Micro HW1

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1 Question 1

The first firm to enter the market will be the firm with the lowest fixed cost, i.e. firm 1. This firm will choose to enter when profits are nonnegative:

$$pq - 1 - q - q^2 \geq 0$$

The firm maximizes profit:

$$\max_q pq - 1 - q - q^2$$
$$\Rightarrow p = 1 + 2q$$
$$\Rightarrow q = \frac{p-1}{2}$$

Our nonnegative profit condition then yields:

$$\frac{p^2 - p}{2} - 1 - \frac{p-1}{2} - \left(\frac{p-1}{2}\right)^2 \geq 0$$
$$p^2 - 2p - 1 - \frac{p^2 - 2p + 1}{2} \geq 0$$
$$p^2 - 2p - 3 \geq 0$$
$$(p-3)(p+1) \geq 0$$

The p^* which is the minimum price to have production is the positive price which satisfies the above expression with equality. Therefore, $p^* = 3$.

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[*]I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, Ryan Mather, and Tyler Welch. I have also discussed problem(s) with Emily Case, Sarah Bass, Katherine Kwok, and Danny Edgel.

2 Question 2

2.1 Part A

Given a tax level τ, a firm will choose quantity to produce q which maximies profits, and will enter the market if their profits are nonnegative.

$$\max_x (1-\tau)2\theta x - x^2 - 1$$
$$\Rightarrow (1-\tau)2\theta = 2x$$
$$\Rightarrow x = (1-\tau)\theta.$$

The firms enter if their profits are nonnegative:

$$(1-\tau)2\theta x - x^2 - 1 \geq 0$$
$$(1-\tau)^2 2\theta^2 - (1-\tau)^2\theta^2 - 1 \geq 0$$
$$\theta \geq \frac{1}{1-\tau}$$

The aggregate supply of the firms is the integral across their distribution. Note that we are provided with a functional form for the mass of firms with ideas above a value of theta. We can convert this to a density by taking the negative of the derivative of this function: $f(\theta) = \beta\theta^{-\beta-1}$. Then, our supply is the following:

$$Q_s(\tau) = \int_{\frac{1}{1-\tau}}^{\infty} 2(1-\tau)\theta^2 f(\theta)d\theta$$
$$= \int_{\frac{1}{1-\tau}}^{\infty} 2(1-\tau)\theta^2 \beta\theta^{-\beta-1}d\theta$$
$$= \frac{1(1-\tau)\beta}{2-\beta}[\theta^{2-\beta}]_{\frac{1}{1-\tau}}^{\infty}$$
$$= \frac{2\beta(1-\tau)^{\beta-1}}{\beta-2}$$

2.2 Part B

When Apple raises its tax rate, the cutoff moves up (i.e. fewer firms are developers). Furthermore, the amount of code each producer produces falls.

2.3 Part C

Apple maximizes their revenue:

$$\max_\tau \tau \frac{2\beta(1-\tau)^{\beta-1}}{\beta-2}$$

$$\Rightarrow \frac{2\beta(1-\tau^*)^{\beta-1}}{\beta-2} = \tau^* \frac{2\beta(1-\tau^*)^{\beta-2}}{\beta-2}(\beta-1)$$

