Computational Problem Set 8

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1 Results

First we compare the score with the numerical first derivative, then the hessian with the numerical second derivative:

Score	Numerical derivative
-2.61e+03	-2.61e+03
-556	-556
-1.16e+03	-1.16e+03
-223	-223
-933	-933
-1.22e+03	-1.22e+03
-2.11e+03	-2.11e+03
-948	-948
-5.05e+03	-5.05e+03
-4.53e+03	-4.53e+03
-1.94e+04	-1.94e+04
-1.92e+04	-1.92e+04
-919	-919
-352	-352
-467	-467
-582	-582
-546	-546

Hessian																
-3.22e+03	-880	-1.43e+03	-388	-1.31e+03	-1.55e+03	-2.62e+03	-1.21e+03	-6.3e+03	-5.76e+03	-2.38e+04	-2.36e+04	-1.4e+03	-664	-682	-674	-583
-880	-880	0	-10.1	-404	-421	-686	-332	-1.72e+03	-1.66e+03	-6.61e+03	-6.56e+03	-390	-164	-189	-212	-170
-1.43e+03	0	-1.43e+03	-165	-560	-676	-1.19e+03	-545	-2.8e+03	-2.55e+03	-1.05e+04	-1.04e+04	-587	-284	-309	-299	-267
-388	-10.1	-165	-716	-186	-188	-325	-152	-784	-694	-2.74e+03	-2.72e+03	43.9	-59.8	-105	-92.4	-77.3
-1.31e+03	-404	-560	-186	-1.31e+03	-694	-973	-501	-2.56e+03	-2.59e+03	-9.55e+03	-9.59e+03	-502	-215	-291	-299	-193
-1.55e+03	-421	-676	-188	-694	-806	-1.23e+03	-585	-3.02e+03	-2.84e+03	-1.14e+04	-1.14e+04	-660	-312	-326	-326	-283
-2.62e+03	-686	-1.19e+03	-325	-973	-1.23e+03	-2.22e+03	-992	-5.13e+03	-4.62e+03	-1.92e+04	-1.91e+04	-1.23e+03	-545	-551	-540	-477
-1.21e+03	-332	-545	-152	-501	-585	-992	-529	-2.37e+03	-2.17e+03	-8.87e+03	-8.8e+03	-557	-248	-257	-253	-220
-6.3e+03	-1.72e+03	-2.8e+03	-784	-2.56e+03	-3.02e+03	-5.13e+03	-2.37e+03	-1.25e+04	-1.13e+04	-4.65e+04	-4.61e+04	-2.72e+03	-1.3e+03	-1.33e+03	-1.32e+03	-1.13e+03
-5.76e+03	-1.66e+03	-2.55e+03	-694	-2.59e+03	-2.84e+03	-4.62e+03	-2.17e+03	-1.13e+04	-1.08e+04	-4.26e+04	-4.23e+04	-2.42e+03	-1.17e+03	-1.22e+03	-1.21e+03	-1.03e+03
-2.38e+04	-6.61e+03	-1.05e+04	-2.74e+03	-9.55e+03	-1.14e+04	-1.92e+04	-8.87e+03	-4.65e+04	-4.26e+04	-1.77e+05	-1.75e+05	-1.01e+04	-4.91e+03	-5.02e+03	-4.97e+03	-4.27e+03
-2.36e+04	-6.56e+03	-1.04e+04	-2.72e+03	-9.59e+03	-1.14e+04	-1.91e+04	-8.8e+03	-4.61e+04	-4.23e+04	-1.75e+05	-1.74e+05	-9.98e+03	-4.85e+03	-4.98e+03	-4.94e+03	-4.25e+03
-1.4e+03	-390	-587	43.9	-502	-660	-1.23e+03	-557	-2.72e+03	-2.42e+03	-1.01e+04	-9.98e+03	-1.4e+03	-293	-307	-306	-268
-664	-164	-284	-59.8	-215	-312	-545	-248	-1.3e+03	-1.17e+03	-4.91e+03	-4.85e+03	-293	-664	0	0	0
-682	-189	-309	-105	-291	-326	-551	-257	-1.33e+03	-1.22e+03	-5.02e+03	-4.98e+03	-307	0	-682	0	0
-674	-212	-299	-92.4	-299	-326	-540	-253	-1.32e+03	-1.21e+03	-4.97e+03	-4.94e+03	-306	0	0	-674	0
-583	-170	-267	-77.3	-193	-283	-477	-220	-1.13e+03	-1.03e+03	-4.27e+03	-4.25e+03	-268	0	0	0	-583

