Macro PS2

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November 29, 2020

1 Question 1

We will construct the sequential market structure equilibrium. Each period, we our bond markets contain claims to each "tree".

1.1 Part A

Define q_t^i as the price in period t of a consumption good in period $t+1$ on the condition that consumer i receives an endowment in period $t+1$. $b_t^{i,j}$ is the quantity of that bond demanded by person j.

Each agent maximizes expected utility:

$$\max_{\{c_t^1, b_t^{1,1}, b_t^{2,1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t log c_t^1 \tag{1}$$

$$\text{s.t. } c_t^1 + q_t^1 b_t^{1,1} + q_t^2 b_t^{2,1} \leq e_t^1 + b_{t-1}^{1,1} 1\{e_t^1 = 1\} + b_{t-1}^{2,1} 1\{e_t^2 = 1\}$$

$$\max_{\{c_t^2, b_t^{1,2}, b_t^{2,2}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t log c_t^2 \tag{2}$$

$$\text{s.t. } c_t^2 + q_t^1 b_t^{1,2} + q_t^2 b_t^{2,2} \leq e_t^2 + b_{t-1}^{1,2} 1\{e_t^1 = 1\} + b_{t-1}^{2,2} 1\{e_t^2 = 1\}$$

Market clearing implies the following conditions:

$$b_t^{1,1} + b_t^{1,2} = 0 \tag{3}$$

$$b_t^{2,1} + b_t^{2,2} = 0 \tag{4}$$

$$c_t^1 + c_t^2 = e_t^1 + e_t^2 \tag{5}$$

The competitive equilibrium is a set of prices $\{q_t^1, q_t^2\}_{t=0}^{\infty}$ and allocations $\{b_t^{1,1}, b_t^{2,1}, b_t^{1,2}, b_t^{2,2}, c_t^1, c_t^2\}_{t=0}^{\infty}$ such that agents optimize (1),(2) and markets clear (3),(4),(5).

*I worked on this assignment with my study group: Alex von Hafften, Andrew Smith, and Ryan Mather. I have also discussed problem(s) with Emily Case, Sarah Bass, and Danny Edgel.

1.2 Part B

Our bellman equation takes the following form:

Once the future endowment has shifted, it will remain shifted forever. So,

1.3 Part C

2 Question 2

3 Question 3

3.1 Part A

Our bellman equation takes the following form:

$$\begin{aligned} V(a,l) &= \max_{a'} \frac{(wl+(1+r)a-a')^{1-\gamma}}{1-\gamma} + \beta E[V(a',l')] \\ &= \max_{a'} \frac{(wl+(1+r)a-a')^{1-\gamma}}{1-\gamma} + \beta(V(a',l_h)P(l'=l_h|l) + V(a',l_h)P(l'=l_h|l)) \end{aligned}$$

Taking FOCs and applying the envelope conditions,

$$\begin{aligned} (wl+(1+r)a-a')^{-\gamma} &= \beta(V'(a',l_h)P(l'=l_h|l) + V'(a',l_h)P(l'=l_h|l)) \\ V'(a,l) &= (1+r)(wl+(1+r)a-a')^{-\gamma} \\ \Rightarrow c^{-\gamma} &= \beta(1+r)((c_h')^{-\gamma}P(l'=l_h|l) + (c_l')^{-\gamma}P(l'=l_h|l)) \end{aligned}$$

The above equation forms our optimality conditions.

