

# Macro PS7

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## 1 Question 1

The planner maximizes utility subject to the resource constraint:

$$\begin{aligned} \max_{c_t^t, h_t, c_t^{t-1}} \quad & \ln(c_t^t) + \alpha h_t + \beta c_t^{t-1} \\ \text{s.t.} \quad & c_t^t + c_t^{t-1} = y \\ & \text{and } h_t = H^s \end{aligned}$$

Clearly we can use the resource constraints to find that  $h_t = H^s = 1$ , and then solve for  $c_t^t = y - c_t^{t-1}$ . We then rewrite our optimization problem as:

$$\max_{c_t^{t-1}} \ln(y - c_t^{t-1}) + \alpha + \beta c_t^{t-1}$$

Taking FOCs:

$$\begin{aligned} \frac{du}{dc_t^{t-1}} = 0 &\Rightarrow \beta = \frac{1}{y - c_t^{t-1}} \Rightarrow c_t^{t-1} = y - \frac{1}{\beta} \\ &\Rightarrow c_t^t = y - (y - \frac{1}{\beta}) = \frac{1}{\beta} \end{aligned}$$

## 2 Question 2

### 2.1 Part A

The young agents face the following optimization problem:

$$\begin{aligned} \max_{c_t^t, h_t, c_t^{t-1}} \quad & \ln(c_t^t) + \alpha h_t + \beta c_t^{t-1} \\ \text{s.t.} \quad & c_t^t + p_t h_t \leq y \\ & \text{and } c_{t+1}^t \leq p_{t+1} h_t \end{aligned}$$

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\*I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, and Ryan Mather. I have also discussed problem(s) with Emily Case, Sarah Bass, and Danny Edgel.

## 2.2 Part B

Markets clearing in the goods and housing market implies the following:

$$\begin{aligned}c_t^t + c_t^{t-1} &= y \\ h_t &= H^s = 1\end{aligned}$$

## 2.3 Part C

A competitive equilibrium is a set of allocations and prices such that agents optimize and markets clear.

## 2.4 Part D

Utility is strictly increasing in consumption, so the budget constraints will hold with equality and we can substitute  $c_t^t = y - p_t h_t$ ,  $c_{t+1}^t = p_{t+1} h_t$  into the maximization problem and take first order conditions:

$$\begin{aligned}\max_{h_t} & \ln(y - p_t h_t) + \alpha h_t + \beta p_{t+1} h_t \\ \frac{du}{dh_t} = 0 & \Rightarrow \frac{p_t}{y - p_t h_t} = \alpha + \beta p_{t+1} \\ \Rightarrow h_t &= \frac{y}{p_t} - \frac{1}{\alpha + \beta p_{t+1}} \\ \Rightarrow c_t^t &= \frac{p_t}{\alpha + \beta p_{t+1}} \\ \Rightarrow c_{t+1}^t &= \frac{y p_{t+1}}{p_t} - \frac{p_{t+1}}{\alpha + \beta p_{t+1}}\end{aligned}$$

We should verify that consumption and housing are nonnegative. This will be the case so long as  $p_t, p_{t+1}$  are positive and  $\frac{y}{p_t} > \frac{1}{\alpha + \beta p_{t+1}}$ .

## 2.5 Part E

From the market clearing conditions,

$$\begin{aligned}\frac{y}{p_t} - \frac{1}{\alpha + \beta p_{t+1}} &= 1 \Rightarrow p_{t+1} = \frac{p_t}{\beta(y - p_t)} - \frac{\alpha}{\beta} \\ \Rightarrow c_t^t &= y - p_t, c_{t+1}^t = p_{t+1}\end{aligned}$$

Note that this also implies the second condition for consumption to be nonnegative. Now we should check the price condition:

$$0 \leq p_{t+1} = \frac{p_t}{\beta(y - p_t)} - \frac{\alpha}{\beta} \Rightarrow p_t \geq \frac{\alpha}{1 + \alpha} y.$$

We are given that  $p_t < y$  so  $\frac{\alpha}{1 + \alpha} y \leq p_t \leq y$ .

## 2.6 Part F

In the steady state,  $\bar{p} = p_t = p_{t+1} \Rightarrow \bar{p} = \frac{\bar{p}}{\beta(y-\bar{p})} - \frac{\alpha}{\beta}$

$$\Rightarrow 0 = \bar{p} - \alpha(y - \bar{p}) - \bar{p}\beta(y - \bar{p})$$

$$\Rightarrow 0 = \beta\bar{p}^2 + (1 + \alpha - \beta y)\bar{p} - \alpha y$$

$$\Rightarrow \bar{p} = \frac{-(1 + \alpha - \beta y) \pm \sqrt{(1 + \alpha - \beta y)^2 + 4\beta\alpha y}}{2\beta}$$

The nonnegativity constraint rules out the negative  $\bar{p}$  value so  $\bar{p} = \frac{-(1+\alpha-\beta y) + \sqrt{(1+\alpha-\beta y)^2 + 4\beta\alpha y}}{2\beta}$ .

## 2.7 Part G

Since in the steady state  $p_t = \bar{p} = \frac{-(1+\alpha-\beta y) + \sqrt{(1+\alpha-\beta y)^2 + 4\beta\alpha y}}{2\beta}$ ,  $\bar{c}_t^t = \bar{p} = \frac{-(1+\alpha-\beta y) + \sqrt{(1+\alpha-\beta y)^2 + 4\beta\alpha y}}{2\beta}$ ,  $\bar{c}_{t+1}^t =$

$y - \bar{p} = y - \frac{-(1+\alpha-\beta y) + \sqrt{(1+\alpha-\beta y)^2 + 4\beta\alpha y}}{2\beta}$  which is not the planner's solution of  $c_t^t = \frac{1}{\beta}$ ,  $c_{t+1}^t = y - \frac{1}{\beta}$ . However, in both equilibria the housing is the same due to market clearing in the housing market:  $\bar{h} = 1 = h_t$ .