Exam Study Guide

Arrow Debreu:

General Setup:

$$\max \sum_{t,s^t} \beta^t u(c(s^t)) \pi(s^t)$$

$$\text{s.t.} \sum_{t,s^t} q(s^t) c(s^t) \le \sum_{t,s^t} q(s^t) e(s^t) \qquad (\lambda_i)$$

NOT $\lambda_{\underline{t}}$

Solving:

1) Take FOC
2) Divide types of agents $FOC_A | FOC_B$
   - show constant ratio over states|times
3) Divide times $FOC_{t+1} | FOC_t$
   - shows progression of prices
4) Divide over states
   - shows ratio of prices by state

Examples:

- Lectures 1 & 2
- HW 1, Question 8.3

Sequential markets - solve using Bellman

see PS1

Limited Commitment (Autarky):

① If planner can observe types:

$$\max \sum_t \beta^t [u(c_1) + u(c_2)]$$
s.t. $c_1 + c_2 \leq e_1 + e_2$

BC could incorporate savings, production, labor, capital, etc.

This usually won't hold if types hidden.

FOC w.r.t c_1, c_2

② Find value of punishment (such as autarky) for deviating at $t=s$

$$V_{it} = \max \sum_{t=s}^{\infty} \beta^{t-s} u(c_i)$$

See 2020 Exam Q2

s.t. HHBC: $c_{it} = e_{it} \pm b$

s.t. $b \leq \phi$ Borrowing constraint

③ Create incentive compatibility constraint:

$$\max \sum_{t=s}^{\infty} \beta^{t-s} u(c_i) \geq V_{it}$$

- usually holds w/ = for high type

④ Resolve planner's problem w/ IC cons.

$$\max \sum_t \beta^t [u(c_1) + u(c_2)]$$
s.t. $c_1 + c_2 \leq e_1 + e_2$

- No distortion at the top

& $\max \sum_{t=s}^{\infty} \beta^{t-s} u(c_{Ht}) = V_{Ht}$

See lecture 3 for spot market supporting ECa, w/ prices

⑤ This generally holds in CE with a borrowing constraint in HHBC - bonds between HHs

Ramsey Problem:

① Solve HH problem to find policy wedge:

$$\max \sum_t \beta^t u(c_t) + v(\ell_t) \quad - v(n_t)$$

s.t. $(1+\tau_{ct})c_t + k_{t+1} + b_{t+1} \leq w(1-\tau_{nt})n_t + R^b_t b_t + R^k_t k_t$

where $R^k_t = 1 + (1-\tau_{kt})(r_t + \delta)$

- could include diff. wedges

Solve for labor supply & Euler to find wedges. See Lecture 7! FOC w.r.t c, k, b, ℓ, n

For the CE:

alloc: $c_t, k_{t+1}, b_{t+1}, \ell_t, n_t$

prices: w, r

policy: τ_i, R^b_t

See 2020 Exam Q1.

② Find Implementability Constraint:

$$\sum_t \beta^t [u_{c_t} \cdot c_t + u_\ell \cdot \ell + u_n \cdot n] = \frac{u_{c_0}}{1+\tau_{c0}} [R_{b_0} b_{-1} + R_{k_0} k_{-1}]$$

③ Solve planners problem:

$$\max \sum_t \beta^t u(c) + v(\ell)$$

s.t. $c_t + g_t + k_{t+1} \leq F(k_t, n_t) + (1-\delta)k_t$ No bonds! RC

and IC above

1) Take FOCs w.r.t c, k, ℓ, b, n

2) Compare Euler/LS for eqn with specified wedge to determine τ_i value

Cash|Credit Good:

① Solve HH problem:

$$\max \sum_t \beta^t u(c_1, c_2, n_t)$$

c_1 - cash
c_2 - credit

s.t $M_t + B_t = M_{t-1} - p_{t-1} c_{1,t-1} - p_{t-1} c_{2,t-1} + w_{t-1}(1-\tau_{nt}) n_{t-1} + R_{t-1} B_{t-1} - T_t$

$p_t c_{1,t} \leq M_t$

FOCs w.r.t c_1, c_2, n, M, B See PS4

② Find IC:

$$\sum_t \beta^t \left[u_1 \cdot c_{1t} + u_2 \cdot c_{2t} + u_3 \cdot n_t\right] = \overset{\text{non-distorted}}{\frac{u_{20}}{p_0}} \cdot \left[M_{-1} + R_{b_0} B_{-1}\right] \Bigg\} \text{ensures a C.E.}$$

