Economics 703 : Anwer Key to the Mid-Term Exam

Raymond Deneckere

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1. Let $\{K_n\}$ be a collection of nonempty closed subsets of $\mathbb{R}^N$ such that $K_{n+1} \subset K_n$ for all $n = 0, 1, ...,$ and such that diameter$(K_n) \to 0$ as $n \to \infty$. Give a complete characterization of the set $\cap_{n=0}^{\infty} K_n$.

   Define $S_n = \{d(p, q) : p \in K_n \text{ and } q \in K_n\}$. Then letting δ_n =diameter(K_n) we have δ_n =sup S_n.

2. Find two subsets A and B of $\mathbb{R}^N$ and a point $x_0 \in \mathbb{R}^N$ such that $A \cup B$ is not connected but $A \cup B \cup \{x_0\}$ is connected.

3. Let $f : K \subset \mathbb{R}^N \to \mathbb{R}$ be continuous and K compact. Define

   $$M = \{x \in K : \forall\ y \in f(x) \geq f(y)\ \ \forall\ y \in K\}.$$

   Is M compact? Defend your answer.

4. Let $f : \mathbb{R} \to \mathbb{R}$ be given by the rule

   $$f(x) = \begin{cases} x^2 \text{, if } x \text{ is irrational} \\ 0\text{, if } x \text{ is rational} \end{cases}.$$

   Is $f(\cdot)$ continuous at 0? Is $f(\cdot)$ differentiable at 0? Defend your answer.

5. Consider the equations

   $$\begin{aligned} u(x, y) &= \frac{x^4 + y^4}{x}\text{, if } (x, y) \neq (0, 0) \\ &= 0\text{, if } (x, y) = (0, 0) \end{aligned}$$

   and

   $$v(x, y) = \sin x + \cos x.$$

   Near which points $(x, y) \in \mathbb{R}^2$ can we solve for x, y in terms of u, v? Defend your answer.