# Adversarial procurement for prover market design

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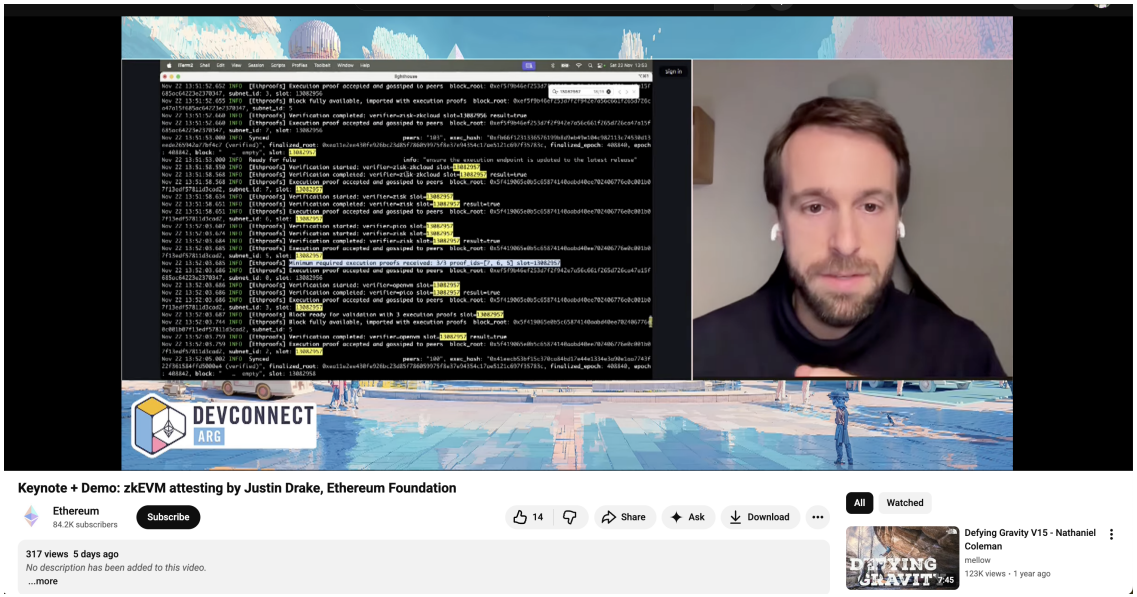
# Motivation

## Ethereum's ZK endgame




- **tl;dr;** Scale the L1 with ZK proofs.
  - ⋆ Validators verify proofs instead of re-executing transactions.
- **Prediction**: proving, especially as we continue to scale, will remain a relatively complex task.
- **Bad scenario**: liveness of the chain depends on a handful of provers.
- **Hopeful scenario**: good market design leads to robust proof acquisition and broader participation.
- We focus on balancing **robustness** and **cost efficiency**.

# Sneak peek 🧐

- The optimal protocol trades-off between **"over-provisioning"** the number of proofs and the risk of **not receiving any** proofs.
- Many considerations change the optimal mechanism.
  1. how many players there are,
  2. how many of them are corruptible,
  3. how costly a liveness fault is,
  4. a byzantine versus crash fault adversary, etc...
- **A word of caution**: this is a subtle task!

# Model (informal)

## Base game

- A one-shot **procurement** game.
- A set of $n$ players have a unit cost for proving.
- The "protocol" sets up a payment rule.
- The players decide whether or not to deliver (i.e., incur the cost).
- If no one delivers, the protocol incurs a **large penalty**, $C$.
- If at least one prover delivers, the protocol simply pays according to its payment rule.
- Naïve solution: tell player 1 you will pay them $1 + \varepsilon$ if they prove.
  - ⋆ This payment rule induces an equilibrium! And achieves the minimal cost! ☺
  - ⋆ ... but it is super vulnerable to attack.

