transformation

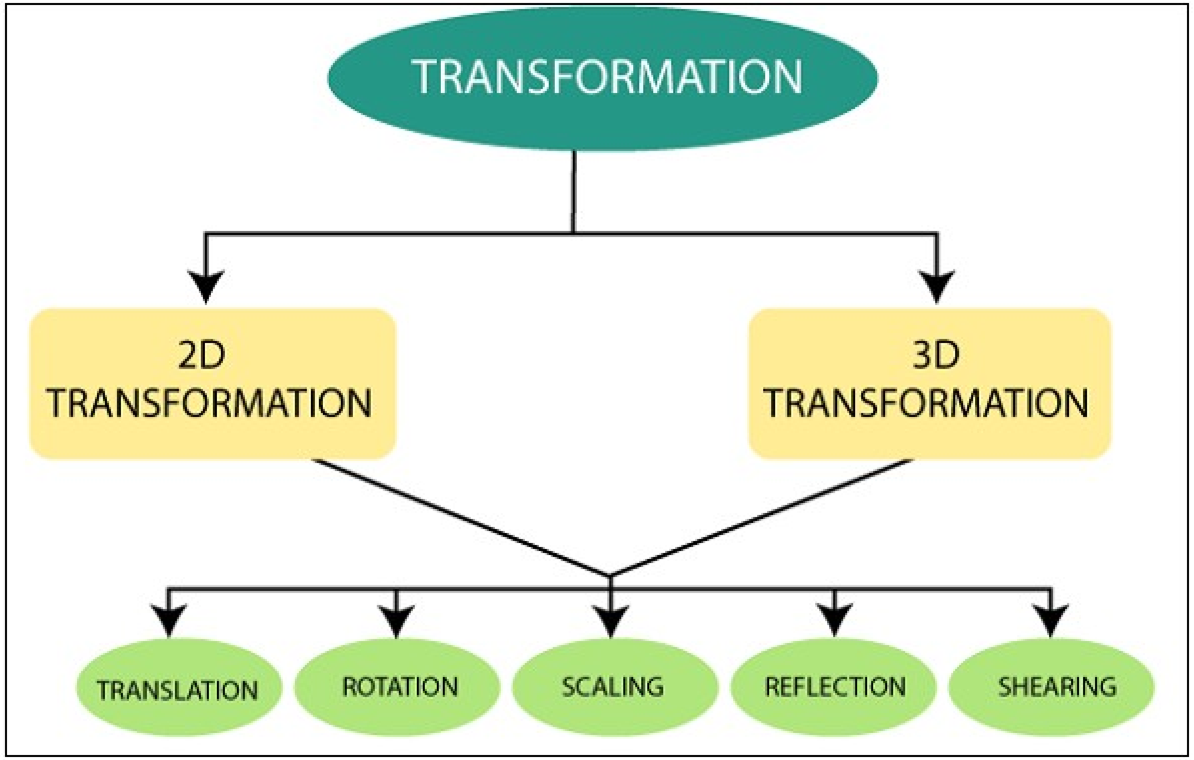
introduction

- The term Transformation is generally referred to as converting a graphic into another graphic by applying some rules or algorithms. Sometimes an image or picture can be a combination of lines, rectangle, circle, and triangle. If we draw and want to combine pictures, then there should be a need to transform these images. Now we can perform the following actions to transform the images-
- We can change the position of an image.
- We can increase or decrease the size of an image.
- We can change the angle of the image.
- By using the above actions, we will find a new image; this process is called Transformation. We can use some algorithms to produce new pictures.

The object transformation includes two important points-

- **Geometric Transformation:** When we are moving the picture, and the background is fixed, then it is a Geometric Transformation.
- **Coordinate Transformation:** When we are moving the background, and the picture is fixed, then it is Coordinate Transformation.

Types of Transformation



Two-Dimensional Transformation

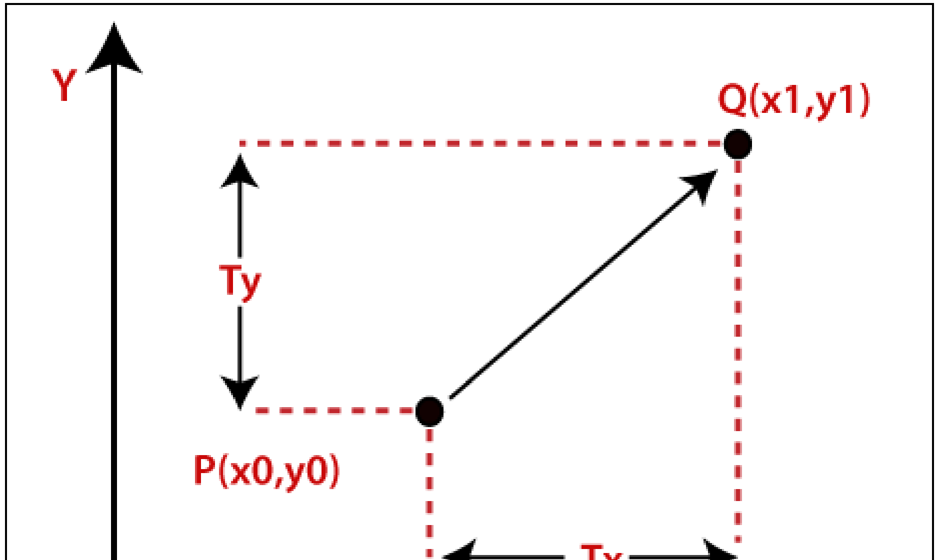
- **2D Translation: “**Translation is a mechanism used to move the object from one position to another position on the screen.**”**
- **2D Rotation: “**Rotation is a process used to rotate the object from origin to a particular angle.**”**
- **2D Scaling: “**Scaling is a process or technique used to resize the object in two-dimensional plane.**”**
- **2D Reflection: “**Reflection is a mechanism or process in which we can rotate the object at the angle of 180°”.
- **2D Shearing: “**Shearing is a process that is used to perform slanting on the object.**”** It is also called **“Skewing.”**
- **2.Three-Dimensional Transformation:**
- **“**When we translate, rotate, and scale object in the three-dimensional plane then, it is called **Three-Dimensional(3D) Transformation”.** A three–dimensional plane consists of x, y, and z-axis.

Three-Dimensional Transformation

- **3D Translation: “**Translation is a mechanism used to move the object from one position to another position on the three-dimensional plane.**”**
- **3D Rotation: “**Rotation is a process used to rotate the object from origin to a particular angle in three-dimensional plane.**”**
- **3D Scaling: “**Scaling is a process or technique used to resize the object in three-dimensional plane**”.**
- **3D Reflection: “**Reflection is a mechanism or process in which we can rotate the object at the angle of 180° in three-dimensional plane.**”**
- **3D Shearing: “**Shearing is a process that is used to perform slanting on the object.**”** It is also called **“Skewing”**. It also includes z-axis.