Hessian (numerical)																
-7.34e+03	-2.12e+03	-3.26e+03	-1.81e+03	-2.79e+03	-2.97e+03	-5.05e+03	-3.35e+03	-8.21e+03	-8.08e+03	-2.51e+04	-2.49e+04	-3.25e+03	-799	-719	-2.8e+03	-2.47e+03
-2.12e+03	-2.12e+03	0	-460	-917	-816	-1.36e+03	-1e+03	-2.36e+03	-2.35e+03	-7.03e+03	-6.97e+03	-960	-199	-215	-862	-722
-3.26e+03	0	-3.26e+03	-800	-1.18e+03	-1.3e+03	-2.32e+03	-1.55e+03	-3.64e+03	-3.58e+03	-1.1e+04	-1.1e+04	-1.37e+03	-335	-327	-1.23e+03	-1.13e+03
-1.81e+03	-460	-800	-757	-668	-115	-960	-1.17e+03	638	198	-2.5e+03	-2.51e+03	-564	3.64	83.7	-1.01e+03	-889
-2.79e+03	-917	-1.18e+03	-668	-2.79e+03	-1.11e+03	-1.61e+03	-1.23e+03	-3.24e+03	-3.25e+03	-1e+04	-1e+04	-1.11e+03	-260	-285	-1.2e+03	-788
-2.97e+03	-816	-1.3e+03	-115	-1.11e+03	-769	-1.88e+03	-1.76e+03	-1.56e+03	-2.08e+03	-1.12e+04	-1.12e+04	-1.28e+03	-200	-214	-1.19e+03	-1.11e+03
-5.05e+03	-1.36e+03	-2.32e+03	-960	-1.61e+03	-1.88e+03	-3.8e+03	-2.55e+03	-4.27e+03	-4.52e+03	-1.97e+04	-1.96e+04	-2.39e+03	-455	-521	-1.93e+03	-1.69e+03
-3.35e+03	-1e+03	-1.55e+03	-1.17e+03	-1.23e+03	-1.76e+03	-2.55e+03	-1.25e+03	-746	-1.27e+03	-9.84e+03	-9.78e+03	-1.66e+03	-261	-361	-1.31e+03	-1.23e+03
-8.21e+03	-2.36e+03	-3.64e+03	638	-3.24e+03	-1.56e+03	-4.27e+03	-746	-1.15e+04	-1.08e+04	-4.5e+04	-4.48e+04	-3.62e+03	-899	-899	-2.8e+03	-2.47e+03
-8.08e+03	-2.35e+03	-3.58e+03	198	-3.25e+03	-2.08e+03	-4.52e+03	-1.27e+03	-1.08e+04	-1.08e+04	-4.17e+04	-4.15e+04	-3.55e+03	-885	-879	-2.8e+03	-2.47e+03
-2.51e+04	-7.03e+03	-1.1e+04	-2.5e+03	-1e+04	-1.12e+04	-1.97e+04	-9.84e+03	-4.5e+04	-4.17e+04	-1.76e+05	-1.75e+05	-1.07e+04	-4.79e+03	-4.91e+03	-5.77e+03	-5.02e+03
-2.49e+04	-6.97e+03	-1.1e+04	-2.51e+03	-1e+04	-1.12e+04	-1.96e+04	-9.78e+03	-4.48e+04	-4.15e+04	-1.75e+05	-1.74e+05	-1.05e+04	-4.76e+03	-4.87e+03	-5.76e+03	-4.97e+03
-3.25e+03	-960	-1.37e+03	-564	-1.11e+03	-1.28e+03	-2.39e+03	-1.66e+03	-3.62e+03	-3.55e+03	-1.07e+04	-1.05e+04	-3.25e+03	-348	-338	-1.23e+03	-1.12e+03
-799	-199	-335	3.64	-260	-200	-455	-261	-899	-885	-4.79e+03	-4.76e+03	-348	-799	-0.909	0	0
-719	-215	-327	83.7	-285	-214	-521	-361	-899	-879	-4.91e+03	-4.87e+03	-338	-0.909	-719	0	0
-2.8e+03	-862	-1.23e+03	-1.01e+03	-1.2e+03	-1.19e+03	-1.93e+03	-1.31e+03	-2.8e+03	-2.8e+03	-5.77e+03	-5.76e+03	-1.23e+03	0	0	-2.8e+03	0
-2.47e+03	-722	-1.13e+03	-889	-788	-1.11e+03	-1.69e+03	-1.23e+03	-2.47e+03	-2.47e+03	-5.02e+03	-4.97e+03	-1.12e+03	0	0	0	-2.47e+03

Now we compute results from numerical log likelihood maximization.

	Newton	Nelder-Mead	BFGS
computation time	1.2663	20.505	12.161
loglikelihood	-5311.5	-5323.6	-5311.5
constant	-6.0564	-5.4357	-6.0563
i_large_loan	0.86759	0.93991	0.86761
i_medium_loan	0.52736	0.5692	0.52737
rate_spread	0.59558	0.59846	0.5956
i_refinance	0.16339	0.0028961	0.1634
age_r	0.87122	1.3247	0.87114
cltv	-0.056624	0.0059927	-0.056576
dti	0.21508	0.0018195	0.21509
cu	1.0079	0.94179	1.0078
first_mort_r	0.3356	0.36558	0.33563
score_0	-0.28419	-0.18642	-0.28418
score_1	0.18943	0.0018143	0.18943
i_FHA	0.75858	0.72328	0.75859
i_open_year2	1.1527	1.0868	1.1527
i_open_year3	0.77016	0.73289	0.77018
i_open_year4	0.37934	0.3476	0.37939
i_open_year5	0.24021	0.18982	0.24023

As we can see above, the computational time of the code I wrote was much lower than the two other Matlab functions, fminsearch (Nelder-Mead) and fminunc (BFGS). This is primarily because the function I wrote is particularly tailored to the application and is quite streamlined, and uses the analytical hessian and gradient. The Matlab functions have no such analytical derivatives so they have to use numerical approximations, which are time consuming to compute.

We can see that the Nelder-Mead results are closer to the Stata results, but actually result in a lower likelihood than the Newton or BFGS methods. The Newton and BFGS methods seem to converge to approximately the same local maximum. I was unable to find any other local maxima that resulted in higher likelihoods than the ones presented.

Computational time is resulted in units of seconds, and measures total runtime from the beginning to end of the estimation procedure.