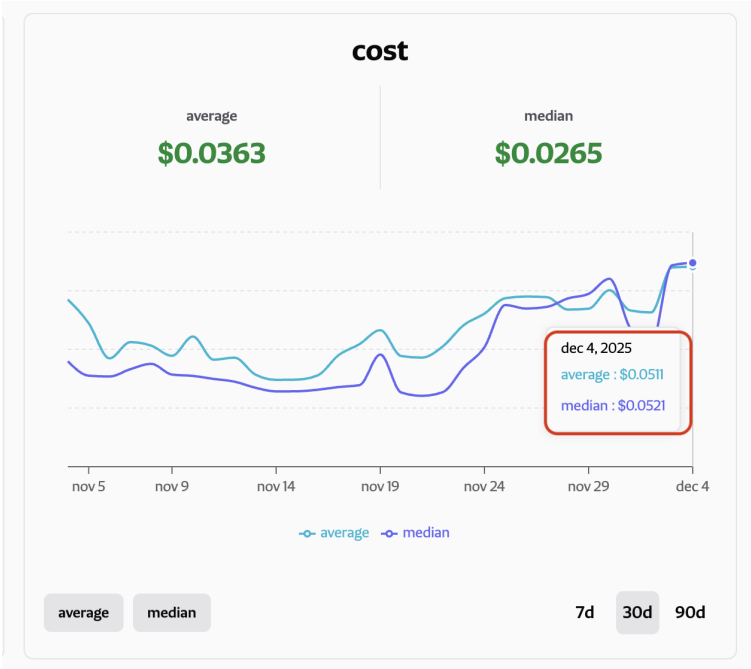
# Model (informal)

## Adversarial game

- A one-shot **procurement** game.
- A set of $n$ players have a unit cost for proving.
- The "protocol" sets up a payment rule.
- An adversary can corrupt up to BYZ of the provers!
- The HON remaining players decide whether or not to prove.
- If no one delivers, the protocol incurs a **large penalty**, $C$.
- If at least one prover delivers, the protocol simply pays according to its payment rule.
- Naïve solution: tell player 1 you will pay them $1 + \varepsilon$ if they prove.
  - ⋆ Under attack, this payment rule achieves a cost of $C$. ☹

# Relative sizes

1. $C$ is big.
2. Proving cost is small.[1]

