

ECII/ECSI 3206:
Artificial Intelligence [and expert systems]
Topic 4: Logic and Truth Tables

By: Edgar Otieno

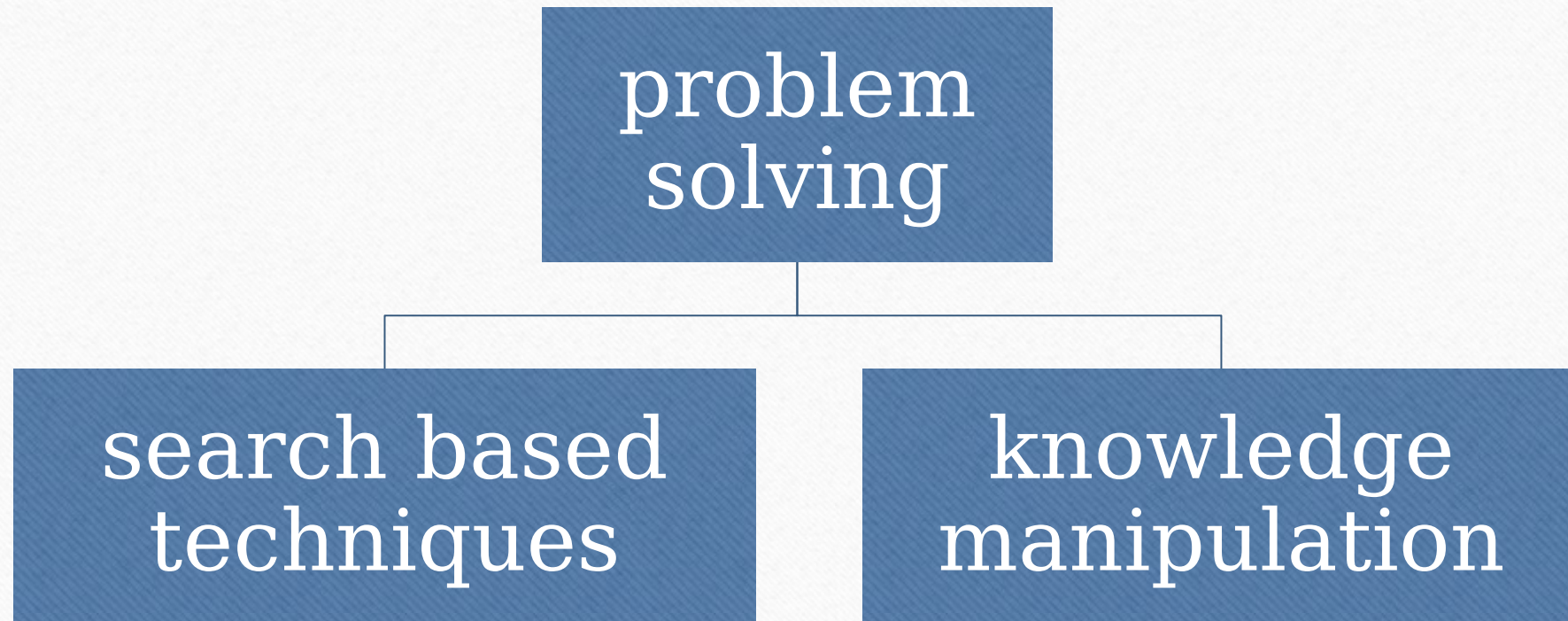
Introduction to Logic

Problem solving could be broadly subdivided into either **Search based techniques** or **Knowledge based techniques**

Knowledge based techniques can be further subdivided into either **knowledge representation** or **Knowledge manipulation**.

The schemes for knowledge representations may be either use of **Logic statements** or use of **Semantic networks**.