2D Translation in Computer Graphics
Translation

- We can move any object from one to another place without changing the shape of the object.
- **For Example-**
- **Translation of a Point:** If we want to translate a point from P (x_0, y_0) to Q (x_1, y_1), then we have to add Translation coordinates (Tx, Ty) with original coordinates.



We can also represent the translation in matrix form-

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} Tx \\ Ty \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

Homogeneous Coordinate Representation:

The above Translation is also shown in the form of 3 x 3 n

$$\begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{pmatrix} x \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$$

Here, Translation coordinates ($\mathbf{T_x, T_y}$) are also called "**Translation** or **Shift Vector.**"

Example– Given a Point with coordinates (2, 4). Apply the translation with distance 4 towards x-axis and 2 towards the y-axis. Find the new coordinates without changing the radius?

Solution: P = (x_0, y_0) = (2,4)

Shift Vector = (T_x, $_{Ty}$) = (4, 2)

Let us assume the new coordinates of P = (x_1, y_1)

Now we are going to add translation vector and given coordinates, then

$x_1 = x_0 + T_x = (2 + 4) = 6$

$y_1 = y_0 + T_y = (4 + 2) = 6$

Thus, the new coordinates = (6,6)

2D Rotation

- The Rotation of any object depends upon the two points.
- **Rotation Point:** It is also called **the Pivot point.**
- **Rotation Angle:** It is denoted by **Theta (?)**.
- We can rotate an object in two ways-
- **Clockwise:** An object rotates clockwise if the value of the Rotation angle is negative (-).
- **Anti-Clockwise:** An object rotates anti-clockwise if the value of the Rotation angle is positive (+).

$$x' = r\cos(\phi + \theta) = r\cos\phi\cos\theta - r\sin\phi\sin\theta$$
$$y' = r\sin(\phi + \theta) = r\cos\phi\sin\theta + r\sin\phi\cos\theta \tag{5-4}$$

The original coordinates of the point in polar coordinates are

$$x = r\cos\phi, \qquad y = r\sin\phi \tag{5-5}$$

Substituting expressions 5-5 into 5-4, we obtain the transformation equations for rotating a point at position (x, y) through an angle θ about the origin:

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta \tag{5-6}$$

With the column-vector representations 5-2 for coordinate positions, we can write the rotation equations in the matrix form:

$$\mathbf{P}' = \mathbf{R} \cdot \mathbf{P} \tag{5-7}$$

where the rotation matrix is

$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{5-8}$$

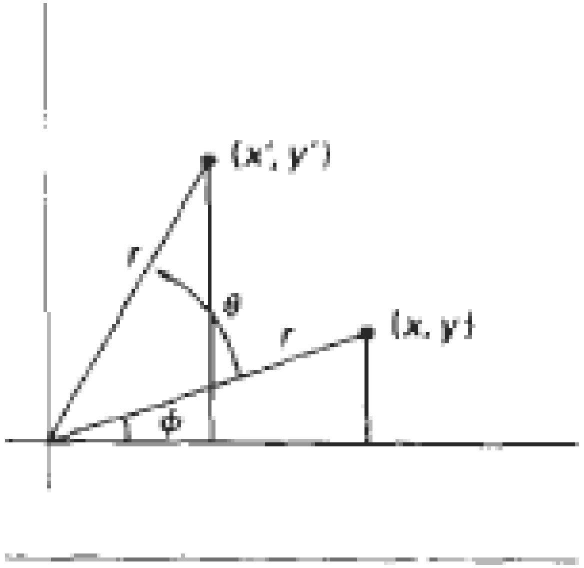


Figure 5-4
Rotation of a point from position (x, y) to position (x', y') through an angle θ relative to the coordinate origin. The original angular displacement of the point from the x axis is ϕ.

Example– A line segment with the starting point (0, 0) and ending points (5, 5). Apply 30-degree rotation anticlockwise direction on the line. Find the new coordinates of the line?

Solution– We can rotate the straight line by its endpoints with the same angle.

We have,

$(P_0, Q_0) = (0, 0)$

Rotation Angle (?) = 30°

Let the new coordinates of line = (P_1, Q_1)

We can apply the rotation matrix, then,

$P_1 = P_0$ x cos? – Q_0 x sin?

= 5 x cos30 – 5 x sin30

= 5 x(?3/2) – 5 x (1/2)

= 4.33 – 2.5

= 1.83

$Q_1 = P_0$ x sin? + Q_0 x cos?

= 5 x sin30 + 5 x cos30

= 5 x (1/2) + 5 x(?3/2)

= 2.5 + 4.33 = 6.83

2D Scaling in Computer Graphics

A **scaling** transformation alters the size of an object. This operation can be carried out for polygons by multiplying the coordinate values (x, y) of each vertex by **scaling factors** s_x and s_y to produce the transformed coordinates (x', y'):

$$x' = x \cdot s_x, \qquad y' = y \cdot s_y \tag{5-10}$$

Scaling factor s_x scales objects in the x direction, while s_y scales in the y direction. The transformation equations 5-10 can also be written in the matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \tag{5-11}$$

or

$$\mathbf{P}' = \mathbf{S} \cdot \mathbf{P} \tag{5-12}$$

Objects transformed with Eq. 5-11 are both scaled and repositioned. Scaling factors with values less than 1 move objects closer to the coordinate origin, while values greater than 1 move coordinate positions farther from the origin. Figure 5-7 illustrates scaling a line by assigning the value 0.5 to both s_x and s_y in Eq. 5-11. Both the line length and the distance from the origin are reduced by a factor of 1/2.

We can control the location of a scaled object by choosing a position, called the **fixed point**, that is to remain unchanged after the scaling transformation. Coordinates for the fixed point (x_f, y_f) can be chosen as one of the vertices, the object centroid, or any other position (Fig. 5-8). A polygon is then scaled relative to the fixed point by scaling the distance from each vertex to the fixed point. For a vertex with coordinates (x, y), the scaled coordinates (x', y') are calculated as

$$x' = x_f + (x - x_f)s_x, \qquad y' = y_f + (y - y_f)s_y \tag{5-13}$$

We can rewrite these scaling transformations to separate the multiplicative and additive terms:

$$\begin{aligned} x' &= x \cdot s_x + x_f(1 - s_x) \\ y' &= y \cdot s_y + y_f(1 - s_y) \end{aligned} \tag{5-14}$$

