

Economics 703 : Mid-Term Exam

Raymond Deneckere

Fall 2005

Please be very explicit in your answers. Carefully state the appropriate definitions and theorems and argue how they apply. Also, make sure that every step in your argument follows logically and directly from the previous step. Each question is worth 20 points.

1. Show that a sequence of real numbers $\{x_n\}$ converges in \mathbb{R} if and only if the “even” and “odd” subsequences $\{x_{2n}\}$ and $\{x_{2n-1}\}$ both converge in \mathbb{R} and they satisfy $\lim x_{2n} = \lim x_{2n-1}$.
2. Let A and B be non-empty subsets of \mathbb{R} . If A and B are closed, is the Cartesian product $A \times B$ a closed subset of \mathbb{R}^2 ? Is the converse true? Prove your claims.
3. If A is a non-empty closed subset of \mathbb{R}^n , and $x \notin A$, is there a point in A that is nearest to x ? Is it unique? Prove your claims.
4. Can the following system of equations
$$\begin{aligned}u(x, y, z) &= x + xyz \\v(x, y, z) &= y + xz \\w(x, y, z) &= z + 2x + 3z^2\end{aligned}$$
be solved for x, y, z near $(0, 0, 0)$? Substantiate your claim.
5. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be given by the rule $f(x, y) = x^3 - 3x^2 + y^2$. Solve the problem of maximizing and minimizing f .