**MICROECONOMIC THEORY, ECONOMICS 713, 2ND QUARTER, SPRING 2012**
**UNIVERSITY OF WISCONSIN-MADISON, PROFESSOR: MARZENA ROSTEK**

**PROBLEM SET 1**
Due in(=before) class April 10, 2012

**Question 1: Weak and strict preferences**
Suppose $\succ$ on X is a weak order, that is asymmetric (if $x \succ y$, then not $y \succ x$, for every x,y in X) and negatively transitive (if (not $x \succ y$, not $y \succ z$), then not $x \succ z$, for every x,y,z in X). (A binary relation $\succ$ on X is called a *preference relation* if it is asymmetric and negatively transitive.)
Let us use $\succ$ to define two other binary relations: $x \succsim y$ if (not $y \succ x$), called *weak preference*; and $x \sim y$ if (not $x \succ y$ and not $y \succ x$), called *indifference*.
Show that

(a) For all x and y, exactly one of $x \succ y$, $y \succ x$ or $x \sim y$ holds;
(b) $\succsim$ is complete and transitive;
(c) $x \succsim y$ if, and only if, $x \succ y$ or $x \sim y$.
(d) Note how the definition of indifference infers/equates the absence of strict preference from/with indifference. Alternatively, what could the absence of strict preference in either direction capture? (A one-line response suffices.)

**Question 2: Equilibria in a Second-Price Auction with Common Values**
Consider a second-price sealed-bid auction with 2 bidders with common values. The valuation of each bidder *i* is given by $v_i = t_i + t_j$, where *j* is the other player. For example, the signal $t_i$ is the number of barrels of oil in a tract. Each bidder knows only his own signal and that the signals come from a uniform distribution on [0,1], which is common knowledge. Show that for any value of $\alpha$>0, there is an *asymmetric* BNE in which player 1 bids $(1+\alpha)t_1$ and player 2 bids $\left(1+\frac{1}{\alpha}\right)t_2$.

**Question 3: Bayesian Nash Equilibria**
Consider a Bayesian game with a finite number of players, and actions.

(a) Prove the equivalence between the set of the *ex ante* and *interim* Bayesian Nash equilibria.
(b) Argue how the set of *ex post* Bayesian Nash equilibria relates to these.
(c) Assume independent private values. Is the equilibrium in the descending/Dutch auction *ex post*? Is the equilibrium in the ascending/English auction *ex post*?

**Question 4: Solving an All Pay Auction via the Revenue Equivalence Theorem**
Consider an auction of a single object with *I* risk-neutral bidders with independent private valuations for the object that are independently drawn from a from a uniform distribution on [0,V]. The auctioneer sells the object through an ‘‘all-pay’’ auction, defined as a simultaneous sealed-bid auction in which the high bidder wins the object, but *every* bidder pays her submitted bid.

(a) What would be the expected payment of a bidder in the second-price (sealed-bid) auction in this market?

(Hint: You might use that, for a uniform distribution F[a,b], the expected kth order statistic for an independent draw of $I$ values is equal to $a + (b-a)((I+1-k)/(I+1))$.

(b) Applying the Revenue Equivalence Theorem, solve for the bidding functions in a symmetric equilibrium in the ''all-pay'' auction.

**Practice problems** (You do not need to turn answers to the question below. Solutions will be provided.)

Other practice problems: Sample midterms will be available at Learn@UW.