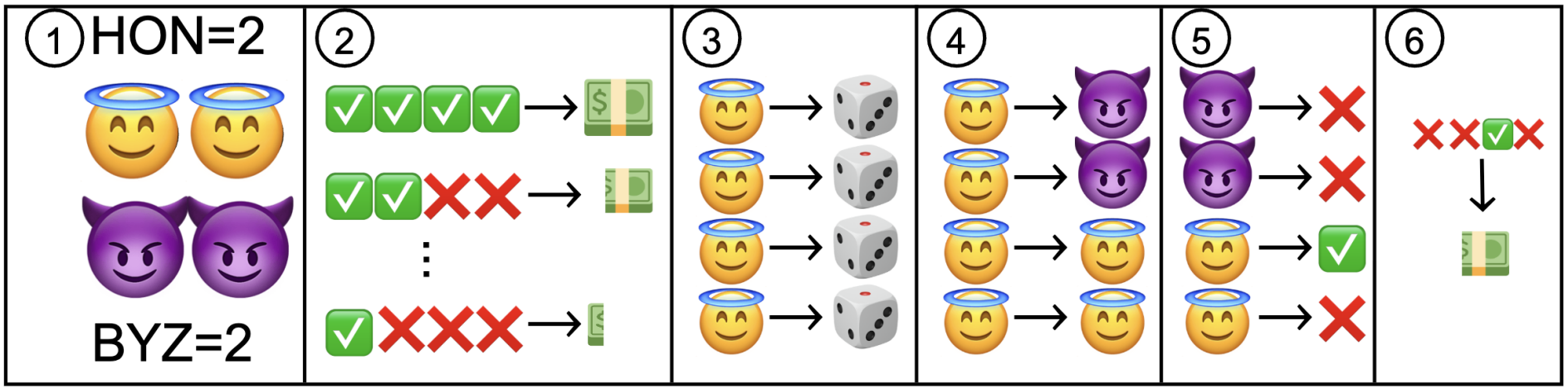




[1] http://ethproofs.org/

# Model (more formal)

## Adversarial game

1. Nature announces HON 😇, BYZ 😈, and $C$ (numbers).
2. The protocol announces a payment rule $p$ : ✅❌ ...✅ $\mapsto \mathbb{R}^n_{\geq 0}$ .
3. Each prover sees $n$ =HON+BYZ and $p$ and decides on $s_i \in [0, 1]$.
4. The attacker sees $p$ and $\mathbf{s}$, and chooses who is BYZ.
5. Each HON decides $a_i \in \{❌, ✅\}$ according to $s_i$. Simultaneously, the attacker selects $a_j \in \{❌, ✅\}$ for $j \in$ BYZ arbitrarily.
6. The protocol pays $p_i$(✅❌ ...✅) and incurs $C$ if all $a_i =$ ❌.



# Example payment rule

## BYZ $+1$

1. HON 😇$= 1$, BYZ 😈$= 2$, and $C = 3$.
2. The protocol announces a payment rule $p_i$ : ✅ $\mapsto 1 + \varepsilon$ for everyone who delivers.
3. Each prover decides ✅.
4. The attacker sees $p$ and $\mathrm{s}$, and chooses who is BYZ.
5. Each HON $a_i =$✅. Simultaneously, the attacker selects $a_j =$✅ for $j \in$ BYZ.
6. The protocol pays $p_i($✅✅✅$) = 1 + \varepsilon$; total payment is $3 + 3\varepsilon$.

- This is **maximally robust**, but expensive.
- If BYZ$= 1000$. Do we really want to pay $BYZ + 1$ *every slot*? Can we be more efficient?

# Protocol cost

## $1^{st}$ of two math slides...

- A payment rule takes an action vector (✅s & ❌s), and outputs per-player payments: $p_i(\mathbf{a}) \mapsto$ player $i$ payment.
  1. Pay a single player $1 + \varepsilon$ if they deliver.
  2. Pay BYZ+1 players $1 + \varepsilon$ each if they deliver.
  3. Choose a lottery prize and give it to one random person who delivers.

**Definition (Single outcome cost)**

The protocol cost under a payment rule $p$ and action profile $\mathbf{a} \in \{0,1\}^n$ is the total payment to provers, plus the penalty $C$ if no proofs are delivered,

$$C_p(\mathbf{a}) := \underbrace{\sum_{i=1}^{n} p_i(\mathbf{a})}_{\text{payments}} + C \cdot \underbrace{\prod_{i=1}^{n} (1 - a_i)}_{1 \text{ if no one delivers}}$$

# The protocol's objective

## $2^{nd}$ of two math slides...

- Let $A$ denote the attacker set and $H$ the honest set.

**Definition (Loss)**

The loss for a payment rule $p$ and an equilibrium strategy profile $\mathbf{s} \in [0,1]^n$ is

$$\ell(p, \mathbf{s}) := \max_{A, \mathbf{a}_A} \left\{ \mathop{\mathbb{E}}_{\mathbf{a}_H \leftarrow \mathbf{s}_H} \left[ C_p\left(\mathbf{a}_A, \mathbf{a}_H\right) \right] \right\}$$

- Protocol's objective is to minimize loss.
- The *optimal* payment rule is, $p^* := \arg\min_p \min_{\mathbf{s}} \{\ell(p, \mathbf{s})\}$.
- Turns out $\uparrow$ is a hard math problem... but **good** news, we can get within $1$ proof-cost (5¢) of $p^*$!

# The optimization problem

## Our "off-by-one" equilibrium

- We find that sometimes, the optimal equilibrium sets $s_1 = 1$.
- **Tiny example**: HON$= 1$, BYZ$= 1$, $C = 3$.
  - ⋆ Optimal equilibrium is $\mathbf{s}^* = (1, 1/2)$.
  - ⋆ Payment rule

$$p_1(10) = 3/2, p_1(11) = 1/2$$
$$p_2(11) = 1.$$

  - ⋆ Optimal loss is $3/2$.
- **Good news**: whenever $s_1 = 1$, we have the optimal payment rule.
- **Bad news**: sometimes $s_1 < 1$.
- **Silver lining**: we can always round $s_1 \to 1$, and implement that version of the payment rule.
- Asymptotically, the optimal loss scales as $\mathcal{O}(\log C)$.

