Backward Euler with MPI

ME471, Spring 2020, Final Project

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```
I was running this on Redhawk, but the nodes were limiting the number of C
PUs that I could use and the cross node communication was doubling the tim
e required to operate the same problem. I tested this situation by running
the spots setup on 1 node, 4 processors on a 128 x 128 grid, and identical
setup on 2 nodes. The 1 node version ran in 1 minute 28.4 seconds, while t
he 2 node version took slightly longer than 3 minutes 17 seconds. So I aba
ndoned Redhawk for our trusty Virtual Machine, and this notebook will only
cover results ran on the VM.
A keen eye will notice that I'm hard coding in timing values for the array
s. This is not because I'm "just guessing the time" that these would take,
rather I'm stuck hard coding it because the run took so long (~ 5 hours) a
nd at 2 in the morning I didn't know exactly how to turn arrays into files
to retain. I was anticipating that the VM would kick me off if my computer
went to sleep so I took a screen shot of the timing results and hard coded
it on my local machine. Not optimal use of technology but a broken clock i
s still right twice a day.
```

Setup of Program and Data

```
I ran this parallel backward Euler solver with the input setup for spots (
from the homework 4 specifications) across square global grids of side len
gths: 32, 64, 128, 256, and 512. These side lengths correspond to p values
in the data (as in 2^p = N; p = 5 -> N = 32). It was ran using across a ra
nge of processors: 1, 2, 4, 8, and 16.
```

In [1]:
```
import matplotlib.pyplot
import scipy
import time, os
%matplotlib notebook
%pylab
```

```
Using matplotlib backend: nbAgg
Populating the interactive namespace from numpy and matplotlib
```

In [2]:
```
# array setup section

#time_1proc[0] corresponds to 1 processor, p = 5
#time_1proc[1] corresponds to 1 processor, p = 6
#time_2proc[1] corresponds to 2 processors, p = 6
time_1proc = [2.45001602055,18.906493884,148.343905799,991.132974659,7
628.0181528739]
time_2proc = [1.90655736811,10.254397049,79.552227533,471.779030761,35
05.6236737109]
time_4proc = [2.49563256209,7.02648843382,40.542972475989,260.38621809
,1764.485852355]
time_8proc = [2.17091187695,5.56852644495,19.158029562095,136.33762141
7,1020.692787826]
time_16proc = [3.0827721408,7.17792607215,13.148246705997,80.894376856
9,621.719308567]

#p5_time[0] corresponds to timing results on 1 proc, p = 5 size grid
#p5_time[1] corresponds to timing results on 2 proc, p = 5 size grid
#pX_time's are strong scaling
p5_time = [2.450016102055,1.906557368114,2.49563256,2.17091187695,3.08
277214086]
p6_time = [18.90649388405,10.25439704908,7.0264884338,5.568526444,7.17
792607215]
p7_time = [148.343905799,79.5522275329,40.5429724759,19.158029562,13.1
4824670599]
p8_time = [991.1312974659,471.77903076,260.3862180931,136.337621416,80
.894376857]
p9_time = [7628.018152873,3505.6236737109,1764.48585235,1020.692787826
,621.719308567]

#weak_scale_time[0]: p5, 1 proc; [1]: p6, 2 proc; [2]: p7, 4 proc; [3]
: p8, 8 procs; [4] p9, 16 procs
weak_scale_time = [time_1proc[0],time_2proc[1],time_4proc[2],time_8pro
c[3],time_16proc[4]]
plist = arange(5,10)
numproc = 2**arange(0,5)

