Microeconomic Theory - 713 II

Midterm

April 11, 2013

Time: 75 minutes

Points: 50 points

Good luck and allocate your time efficiently!

Professor : Marzena Rostek

TA : Jose Antonio Carrasco

1. **3-rd Price Auction - [30 points]** Consider an independent value auction for a single object with N bidders, all of whom place their bids simultaneously. As usual, the rules include the assignment of the object to the player with the highest bid. What is different in this case, is that the winner **only pays the 3-rd highest bid**, whereas all players who lose, pay 0. Valuations v are drawn from a common distribution $F(\cdot)$ with a strictly positive density $f(\cdot)$ on the support $[0, V]$.

   (a) Define the game and the Bayesian Nash Equilibrium.

   (b) Is truthful bidding (i.e $b(v) = v$) a dominant strategy? Explain if there are any incentives to bid higher or lower than the true valuation. *(Hint: Argue by contradiction.)*

   (c) Find the condition to solve for the *symmetric* pure strategy BNE in *strictly increasing strategies* ($b^*(v)$) for the **general 3-rd price auction**. Do *not* solve for equilibrium, just state the condition that we know should hold in equilibrium. *(Hint: You can use the Revenue Equivalence Theorem, for instance, with the all pay auction.).*

   (d) Assume uniform distribution for valuations, namely $f(v) = 1/V$ for all $v \in [0, V]$. Solve for the *symmetric* pure strategy BNE in *strictly increasing strategies* for the **3-price auction**.*(Hint: Use the condition you found in* (c) *and differentiate twice with respect to the valuation.)*

   (e) Compare the bids of the third price auction with the second price auction. Is there overbidding or shading? Explain.

2. **Duopoly with one sided demand information - [20 points]** Consider two firms in the Cournot market, who compete by choosing quantities q_1 and q_2. Firms face the same technology and, hence, their marginal costs are the same $c_1 = c_2 = c$. The two firms serve the same market defined by the inverse demand function $P(Q) = a - Q$, $Q \leq a$ and $P(Q) = 0$ for $Q > a$. However, while firm 2 knows the demand size a, firm 1 only knows that $a \sim F$. Finally, assume all of this is common knowledge.

   (a) Find the Bayesian Nash Equilibrium where both firms produce some positive quantity (i.e., find the conditions such that both produce positive quantity in equilibrium).

   (b) Consider the particular case $c_1 = c_2 = 0$. Compute the expected payoffs $\mathbb{E}[\pi_1]$ and π_2. Which firm gets more profits? Explain.