TECHNICAL UNIVERSITY OF KENYA
FACULTY OF APPLIED SCIENCES AND TECHNOLOGY
SCHOOL OF COMPUTING AND INFORMATION TECHNOLOGY

END OF SEMESTER SEPTEMBER 2017 EXAMINATION SERIES
FIRST SEMESTER EXAMINATIONS 2017/2018
THIRD YEAR EXAMINATIONS FOR THE DEGREE OF
BACHELOR OF TECHNOLOGY IN COMPUTER TECHNOLOGY
BACHELOR OF TECHNOLOGY IN INFORMATION TECHNOLOGY
BACHELOR OF TECHNOLOGY IN COMMUNICATION AND COMPUTER NETWORKS.

NUMERICAL METHODS

SCII 3202

Time: ***2 Hours***

Instructions.

This paper consists of FIVE Questions.
Answer ***Question ONE*** *[30 Marks] and any other* ***TWO Questions*** *[20 Marks Each].*
Write your college number on the answer sheet.
This paper consists of 4 printed pages

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Question One: 30 Marks

(a) Define the shift operator E , average operator μ , and the differential operator D. Hence show that

$$D^2 = \frac{1}{h^2}\{\Delta^2 - \Delta^3 + \frac{11}{12}\Delta^4 - \frac{5}{6}\Delta^5 + \cdots\}$$

[5 marks]

(b) Apply the Newton-Raphson's method to determine a root of the equation $\cos x = xe^x$ correct to three decimal places using the initial approximation $x_0 = 1$

[5 marks]

(c) Find the solution of of the system of equations below using Gauss-Seidel method and perform the first four iterations.

$$x_1 = 0.5 + 0.25x_2 + 0.25x_3$$
$$x_2 = 0.5 + 0.25x_1 + 0.25x_4$$
$$x_3 = 0.25 + 0.25x_1 + 0.25x_4$$
$$x_4 = 0.25 + 0.25x_2 + 0.25x_3$$

[5 marks]

(d) An alternating current i has the following values at equal intervals of $5ms$.

Time $t(ms)$	0	5	10	15	20	25	30
Current $i(A)$	0	4.8	9.1	12.7	8.8	3.5	0

Charge q , in Coulombs is given by

$$q = \int_0^{30\times10^{-3}} i\,dt$$

Use Simpson's rule to determine the approximate charge in the $30ms$ period.

[5 marks]

(e) Using Newton's forward differences formula, compute $f'(0.2)$ and $f''(0)$ from the following tabular data.

x	0.0	0.2	0.4	0.6	0.8	1.0
$f(x)$	1.00	1.16	3.56	13.96	41.96	101.00

[5 marks]

(f) Find y approximately at $x = 0.1$ in five steps using **Euler's** method. Given

$$\frac{dy}{dt} = \frac{y - t}{y + t}$$

with initial condition $y(0) = 1$.

[5 marks]

Question Two: 20 Marks.

(a) Using Richardson's extrapolation limit, find $y'(0.05)$ to the function $y = -\frac{1}{x}$, with $h = 0.0128$, 0.0064 and 0.0032. Use

$$F(h) = \frac{y(x+h) - y(x-h)}{2h}$$

[8 marks]

(b) Use Gauss-Jordan method to solve the system of equations:

$$x + y + z = 7$$
$$3x + 3y + 4z = 24$$
$$2x + y + 3z = 16$$

[7 marks]

(c) Using Lagrange's interpolation, find the polynomial of degree three which takes the values prescribed below.

x	0	1	2	4
y	1	1	2	5

[5 marks]

Question Three: 20 Marks.

(a) Using the power method, find the eigenvalue of largest modulus and the associated eigenvector of the matrix

$$A = \begin{vmatrix} 2 & 3 & 2 \\ 4 & 3 & 5 \\ 3 & 2 & 9 \end{vmatrix}$$

[5 marks]

(b) Evaluate the integral

$$\int_1^2 \frac{dx}{x}$$

Using Romberg's method of integration starting with Trapezoidal rule, taking $h = 1$, 0.5, 0.25, 0.125.

[8 marks]

(c) Solve the following system of equations by Gaussian elimination method.

$$4x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 - 2x_3 = 4$$
$$3x_1 + 2x_2 - 4x_3 = 6$$

[7 marks]

Question Four: 20 Marks.

(a) The following data gives the melting point of an alloy of lead and zinc; where T is the temperature in ^0C and P is the percentage of lead in the alloy. Find the melting point of the alloy containing 84% of lead using Newtons Interpolation polynomial method.

P	60	70	80	90
T	226	250	276	304

[7 marks]

(b) Use Trapezoidal rule with six intervals to evaluate

$$\int_0^{\frac{\pi}{6}} \frac{1}{1+\sin x}\, dx$$

Give the answer correct to four significant figures.

[7 marks]

(c) Find the root of the equation

$$2x - \cos x - 3 = 0$$

correct to three decimal places using iteration method.

[6 marks]

Question Five: 20 Marks

(a) Find $y'(0.25)$ and $y''(0.25)$ using the method based on divided differences.

x	0.21	0.23	0.27	0.32
y	0.3222	0.3617	0.4314	0.5051

[6 marks]

(b) Solve the following differential equation

$$\frac{dy}{dt} = t + y$$

with initial condition $y(0) = 1$, using fourth order Runge-Kutta method from $t = 0$ to $t = 0.4$ taking $h = 0.1$

[9 marks]

(c) (i) State three error sources in numerical computing.

(ii) State three basic properties of an algebraic equation.

(iii) State the major steps involved in problem solving using computers.

[5 marks]

——— *End of Examination* ———