#time_results is a linear version of the above information
time_results = []
for i in range(len(numproc)):
    time_results.append(time_1proc[i])
for i in range(len(numproc)):
    time_results.append(time_2proc[i])
for i in range(len(numproc)):
    time_results.append(time_4proc[i])
for i in range(len(numproc)):
    time_results.append(time_8proc[i])
for i in range(len(numproc)):
    time_results.append(time_16proc[i])
```

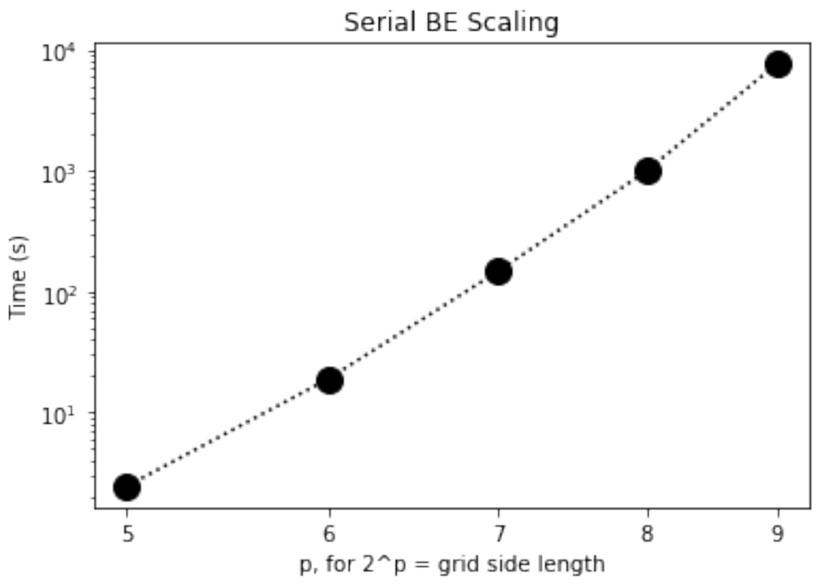
In [3]:
```
def fix_xticks(procs):
    p0 = log2(procs[0])
    p1 = log2(procs[-1])
    xlim([2**(p0-0.5), 2**(p1+0.5)])


    # Make nice tick marks
    pstr = (['{:d}'.format(int(p)) for p in procs])
    xticks(procs,pstr)

# does every other, for large processor setups
def fix_2xticks(procs):
    p0 = log2(procs[0])
    p1 = log2(procs[-1])
    xlim([2**(p0-0.5), 2**(p1+0.5)])
    numList = []
    i=0
    for p in procs:
        if (i%2 == 0):
            numList.append(p)
        i+=1
    pstr = (['{:d}'.format(int(n)) for n in numList])

    #xticks(procs,pstr)
    xticks(numList,pstr)
```

Serial Behavior

To run the code in serial, I still used MPI, but left the number of processors at 1. We see the expected relationship for the serial timings in the figure below.

In [4]:
```
numproc = 2**arange(0,5)
line = arange(0,10,2)
m,b= polyfit(numproc,weak_scale_time,1)
figure(1)
clf()
ylabel("Time (s)")
xlabel("p, for 2^p = grid side length")
title("Serial BE Scaling")
loglog(plist, time_1proc,'ko:', markersize=12)
xticks(plist,[5,6,7,8,9])
```

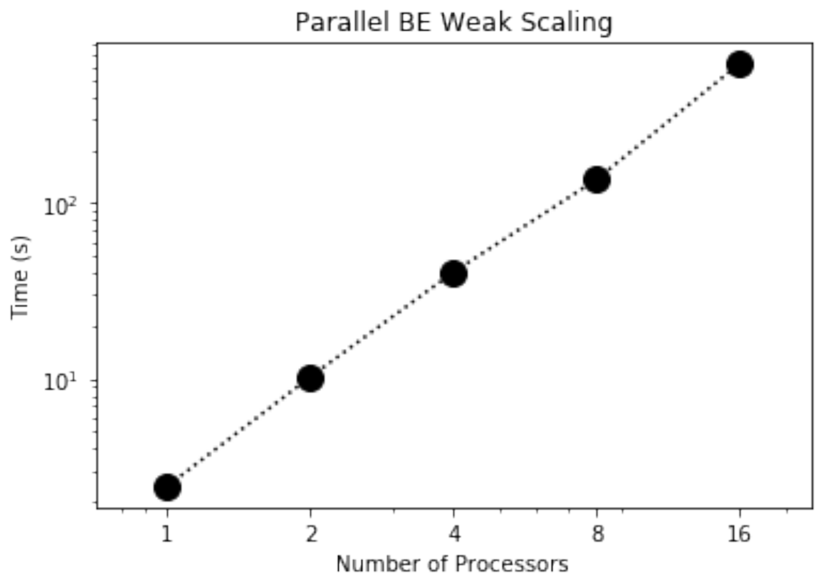
Out[4]:
```
([<matplotlib.axis.XTick at 0x11eb4ad10>,
  <matplotlib.axis.XTick at 0x11f498b10>,
  <matplotlib.axis.XTick at 0x110685cd0>,
  <matplotlib.axis.XTick at 0x11fbe66d0>,
  <matplotlib.axis.XTick at 0x11fbe6c50>],
 <a list of 5 Text xticklabel objects>)
```



Weak Scaling

