Economics 703 : Mid-Term Exam

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Answer three out of four questions. Each question is worth 33 points; the remaining point is free. Please be very explicit in your answers. Be sure to substantiate your answers by citing the proper definitions, and by proving your assertions. Also, make sure that every step in your argument follows logically and directly from the previous step.

1. Which of the following sets are compact? Which are connected? Substantiate your claim.

   (a) A finite set in $\mathbb{R}^n$.

   (b) The rationals in $[0,1]$.

   (c) $\{(x,y) \in \mathbb{R}^2 | xy \geq 1\} \cap \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 5\}$.

2. Let $A \subset \mathbb{R}$ and $M = \sup A$. Prove of disprove the following claim : M is a limit point of A.

3. Let $\{x_k\}$ be a sequence in $\mathbb{R}^n$ satisfying $||x_k - x_l|| \leq \frac{1}{k} + \frac{1}{l}$. Does x_k converge? Why or why not?

4. Determine whether the "curve" described by the equation $x^2+y+\sin(xy) = 0$ can be written in the form $y = f(x)$ in a neighbourhood of $(0,0)$. Can the equation be written in the form $x = h(y)$ in a neighbourhood of $(0,0)$? Prove your claim.