SATB

A Colourful Game of Musical Puzzles

Michael Purcell

1 Introduction

SATB is a game for 1-5 players who work together to create a *composition*. A composition is a two-dimensional array of *colours*. Each row of a composition is called a *voice* while each column is called a *beat*. Players create compositions by choosing which colour each voice will play on each beat.

2 Colours and Rests

There are five colours in an SATB composition: red (R), yellow (Y), green (G), blue (B), and white (W). We impose a structure on these five colours. This structure is depicted on the game board by solid and dashed lines. If two colours are connected by a solid line then we say that they are *adjacent*. If two colours connected by a dashed line then they are not adjacent. Given a composition $\mathbf{C} = [C_{ij}]$, we say that ith voice *plays a colour* on the jth beat if $C_{ij} \in \{R, Y, G, B, W\}$.

There is a sixth space on the game board (X) which represents a rest. Given a composition $\mathbf{C} = [C_{ij}]$, we say that the ith voice *rests* on the jth beat if $C_{ij} = X$.

3 Composition Rules

Each composition must satisfy a set of rules which ensure that all of the voices are both individually interesting and mutually consistent with one another.

3.1 Rhythm

As described in Section 1, each composition is divided up into a sequence of beats. On each beat, each voice must either play a colour or rest. Furthermore, the voices must collectively obey the *continuity* rules:

R1 No more than one voice may rest on each beat.

R2 No voice may rest on more than one consecutive beat.

3.2 Melody

A *melody* is a sequence of colours and rests played by a single voice. A *repeat* is a when a voice plays the same colour on two consecutive beats. A *step* is when a voice plays two adjacent colours on two consecutive beats. A *skip* is when a voice plays two non-adjacent colours on two consecutive beats. The direction of a step or skip is the direction (clockwise or anticlockwise) that a token would travel on the game board when it moves via the shortest path between the two spaces depicting the two colours involved. A melody must obey the *phrasing* rules:

M1 A repeat must be followed by a step.

M2 A skip must be followed by a step in the opposite direction.

A *phrase* is a sequence of colours that obey the phrasing rules. Notice that a phrase may not be interrupted by a rest.

Example 1 *There are two two-beat phrases that start with R:*

$$[R \quad W] \qquad [R \quad Y]$$

and four three-beat phrases that start with R:

$$[R \quad B \quad W] \qquad [R \quad G \quad Y] \qquad [R \quad R \quad Y] \qquad [R \quad R \quad W]$$

3.3 Harmony

Harmony is when several different colours are played on the same beat by different voices. A *chord* is a set of colours that obey the *consonance* rule:

H1 No more than two colours in a chord may be adjacent.

A three-note chord consists of two adjacent colours and a third isolated colour. This isolated colour is called the *root* of the chord.

Example 2 *There are five one-colour chords:*

$$[R] \qquad [W] \qquad [B] \qquad [G] \qquad [Y]$$

ten two-colour chords:

$$\begin{bmatrix} R \\ W \end{bmatrix} \quad \begin{bmatrix} W \\ B \end{bmatrix} \quad \begin{bmatrix} B \\ G \end{bmatrix} \quad \begin{bmatrix} G \\ Y \end{bmatrix} \quad \begin{bmatrix} Y \\ R \end{bmatrix} \quad \begin{bmatrix} R \\ B \end{bmatrix} \quad \begin{bmatrix} W \\ G \end{bmatrix} \quad \begin{bmatrix} B \\ Y \end{bmatrix} \quad \begin{bmatrix} R \\ G \end{bmatrix} \quad \begin{bmatrix} W \\ Y \end{bmatrix}$$

and five three-colour chords:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} \qquad \begin{bmatrix} W \\ Y \\ G \end{bmatrix} \qquad \begin{bmatrix} B \\ Y \\ R \end{bmatrix} \qquad \begin{bmatrix} G \\ R \\ W \end{bmatrix} \qquad \begin{bmatrix} Y \\ W \\ B \end{bmatrix}$$

3.4 Counterpoint

Counterpoint is when several voices play simultaneously. A pair of voices move in *similar motion* if they both step or skip in the same direction. A pair of voices move in *contrary motion* if they step or skip in opposite directions. A pair of voices move in *oblique motion* if one voice repeats a colour while the other voice changes colours. A group of melodies played in counterpoint must satisfy the *voice leading* rules:

C1 At least one voice must move on each beat.

C2 At least on pair of voices must move in contrary or oblique motion on each beat.

Example 3 *This composition follows the rules established above:*

$$\begin{bmatrix} B & Y & G & W & B \\ G & G & B & G & Y \\ R & W & B & W & R \end{bmatrix}$$