```
We see in the weak scaling plot below that the parallel BE shows poor weak
scaling. As we increase p and the number of processors together it scales
just like the serial code, but with a little less of a slope. A quick esti
mation is a net time saving of about a factor of 10 for the parallel weak
scaling over the serial code. Ideally, that black dotted line would be fla
t, showing that the time remains constant as we increase p and the number
of processors each step. Why would this behavior happen for parallel BE? I
t could be my algorithm has some communication issues or inefficiencies, b
ut I'm not sure. It's possible that the number of calculations required fo
r a matrix, by the parallel BE algorithm, has an associated constant facto
r. As we increase the matrix size, this factor grows causing the calculati
on time to increase at aconstant rate for BE.
```

In [5]:
```
numproc = 2**arange(0,5)
line = arange(0,10,2)
figure(2)
clf()
ylabel("Time (s)")
xlabel("Number of Processors")
title("Parallel BE Weak Scaling")
loglog(numproc,weak_scale_time,'ko:', markersize=12)
fix_xticks(numproc)
```

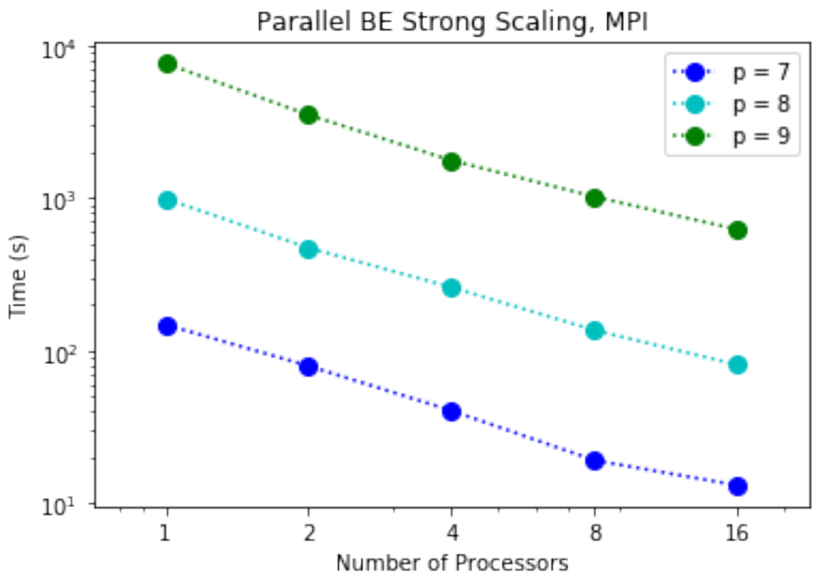


Strong Scaling

```
We see in the strong scaling plot below that the parallel BE shows very go
od strong scaling for the larger grid sizes. A linear best fit line shows
slopes of -44 for p = 8, and -338 for p = 9 which is reflected in the sign
ificant time improvements for p = 8 and p = 9 grids. For smaller grid size
s, the scaling is much flatter. I'm assuming that this is related to the s
implicity of a 32 x 32 grid for this problem. A small grid would require l
ots of processor communication with few calculations thus not maximizing t
ime actually on the processor.
```

In [6]:
```
numproc = 2**arange(0,5)
line = arange(0,10,2)
m,b= polyfit(numproc,p9_time,1)
figure(3)
clf()
#ylim((1,5))
ylabel("Time (s)")
xlabel("Number of Processors")
title("Parallel BE Strong Scaling, MPI")
#loglog(numproc,(m*numproc+b),label = "Best fit line, p = 9, m: %0.3f"
%m)
#loglog(numproc, p5_time,'ko:', markersize=8, label="p = 5")
#loglog(numproc, p6_time,'ro:', markersize=8, label="p = 6")
loglog(numproc, p7_time,'bo:', markersize=8, label="p = 7")
loglog(numproc, p8_time,'co:', markersize=8, label="p = 8")
loglog(numproc, p9_time,'go:', markersize=8, label="p = 9")
legend()
fix_xticks(numproc)
print("m = %0.2f for linear best fit, p = 9"%m)
```

