_1) + (1-t)f(z_2)$ so f is concave.

1.2 Prove that if f is concave, the cost function is convex in q.

We can fix $w \in \mathbb{R}^k$ Let $q_1, q_2 \in \mathbb{R}$ Let z_1, z_2 be $\underset{z:f(x)\geq q_1}{\arg\min}\, w \cdot z, \underset{z:f(x)\geq q_2}{\arg\min}\, w \cdot z$.

By the concavity of f, for $t \in (0, 1)$ we have $f(tz_1 + (1-t)z_2) \geq tf(z_1) + (1-t)f(z_2) \geq tq_1 + (1-t)q_2$. Therefore $tc(q_1, w) + (1-t)c(q_2, w) \leq c(tq_1 + (1-t)q_2, w)$ because we can produce at least $tq_1 + (1-t)q_2$ goods by using $(t(z_1) + (1-t)(z_2))$ inputs.

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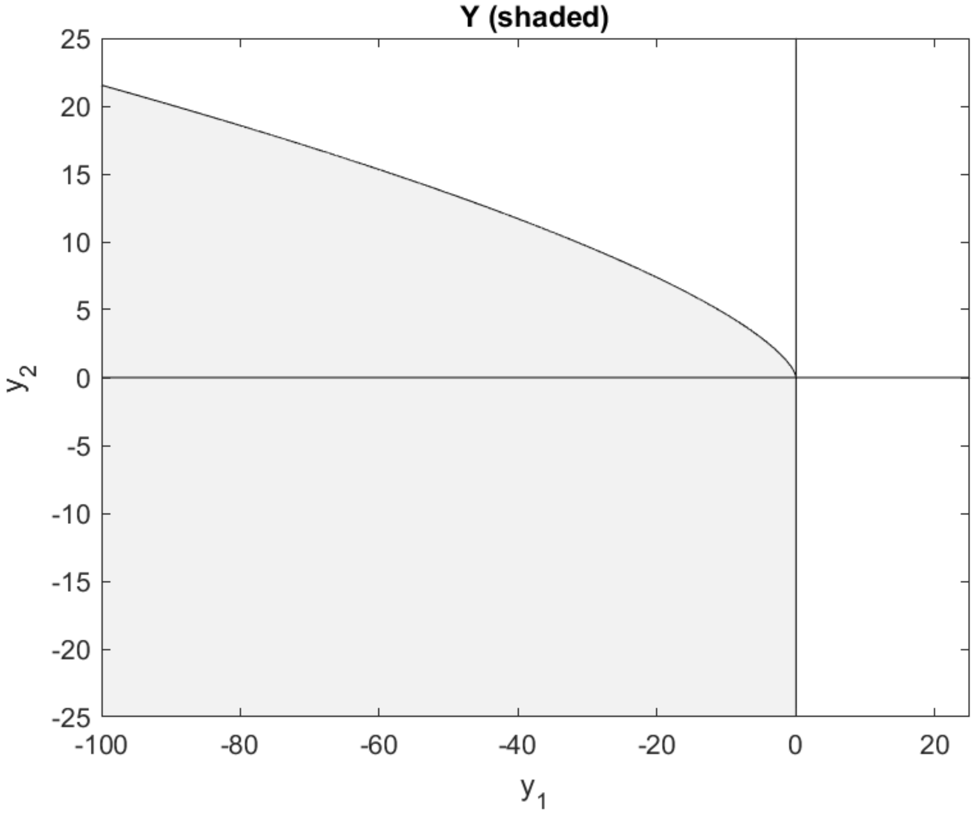
*I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, Ryan Mather, and Tyler Welch. I have also discussed problem(s) with Emily Case, Sarah Bass, and Danny Edgel.

2 Question 2

2.1 Draw Y



The shaded area in the above figure is Y graphed in Matlab, for a sample value of $B = 1$.

2.2 Solve the firm's profit maximization problem to find $\pi(p)$ and $Y^*(p)$.

The firm chooses production to maximize profit: $\max\limits_{-y_1, y_2 \in \mathbb{R}_+} p \cdot (y_1, y_2)'$ s.t. $y_2 \leq B(-y_1)^{2/3}$. Since profits are strictly increasing in y_2 the profit maximizing firm will set $y_2 = B(-y_1)^{2/3}$. We will also write $-y_1 = z$. Our optimization problem thus becomes: $\max\limits_{q \in \mathbb{R}_+} p \cdot (-q, Bq^{2/3})'$. Taking the firm's first order conditions, we find that $0 = \frac{d\pi(q)}{dq} = 0 \Rightarrow \frac{d}{dq}\left(-p_1 q + p_2 B q^{2/3}\right) = 0 \Rightarrow -p_1 + (2/3)p_2 B q^{-1/3} = 0 \Rightarrow q = \left(\frac{Bp_2}{(3/2)p_1}\right)^3$. This production yields the maximum profits given p, which we can compute as:

$\pi(p) = p_1 \left(\frac{Bp_2}{(3/2)p_1}\right)^3 + p_2 B \left(\frac{Bp_2}{(3/2)p_1}\right)^2$, since $Y^*(p) = \left(\left(\frac{Bp_2}{(3/2)p_1}\right)^3, B\left(\frac{Bp_2}{(3/2)p_1}\right)^2\right)'$.

2.3 Verify that $\pi(p)$ is homogeneous of degree 1, and $y(p)$ is homogeneous of degree 0.

$\pi(\lambda p) = \lambda p_1 \left(\frac{B\lambda p_2}{(3/2)\lambda p_1}\right)^3 + \lambda p_2 B \left(\frac{B\lambda p_2}{(3/2)\lambda p_1}\right)^2 = \lambda(p_1 \left(\frac{Bp_2}{(3/2)p_1}\right)^3 + p_2 B \left(\frac{Bp_2}{(3/2)p_1}\right)^2) = \lambda\pi(p)$ so $\pi(p)$ is homogeneous of degree 1.

$y(\lambda p) = \left(\left(\frac{B\lambda p_2}{(3/2)\lambda p_1}\right)^3, B\left(\frac{B\lambda p_2}{(3/2)\lambda p_1}\right)^2\right)' = \left(\left(\frac{Bp_2}{(3/2)p_1}\right)^3, B\left(\frac{Bp_2}{(3/2)p_1}\right)^2\right)' = y(p)$ so $y(p)$ is homogeneous of degree 0.

2.4 Verify that $y_1(p) = \frac{\partial \pi}{\partial p_1}$ and $y_2(p) = \frac{\partial \pi}{\partial p_2}$.

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\begin{aligned}
\frac{\partial \pi}{\partial p_1} &= \frac{\partial}{\partial p_1}\left(p_1\left(\frac{Bp_2}{(3/2)p_1}\right)^3 + p_2 B\left(\frac{Bp_2}{(3/2)p_1}\right)^2\right) \\
&= \left(\frac{Bp_2}{(3/2)p_1}\right)^3 - 3p_1\left(\frac{Bp_2}{(3/2)p_1}\right)^2\left(\frac{Bp_2}{(3/2)p_1^2}\right) - 2p_2 B\left(\frac{Bp_2}{(3/2)p_1}\right)\left(\frac{Bp_2}{(3/2)p_1^2}\right)
\end{aligned}
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3 Question 3