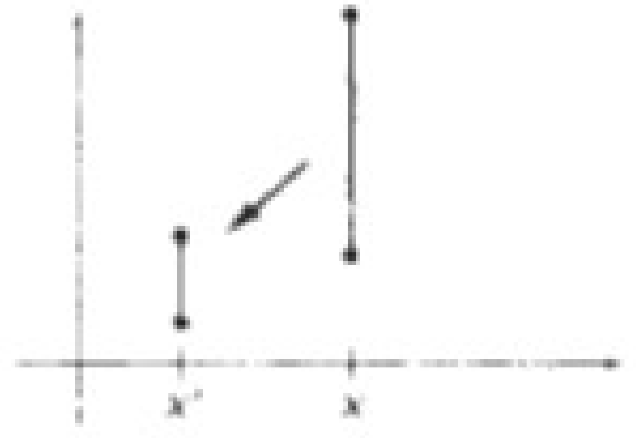
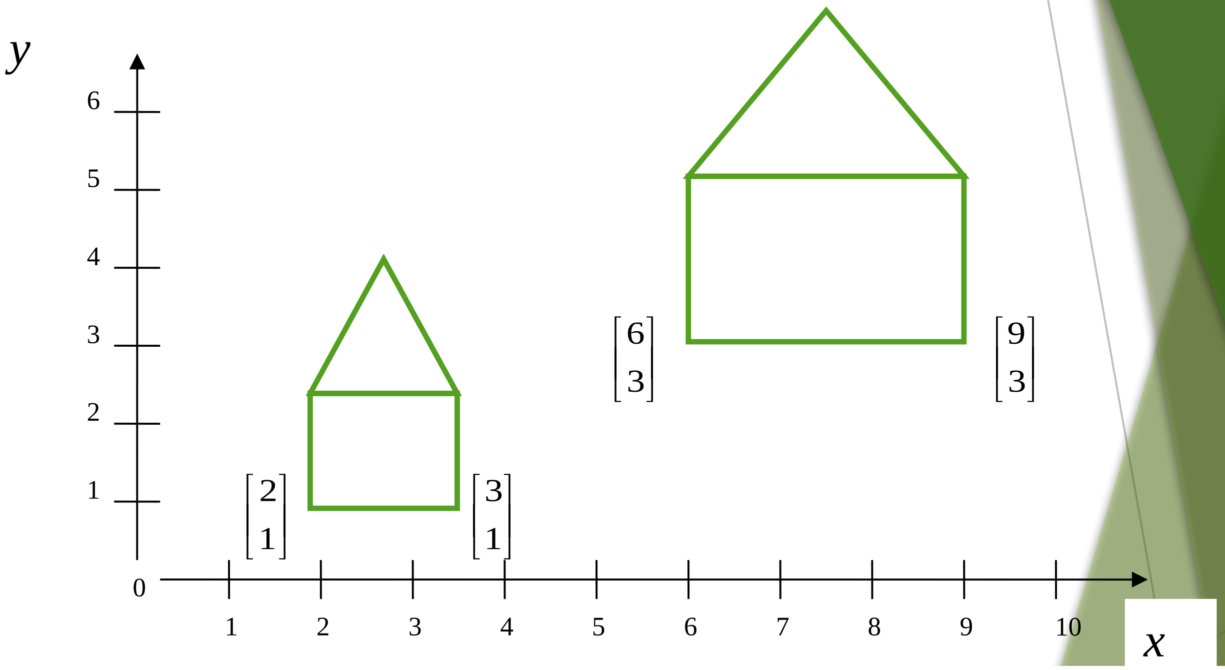


Figure 5-7
A line scaled with Eq 5-12 using $s_x = s_y = 0.5$ is reduced in size and moved closer to the coordinate origin.



Note: House shifts position relative to origin

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We can rewrite these scaling transformations to separate the multiplicative and additive terms:

$$\begin{aligned} x' &= x \cdot s_x + x_f(1 - s_x) \\ y' &= y \cdot s_y + y_f(1 - s_y) \end{aligned} \tag{5-14}$$

where the additive terms $x_f(1 - s_x)$ and $y_f(1 - s_y)$ are constant for all points in the object.

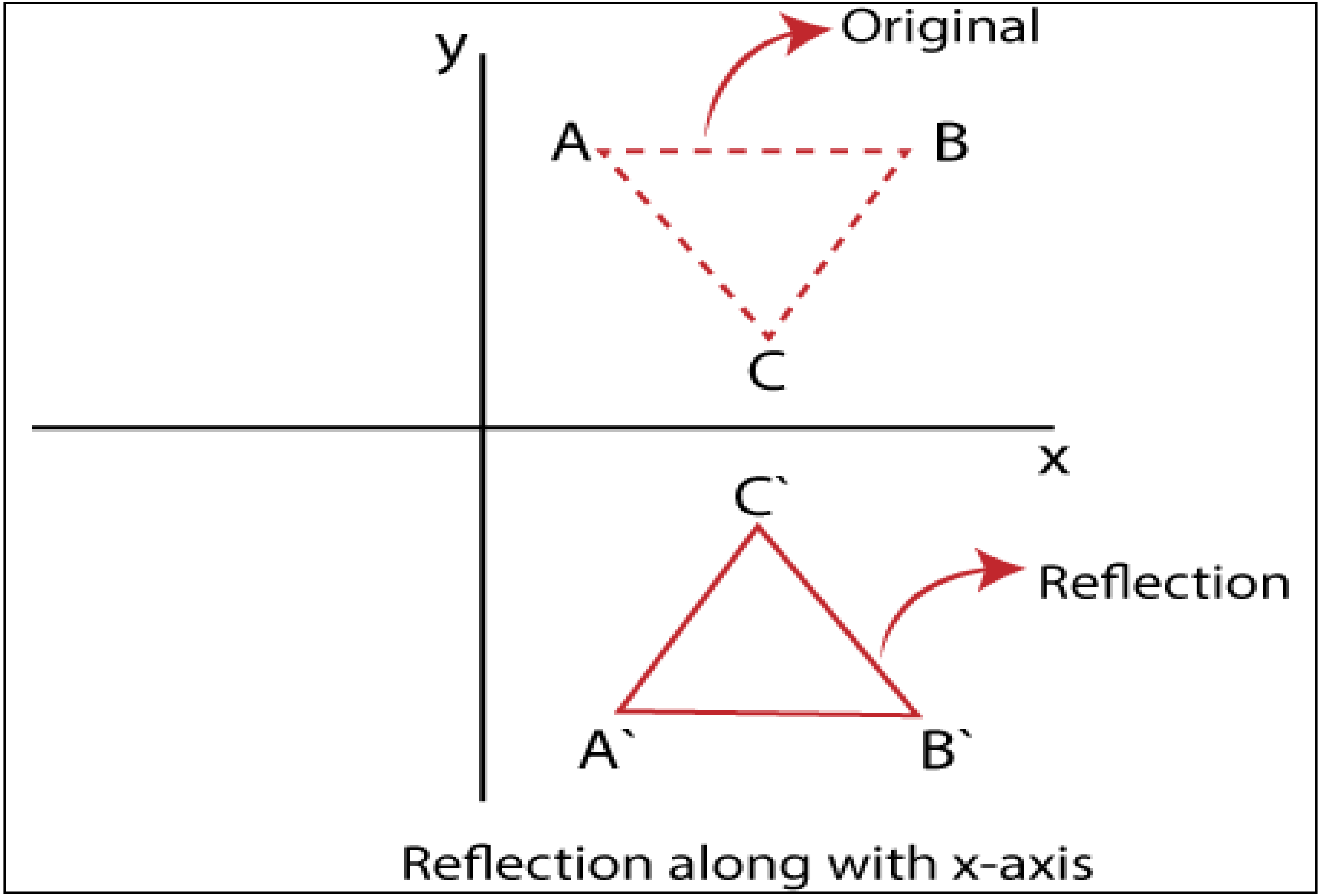
2D Reflection

- **Reflection along X-axis:** In this kind of Reflection, the value of X is positive, and the value of Y is negative.
- We can represent the Reflection along x-axis by following equation-
- $\mathbf{X_1 = X_0}$
- $\mathbf{Y_1 = -Y_0}$
- **We can also represent Reflection in the form of matrix**

$$\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} \begin{matrix} = \\ = \end{matrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{matrix} x \\ x \end{matrix} \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}$$