# Generalizing our nearly optimal payments

- **Payment rule recipe**:
  1. Pre-commit to paying the same amount (or nothing) **no matter how many proofs were delivered**.
  2. Tell player 1 to deliver **100% of the time**.
  3. Tell a **committee** of the remaining players to **mix**, but they only get paid if player 1 delivers.
  4. Solve a well-defined math problem to determine the **committee size** and their **mixing probability**.
- **What this gets us**:
  - ⋆ The attacker only has two options:
  1. Corrupt $s_1$ and try to get the committee to fail, or
  2. Not corrupt $s_1$ and make us pay (a constant).
  - ⋆ So we can just **balance** the two.
- **Counter-intuitive**: we only pay when we *least* need extra proofs.
- **Counter-counter intuition**: we *detect* when we are being attacked!

# Takeaways for practitioners

## Designate and committee sizing

- **Key takeaway!** Designate to a single player.
  - ⋆ Can think of it as a rotating leader election, with backup provers delivering some of the time.
  - ⋆ If there is an attacker, this limits the damage they can do.
- Committee **sizing** and **delivery probability** are key factors.
  - ⋆ Consider HON$= 4$, BYZ$= 3$. With one designate, there are six potential committee members.
    - $C = 10 \implies$ opt. committee **size** $6$, each **delivers** $28\%$ of the time. **Intuition:** the attacker corrupts fewer mixers.
    - $C = 40 \implies$ opt. committee **size** $4$, each **delivers** $70\%$ of the time. **Intuition:** drop two of the mixers to give everyone else more mass and reduce cross-term failures.
    - $C = 200 \implies$ opt. committee **size** $3$, each **delivers** $98\%$ of the time. **Intuition:** we converge to BYZ$+1 = 4$ designates.

# A natural objection

- Everyone's payment depends on one player!
- **Counter-point**: Ethereum *already* correlates attester rewards to proposer behavior.
- We really think this payment rule is cost-efficient, implementable, and robust. But if it's not feasible for your use-case, then...
  1. You can condition payments on more than one player.
     - **Ethereum's inactivity leak**: Validators are only paid if $2/3$ of the chain is online.
  2. You can randomly select a set of provers to pay $1 + \varepsilon$ on delivery.
     - This weakens the attacker, who now only corrupts a random subset.
     - **Ethereum's secret leader election**: Proposers selected in private.
  3. You might only be concerned with the crash fault adversary.
     - This weakens the attacker, who can only cause $a_i = 0$.
     - Under this adversary, we can always achieve optimal loss.

# Takeaways for practitioners

## Staking & slashing

- Thought experiment:
  - ⋆ Let people signal, **with stake**, the **provers** that they think are the **most reliable** and **offer the best revenue-share**.
  - ⋆ If the **prover shirks** when the protocol designates it, the stake is **slashed** and the loss is socialized among those who staked with that **prover**.
- Does this sound weird? Or familiar? ... What do staking pools do?
  - ⋆ Let people signal, **with stake**, the **validator** that they think are the **most reliable** and **offer the best revenue-share**.
  - ⋆ If the **validator equivocates** when the protocol designates it, the stake is **slashed** and the loss is socialized among those who staked with that **validator**.
- A pretty reasonable way to "insure" against attacks!
- With stake $\geq C - 1$, the protocol can procure with a **cost of** $1$. ☺
  *Can you think of a payment rule that works?* $\begin{cases} p_1 = 1 & \text{if } a_1 = 1 \\ p_1 = -(C-1) & \text{otherwise} \end{cases}$

# Takeaways for practitioners

## This is a delicate task – YMMV!

- Many considerations change the optimal mechanism
  1. how many players there are,
  2. how many of them are corruptible,
  3. how costly a liveness fault is,
  4. a byzantine versus crash fault adversary,
  5. **(new)** importance of uncorrelated payments,
  6. **(new)** the attackers objective,
  7. **(new)** when the equilibrium is revealed,
  8. **(new)** whether you can slash, etc...
- Also *this project is WIP!* More details to follow soon™ ☺

# thanks :)