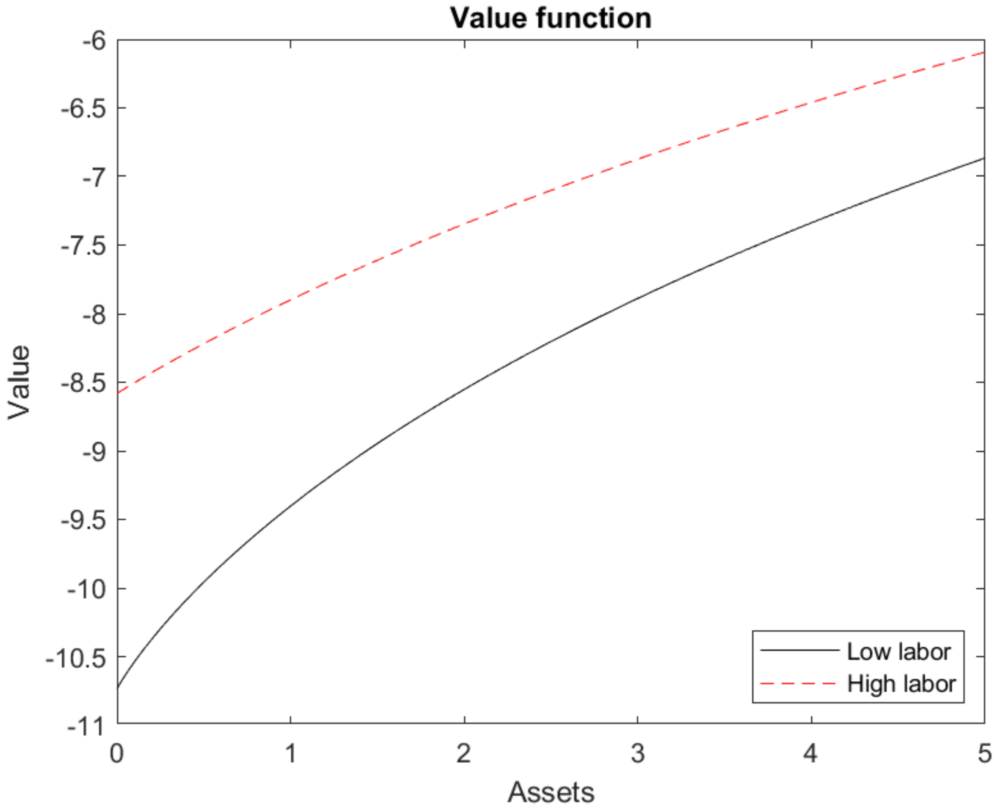
3.2 Part B

In the stationary distribution, $PQ = P$ where $P = [P_l P_h]$. We can easily solve for this distribution numerically from an arbitrary starting point. As this is a very simple markov process, markov-chain monte carlo methods will converge to the stationary distribution by standard ergodic properties which Q easily satisfies. We can therefore start with an arbitrary initial distribution P_0 and iterate through the transition matrix Q, with product $P := P_0 Q$ becoming next iteration's P_0, until P and P_0 have converged to some tolerance.

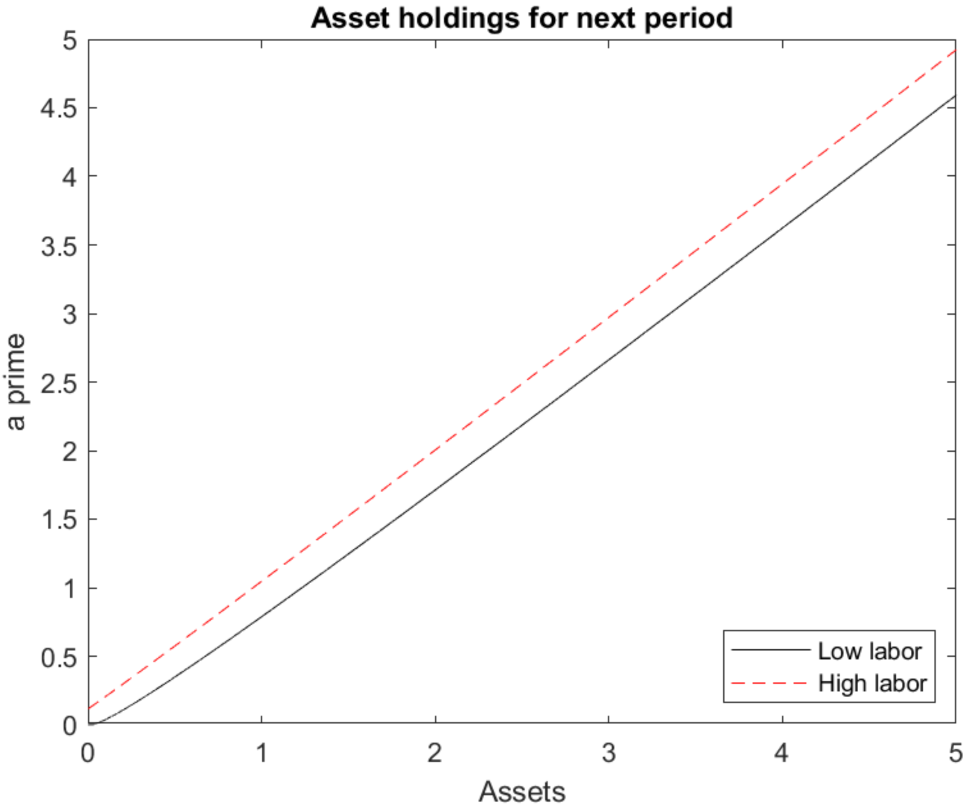
The numerical solution is $P = [0.250.75]$ so the stationary distribution has 3/4 of the weight on the high-labor distribution, and the rest on the low-labor distribution. Therefore, the unconditional mean of the labor endowment is $0.25(0.7) + 0.75(1.1) = 1$.

