Economics 703 : Mid-Term Exam

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Fall 2003

Please be very explicit in your answers. Carefully state the appropriate definitions and theorems and argue how they apply. Also, make sure that every step in your argument follows logically and directly from the previous step. Each question is worth 20 points.

1. Let $f : A \subset R^n \to R$ be a differentiable function, where A is convex. Suppose that $\| Df(x) \| \leq M$ for all $x \in A$.

   Mean Value thm

   (a) Prove that $| f(x) - f(y) | \leq M \| x - y \|$, for all $x, y \in A$.

   (b) Does the above formula hold when A is not convex? Prove your claim.

   $g(t) = (f(x) - f(y)) \cdot t - f(tx + (1-t)y)$

2. Let $A \subset R^n$ and $x \in R^n$. Define $d(x, A) = \inf\{d(x, y) \mid y \in A\}$, where $d(x, y)$ is the Euclidean distance between x and y. Must there exist a $z \in A$ such that $d(x, A) = d(x, z)$? (If the answer is in the affirmative, provide a proof. If the answer is not in the affirmative, then delineate the different ways in which the statement can fail to hold).

3. Let $\{x_n\}$ be a sequence in R^n such that $x_n \to x$.

   (a) Show that $x \in cl\{x_1, x_2, ...\}$.

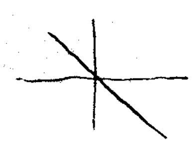
   (b) Is x a limit point of $\{x_1, x_2, ...\}$?

4. Let $A \subset R^n$ be connected, and let $f : A \to R$ be continuous with $f(x) \neq 0$ for all $x \in A$. Prove or disprove the following claim : $f(x)f(y) > 0$ for all $x, y \in A$.

   Intermediate

5. Determine the second order Taylor formula of the function $f : R^2 \to R$ given by the rule $f(x, y) = (x + y)^2$ around the point $(x_0, y_0) = (0, 0)$.

多变元Taylor公式.



$2x+2y \quad 2x+2y$

$x \quad y$

$2x^2 + 2xy + 2xy + 2y^2$

$= 2(x+y)^2$