$$\Rightarrow \tau^* = \frac{1}{\beta}.$$

2.4 Part D

If β rises then the firms become more concentrated at lower levels of ideas. Fewer firms produce, and total production falls.
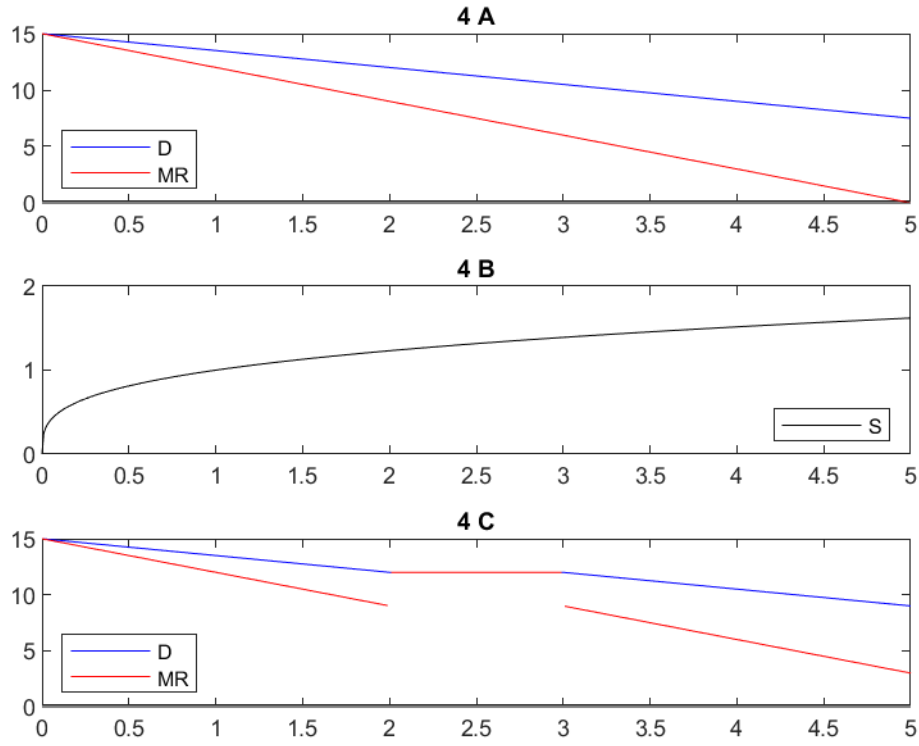
3 Question 3

Our two inverse demand curves satisfy $P_i(Q) = a_i - b_i Q$ for $i \in \{3\mathrm{C}, \mathrm{FB}\}$.

Ben and Jerry maximize profits:

$$\max_{Q_i} a_i Q_i - b_i Q_i^2 - M Q_i$$

$$\Rightarrow a_i - 2b_i Q_i - M = 0$$

$$\Rightarrow Q_i = \frac{a_i - M}{2b_i}$$

$$\Rightarrow P_i = \frac{a_i + M}{2}$$

Therefore, Fruit Bowl will be set to the higher price as it has the higher intercept.

4 Question 4

5 Question 5

For the initial part of this question, we have that $P_i(Q_i) = a_i - Q_i$

$$\max_{Q_i} a_i Q_i - Q_i^2 - Q_i$$
$$\Rightarrow a_i - 2Q_i - 1 = 0$$
$$\Rightarrow Q_i = \frac{a_i - 1}{2}$$
$$\Rightarrow P_i = \frac{a_i + 1}{2}$$

This implies that $P_1 = 2, P_2 = 3$. The store can make this happen by offering the higher price online and the lower price in person. Type 1 people will come in to the store and get the lower price, while Type 2 people will order online and be charged the higher price.

When the marginal cost varies by total quantity sold, the two problems become

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linked by the cost function and, therefore, we have a single optimization to solve.

$$\max_{Q_1, Q_2} P_1 Q_1 + P_2 Q_2 - (Q_1 + Q_2)^2$$

$$\max_{Q_1, Q_2} 3Q_1 - Q_1^2 + 5Q_2 - Q_2^2 - (Q_1 + Q_2)^2$$

$$\Rightarrow 3 = 2Q_1 + 2Q_1 + 2Q_2,$$

$$\Rightarrow 5 = 2Q_2 + 2Q_2 + 2Q_1,$$

$$\Rightarrow \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = \begin{pmatrix} 1/6 \\ 7/6 \end{pmatrix},$$

$$\Rightarrow \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} 17/6 \\ 23/6 \end{pmatrix}.$$