Econometrics HW3

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December 2, 2020

1 7.28

1.1 Part A

	Edu	Exp	Exp^2/100	Constant
Coefficient	0.14431	0.042633	-0.095056	0.53089
Robust SE	0.011726	0.012422	0.033796	0.20005

1.2 Part B

The derivative of log wage with respect to education is β_1 and the derivative of log wage with respect to experience is $\beta_2 + \beta_3 exp/50$ so $\theta = \frac{\beta_1}{\beta_2+\beta_3 exp/50}$. Therefore, for 10 experience, our estimate implied by our regressions is the following:

$$\hat{\theta} = \frac{\hat{\beta}_1}{\hat{\beta}_2 + \hat{\beta}_3 exp/50} \tag{1}$$
$$= \frac{0.1443}{0.0426 - 0.0951(10)/50} \tag{2}$$
$$= 6.109 \tag{3}$$

1.3 Part C

We can find the asymptotic standard error as the square root of the asymptotic variance of the $\hat{\theta}$ estimator, which we can calculate through the delta method:

$$s(\hat{\theta}) = \sqrt{g'(\beta)'Vg'(\beta)},$$

*I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, and Ryan Mather. I have also discussed problem(s) with Emily Case, Sarah Bass, and Danny Edgel.

where V is the asymptotic covariance matrix of the non-intercept coefficients, and $g(\beta) = \frac{\hat{\beta}_1}{\hat{\beta}_2 + \hat{\beta}_3 exp/50}$. Then,

$$g'(\beta) = \begin{pmatrix} \frac{1}{\beta_2 + \beta_3 exp/50} \\ \frac{-\beta_1}{(\beta_2 + \beta_3 exp/50)^2} \\ \frac{-\beta_1 exp/50}{(\beta_2 + \beta_3 exp/50)^2} \end{pmatrix}$$

We can calculate an estimate for $s(\hat{\theta})$ by plugging in OLS estimates of β and our robust standard error matrix we used in Part A. Our 90% CI is $[\hat{\theta} - 1.645 s(\hat{\theta}), \hat{\theta} + 1.645 s(\hat{\theta})]$.

1.4 Part D

Our computed $\hat{\theta}$, $s(\hat{\theta})$, and confidence interval are the following:

$$\begin{aligned} \hat{\theta} &= 6.109 \\ s(\hat{\theta}) &= 1.6178 \\ CI &= [4.4912, 7.7269] \end{aligned}$$

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2 8.1

Let $\beta = [\beta_1, \beta_2]$ be the CLS estimator of $Y = X_1'\beta_1 + X_2'\beta_2 + e$ subject to the constraint that $\beta_2 = 0$. From definition (8.3),

$$\begin{aligned} \beta &= \underset{\beta_2 = 0}{\arg\min} (Y - X_1\beta_1 - X_2\beta_2)'(Y - X_1\beta_1 - X_2\beta_2) \\ \Rightarrow \mathcal{L} &= (Y - X_1\beta_1 - X_2\beta_2)'(Y - X_1\beta_1 - X_2\beta_2) + \lambda'(\beta_2 - 0) \\ \Rightarrow 0 &= -2X_1'(Y - X_1\beta_1 - X_2\beta_2) \\ \Rightarrow X_1'Y &= (X_1'X_1)\beta_1 \\ \Rightarrow \beta_1 &= (X_1'X_1)^{-1}X_1'Y. \end{aligned}$$

3 8.3

$$
\begin{aligned}
\beta &= \underset{\beta_1=-\beta_2}{\arg\min}(Y - X_1\beta_1 - X_2\beta_2)'(Y - X_1\beta_1 - X_2\beta_2) \\
\Rightarrow \mathcal{L} &= (Y - X_1\beta_1 - X_2\beta_2)'(Y - X_1\beta_1 - X_2\beta_2) + \lambda'(\beta_2 + \beta_1) \\
\Rightarrow 0 &= -2X_1'(Y - X_1\beta_1 - X_2\beta_2) + \lambda \\
\Rightarrow 0 &= -2X_2'(Y - X_1\beta_1 - X_2\beta_2) + \lambda \\
\Rightarrow 0 &= (X_1 - X_2)'(Y - X_1\beta_1 + X_2\beta_1) \\
\Rightarrow \beta_1 &= -\beta_2 = ((X_1 - X_2)'(X_1 - X_2))^{-1}(X_1 - X_2)'Y
\end{aligned}
$$

4 8.4(a)

Let $Z = X$

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\begin{aligned}
\alpha &= \underset{\beta=0}{\arg\min}(Y - X\beta - \alpha)'(Y - X\beta - \alpha) \\
\Rightarrow \mathcal{L} &= (Y - X\beta - \alpha)'(Y - X\beta - \alpha) + \lambda'(\beta) \\
\Rightarrow 0 &= -\vec{1}(Y - X\beta - \alpha) \\
\Rightarrow \alpha &= \frac{1}{n}\vec{1}'Y = \frac{1}{n}\sum_i Y_i
\end{aligned}
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5 8.22

5.1 Part A

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\begin{aligned}
\tilde{\beta} &= \underset{2\beta_2=\beta_1}{\arg\min}(Y - X_1\beta_1 - X_2\beta_2)'(Y - X_1\beta_1 - X_2\beta_2) \\
\Rightarrow \mathcal{L} &= (Y - X_1\beta_1 - X_2\beta_2)'(Y - X_1\beta_1 - X_2\beta_2) + \lambda'(2\beta_2 - \beta_1) \\
\Rightarrow 0 &= -2X_1'(Y - X_1\beta_1 - X_2\beta_2) + \lambda \\
\Rightarrow 0 &= -2X_2'(Y - X_1\beta_1 - X_2\beta_2) + 2\lambda \\
\Rightarrow 0 &= (2X_1 + X_2)'(Y - X_1 2\beta_2 - X_2\beta_2) \\
\Rightarrow \tilde{\beta}_2 &= ((2X_1 + X_2)'(2X_1 + X_2))^{-1}(2X_1 + X_2)'Y \\
\Rightarrow \tilde{\beta}_1 &= 2\tilde{\beta}_2
\end{aligned}
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5.2 Part B

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\begin{aligned}
\sqrt{n}(\tilde{\beta}_2 - \beta_2) &= 2\sqrt{n}((2X_1 + X_2)'(2X_1 + X_2))^{-1}(2X_1 + X_2)'e \\
&= 2(\frac{1}{n}\sum_i (2X_{1,i} + X_{2,i})^2)^{-1}\frac{1}{\sqrt{n}}\sum_i (2X_{1,i} + X_{2,i})e_i \\
&\Rightarrow N\left(0, \frac{E[(2X_{1,i} + X_{2,i})^2 e_i^2]}{E[(2X_{1,i} + X_{2,i})^2]^2}\right)
\end{aligned}
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6 9.1

7 9.2

8 9.4

9 9.7