University of Wisconsin-Madison
Department of Economics

Econ 703 | Prof. R. Deneckere
Fall 2001

Midterm Exam

Except for question #1, where you are specifically directed to do otherwise, be very explicit in your answers. Carefully state the appropriate definitions and argue how they apply. Also, make sure that every step in your argument follows logically and directly from the previous one.

1. (20 points) Find the lim sup and the lim inf of the sequence $\{s_n\}$, inductively defined by :

   $s_1 = 0$; $\quad s_{2m} = s_{2m-1}/2$; $\quad s_{2m+1} = s_{2m} + 1/2$; $\quad m \geq 1$.

   A numerical answer suffices.

2. (25 points) Determine which of the following sets are open, closed, convex, compact, and connected :
   (a) $\{(x,y)\in\mathbb{R}^2: x^2 + y^2 = 1\}$
   (b) $\{(x,y)\in\mathbb{R}^2: 0 < x < 1 \text{ and } y = x\}$
   (c) $\{(x,y)\in\mathbb{R}^2: y = \sin(1/x),\ x > 0\} \cup \{0,0\}$

3. (25 points) Let $f:[-1,+1]\to\mathbb{R}$ be defined by $f(0)=0$ and $f(x) = x^a\sin(1/x)$ for $x \neq 0$. For which values of a is the function f :
   (a) continuous?
   (b) differentiable?
   (c) continuously differentiable?

4. (15 points) Consider $f: [-1,+1]\to\mathbb{R}^2$ defined by $f_1(t) = t$ and $f_2(t) = t^2$. Is f(.) differentiable at t = 0?

5. (15 points) Is every compact metric space closed and bounded? Is the converse true? (You may cite any theorems we proved in class, but you must defend your answer).