Money-in-the-Utility Model

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Primitives of the endowment model:

1. preferences: $U = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\Big(\frac{C_t^{1-\sigma}-1}{1-\sigma}+\kappa\frac{(M_t/P_t)^{1-\phi}-1}{1-\phi}\Big)$,
2. endowment: Y_t is given.

Competitive equilibrium boils down to the household problem

$$\max \mathbb{E}\sum_{t=0}^{\infty}\beta^t\Big(\frac{C_t^{1-\sigma}-1}{1-\sigma}+\kappa\frac{(M_t/P_t)^{1-\phi}-1}{1-\phi}\Big)$$

$$\text{s.t. } C_tP_t + B_t + M_t = Y_tP_t + (1+i_{t-1}^b)B_{t-1} + (1+i_{t-1}^m)M_{t-1}.$$

Take the FOC wrt C_t, B_t and M_t respectively:

$$\beta^t C_t^{-\sigma} = \lambda_t P_t,$$

$$\lambda_t = \mathbb{E}_t\lambda_{t+1}(1+i_t^b),$$

$$\kappa\beta^t\frac{M_t^{-\phi}}{P_t^{1-\phi}} - \lambda_t + \mathbb{E}_t\lambda_{t+1}.(1+i_t^m) = 0,$$

where B_t and M_t are the amount of bonds and money held by a representative household, i_t^b and i_t^m are the respective nominal interest rates. Substitute in the Lagrange multiplier from the first equation into the second one to obtain a standard Euler equation:

$$\beta\mathbb{E}_t\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}}(1+i_t^b) = 1. \tag{1}$$

Substitute the first two FOCs into the third one to obtain the optimal money demand:

$$\left(\frac{M_t}{P_t}\right)^{\phi} = \kappa\frac{1+i_t^b}{i_t^b - i_t^m}C_t^{\sigma}. \tag{2}$$