Market Exchange

- Consumer Surplus: $CS = U(x) - p \cdot x$
  - take FOC w.r.t x for $X(p)$
  - integrate $x(p)$ over pdf for aggregate demand $Q_D(p)$
- Aggregate demand: $Q_D(p)$
- Inverse demand: $P_D(Q)$ rearrange agg demand
- consumers buy if valuation ≥ price, valuation ≠ benefit/utility
  - use $v \geq p$ as bounds, integrate over quantity dem.
    $D(p) = \int_{p(x)}^{1} x \, f(x)\, dx.$

- Producer surplus = profit: $\pi = P \cdot x - TC(x)$
  - see market power section for setups
  - take FOC w.r.t x for $X(p)$
  - integrate $x(p)$ over pdf for aggregate supply $Q_S(P)$
  - marginal cost = dTC/dx
  - average cost = TC/x
- Total surplus = $CS + PS = CS + \pi$
- In the LR, firms produce if $P \geq AC$
- In the SR, firms produce if $P \geq$ average variable cost
  - In SR produce where $p = MC$ on graph
  - SR supply: $p \cdot q = \int_0^q MC \, dq$, solve with $q(p)$
  - LR supply: $p \cdot q = \int_0^q MC \, dq$, solve with $q(p)$
- Price takers: $P = MC$
- Price setters: $MR = MC$, $MR = P(Q) + qP'(Q)$
- If there exists another allocation/price combination with higher utility, then the current equilibrium is not efficient or stable

Stability and Elasticity

- Stability: when we are away from an equilibrium, the supply & demand dynamics push us to eq.
  - equilibrium may not be stable with a backwards bending supply curve
- Walrasian price stability: raise price if net demand is positive and lower if net demand is negative
- Marshallian quantity stability: raise supply quantity if demand price exceeds supply price
- Elasticity of Demand: $\epsilon = \frac{\Delta Q_D}{\Delta P} \cdot \frac{P}{Q_D}$
- Elasticity of Supply: $\eta = \frac{\Delta Q_S}{\Delta P} \cdot \frac{P}{Q_S}$

Costs

- escapable: can be avoided
- sunk: not escapable
- fixed: inescapable in SR, escapable in LR, invariant to Q
- variable: escapable always, varies with Q

Merging Markets

- sum supply and demand curves vertically for public goods
- sum supply and demand horizontally for private goods

Market Power

- consumer surplus = benefit − cost = $B(x) - px$.
  - Take FOC for w.r.t x for $x(p)$
  - solve for bounds on distribution w.r.t p
  - integrate $x(p)$ over bounds for the aggregate demand $Q^D(p)$
- without MC, monopolists set price where elasticity of demand = 1.

Types of Markets:

- Competitive Markets: $\pi = P \cdot q - TC(q)$
  - take price as given, find $Q(P)$
- Monopoly: $\pi = P(Q) \cdot Q - TC(Q)$
  - price setters, price is a function of Q.
- Cartel: $\pi = \underbrace{P(q_1+q_2)}_{P(Q)} \underbrace{(q_1+q_2)}_{Q} - \underbrace{tc(q_1) - tc(q_2)}_{TC(Q)}$
  - multiple production units, think about production allocation, $q_1 = q_2$ if same costs
- Cournot Duopoly: $\pi_1 = P(q_1+q_2)\,q_1 - TC(q_1)$
  $\pi_2 = P(q_1+q_2)\,q_2 - TC(q_2)$
  - set quantities <u>simultaneously</u>
  - solve dual maximization, use symmetry
- Stackleberg competition: same πs as Cournot
  - timing matters: $q_1 \to q_2 \to P$
  - solve backwards: max π_2, get $q_2(q_1)$, plug $q_2(q_1)$ into π_1, max π_1, solve q_1, solve q_2

Externalities:

- Negotiation: social planner max $\sum_i \pi_i$

- Coasian Negotiation: $\pi_1(Q_A) - \pi_1(Q_B)$, $\pi_2(Q_A) - \pi_2(Q_B)$ = payment bounds
  - property rights well assigned, how much would one firm pay another firm.
  - Q_A - individual equilibrium for firm w/ rights
  - Q_B - SPP equilibrium
  - Limitations: 1) costly bargaining
    2) property rights should be defended
    3) budget constraint
- Pigouvian Tax: $t(x)$ = cost of externality
  $\pi_1' = \pi_1 - t(x)$ firm causing externality
  $\pi_2' = \pi_2$ firm w/ externality
  - Limitations: 1) how can we determine cost of externality
- Permits: price of permit = cost of externality, decrease in welfare
  - produce s.t. $MB(x) = P_p$ for externality causing firm
  - optimal quantity of permits $\in (Q_A, Q_B)$ (defined above)
  - Limitations: 1) need to know how many permits to issue