3.3 Part C

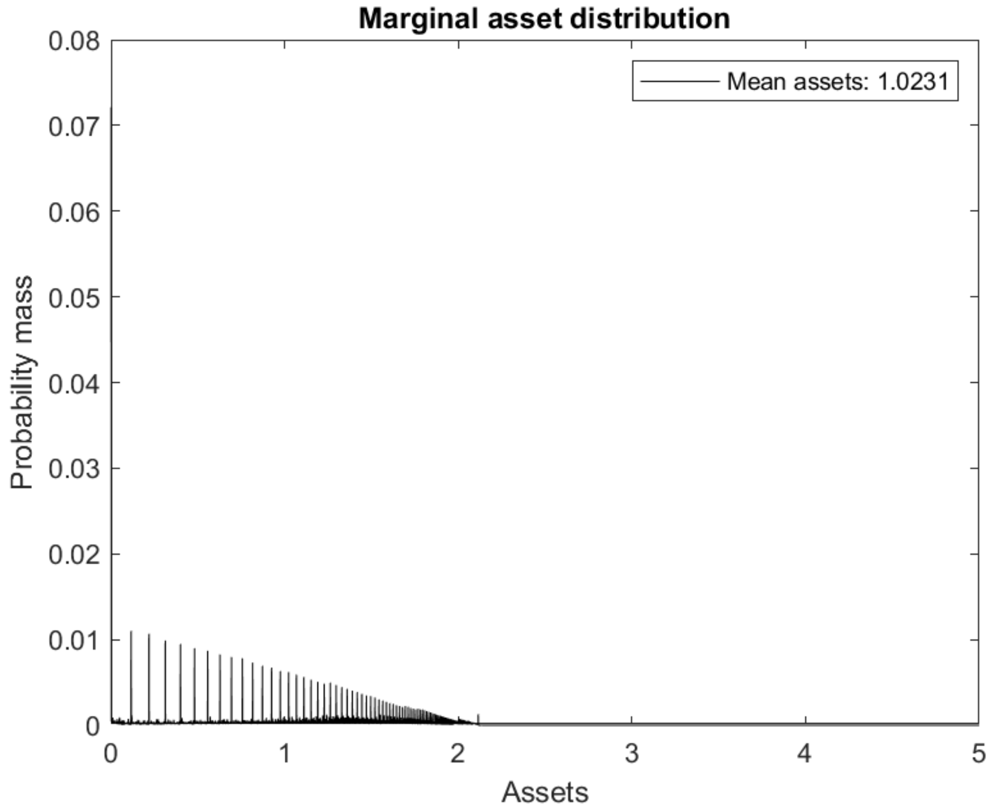
I solved for the value function numerically. Results are plotted below.



The value function solution does appear to be continuous, increasing, concave, and differentiable. The value from having a high labor draw is higher than the value from having a low labor draw. All of these features are as predicted by theory.


Asset holdings for next period
a prime
Assets
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
1
2
3
4
5
Low labor
High labor

3.4 Part D


Marginal asset distribution
Mean assets: 1.0231
Probability mass
Assets
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
1
2
3
4
5