```
m = -338.31 for linear best fit, p = 9
```

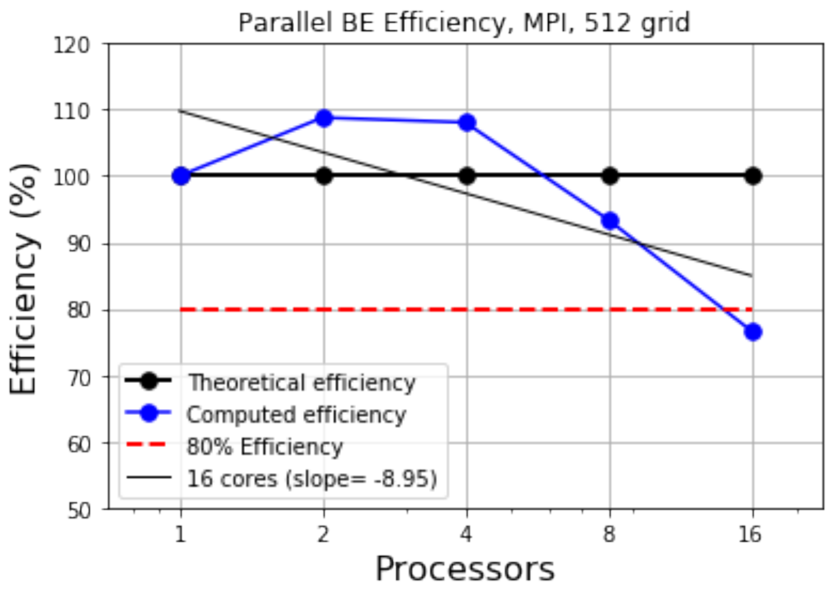


Efficiency

```
 Even at the highest grid size, 512 x 512, we see incredible efficiency fo
r this algorithm and calculation. As we've mentioned before, the Virtual M
achine's parallel behavior is unknown/inconsistant around 16 processors. E
ven with the possible threading behavior at 16 processors we see an effici
ency of near 80% which is very good. For 2 processors and 4 processors we
even see an efficiency over 100%, with 8 processors coming in above 90% as
well. All in all, I think this implicit scheme seems well suited to "stron
g parallelization", but for large matrices, the weak scaling may dictate t
hat a different algorithm should be sought.
```

In [7]:
```
pAdj = 4 # pAdj=0 -> p = 5; pAdj=1 -> p = 6; pAdj=2 -> p = 7; pAdj=3 -
> p = 8; pAdj=4 -> p = 9
speed_up = []
for i in range(len(numproc)):
    speed_up.append(time_results[pAdj]/time_results[pAdj+i*len(numproc
)])
efficiency = []
for i in range(len(numproc)):
    efficiency.append(100*speed_up[i]/numproc[i])
```

In [8]:
```
figure(4)
clf()
title("Parallel BE Efficiency, MPI, 512 grid")
hundred = [100,100,100,100,100]
eighty = [80,80,80,80,80]

semilogx(numproc,hundred,'k.-',markersize=15,linewidth=2,label='Theore
tical efficiency')

semilogx(numproc,efficiency,'.-',color='b',markersize=15,label='Comput
ed efficiency')


# Let's also add a line indicating "good" efficiency.
semilogx(numproc,eighty,'r--',markersize=15,linewidth=2,label='80% Eff
iciency')
# Add a best-fit line through first few cores.
t_eff = efficiency

# Fit line through all 16 cores
proc_max = 16    # number of processors on VM, supposedly
imax = int(log2(proc_max))+1
c = polyfit(log(numproc[:imax]),t_eff[:imax],1)
semilogx(numproc,polyval(c,log(numproc)),'k-', \
         label='{:d} cores (slope={:6.2f})'.format(proc_max,c[0]),line
width=1)


