

Ex0909A3: The generalized canonical transformation

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The problem: A system is described by the Hamiltonian $H(q,p,t)$. The coordinate q is transformed $q \rightarrow Q = \psi(q,t)$

1. Find the most general transformation $p \rightarrow P(q,p,t)$ so that $(q,p) \rightarrow (Q,P)$ will be canonical.
2. Calculate the appropriate generating function (explain why $F_1(q,Q,t)$ isn't relevant).
3. Given the new Hamiltonian $H'=0$. Find the original Hamiltonian.
4. Prove that for $\psi(q+\omega t)$, the original Hamiltonian is a function of p alone. What does it mean?

The solution:

1. We will derive here the transformation for P :

$$[Q, P] = \frac{\partial Q}{\partial q} \cdot \frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p} \frac{\partial P}{\partial q} = \frac{\partial Q}{\partial q} \cdot \frac{\partial P}{\partial p} = \frac{\partial \psi(q,t)}{\partial q} \cdot \frac{\partial P}{\partial p} = 1 \quad (1)$$

$$\frac{\partial P}{\partial p} = \frac{\partial q}{\partial \psi} = \frac{\partial q}{\partial Q} \quad (2)$$

$$\Rightarrow \boxed{P = p \cdot \frac{\partial q}{\partial Q} + f(q,t)} \quad (3)$$

2. Finding the generative function:

We will choose the third class of generative function, And we will define q with (Q,t) , leading to $q=g(Q,t)$, because we have $Q=\psi(q,t)$. So f can be expressed with (Q,t) .

$$F_3(p, Q, t) \quad (4)$$

$$-P = \frac{\partial F_3}{\partial Q} \quad (5)$$

$$-q = \frac{\partial F_3}{\partial p} \quad (6)$$

$$(5) : \frac{\partial F_3}{\partial Q} = -p \cdot \frac{\partial q}{\partial Q} - f(q,t) \quad (7)$$

$$F_3 = -pq - \int f(Q,t)dQ + h(p,t) \quad (8)$$

$$(6) : \frac{\partial F_3}{\partial p} = -q + \frac{\partial h}{\partial p} = -q \Rightarrow h = l(t) \quad (9)$$

$$F_3 = -pq - \int f(Q,t)dQ + l(t) \quad (10)$$

$$\Rightarrow \boxed{F_3 = -p \cdot g(Q,t) - \int f(Q,t)dQ} \quad (11)$$

Where we dropped the function that depends only on time because it cant affect the equation of motion.

3. Here we will find the original Hamiltonian.

we know that :

$$0 = \dot{\mathcal{H}} = \mathcal{H} + \frac{\partial F_3}{\partial t} \quad (12)$$

$$\mathcal{H} = -\frac{\partial F_3}{\partial t} \quad (13)$$

$$\Rightarrow \boxed{\mathcal{H} = \frac{\partial(p \cdot g(Q, t))}{\partial t} + \frac{\partial(\int f(g(q, t), t)dQ)}{\partial t}} \quad (14)$$

4. answer 4

5. This is how you write an equation

$$\mathcal{I} = \int_0^z d^4x \sqrt{-g} \left[\varphi^2 (\mathcal{R} - 6sg^{\mu\nu} \kappa_\mu \kappa_\nu) + 4\omega g^{\mu\nu} D_\mu \varphi D_\nu \varphi + \lambda \varphi^4 + \frac{1}{4} g^{\mu\nu} g^{\lambda\sigma} X_{\mu\lambda} X_{\nu\sigma} \right]. \quad (15)$$

6. If you wish to add a set of equations (add the ampersands to align the two equations):

$$s = \frac{3+2\omega}{2\omega} \neq 1, \quad (16)$$

$$D_\mu \varphi = \varphi_{;\mu} + s \kappa_\mu \varphi, \quad (17)$$

If you wish to have equations with no numbers add an asterisk

$$g^{\mu\nu} (\varphi^2_{;\mu} + 2\kappa_\mu \varphi^2)_{;\nu} = \frac{\partial W_{eff}(\varphi^2)}{\partial \varphi^2},$$

$$\frac{\partial W_{eff}(\varphi^2)}{\partial \varphi^2} = \frac{1}{3+2} \omega \left(\frac{1}{2} \varphi V'(\varphi) - 2V(\varphi) \right).$$

To write a vector

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z} \quad ; \quad \vec{r}_i = |\vec{r}| \hat{r} \quad (18)$$

To write a matrix

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \text{“\quad” makes space in equations and this is how you add text.} \quad (19)$$

You can also use “align” to tightly align the equal sign, i.e “=” is aligned: see Eq. (19) and Eq. (20), also note that **every end** of an equation needs to be punctuated, i.e : “,” or “.”, according to the sentence.

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad (20)$$

$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 0 \end{pmatrix}, \quad (21)$$

and centered

$$\left[\frac{\lambda_i}{2}, \frac{\lambda_j}{2}\right] = i \sum_{k=1}^n f^{ijk} \frac{\lambda_k}{2},$$

$$f^{147} = f^{165} = f^{246} = f^{257} = f^{345} = f^{376} = \frac{1}{2}.$$

7. Known arguments are **not** *written in italic mode*, some have special syntax in LaTeX otherwise just use “\mathrm”:

$$\cos(\omega t); \quad \sin(\omega t); \quad \mathrm{Tr} \left[\hat{A} \right]; \quad \det \left[\hat{A} \right]; \quad \tan(\omega t); \quad \mathrm{e}^{\frac{t}{\tau}}; \quad \exp \left(\frac{t}{\tau} \right); \quad \log \left(\frac{t}{\tau} \right). \quad (22)$$

For all Latex related knowledge go to <https://en.wikibooks.org/wiki/LaTeX>

To include a picture use, note that the picture file has to be in the same folder as the *.tex file.