③ Solve Planner problem: Fiscal|monetary pol notes

$$\max \sum \beta^t u(c_1, c_2, n)$$

~~s.t. $M_t - M_{t-1} + B_t = R_{t-1} B_{t-1} + P_{t-1} g_{t-1} - P_{t-1} \tau_{t-1} n_{t-1} - T_t$~~

and IC
and $c_{1t} + c_{2t} + g_t \leq l_t$ } primal approach

FOC w.r.t c_1, c_2, n, M, B

Friedman rule: return on bonds $= 1$

Mirleesian: Discrete types

① Types θ are known, solve utilitarian planner problem:

$$\max \sum_t \beta^t \Big[\pi(\theta_H)\big[u(c(\theta_H)) - v(y(\theta_H)/\theta_H)\big] + \pi(\theta_L)\big[u(c(\theta_L)) - v(y(\theta_L)/\theta_L)\big]\Big]$$

$$\text{s.t } \pi(\theta_H)c(\theta_H) + \pi(\theta_L)c(\theta_L) \leq \pi(\theta_H)y(\theta_H) + \pi(\theta_L)y(\theta_L)$$

FOC w.r.t c, y

Usually get that everyone consumes equally.

② IC constraint:

$$u(c(\theta_H)) - v(y(\theta_H)/\theta_H) \geq u(c(\theta_L)) - v(y(\theta_L)/\theta_H) \quad \text{Binds!}$$

$$u(c(\theta_L)) - v(y(\theta_L)/\theta_L) \geq u(c(\theta_H)) - v(y(\theta_H)/\theta_L)$$

③ Planner's Problem with unknown types:

$$\max \sum_t \beta^t \Big[\pi(\theta_H)\big[u(c(\theta_H)) - v(y(\theta_H)/\theta_H)\big] + \pi(\theta_L)\big[u(c(\theta_L)) - v(y(\theta_L)/\theta_L)\big]\Big]$$

$$\text{s.t } \pi(\theta_H)c(\theta_H) + \pi(\theta_L)c(\theta_L) \leq \pi(\theta_H)y(\theta_H) + \pi(\theta_L)y(\theta_L) \quad \text{RC}$$

$$\text{and } u(c(\theta_H)) - v(y(\theta_H)/\theta_H) = u(c(\theta_L)) - v(y(\theta_L)/\theta_H) \quad \text{IC}$$

FOC w.r.t c, y. No distortion for θ_H.

Let these be c^*, y^*

④ Tax structures:

$$T(y) = \begin{cases} y - c & \text{if } y \in \{y^*\} \\ y & \text{otherwise} \end{cases}$$

- Tax so that wealth for each group = SPP consumption

HH solves:

$$\max u(c) - v(y/\theta)$$

$$\text{s.t } c \leq y - T(y)$$

- use lump sum unless someone wants to overproduce

Mirleesian: continuum of types

Local incentive compatibility:

1. $y(\theta)$ increasing in θ
2. $u'(\theta) = \frac{y(\theta)}{\theta^2} v'\left(\frac{y(\theta)}{\theta}\right)$

Globally incentive compatible:

$$u(\theta) \equiv u\left(c(\theta) - v\left(\frac{y(\theta)}{\theta}\right)\right)$$
$$\geq u\left(c(\hat{\theta}) - v\left(\frac{y(\hat{\theta})}{\theta}\right)\right)$$ utility of θ pretending to be $\hat{\theta}$
$$= u(\hat{\theta}, \theta)$$

GIC $\longleftrightarrow$ LIC   See lecture 10!

Planner's Problem:

$$\max_{c,y} \int W\left(u\left(c(\theta) - v\left(\frac{y(\theta)}{\theta}\right)\right)\right) dF(\theta)$$

s.t. $\int c(\theta)\, dF(\theta) \leq \int y(\theta)\, dF(\theta)$ RC

and $u(\theta) = u(c(\theta) - v(y(\theta)/\theta))$

$\left.\begin{array}{l} \text{and } u'(\theta) = \frac{y(\theta)}{\theta^2} v'\left(\frac{y(\theta)}{\theta}\right) \\ \text{and } y(\theta) \text{ increasing in } \theta \end{array}\right\}$ LIC constraint