Problem Solving approaches



1.Introduction to Logic/Logic statements

- **Logic**-It's a truth perceiving system of inference.
- **Proposition**-defines a statement that valuates to either true or false but not both .
- **Propositional Logic**-Defines rules of mathematical logic used to specify methods of reasoning.
- Which one is a proposition and which is not out of the statements below?
 - This is AI class
 - Today is Monday
 - B.Tech CN students are doing AI course
 - A is less than 14

Connectors and logic operators

- AND/CONJUNCTION [\wedge]
- OR/DISJUNCTION [\vee]
- EXCLUSIVE DISJUNCTION/EXCLUSIVE OR [\oplus] or [$\underline{\vee}$]
- NOT/NEGATION [\neg] or [\sim] or [$!$]
- IMPLICATION/CONDITIONAL IMPLICATION [\rightarrow]
- BICONDITIONAL IMPLICATION [\leftrightarrow]

Propositional logic connectives

Meaning summary



Name	Symbol	Connection	Meaning
AND (Conjunction)	\wedge	$P \wedge Q$	$(P \wedge Q)$ is true if both P and Q are true otherwise false.
OR (Disjunction)	\vee	$P \vee Q$	$(P \vee Q)$ is true if either P or Q is true (or both) otherwise false.
NOT (Negation)	\neg	$\neg P$	$\neg P$ is the opposite of P. If P is true, $\neg P$ will be false and vice versa.
Exclusive OR	\oplus	$P \oplus Q$	Either P or Q but not both. If both are different, then $P \oplus Q$ will be true otherwise false.
Implication	\rightarrow	$P \rightarrow Q$	If P happens then Q happens.
Double Implication	\leftrightarrow	$P \leftrightarrow Q$	P happens if and only if Q happens.



Truth tables

- Truth tables-are diagrams used to show every possible truth value

p	q	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \rightarrow q$	$p \leftrightarrow q$
0	0	0	0	0	1	1
0	1	0	1	1	1	0
1	0	0	1	1	0	0
1	1	1	1	0	1	1

Truth tables terminologies.

Compound truth tables-These are those truth tables that use multiple connectors e.g $((P \vee Q) \wedge R)$

Tautology- It is a compound proposition that always evaluates to TRUE e.g $((P \vee \neg P))$

Contradiction-It is a compound proposition that always evaluates to FALSE e.g $((P \wedge \neg P))$

Contingency-any proposition that is neither a Contradiction or a Tautology

Cont..

- **Well formed formulae**-This is an sentence/expression that is constructed correctly according to the rules of propositional calculus
- **Logical equivalence**-two compound propositions are said to be logical equivalent iff $P \leftrightarrow Q$ is a tautology[always evaluates to TRUE]
 - e.g $\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$
 - $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$

[The above two statements are Defined as De'morgans Laws]

English statements and their equivalent propositional representation

- It is raining in Nairobi: **$R(N)$** // **$RAINING(NAIROBI)$**
- It is raining in Nairobi and I'm either sick or very tired:
 $R(N) \wedge (S(I) \vee VT(I))$
- It is Not Raining in Nairobi: **$\neg R(N)$**
- If it is Raining in Nairobi then I will get wet: **$R(N) \rightarrow W(I)$**

Predicate Logic/Predicate Calculus

- Some statements cannot be represented using propositional logic e.g $X > 3$, all men die..etc
- **Predicate logic**-It is an expression of one or more Variables defined within some specific domain.

Structure of Predicate statements

- Consider the statement $P(X,Y)$
 - P is the predicate describing the relationship
 - X and Y are Variables/arguments that the relationship P applies upon

e.g.

Likes(edgar,chapati)

Father(sonko,saumu)

Mother(X,Y):-son(Y,X) \vee daughter(Y,X) \wedge female(X)

Quantifiers used in Predicate Logic

- These are symbols used to express the extent to which a predicate is True over a set of elements.
- They include:-
 - Universal/for all: \forall
 - Existential/for some: \exists

Predicate Logic examples

- Everyone likes Chapati: $\forall(\mathbf{x}) \mathbf{P}(\mathbf{x}) \rightarrow \mathbf{L}(\mathbf{x},\mathbf{c})$
- There exists a person that likes Chapati : $\exists(\mathbf{x}) \mathbf{L}(\mathbf{x},\mathbf{c})$
- For everyone, there exist a Y where X loves Y: $\forall(\mathbf{x}) \exists(\mathbf{y}) \mathbf{L}(\mathbf{x},\mathbf{y})$
- Some persons in our class have visited Mombasa: $\exists(\mathbf{x}) (\mathbf{C}(\mathbf{x}) \wedge \mathbf{M}(\mathbf{x}))$
- No one in our class is a musician?

