# Dixit-Stiglitz Model

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**Primitives** of the static model:

1. preferences: $U = C = \left(\int C_i^{\frac{\theta-1}{\theta}} \mathrm{d}i\right)^{\frac{\theta}{\theta-1}}$,
2. technology: $Y_i = AL_i$,
3. endowment: $L = \int L_i = 1$.

In contrast to the growth and RBC model, assume that only one firm can produce each variety and hence, is a monopoly in the market of that product. At the same time, the firm takes GE prices as given.

**Households** choose consumption of each product:

$$\max_{\{C_i\}} \left(\int C_i^{\frac{\theta-1}{\theta}} \mathrm{d}i\right)^{\frac{\theta}{\theta-1}}$$

$$\text{s.t.} \quad \int P_i C_i \mathrm{d}i = W + \int \Pi_i \mathrm{d}i \equiv E,$$

where $E$ is the total income of a representative consumer. It is more convenient, however, to solve the dual problem of minimizing expenditures:

$$\min_{\{C_i\}} \int P_i C_i \mathrm{d}i$$

$$\text{s.t.} \quad \left(\int C_i^{\frac{\theta-1}{\theta}} \mathrm{d}i\right)^{\frac{\theta}{\theta-1}} = C.$$

Denote the Lagrange multiplier with $P$ and take the FOC wrt $C_i$:

$$P_i = P \left(\int C_i^{\frac{\theta-1}{\theta}} \mathrm{d}i\right)^{\frac{1}{\theta-1}} C_i^{-\frac{1}{\theta}}.$$

This implies that demand for product $i$ is equal

$$C_i = \left(\frac{P_i}{P}\right)^{-\theta} C. \tag{1}$$

Substitute into the constraint to solve for the Lagrange multiplier:

$$P = \left(\int P_i^{1-\theta} \mathrm{d}i\right)^{\frac{1}{1-\theta}}.$$

Note that $\int P_i C_i \mathrm{d}i = PC$, so it makes sense to call it the aggregate (ideal) price index, i.e. the price of one unit of consumption bundle.

**Firms** maximize profits subject to household demand (1), production technology and taking decisions of other firms as given:

$$\max_{C_i, P_i} P_i C_i - W L_i$$

$$\text{s.t. } C_i = \left(\frac{P_i}{P}\right)^{-\theta} C,$$
$$C_i = A L_i.$$

Substitute constraints in the objective function and take the FOC:

$$C_i - \theta \left(P_i - \frac{W}{A}\right) \left(\frac{P_i}{P}\right)^{-\theta} \frac{C}{P_i} = 0,$$

which can be solved for the optimal price:

$$P_i = \frac{\theta}{\theta - 1} \frac{W}{A}. \tag{2}$$

Given symmetry across firms, we obtain $P = \frac{\theta}{\theta-1}\frac{W}{A}$. Note that the equilibrium conditions only describe the real wage and relative prices of products, while nominal prices and wages are undetermined. Finally, the market clearing condition

$$\int \frac{C_i}{A} \mathrm{d}i = \frac{C}{A} = 1 \tag{3}$$

then pins down the output $C_i = C = 1$, so that the aggregate welfare is equal $U = C = 1$.

**SPP** is to allocate labor across firms in an optimal way:

$$\max_{\{C_i\}} \left( \int C_i^{\frac{\theta-1}{\theta}} \mathrm{d}i \right)^{\frac{\theta}{\theta-1}}$$

$$\text{s.t.} \int \frac{C_i}{A} \mathrm{d}i = 1.$$

The FOC implies $C_i = C = 1$. Therefore, the monopolistically competitive equilibrium coincides with the first-best allocation and there is no room for government interventions.