fix_xticks(numproc)
ylim(50,120)
xlabel('Processors',fontsize=16)
ylabel('Efficiency (%)',fontsize=16)
legend(loc=3)
grid()
```

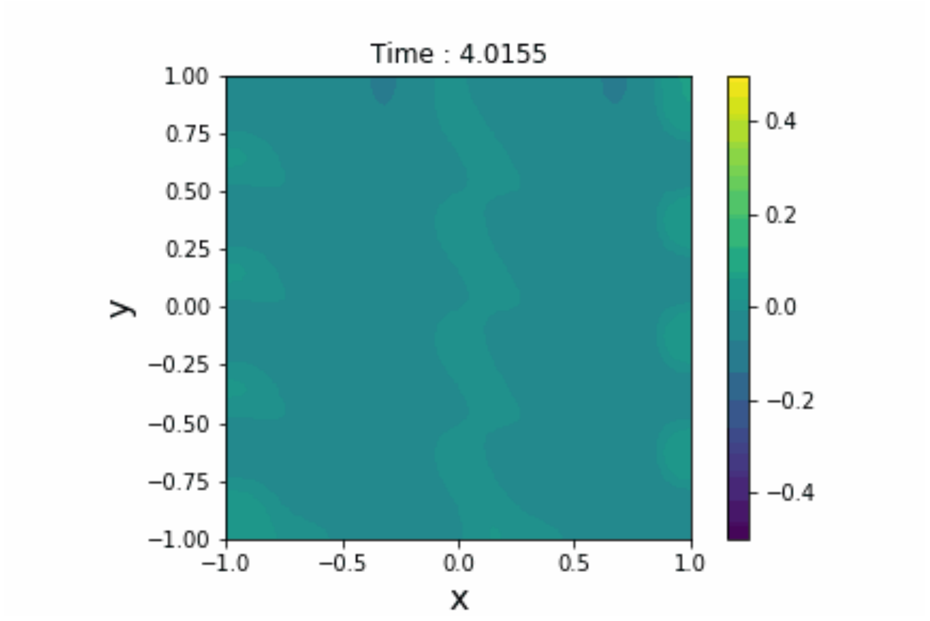


In [13]:
```
figure(1)
savefig('serial.png', facecolor=None, edgecolor=None,
        format='png',transparent=False, bbox_inches='tight', pad_inche
s=0.1,metadata=None)

figure(2)
savefig('weakScaling.png', facecolor=None, edgecolor=None,
        format='png',transparent=False, bbox_inches='tight', pad_inche
s=0.1,metadata=None)

figure(3)
savefig('strongScaling.png', facecolor=None, edgecolor=None,
        format='png',transparent=False, bbox_inches='tight', pad_inche
s=0.1,metadata=None)

figure(4)
savefig('efficiency.png', facecolor=None, edgecolor=None,
        format='png',transparent=False, bbox_inches='tight', pad_inche
s=0.1,metadata=None)
```

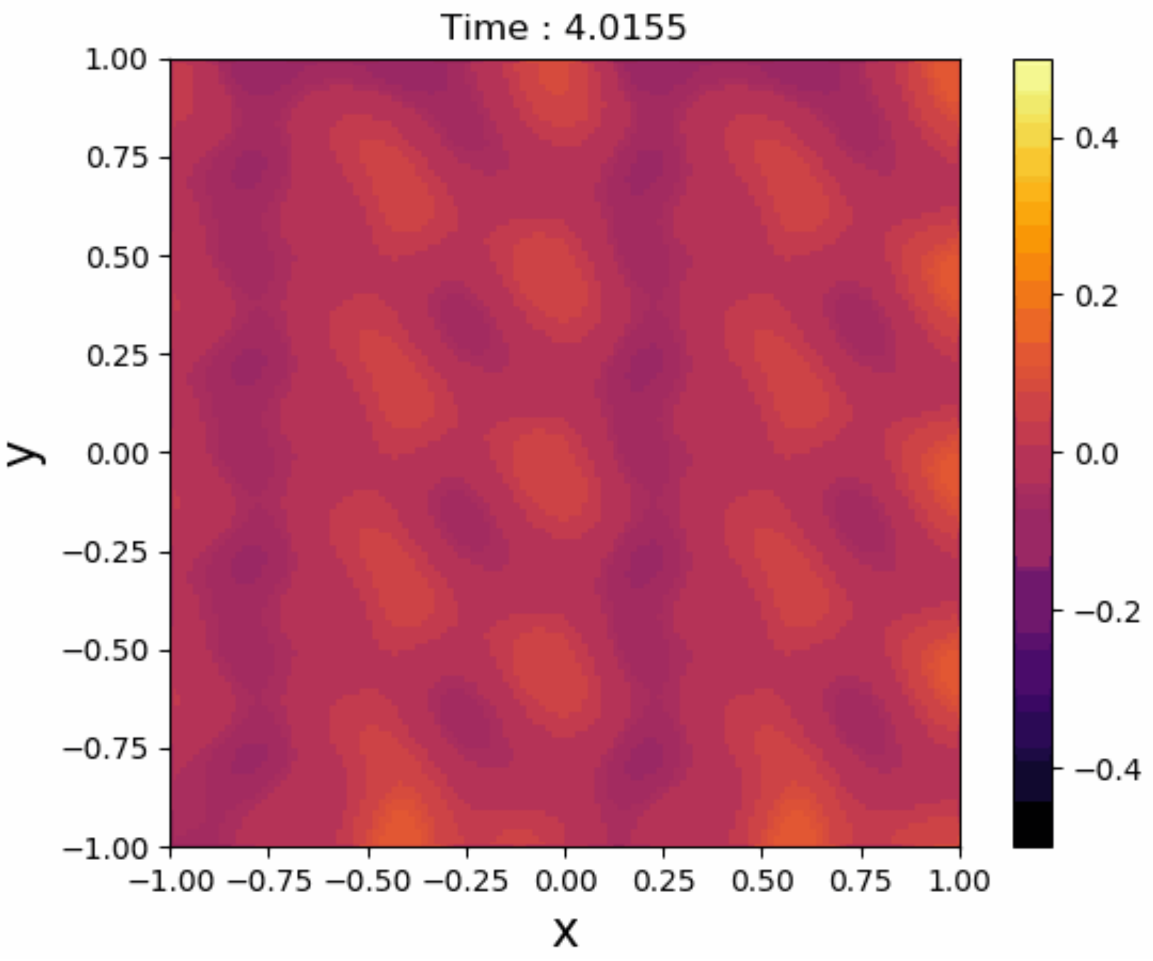
```
<Figure size 432x288 with 0 Axes>

