## MICROECONOMIC THEORY (ECON 713) UNIVERSITY OF WISCONSIN-MADISON, PROF. MARZENA ROSTEK

## MIDTERM April 14, 2011

Time: **75** minutes Number of questions: 2 Number of points: **40** Rules: **Closed-book** exam

## Good luck!

**Question 1: An auction [25 points]** Consider a second price auction with two bidders and an entrance fee f. The valuation for each bidder is uniformly distributed on [0, 1]. The timing is as follows. First, the seller offers a second price auction with an entrance fee f > 0, which is a fee that the bidders have to pay if they participate in the auction, independently of the outcome of the auction. Then, given the auction format, the bidders have to simultaneously decide whether or not to participate, and if so, how much to bid. If they don't bid at all (or equivalently bid zero) then they don't have to pay the fee but will also never get the object.

- (a) Describe the payoff function of each bidder as a function of an arbitrary pair of bids.
- (b) Define the notion of a strategy and the notion of a Bayesian Nash equilibrium for the auction game with the entry fee. Describe the expected payoff of the auctioneer as a function of the bidding strategies and the reserve price.
  - (c) Does the imposition of an entrance fee change the bidding strategy of the agents?
  - (d) Compute the symmetric Bayesian Nash equilibrium.
  - (e) Compute the seller's expected revenue from the auction with an arbitrary entrance fee f > 0.
- (f) Which entrance fee maximizes the expected revenue of the auctioneer? Is the resulting allocation ex-post efficient (assume the seller's valuation of the good is zero)?

Question 2: A duopoly with one-sided private information [15 points] Consider a Cournot market (quantity competition) with two firms and the inverse demand function given by P(Q)=a-Q,  $Q \le a$  and P(Q)=0 for Q > a. Both firms have a constant marginal cost. However, only firm 2 knows its own marginal cost. Firm 1 knows that firm 2's cost is  $c_L$  with probability  $\theta$  and  $c_H$  with probability 1- $\theta$ ,  $0 < \theta < 1$ ,  $c_H > c_L$ . Firm 1's marginal cost is  $c_R$  and this is known to both firms. The cost function of firm e is  $C_{e}(q_{e})=c_{e}(q_{e})$ , for  $c_{e}=c$ ,  $c_L$ ,  $c_H$ . Find the Bayesian Nash Equilibrium. (Assume that values of  $c_H$  and  $c_L$  are close enough that there is an equilibrium in which both outputs are positive).