- **Homogeneous Coordinate Representation:** We can also represent Reflection along x-axis in the form of 3 x 3 matrix-

$$\begin{pmatrix} X_1 \\ Y_1 \\ 1 \end{pmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{matrix} x \\ x \\ x \end{matrix} \begin{pmatrix} X_0 \\ Y_0 \\ 1 \end{pmatrix}$$


Original
y
A
B
C
x
C`
Reflection
A`
B`
Reflection along with x-axis

Reflection along Y-axis:

In this kind of Reflection, the value of X is negative, and the value of Y is positive. We can represent the Reflection along y-axis by following equation-

$$X_1 = -X_0$$

$$Y_1 = Y_0$$

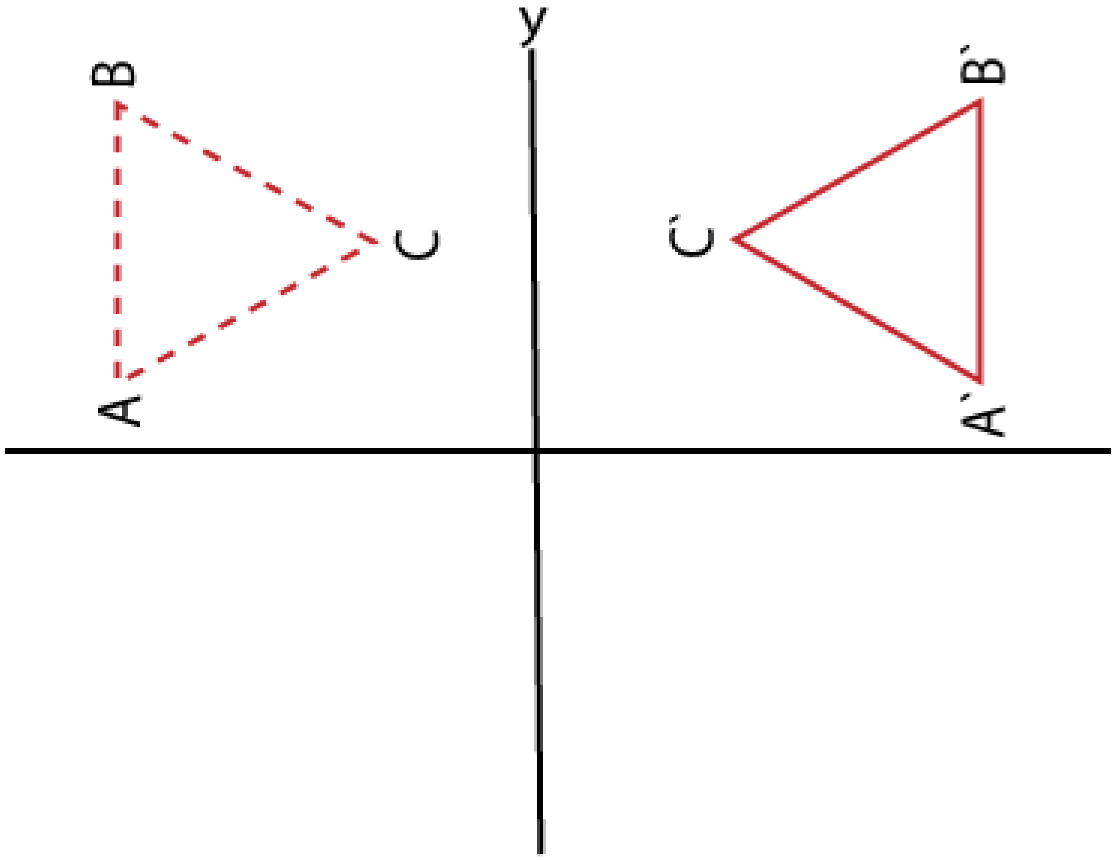
We can also represent Reflection in the form of matrix–

$$\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} x \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}$$

Homogeneous Coordinate Representation: We can also represent the Reflection along x-axis in the form of 3 x 3 matrix-

$$\begin{pmatrix} X_1 \\ Y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x \begin{pmatrix} X_0 \\ Y_0 \\ 1 \end{pmatrix}$$