<Figure size 432x288 with 0 Axes>

<Figure size 432x288 with 0 Axes>

<Figure size 432x288 with 0 Axes>
```

Visuals

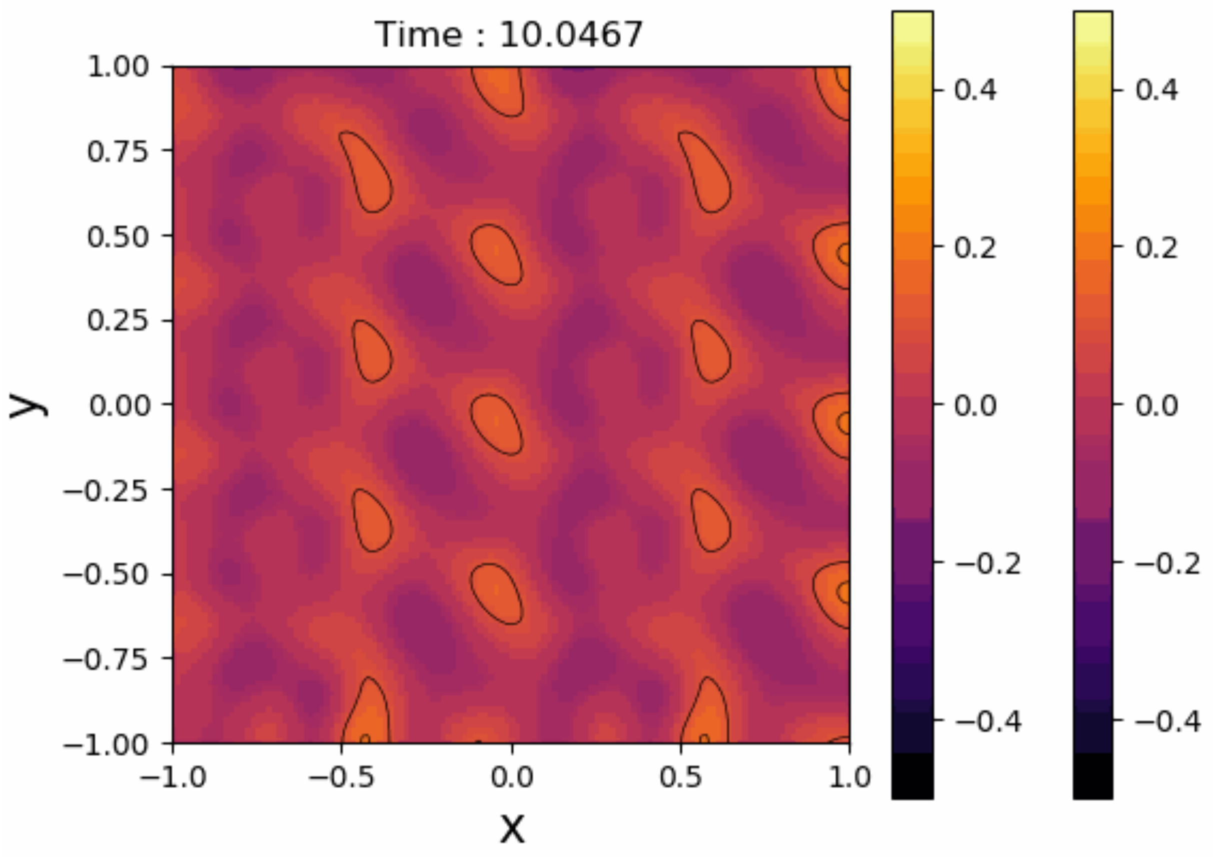
Spots 256 grid

