

Economics 703 : Mid-Term Exam

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Please be very explicit in your answers. Carefully state the appropriate definitions and theorems and argue how they apply. Also, make sure that every step in your argument follows logically and directly from the previous step. Each question is worth 20 points.

1. Which of the following sets are compact? Which are connected? Substantiate your claim.
 - (a) A finite set in \mathbb{R}^n .
 - (b) The rationals in $[0, 1]$.
 - (c) $\{(x, y) \in \mathbb{R}^2 | xy \geq 1\} \cap \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 5\}$.
2. Let $A \subset \mathbb{R}$ and $M = \sup A$. Prove or disprove the following claim : M is a limit point of A .
3. Let $\{x_k\}$ be a sequence in \mathbb{R}^n satisfying $\|x_k - x_l\| \leq \frac{1}{k} + \frac{1}{l}$. Does x_k converge? Why or why not?
4. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by the rule $f(x) = ax^3 + bx^2 + cx + d$, where $a > 0$. Show that f has a real root, i.e. there exists $x_0 \in \mathbb{R}$ such that $f(x_0) = 0$.
5. Determine whether the “curve” described by the equation $x^2 + y + \sin(xy) = 0$ can be written in the form $y = f(x)$ in a neighbourhood of $(0, 0)$. Can the equation be written in the form $x = h(y)$ in a neighbourhood of $(0, 0)$? Prove your claim.