REFRECTION ALON Y-AXIS



Reflection along with y-axis

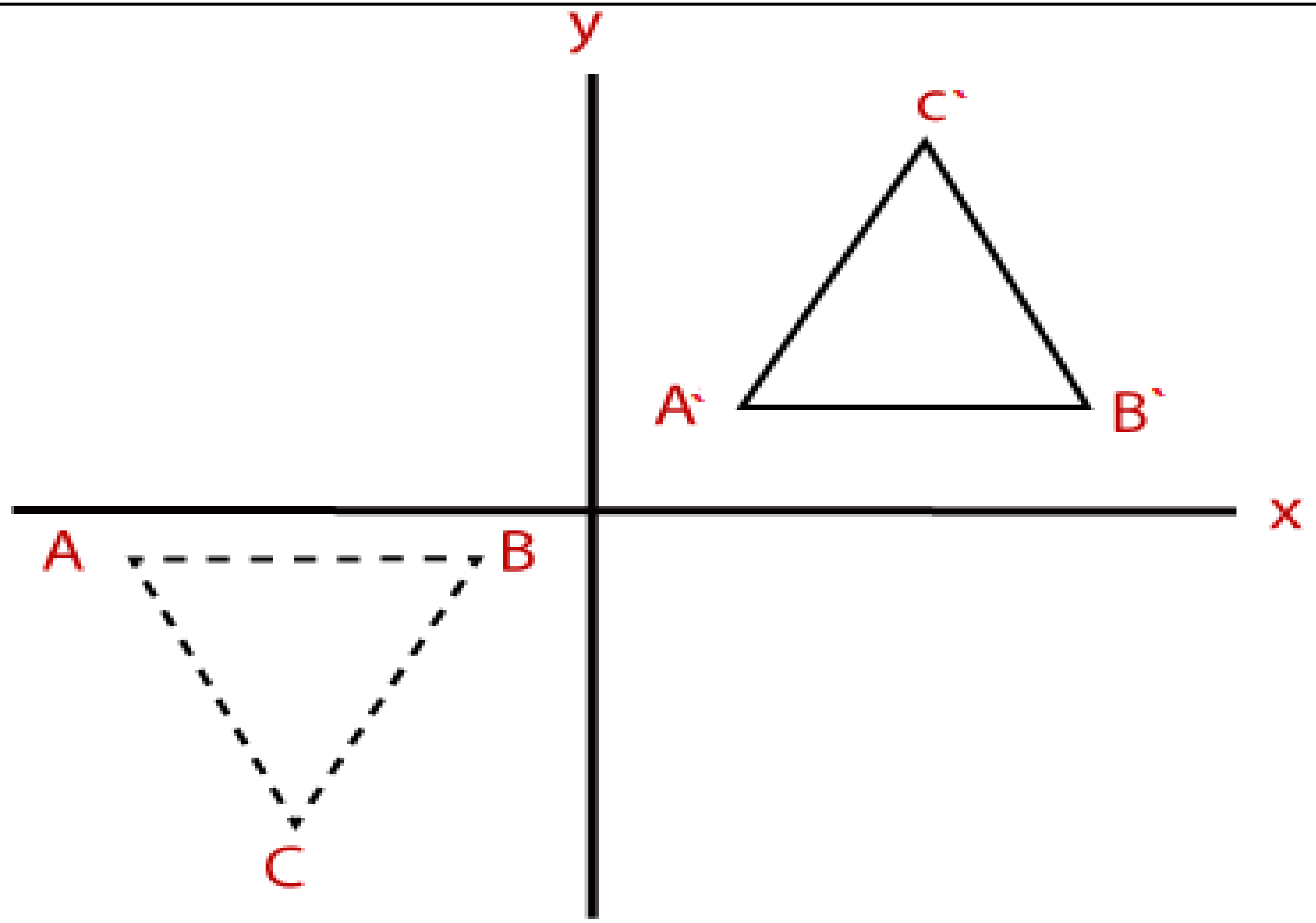
3. Reflection perpendicular to XY plane:

- In this kind of Reflection, the value of both X and Y is negative.
- We can represent the Reflection along y-axis by following equation-
- $\mathbf{X_1 = -X_0}$
- $\mathbf{Y_1 = -Y_0}$
- **We can also represent Reflection in the form of matrix–**

$$\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \times \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}$$

Homogeneous Coordinate Representation: We can also represent the Reflection along with x-axis in the form of 3 x 3 matrix-

$$\begin{pmatrix} X_1 \\ Y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} X_0 \\ Y_0 \\ 1 \end{pmatrix}$$



REFLECTION
perpendicular to xy Plane

Reflection along with the line

In this kind of Reflection, the value of X is equal to the value of Y.

We can represent the Reflection along y-axis by following equation-

Y=X, then the points are **(Y, X)**

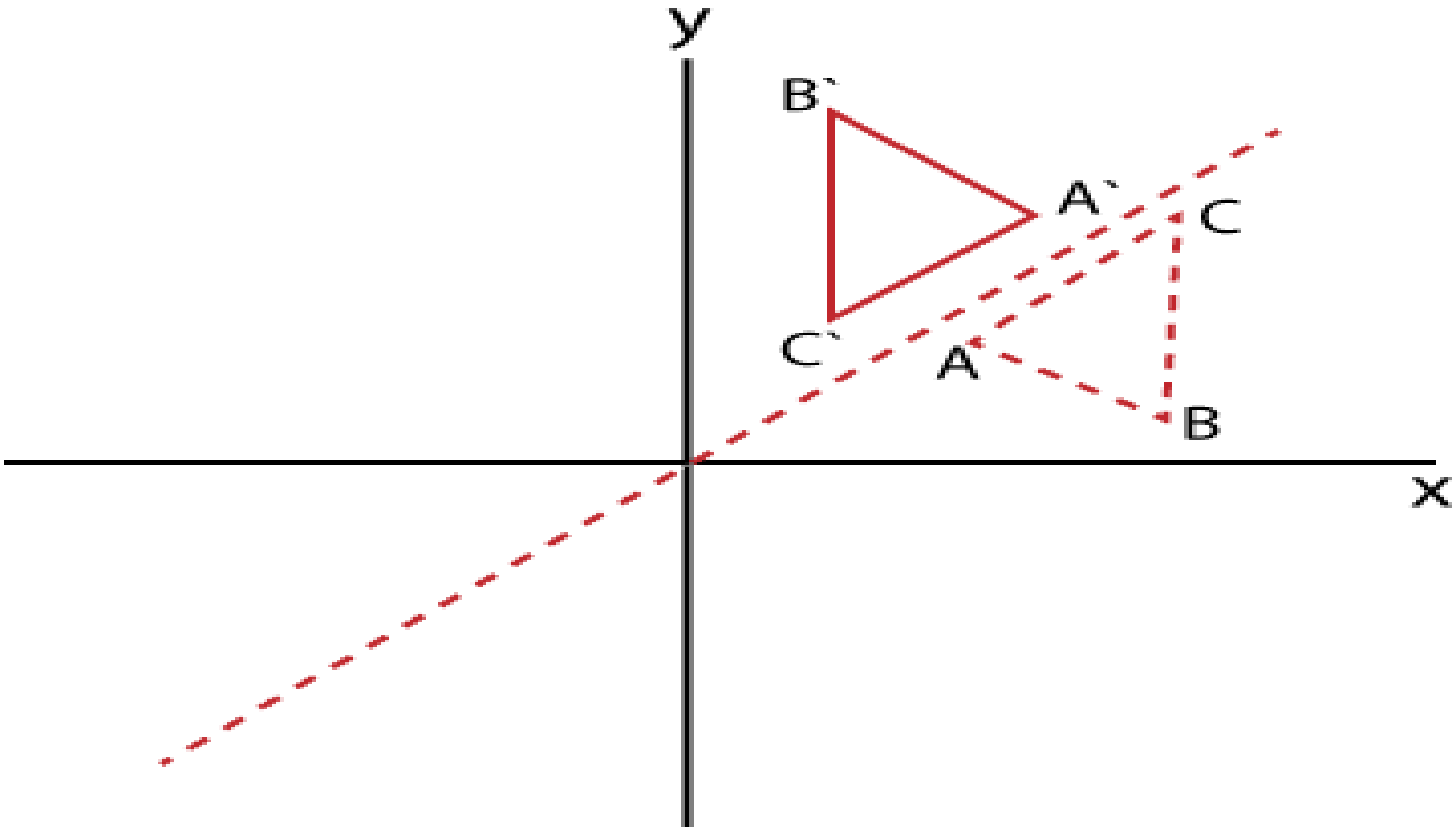
Y= –X, then the points are **(–Y, –X)**

We can also represent Reflection in the form of matrix–

$$\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}$$

Homogeneous Coordinate Representation: We can also represent the Reflection along with x-axis in the form of 3 x 3 matrix-

$$\begin{pmatrix} X_1 \\ Y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} X_0 \\ Y_0 \\ 1 \end{pmatrix}$$