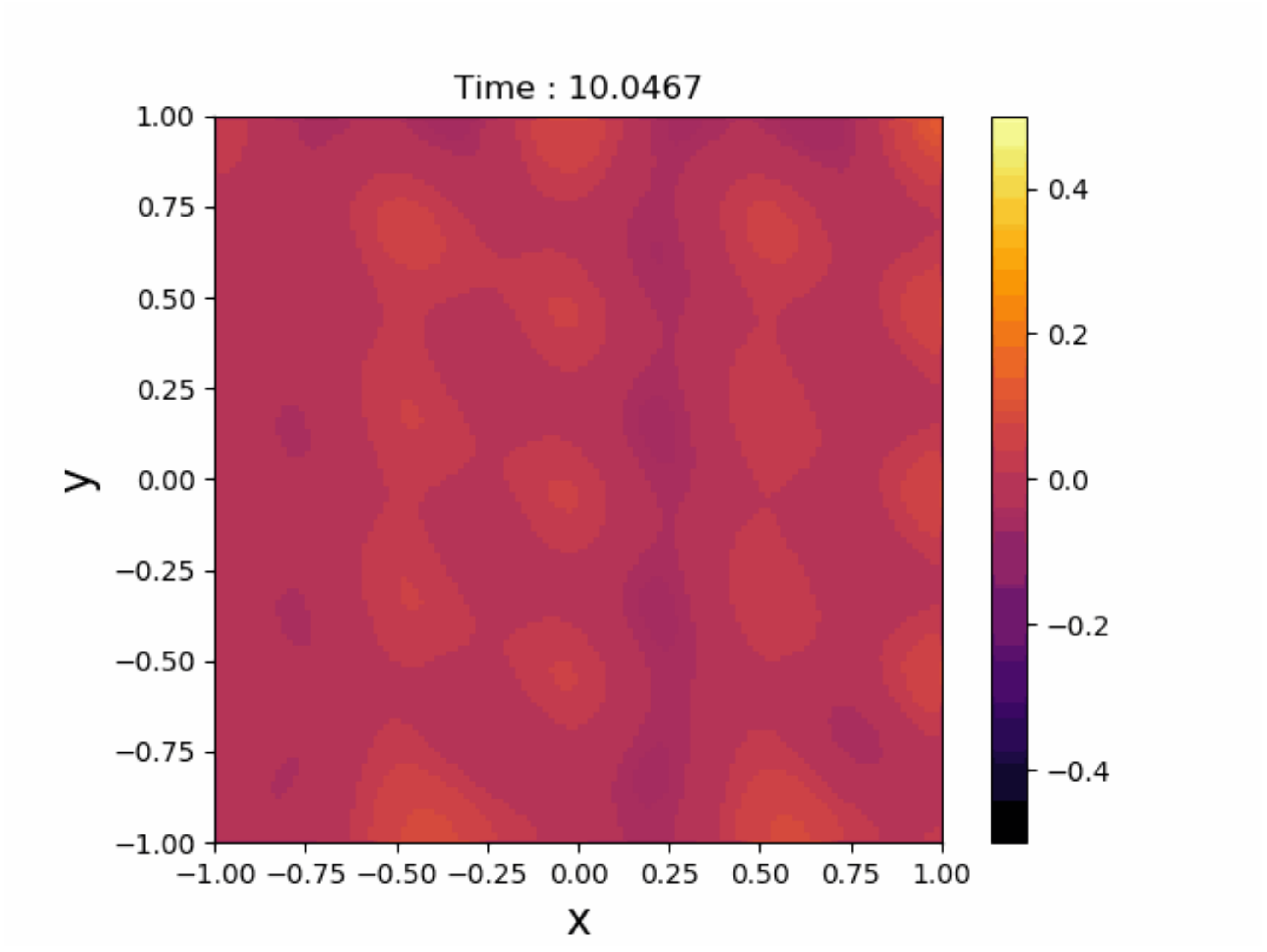


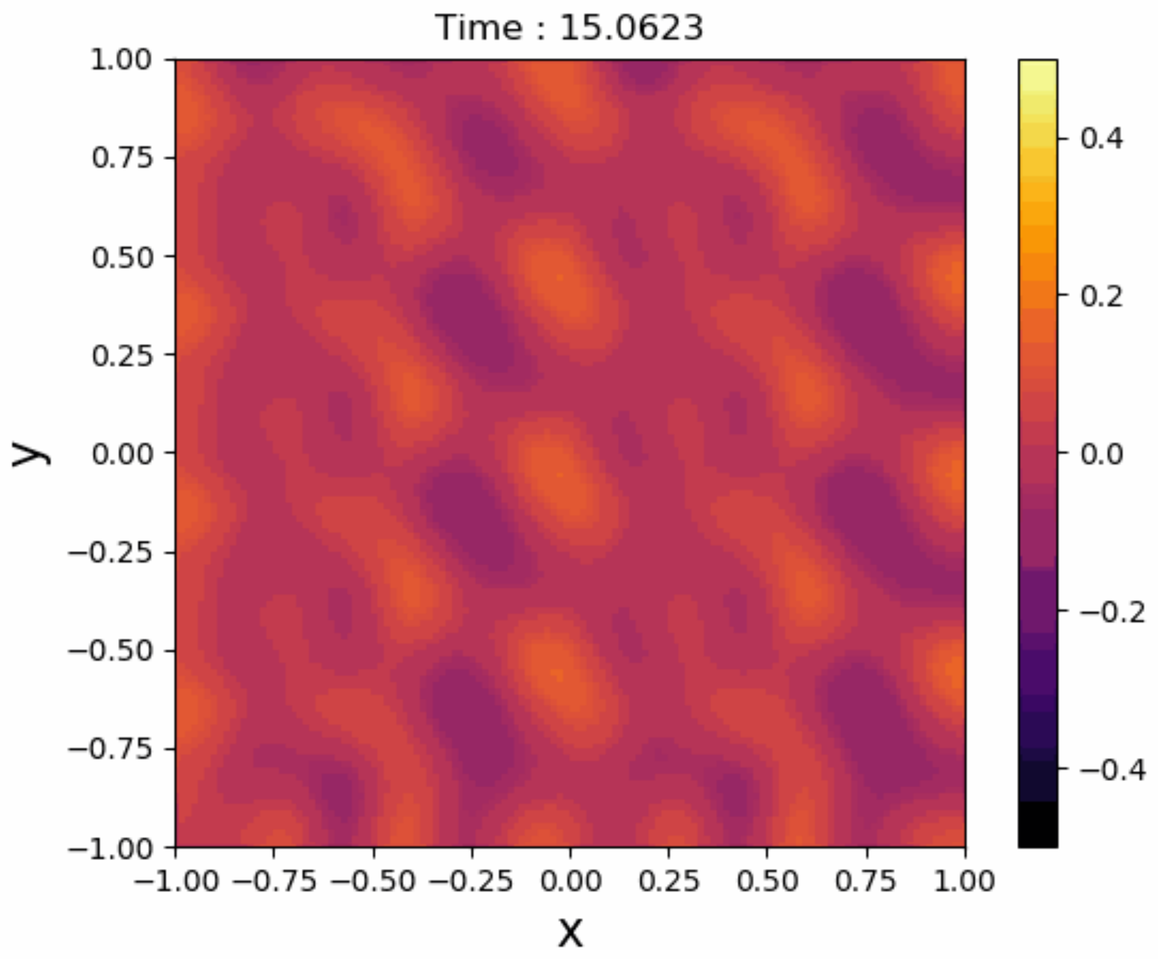
Stripes 512 grid



In Between Conditions 256 grid


Time : 10.0467
y
x
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.0
−0.5
0.0
0.5
1.0
0.4
0.2
0.0
−0.2
−0.4
0.4
0.2
0.0
−0.2
−0.4


Time : 10.0467
y
x
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.4
0.2
0.0
−0.2
−0.4



```
In [ ]:
```