Midterm Exam

Raymond Deneckere

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Answer three out of four questions. Each question is worth 33 points; the remaining point is free. Be sure to substantiate your answers by citing the proper definitions, and by proving your assertions.

1. Let $f : K \subset \mathbb{R}^N \to \mathbb{R}$ be continuous and K compact. Define

$$M = \{x \in K : f(x) \geq f(y),\ \forall\, y \in K\}.$$

   Is M compact? Defend your answer.

2. Find the lim sup and lim inf of the sequence $\{s_m\}$, inductively defined by:

$$s_1 = 0; \quad s_{2m} = s_{2m-1/2}; \quad s_{2m+1} = s_{2m} + 1/2; \quad m \geq 1$$

3. Solve the following problem:

$$\max_{(x,y)\in\mathbb{R}^2} \{3x^3 - y^2(x-1)\}$$

4. Can the following system of equations

$$u(x,y,z) = x + xyz$$

$$v(x,y,z) = y + xz$$

$$w(x,y,z) = z + 2x + 3z^2$$

   be solved for x, y, z near $(0,0,0)$? Substantiate your claim.