Econ 713 Midterm (Part 2)

April 7, 2020

Carefully justify all your responses. You may email Jonathan with clarifying questions. You have 24 hours for 60 points. Good luck!

Question 1: The Careless Auctioneer[1] **[25pts]**

Suppose that we have $I = 2$ bidders with valuations for the auctioned good $v_i = \theta_i$, with $\theta_i \overset{i.i.d.}{\sim} U[0,1]$. Bidders submit sealed bids in an FPA format. Only the higher bidder pays their bid, but the auctioneer is a bit scatterbrained and will mistakenly give the good to the *lower* bidder with probability $a \in [0,1]$. If both players bid the same amount then a fair coin flip determines who gets and pays for the good.

(a) Define a Bayesian Game for this problem. [5]

(b) Without using any algebra, describe how the bidders will behave if $a \geq 1/2$ and why. [5]

For the remainder of the problem you may assume that $a \in [0, 1/2]$.

(c) How should a $0-$valuation bidder behave? Use your insight from the $0-$valuation case to find a symmetric BNE. [10]

(d) What is the expected revenue of this auction? If it is different from that of a standard FPA (that is, one without such an incompetent auctioneer), explain why the two auctions don't satisfy the criteria specified in the Revenue Equivalence Theorem. [5]

Question 2: The Bidding Cartel **[35pts]**

Consider a single-unit second-price sealed-bid auction between $N > 2$ bidders, with types distributed $\theta_i \overset{i.i.d.}{\sim} U[0,1]$. Payoffs are quasilinear in types–if player i wins the object, they receive utility

$$u_i = m_i + \theta_i$$

where m_i is the (remaining) wealth of player i. Suppose that $M < N$ of these bidders form a bidding cartel in which all participants coordinate on an enforceable collusive set of strategies to maximize the total utility of the cartel members.

(a) Does the optimal bidding strategy for non-cartel members change from the strategy in the generic second-price auction? [5]

(b) Find the optimal bidding strategies for all cartel members. [5]

(c) Find the expected value of the winning bid. Does this change from the second-price sealed bid auction without collusion? [5]

(d) If an *additional* bidder joins the auction, do the non-cartel bidders prefer this new bidder to join as part of the cartel or as an independent bidder? [5]

(e) Assume that there is only a single non-cartel member. Compute the expected revenue of this auction. Is it the same as or different than the expected revenue in the second price auction without collusion? Explain. [15]

[1] Thanks to Garrett Anstreicher for providing this question.