Public Goods

- nonexcludable: public parks, roads, etc
- rival: too many people at the park, traffic, etc
- pure public goods: no competition for use

Samuelson Condition

$$\sum_{i=1}^{n} MRS^{i}_{G,w} = \sum_{i=1}^{n} \frac{u^{i}_{G}}{u^{i}_{m}} = \frac{1}{f'(t)} = MRT_{Gw}$$

MRS: $[du/dG] / [du/dm]$

MRT: $1/f'(t)$ conversion of private/public goods

m: private goods (money)

G: public goods

$G = f(t)$: production of public goods

- Resource constraint must still hold $\sum m + G = \sum w$
- Quasilinearity + concavity → unique efficient allocation
  - concave: du/dG decreasing as G↑ [not req. for uniqueness]
  - quasilinear: du/dm = constant

Lindahl Equilibrium

charging different individuals different prices for units of a public good, SPP solution, decentralized

Solve: $\max_{x_i, G} u^i(x_i, G)$ s.t. $x_i + p_i G = w_i$

x_i: private good consumed by person i

p: cost of producing 1 public good G

1: cost of private good (normalized)

w_i: endowment for person i

- everyone gets same amount of $G_i = \bar{G}$
- Sum of $G_i = G$ (often normalized to 1)

Other notes

- efficiency: maximize aggregate utility subject to constraints

Hotelling Model

1. choose location
2. choose prices

- use backwards induction
- spatial competition

$U(t) = \mu_0 - p_1 - t$ from firm 1

$U(t) = \mu_0 - p_2 - (1-t)$ from firm 2

Step 1: find indifference point

t indifferent

$\rightarrow \mu_0 - p_1 - t = \mu_0 - p_2 - (1-t)$

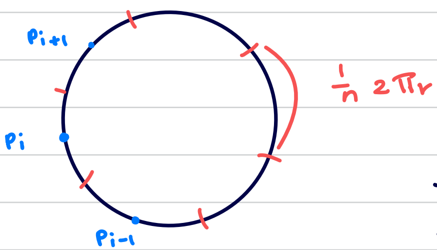
$\rightarrow t = \frac{1}{2} + \frac{p_2 - p_1}{2}$ } quantity $= t$

$\pi_1 = p_1 t = p_1\left(\frac{1}{2} + \frac{p_2 - p_1}{2}\right) \rightarrow$ max p_1 for p_1^*

$\pi_2 = p_2 t = p_2\left(\frac{1}{2} + \frac{p_1 - p_2}{2}\right) \rightarrow$ max p_2 for p_2^*

) assumes costs are zero

Salop Circle



1. entry decision
2. location
3. price

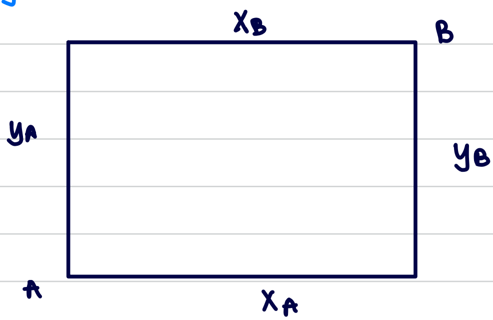
- use backwards induction
- search for symmetric eq. $p_j^* = p_i^* \; \forall i,j$

1) $\pi(p_i, p_{i+1}, p_{i-1})$ max p_j
2) $\pi(N) \geq FC$

   $\pi(N+1) < FC$

General Equilibrium

Edgeworth Box



- size of box is size of total endowment, Σx, Σy

1) Identify initial endowment (IE)
2) Draw indifference curves through IE
   - $x^{1/2}y^{1/2}$ Cobb Douglas
   - $\min\{x, y\}$ offer curve goes through points here
3) Find demand functions:

   $\max U_i$ s.t $\underbrace{P_x x + P_y y}_{\text{buying}} \leq \underbrace{P_x x_{IE} + P_y y_{IE}}_{\text{value of endowment}}$

   - solve for x, y as functions of P_x, P_y
4) solve for prices using $x_i + x_j = X$, $y_i + y_j = Y$

Walras Law: solve for price ratio using only $x_i + x_j = X$

Economy with production

- production possibilities set $\langle$(inputs ℓ, y, etc): bounds $0 \leq \ell \leq 1$, $y \leq 2\ell \rangle$
- producer demand: $\pi = p \cdot y(\ell) - w \cdot \ell \rightarrow$ max w.r.t ℓ
  - take p, w as given
  - max w.r.t ℓ, find ℓ^*, $y(\ell^*)$, π^* as functions of w, p
- consumer demand: $\max U_i$ s.t $p \cdot y \leq \ell w + \pi(w,p)$
  - max w.r.t ℓ, find ℓ^*, $y(\ell^*)$ as functions of w, p
- solve for prices: set $\ell^* = \ell^*$ and $y^* = y^*$