Econ 703 Prof. R. Deneckere
Fall 2002

Midterm Exam

1. (15 points) Compute the liminf and limsup of the following sequences (a numerical answer suffices):
   (a) $(-1)^{n-1}(1+ 1/n)$
   (b) Let $g(n)= n(-1)^n$

2. (25 points) For each of the following sets, determine if they are open, closed, compact and connected :
   (a) $\{x \in \mathbb{R}^n: 1 \leq \|x\| \leq 2\}$
   (b) $\{x \in \mathbb{R}^n: \|x\| = 1\}$
   (c) A finite subset set of $\mathbb{R}$

3. (20 points) Let $f: \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x,y) = x^2y^2/(x^2+y^2)^{1/2}$. Determine if f differentiable at (0,0).

4. (20 points) Investigate the nature of the critical point (0,0) of the function $f:\mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y)=x^2 + 2xy +y^2 +6$.

5. (20 points) Consider the function $f:\mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x,y)=((x^2 -y^2)/(x^2+y^2),xy/(x^2+y^2))$. Does it have a local inverse near (0,1)?