Macro PS2

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1 Question 1

1.1 Part A

We will write down the equations to characterize the system, normalize by A, and then solve for our difference equations.

Households maximize utility subject to their budget constraints:

$$
\begin{aligned}
&\max_{\{C_t,I_t,N_t,K_t\}_{t=0}^{\infty}} \sum_{i=0}^{\infty} \beta^t u(C_t, 1-N_t) \\
&\text{s.t. } \sum_{t=0}^{\infty} p_t(C_t + I_t) = \sum_{t=0}^{\infty} p_t(r_t K_t + w_t A_t N_t) + \Pi_0 \\
&\text{and } K_{t+1} = (1-\delta)K_t + I_t
\end{aligned}
$$

Firms maximize profits:

$$
\begin{aligned}
&\max \Pi_0 = \sum_{t=0}^{\infty} p_t(Y_t - r_t K_t^d - w_t A_t N_t^d) \\
&\text{s.t. } Y_t = F(K_t, A_t N_t^d)
\end{aligned}
$$

Now we normalize. Let lowercase $x_t = X_t/A_t$:

HH:

$$
\begin{aligned}
&\max_{\{c_t,i_t,n_t,k_t\}_{t=0}^{\infty}} \sum_{i=0}^{\infty} \beta^t u(c_t A_t, 1-N_t) \\
&\text{s.t. } \sum_{t=0}^{\infty} p_t(c_t + i_t) = \sum_{t=0}^{\infty} p_t(r_t k_t + w_t N_t) + \Pi_0 \\
&\text{and } k_{t+1}(1+g) = (1-\delta)k_t + i_t
\end{aligned}
$$

*I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, and Ryan Mather. I have also discussed problem(s) with Emily Case, Sarah Bass, and Danny Edgel.

Firms:

$$\begin{aligned}\max \Pi_0 &= \sum_{t=0}^{\infty} p_t(y_t - r_t k_t^d - w_t N_t^d) \\ \text{s.t. } y_t &= F(k_t^d, N_t^d)\end{aligned}$$

Now we begin to solve. We will start with the firm side:

$$\begin{aligned}F_k(k_t^d, N_t^d) &= r_t \\ F_N(k_t^d, N_t^d) &= w_t \\ \Rightarrow F(k_t^d, N_t^d) - F_k(k_t^d, N_t^d) r_t - F_N(k_t^d, N_t^d) w_t &= 0 \\ \Rightarrow \Pi_0 &= 0.\end{aligned}$$

Now we move to the HH side. Our HH problem can be reduced to the following:

$$\begin{aligned}&\max_{c_t, n_t} \sum_{i=1}^{\infty} \beta^t u(A_t c_t, 1 - N_t) \\ &\text{s.t. } \sum_{t=0}^{\infty} p_t(c_t + k_{t+1} - [r_t + (1-\delta)]k_t - w_t n_t) = 0\end{aligned}$$

Taking first order conditions of our lagrangian:

$$\begin{aligned}A_t \beta^t u_c(c_t A_t, 1 - N_t) &= \lambda p_t \\ A_{t+1} \beta^{t+1} u_c(c_{t+1} A_{t+1}, 1 - N_{t+1}) &= \lambda p_{t+1} \\ \Rightarrow (1+g) \frac{\beta u_c(c_{t+1} A_{t+1}, 1 - N_{t+1})}{u_c(c_t A_t, 1 - N_t)} &= \frac{p_{t+1}}{p_t} \\ -\beta^t u_n(c_t A_t, 1 - N_t) &= \lambda p_t w_t \\ -\beta^{t+1} u_n(c_{t+1} A_{t+1}, 1 - N_{t+1}) &= \lambda p_{t+1} w_{t+1} \\ \Rightarrow \frac{\beta u_n(c_{t+1} A_{t+1}, 1 - N_{t+1})}{u_n(c_t A_t, 1 - N_t)} &= \frac{p_{t+1} w_{t+1}}{p_t w_t}\end{aligned}$$

In the competitive equilibrium, markets clear:

$$\begin{aligned}k_t^d &= k_t \\ N_t^d &= N_t \\ c_t + k_{t+1}(1+g) - (1-\delta) K_t &= F(k_t^d, N_t^d)\end{aligned}$$

The equations that give us C_0, N_0, w_0, r_0 are the following four equations:

$$F_k(k_t, N_t) = r_t \tag{1}$$

$$F_N(k_t, N_t) = w_t \tag{2}$$

$$A_t u_c(c_t A_t, 1 - N_t) = -u_n(c_t A_t, 1 - N_t)/w_t \tag{3}$$