

**MICROECONOMIC THEORY (ECON 713)**  
**UNIVERSITY OF WISCONSIN-MADISON, PROF. MARZENA ROSTEK**

**MIDTERM**  
April 13, 2010

Time: 75 minutes  
Number of questions: 3  
Number of points: 26  
Rules: **Closed-book** exam

**Good luck!**

**Question 1: Equilibria in a First-Price Auction with Common Values [10 points]**

Consider a first-price sealed-bid auction with 2 bidders with common values. (The allocation rules, including tie-breaking are standard, as in class.) The valuation of each bidder  $i$  is given by  $v_i = \alpha t_i + \gamma t_j$ , where  $j$  is the other player,  $t_h$  is the signal received by player  $h=i,j$  (e.g., the number of barrels of oil in a tract) and  $\alpha, \gamma > 0$ . Each bidder knows only his own signal and that the signals come from a uniform distribution on  $[0,1]$ , which is common knowledge.

- (i) Define a Bayesian Game induced by this auction.
- (ii) Define a Bayesian Nash Equilibrium of this game.
- (iii) Derive the linear Bayesian Nash Equilibrium of the game.

**Question 2: Adverse Selection [12 points]**

Consider a market for second-hand cars with a continuum of sellers, each of which owns a car of value  $v$  in  $[0,1]$  and values the car at  $u_s = \theta v$  and a continuum of buyers who derive utility  $u_b = \theta v$  from driving a car,  $\theta$  in  $[0,1]$ ;  $\theta$  is common to all sellers. The values  $v$  are distributed according to a continuous c.d.f.  $F(v)$ , which are independent across sellers and buyer characteristics  $\theta$  are distributed according to a continuous c.d.f.  $G(\theta)$ . Assume that the value  $v$  is known to the seller and the buyer knows only the distribution.

- (i) Which buyers should buy? (i.e., what is the efficient volume of trade (in terms of  $G()$ )?) Which sellers ( $v$ 's) should sell?
- (ii) Find the demand and supply functions (in terms of  $G()$  and  $F()$ , respectively). Is the demand necessarily downward-sloping? Explain.
- (iii) Let  $f$  and  $g$  be densities of  $F$  and  $G$ , respectively. Solve for the competitive equilibrium assuming that  $f$  and  $g$  are uniform on  $[0,1]$ . Is trade efficient?
- (iv) Suppose that a minimal quality standard  $s_0 > 0$  is introduced (i.e., selling a car of quality lower than  $s_0$  is prohibited). Show that the standard may improve welfare.
- (v) Ignore the quality standard and go back to the general specification in (2). Show that a higher-price equilibrium Pareto dominates a lower price equilibrium.

**Question 3: Axiomatization [4 points]**

What are the benefits of axiomatization? In what sense is axiomatization useful outside of decision theory?