REFLECTION
along with line y=x

2D Shearing

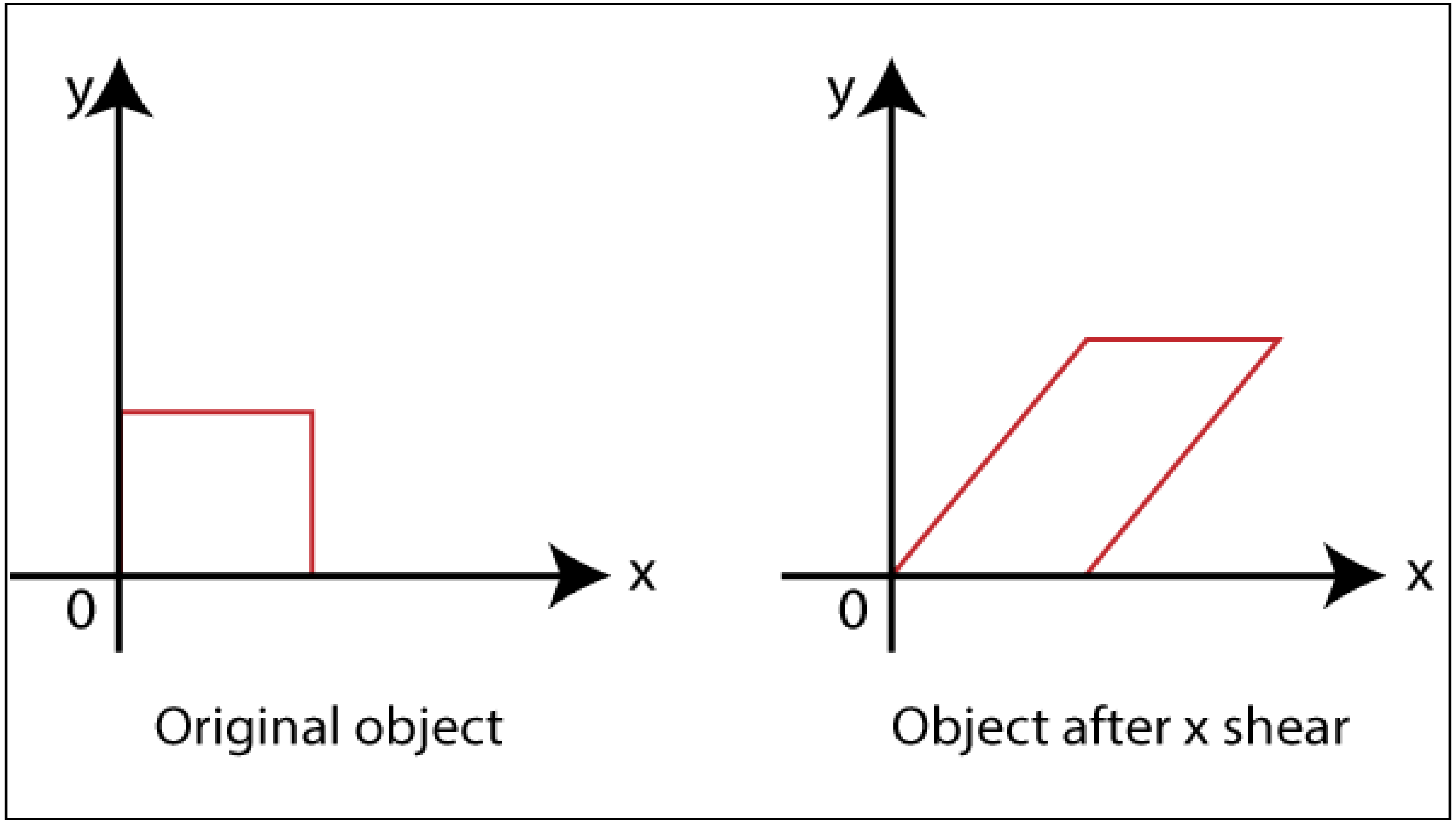
- We can denote shearing with **'SH_x'** and **'SH_y.'** These '**SH_x**' and **'SH_y'** are called **"Shearing factor."**
- We can perform shearing on the object in two ways-
- **Shearing along x-axis:** In this, wecan store the y coordinate and only change the x coordinate. It is also called **"Horizontal Shearing."**
- We can represent Horizontal Shearing by the following equation-
- $X_1 = X_0 + SH_x . Y_0$
- $Y_1 = Y_0$
- **We can represent Horizontal shearing in the form of matr**

$$\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} = \begin{pmatrix} 1 & SH_x \\ 0 & 1 \end{pmatrix} \mathrm{x} \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}$$

- **Homogeneous Coordinate Representation:** The 3 x 3 matrix for Horizontal Shearing is given below-

$$\begin{pmatrix} X_1 \\ Y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ SH_x & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathrm{x} \begin{pmatrix} X_0 \\ Y_0 \\ 1 \end{pmatrix}$$

Shearing along x-axis



Shearing along y-axis:

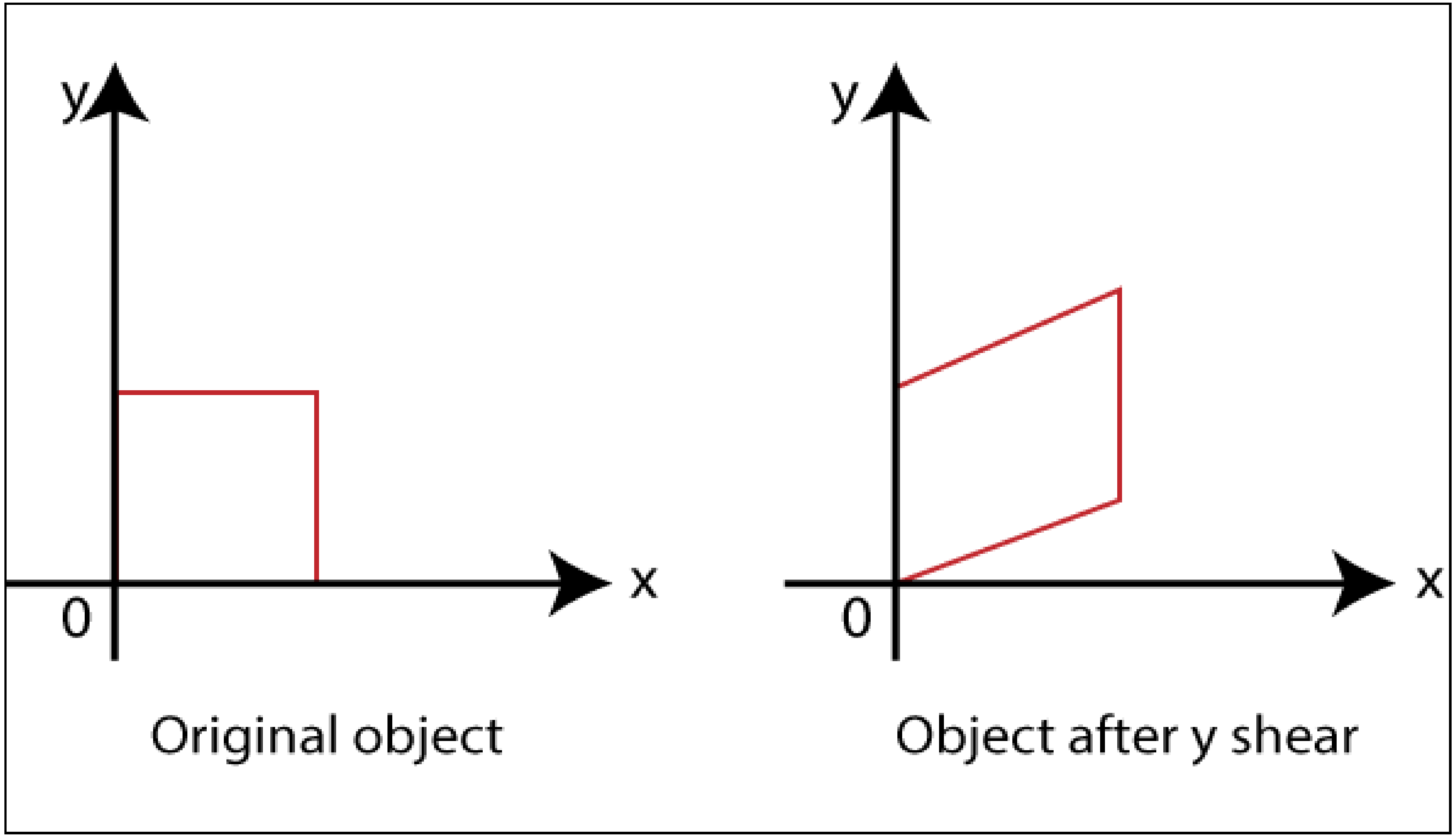
- In this, wecan store the x coordinate and only change the y coordinate. It is also called **"Vertical Shearing."**
- We can represent Vertical Shearing by the following equation-
- $\mathbf{X_1 = X_0}$
- $\mathbf{Y_1 = Y_0 + SH_y \cdot X_0}$
- **We can represent Vertical Shearing in the form of matrix–**

$$\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ SH_y & 1 \end{pmatrix} x \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}$$

- **Homogeneous Coordinate Representation:** The Vertical Shearing is given below-

$$\begin{pmatrix} X_1 \\ Y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & SH_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x \begin{pmatrix} X_0 \\ Y_0 \\ 1 \end{pmatrix}$$

Shearing along y-axis



- **Example:** A Triangle with (2, 2), (0, 0) and (2, 0). Apply Shearing factor 2 on X-axis and 2 on Y-axis. Find ou the new coordinates of the triangle?
- **Solution:** We have,
- The coordinates of triangle = P (2, 2), Q (0, 0), R (2, 0)
- Shearing Factor for X-axis = 2
- Shearing Factor for Y-axis = 2
- Now, apply the equation to find the new coordinates.

Shearing for X-axis:

For Coordinate P (2, 2)-

Let the new coordinate for P = (X_1, Y_1)

$X_1 = X_0 + SH_x . Y_0 = 2 + 2 \times 2 = 6$

$Y_1 = Y_0 = 2$

The New Coordinates = (6, 2)

For Coordinate Q (0, 0)-

Let the new coordinate for Q = (X_1, Y_1)

$X_1 = X_0 + SH_x . Y_0 = 0 + 2 \times 0 = 0$

$Y_1 = Y_0 = 0$

The New Coordinates = (0, 0)

For Coordinate R (2, 0)-

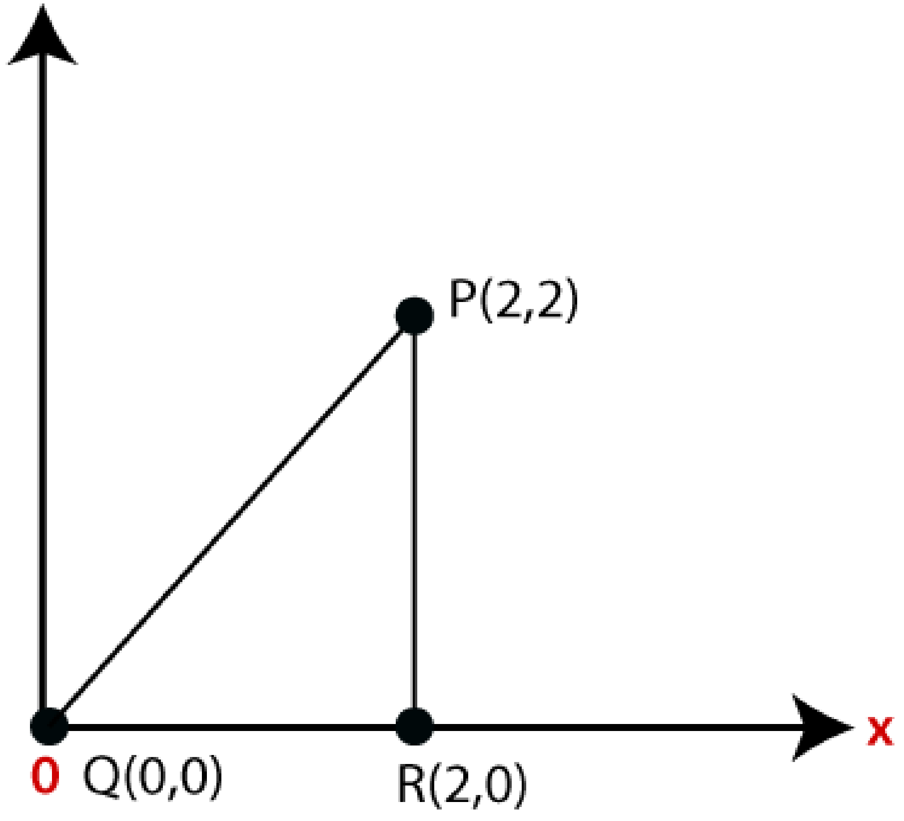
Let the new coordinate for R = (X_1, Y_1)