Using De'morgan rules to evaluate statements

- Consider the expression for : Not everyone likes Chapati.

$$\neg \forall (\mathbf{x}) (\mathbf{P}(\mathbf{x}) \rightarrow \mathbf{L}(\mathbf{x}, \mathbf{c}))$$

- Using De'morgan rules, we can rewrite the statement as follows to eliminate implication symbol[\rightarrow].
- **De'morgan rules:**
 - $\neg (\mathbf{P} \wedge \mathbf{Q}) \equiv (\neg \mathbf{P} \vee \neg \mathbf{Q})$
 - $\neg (\mathbf{P} \vee \mathbf{Q}) \equiv (\neg \mathbf{P} \wedge \neg \mathbf{Q})$

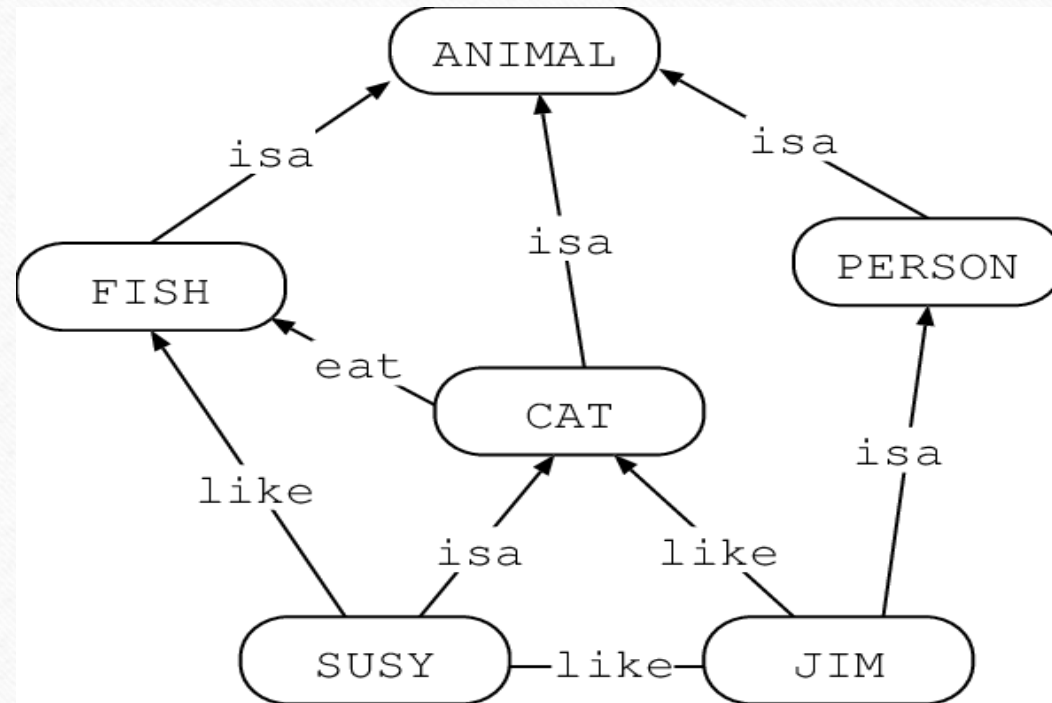
Cont...

- $A \rightarrow B \equiv \neg A \vee B$ and also $\neg A \vee B \equiv \neg (A \wedge \neg B)$
- So let $P(x)$ be represented by A and $L(x,c)$ be represented by B
- Therefore from the first statement we can re write the part $(P(x) \rightarrow L(x,c))$ as follows: $\neg (P(x) \wedge \neg (L(x,c)))$
- The final expression evaluates to : $\neg \forall (x) \neg (P(x) \wedge \neg (L(x,c)))$

2. Semantic Networks(Semantic nets)

- **Semantic nets**-are structures used to represent knowledge as a set of interconnected nodes and arcs.
- **Nodes**-Represent Entities , attributes and Events
- **Arcs**-Represent the relationship between Nodes

Sample Semantic Net



Advantages of Semantic Nets

- Easy to understand
- We can identify relationships
- Easy categorization of knowledge
- Nodes and objects are represented only once

Disadvantages of Semantic Nets

- It has limited handling of quantifiers
- Cannot express connectives
- Difficult to represent some statements e.g how would we represent the statement “Khadija and Adannor really Caused trouble during the last school trip to Mombasa”