Macro PS7

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1 Question 1

The collateral constraint is binding, i.e. $R_t B_t = Q^{min}_{t,t+1} K_t$. Our arbitrage condition is the following:

$$R_t = \frac{E_t[Q_{t+1}] + \frac{1}{2}(\bar{K} - K_t)}{Q_t},$$

where the expectation is taken with respect to the realization of a_{t+1}. The budget constraint of the constrained household is the following:

$$\begin{aligned}
Q_t K_t &= (a_t + Q_t)K_{t-1} + B_t - R_{t-1}B_{t-1} \\
Q_t K_t &= (a_t + Q_t)K_{t-1} + \frac{Q^{min}_{t,t+1} K_t}{R_t} - R_{t-1}B_{t-1} \\
Q_t K_t &= \frac{1}{1 - \frac{Q^{min}_{t,t+1}}{R_t Q_t}} \left[(a_t + Q_t)K_{t-1} - R_{t-1}B_{t-1}\right]
\end{aligned}$$

Finally, optimality from the unconstrained household gives us that $R_t = \beta_2^{-1}$. Now we can rearrange to find something we can compute in Matlab:

$$Q_t = \beta_2 E_t[Q_{t+1}] + \frac{\beta}{2}(\bar{K} - K_t) \tag{1}$$

$$Q_t K_t = \frac{1}{1 - \frac{Q^{min}_{t,t+1}}{R_t Q_t}} \left[(a_t + Q_t)K_{t-1} - R_{t-1}B_{t-1}\right] \tag{2}$$

The state variables in this case are K_{t-1} and our realization of a.

2 Question 2

Across a grid of capital and prices we can initialize our guess for a function $Q(K_{t-1}, a_t, B_{t-1})$, for a grid of K_{t-1} levels, and for each grid point calculate K_t using (2) and then update $Q(K_{t-1}, a_t, B_{t-1})$ using equation (1) and our guess of $E[Q(K_t, a_{t+1})]$. We continue to iterate until this function has converged.

3 Question 3

4 Question 4