1 Project 1: Part 1

1.1 X^{UNSAT} Results

1.1.1 Asymm. and Symm.

Let us assume that it is possible for an object property R to be constrained by both Asymmetry and Symmetry. Assume that $(x, y) \in R$. By Symmetry, this means that $(y, x) \in R$. By Asymmetry, this means that $(y, x) \notin R$; however, we just stated that $(y, x) \in R$, so we have a contradiction.

1.1.2 Ref. and Asymm.

Let us assume that it is possible for an object property R to be constrained by both Reflexivity and Asymmetry. By Reflexivity, this means that $(x, x) \in R$ for any x. So, let us assume that $(x, x) \in R$ for some arbitrary x. Asymmetry states that if $(y, z) \in R$, then $(y, z) \notin R$ for any y and z. Applying this to our assumption that $(x, x) \in R$ gives us the conclusion that $(x, x) \notin R$, so we have a contradiction.

1.1.3 Irref. and Ref.

Let us assume that it is possible for an object property R to be constrained by both Irreflexivity and Reflexivity. Reflexivity says that $(x, x) \in R$ for any x, so assume that $(x, x) \in R$ for some arbitrary x. Irreflexivity, on the other hand, states that $(y, y) \notin R$ for all y. From this, it follows that $(x, x) \notin R$, but we already stated that $(x, x) \in R$, so we have a contradiction.

1.2 X^{NS} Results

1.2.1 Trans. and Funct.

1.2.2 Trans. and iFunct.

1.2.3 Trans. and Asymm.

To show that the Transitive and Asymmetric constraints together may lead to undecidability, let us suppose that we have the object properties `descendantOf` and `descendantOfOrSelf`, as well as the class `Person`. `descendantOf` has a domain and range of `Person` and is constrained by the Transitive and Asymmetric constraints.

Now, `descendantOf` $\sqsubseteq$ `descendantOfOrSelf` and `descendantOfOrSelf` is identical to `descendantOf`, except that it also relates every individual in `Person` to itself.

Now consider the following object property chain:

`descendantOf` $\circ$ `descendantOfOrSelf` $\circ$ `descendantOf` $\sqsubseteq$ `descendantOf`

This object property chain does not exhibit a strict partial order, so it may result in undecidability.

1.2.4 Trans. and Irref.

The previous example also works for this case, except that `descendantOf` is constrained by Transitivity and Irreflexivity.