$X_1 = X_0 + SH_x . Y_0 = 2 + 2 \times 0 = 2$

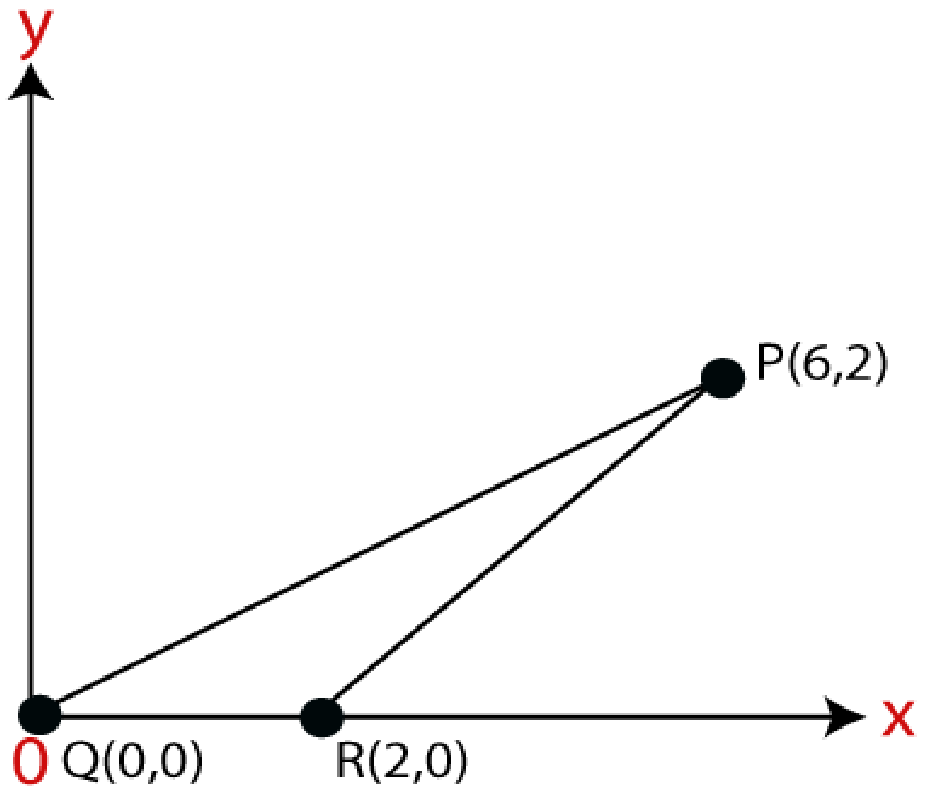
$Y_1 = Y_0 = 0$

The New Coordinates = (2, 0)

The New coordinates of triangle for x-axis = (6, 2), (0, 0), (2, 0)



Original Object



Shearing X-axis

Shearing for y-axis:

For Coordinate P (2, 2)-

Let the new coordinate for P = (X_1, Y_1)

$X_1 = X_0 = 2$

$Y_1 = Y_0 + Sh_y.X_0 = 2 + 2 \times 2 = 6$

The New Coordinates = (2, 6)

For Coordinate Q (0, 0)-

Let the new coordinate for Q = (X_1, Y_1)

$X_1 = X_0 = 0$

$Y_1 = Y_0 + Sh_y.\ X_0 = 0 + 2 \times 0 = 0$

The New Coordinates = (0, 0)

For Coordinate R (2, 0)-

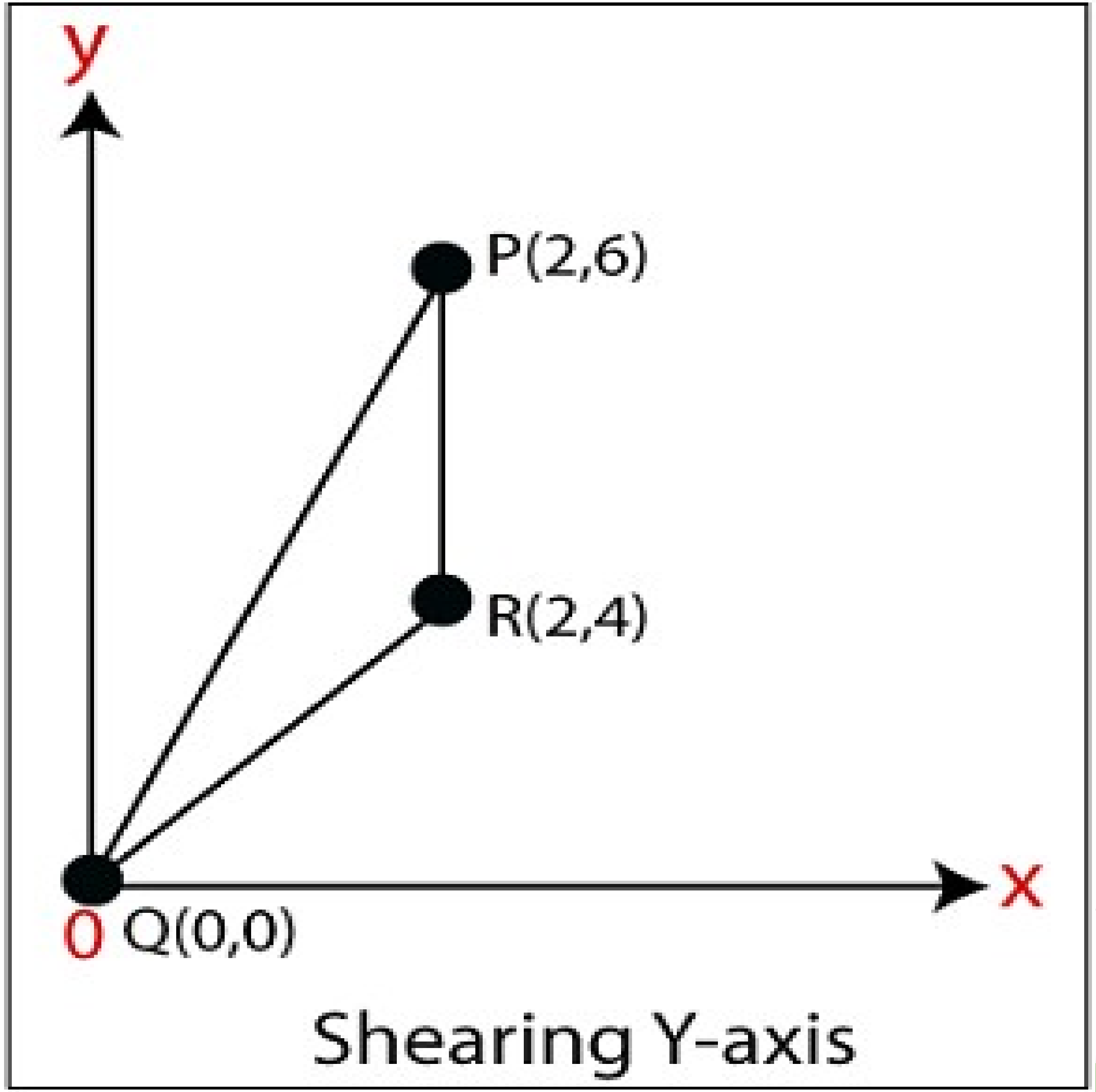
Let the new coordinate for R = (X_1, Y_1)

$X_1 = X_0 = 2$

$Y_1 = Y_0 + Sh_y.\ X_0 = 0 + 2 \times 2 = 4$

The New Coordinates = (2, 4)

The New coordinates of triangle for y-axis = (2, 6), (0, 0), (2, 4)


y
P(2,6)
R(2,4)
x
O
Q(0,0)
Shearing Y-axis