

Econometrics HW2

Michael B. Nattinger*

February 10, 2021

1 Question 1

1.1 Part i

$$\begin{aligned} E[ZX'] &= E \left[\begin{pmatrix} Z_1 \\ X_2 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}' \right] \\ &= \begin{pmatrix} E[Z_1X_1] & E[Z_1X_2'] \\ E[X_2X_1] & E[X_2X_2'] \end{pmatrix} \\ E[ZZ'] &= E \left[\begin{pmatrix} Z_1 \\ X_2 \end{pmatrix} \begin{pmatrix} Z_1 \\ X_2 \end{pmatrix}' \right] \\ &= \begin{pmatrix} E[Z_1^2] & E[Z_1X_2'] \\ E[X_2Z_1] & E[X_2X_2'] \end{pmatrix} \end{aligned}$$

Note that $E[X_2X_2']$ must be invertible for either $E[ZX']$ or $E[XX']$ to be invertible.

Block inversion implies that $E[ZX']$ is invertible iff $E[Z_1X_1] - E[Z_1X_2']E[X_2X_2']^{-1}E[X_2X_1] \neq 0$, and similarly $E[ZZ']$ is invertible iff $E[Z_1^2] - E[Z_1X_2']E[X_2X_2']^{-1}E[X_2Z_1] \neq 0$

*I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, and Ryan Mather. I have also discussed problem(s) with Emily Case, Sarah Bass, Katherine Kwok, and Danny Edgel.

1.2 Part ii

1.3 Part iii

1.4 Part iv

1.5 Part v

2 Question 2

2.1 Part i

2.2 Part ii

2.3 Part iii

2.4 Part iv

2.5 Part v

3 Question 3

3.1 Part i

3.2 Part ii

3.3 Part iii

3.4 Part iv

3.5 Part v