Midterm
ECON 713 Part II
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Good luck and allocate your time efficiently!

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1. (**A Bundle Auction** (**30 points**)) The Wisconsin Government owns two pieces of land. The commercial values of Land 1 and Land 2 are v_1 and v_2, respectively, which are common to every potential bidder. Assume that v_1 and v_2 are independent and uniformly distributed, $U[0, 1]$ (which is the common prior). There are two bidders: bidder 1 and bidder 2. Bidder 1 knows v_1 but not v_2 and bidder 2 knows v_2 but not v_1.

   Suppose that the Wisconsin Government bundles these two pieces of land together and sells them in a *single second price auction for the bundled good.* Each bidder's valuation of the bundled good in the auction is the sum of v_1 and v_2.

   (a) (**5 points**) Define a Bayesian Game for this problem.

   (b) (**10 points**) Find a symmetric BNE (if any) in which each bidder is using a strictly increasing linear bidding strategy *without a constant term.* Remember to check the second-order condition.

   (c) (**5 points**) Is the equilibrium you found in (b) in weakly dominant strategies? Prove or disprove that it is.

   (d) (**2 points**) Find the Government's ex-ante expected revenue given the equilibrium in (b).

   Now, suppose that the government adopts an all-pay auction, i.e., the highest bid wins and each bidder pays his/her bid no matter whether s/he wins or not.

   (e) (**8 points**) Find a symmetric BNE (if any) in which each bidder uses a bidding strategy $b_i = av_i^2$ for some $a > 0$. Remember to check the second-order condition.

2. (**A Gamble** (**16 points**)) Two players gamble with each other. The rules are as follows:

   - First, two numbers, t_1 and t_2, are independently drawn from $U[0, 1]$ and assigned to player 1 and player 2, respectively. Each player's number is her private information.
   - After seeing their numbers, they simultaneously decide to *raise* or to *fold.*
   - The final payoffs are determined as follows: if both players raise, the player with a higher number t_i gets 2 while the other player gets -2; if one of them raises, the person who raises gets 1 and her opponent gets -1; if both of them fold, they both receive payoff 0.

   (a) (**7 points**) Show that given any strategy used by player 2, if it is optimal for player 1 to raise when $t_1 = \bar{t}$, it is also optimal for player 1 to raise when $t_1 > \bar{t}$.

   (b) (**9 points**) Motivated by (a), let's assume that each player is using a threshold strategy, i.e., player i raises if and only if $t_i > a$ for some a. Solve for a symmetric BNE in such strategies.